Momentum transfer dependence of kaon semileptonic form factor on (10 fm)^4 at the physical point

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Collaborators

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Refs: PACS:PRD101,9,094504(2020), arXiv:2206.08654

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Outline

1. Introduction
2. Simulation parameters
3. Results
   - Shape of K_{l3} form factors $f_+(q^2), f_0(q^2)$
   - Phase space integral I_K^ℓ
   - $|V_{us}|$ using I_K^ℓ
4. Summary

Introduction

Urgent task: search for signal beyond standard model (BSM)

Muon $g-2$ @ FNAL 2021 : 4.2σ away from SM

$|V_{us}|$: a candidate of BSM signal

Most accurate $|V_{us}|$ from K_{l3} decay

['19 FNAL/MILC]

$\sim 5\sigma$ from CKM unitarity (cyan band)

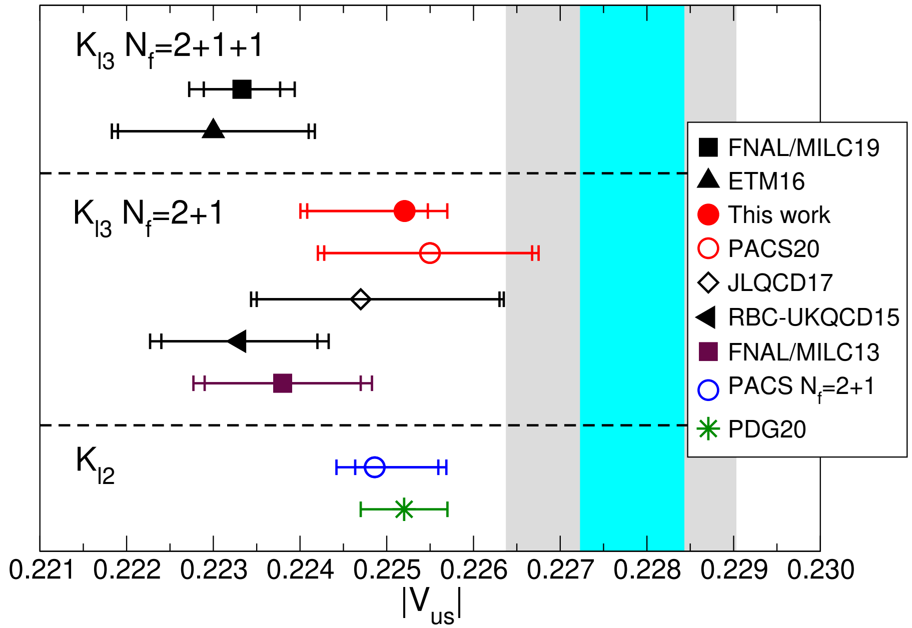
$|V_{us}| \approx \sqrt{1-|V_{ud}|^2}$ w/ $|V_{ud}|$ ['18 Seng *et al.*]

$\sim 3\sigma$ (grey band) w/ $|V_{ud}|$ ['20 Hardy, Towner]

Important to confirm by

several independent calculations

[PACS, arXiv:2206.08654]



K_{l3} form factors with PACS10 configurations [PACS20, arXiv:2206.08654]

$L \gtrsim$ 10[fm] at physical point in $a = 0$ with two a

Negligible finite L effect, tiny q^2 region, without chiral extrapolation

Largest uncertainty: fit form dependence of finite a effect

Reasonably agree with previous results

Simulation parameters

[PACS:PRD101,9,094504(2020), arXiv:2206.08654]

PACS10 configurations: $L \gtrsim 10$[fm] at physical point

$N_f = 2+1$ six-stout-smeared non-perturbative $O(a)$ Wilson action
+ Iwasaki gauge action

β	$L^3 \cdot T$	L[fm]	a^{-1}[GeV]	M_π[MeV]	M_K[MeV]	N_{conf}
1.82	128^4	10.9	2.3162	135	497	20
2.00	160^4	10.2	3.1108	137	501	20

K_{l3} form factors $f_+(q^2), f_0(q^2)$ from 3-point function

$$C_{V_\mu}(t,p) = \langle 0|O_K(t_{\text{sep}},\mathbf{0})V_\mu(t,\mathbf{p})O_\pi^\dagger(0,\mathbf{p})|0\rangle, \quad p \equiv |\mathbf{p}| = \left|\frac{2\pi}{L}\mathbf{n}\right|, \ |\mathbf{n}| = 0 \sim 6$$

in periodic BC

V_μ: local and conserved vector currents

$$\langle \pi(p)|V_\mu|K(0)\rangle = (p_K + p_\pi)_\mu f_+(q^2) + (p_K - p_\pi)_\mu f_-(q^2)$$

$$f_0(q^2) = f_+(q^2) - \frac{q^2}{M_K^2 - M_\pi^2} f_-(q^2)$$

$p_K = (M_K, \mathbf{0}), p_\pi = (E_\pi, \vec{p})$
$q^2 = -(M_K - E_\pi)^2 + p^2$

$f_+(q^2), f_0(q^2)$ at finite q^2 around $q^2 = 0$

$\rightarrow$ observable from q^2 dependence of $f_+(q^2), f_0(q^2)$

Resource: HPCI System Research Project (hp200062, hp200167, hp210112, hp220079 + $\cdots$)

q^2 interpolation $+\ a \to 0$ extrapolation

Fit based on SU(3) NLO ChPT with $f_+(0) = f_0(0)$ [PACS, arXiv:2206.08654]

$$f_+(q^2) = 1 - \frac{4}{F_0^2} L_9(\mu) q^2 + K_+(q^2, M_\pi^2, M_K^2, F_0, \mu) + c_0 + c_2^+ q^4 + g_+^{\text{cur}}(a, q^2)$$

$$f_0(q^2) = 1 - \frac{8}{F_0^2} L_5(\mu) q^2 + K_0(q^2, M_\pi^2, M_K^2, F_0, \mu) + c_0 + c_2^0 q^4 + g_0^{\text{cur}}(a, q^2)$$

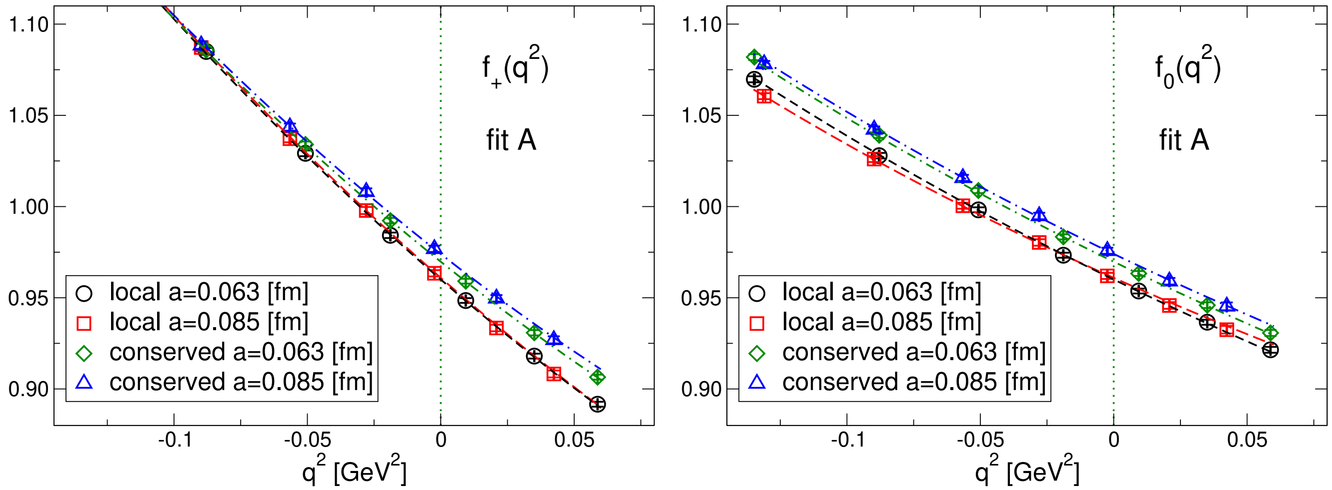
K_+, K_0: known functions ['85 Gasser, Leutwyler]

$g_{+,0}^{\text{cur}} = \sum_{n,m} e_{+,0}^{\text{cur},nm} a^n q^{2m}$, cur = local, conserved: 3 types (fit A,B,C) investigated

free parameters: $L_5(\mu), L_9(\mu), c_0, c_2^+, c_2^0 + e_{+,0}^{\text{cur,nm}}$

fixed parameters: $\mu = 0.77$ GeV, $F_0 = 0.11205$ GeV

F_0 estimated from FLAG $F^{\text{SU(2)}}/F_0$ w/ $F^{\text{SU(2)}} = 0.129$ GeV



Simultaneous fit for (f_+, f_0) with (local,conserved) works well.

Tiny extrapolation to physical M_{π^-} and M_{K^0} using same formulas

q^2 interpolation $+$ $a \to 0$ extrapolation

Fit based on SU(3) NLO ChPT with $f_+(0) = f_0(0)$ [PACS:arXiv:2206.08654]

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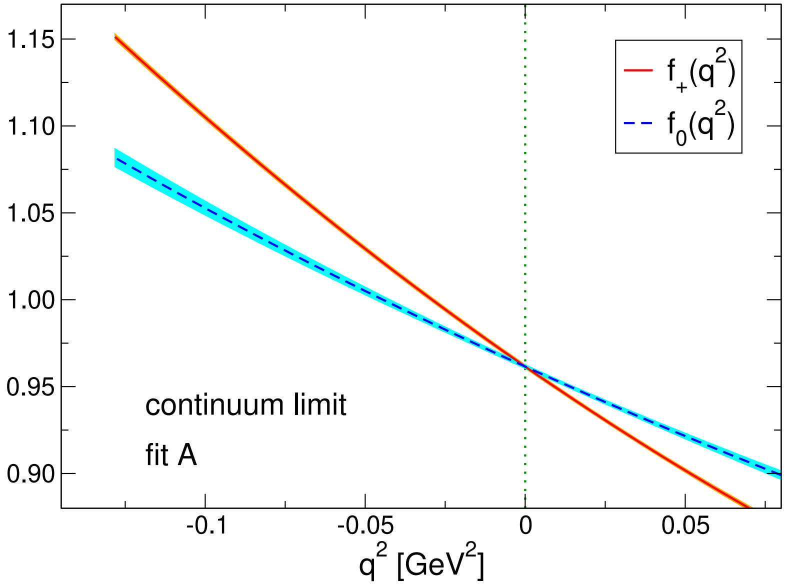
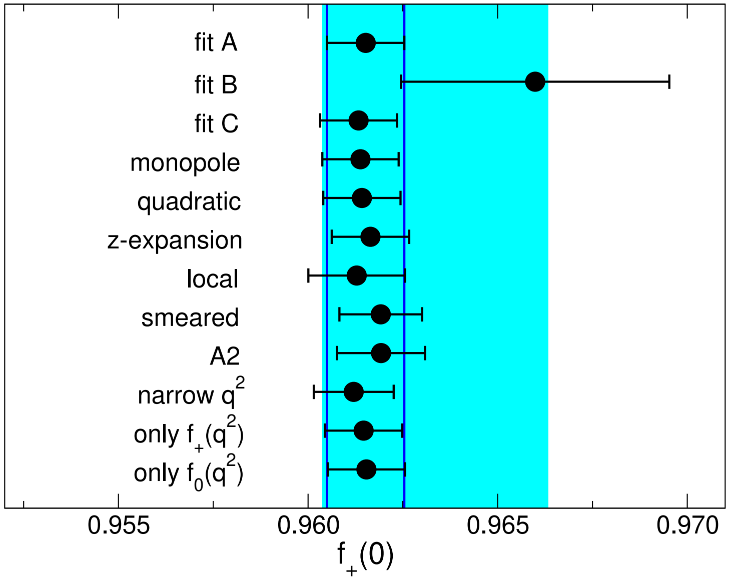




$$f_+(0) = 0.9615(10)(^{+47}_{-2})(5)$$

uncertainty: 1st statistical, 2nd fit form + data, 3rd isospin breaking

Using experimental value $|V_{us}|f_+(0) = 0.21654(41)$ ['17 Moulson]

$$|V_{us}| = 0.22521(24)(^{+6}_{-109})(11)(43)$$

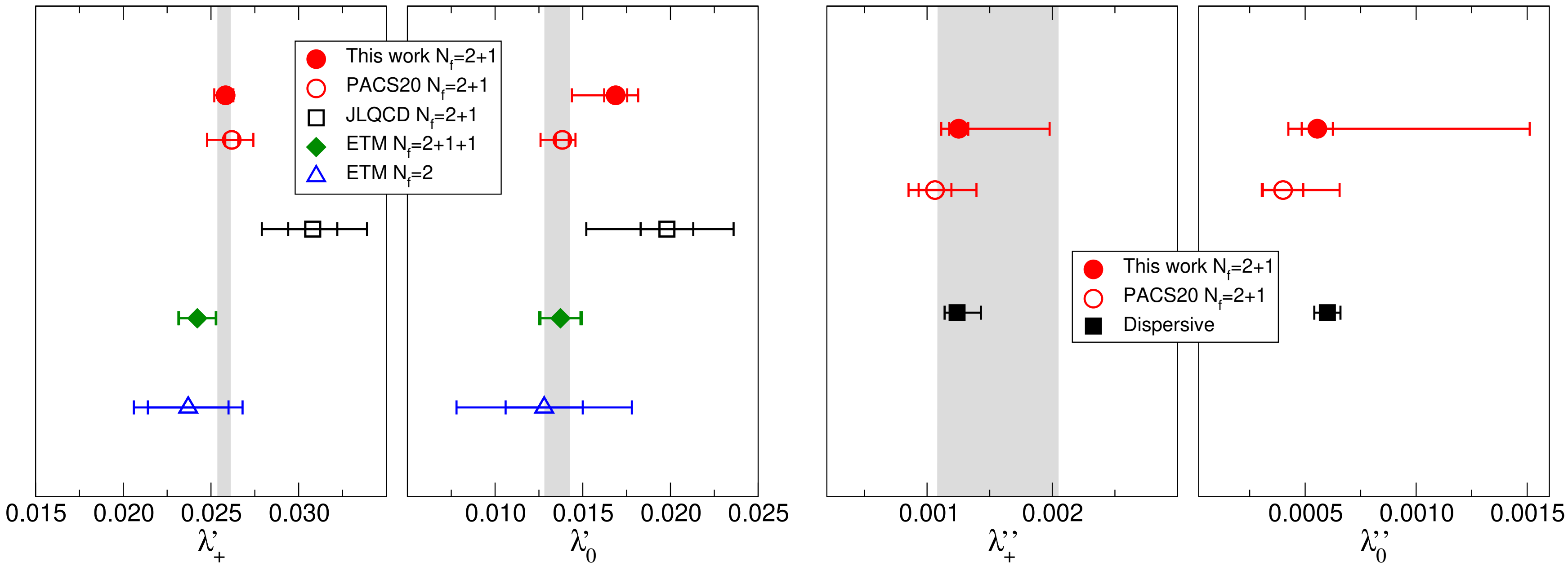
Shape of $f_+(q^2), f_0(q^2)$ at $q^2 = 0$

slope

$$\lambda'_{+,0} = \frac{M_{\pi^-}^2}{f_+(0)} \frac{df_{+,0}(q^2)}{d(-q^2)}$$

curvature

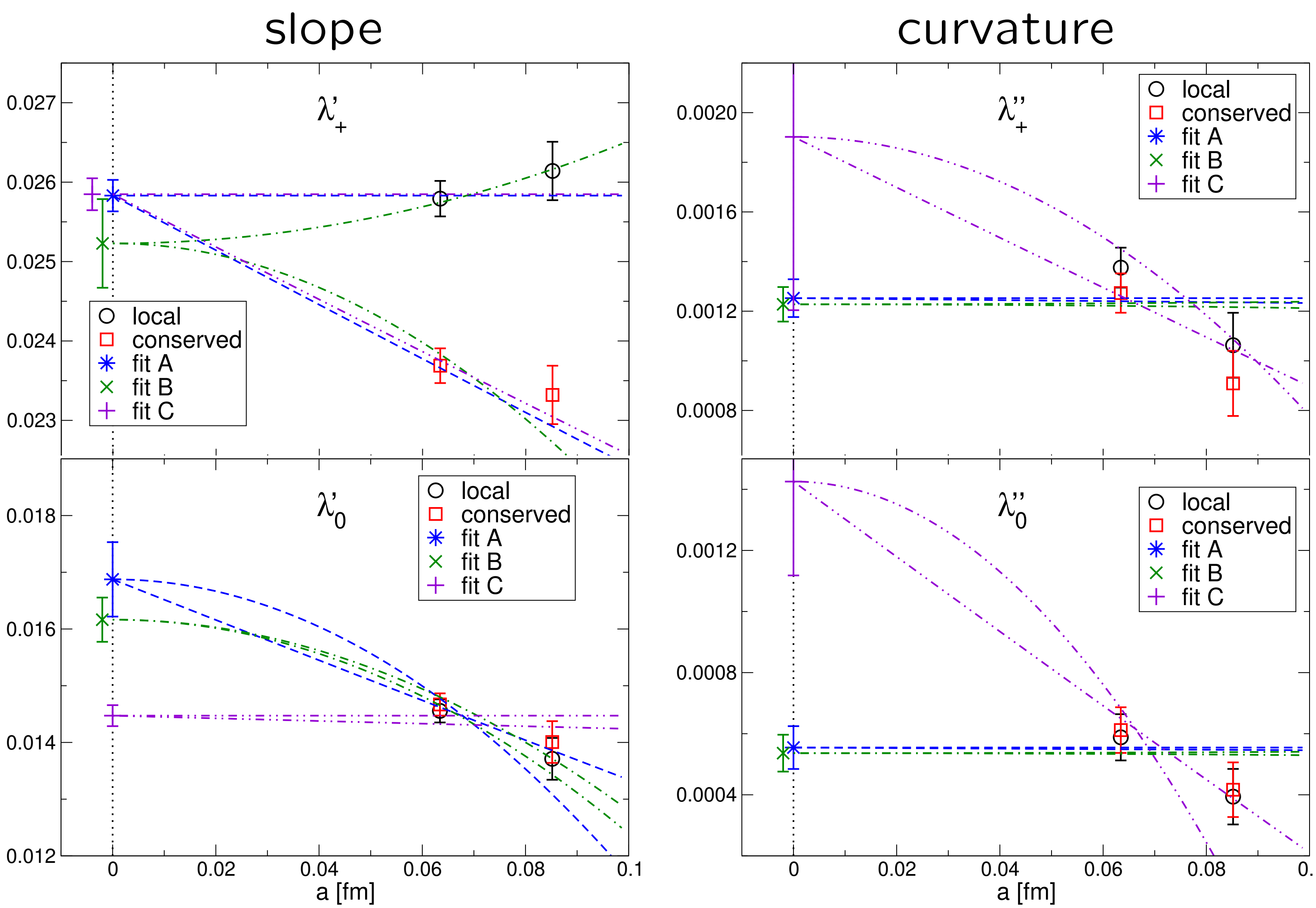
$$\lambda''_{+,0} = \frac{M_{\pi^-}^4}{f_+(0)} \frac{d^2 f_{+,0}(q^2)}{d(-q^2)^2}$$



inner: stat. error; outer: total error = stat. + sys. errors in quadrature

Larger error than our previous work, except for λ'_+

Shape of $f_+(q^2), f_0(q^2)$ at $q^2 = 0$



local and conserved data degenerated at each a, except for λ'_+

$\rightarrow$ large dependence on choice of $g^{\mathrm{cur}}_{+,0}$

Smaller a data will improve $a \rightarrow 0$ extrapolation.

Shape of $f_+(q^2), f_0(q^2)$ at $q^2 = 0$

slope

$$\lambda'_{+,0} = \frac{M_{\pi^-}^2}{f_+(0)} \frac{df_{+,0}(q^2)}{d(-q^2)}$$

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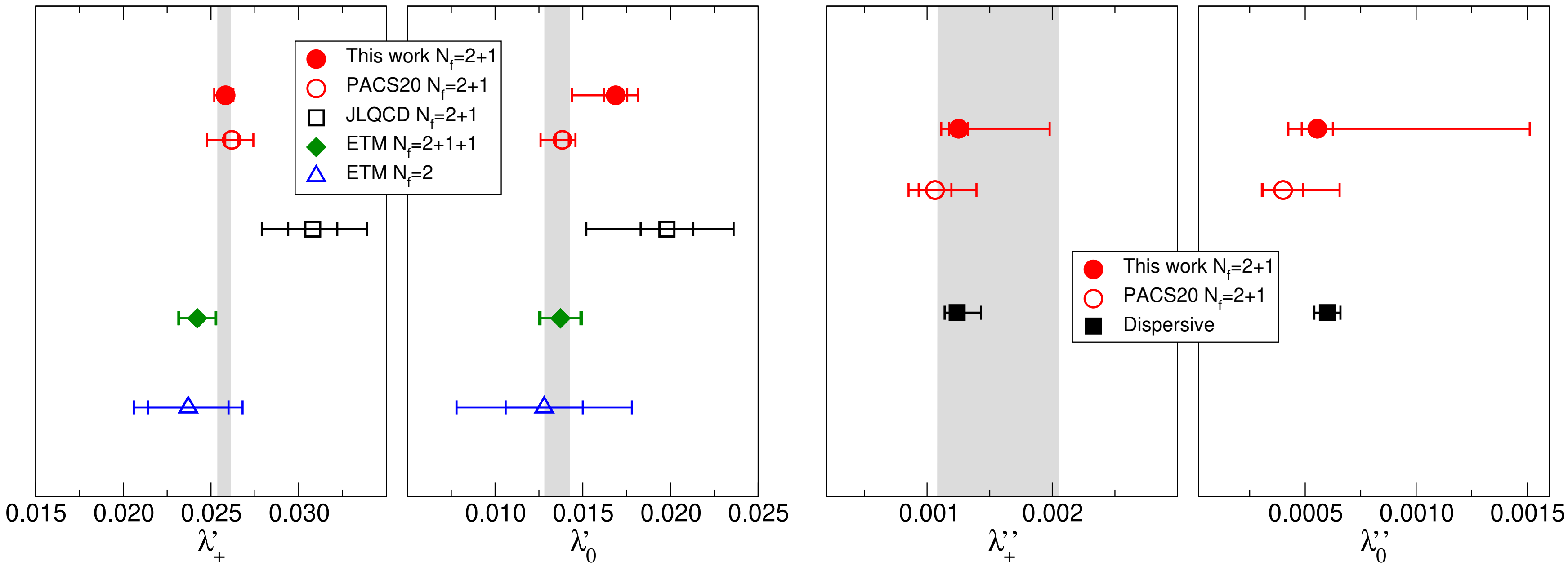


Large uncertainty from choice of $g^{\text{cur}}_{+,0}$

Comparable with experiment (grey band), dispersive representation,

['10 Antonelli *et al.*; '17 Moulson; '09 Bernard *et al.*]

and also previous lattice calculations ['09, '16 ETM; '17 JLQCD, '20 PACS]

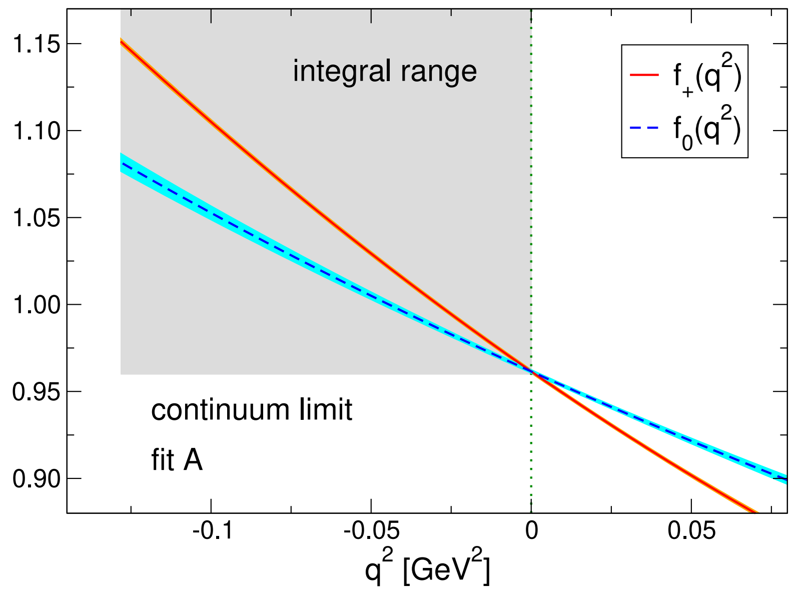
Phase space integral I_K^ℓ

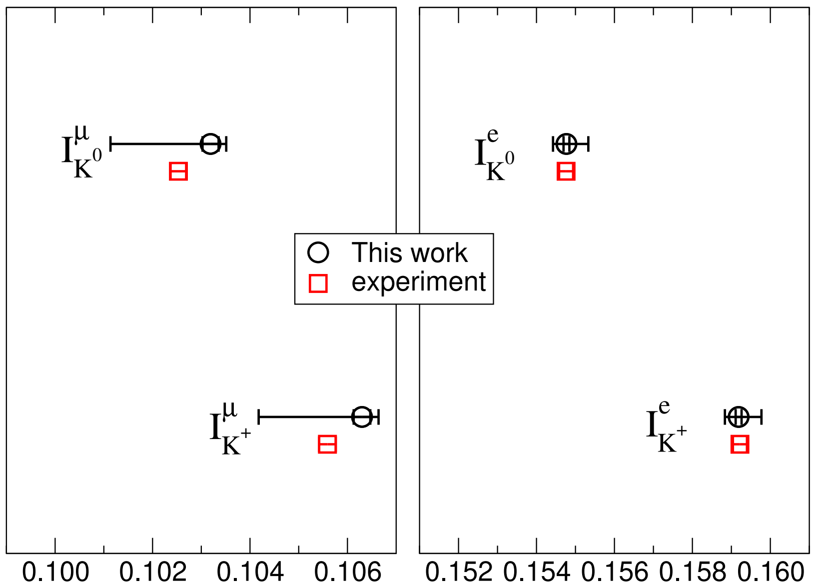
$\Gamma_{K_{\ell 3}} = C_{K_{\ell 3}} (|V_{us}| f_+(0))^2 I_K^\ell$ $\Gamma_{K_{\ell 3}}$: decay width, $C_{K_{\ell 3}}$: known factor, $\ell = e, \mu$

averaging 6 decay processes $\to |V_{us}| f_+(0) = 0.21654(41)$ ['17 Moulson]

$$I_K^\ell = \int_{m_\ell^2}^{(M_K - M_\pi)^2} dt \left(J_+(t) \overline{F}_+^2(t) + J_0(t) \overline{F}_0^2(t) \right), \quad \overline{F}_{+,0}(t) = \frac{f_{+,0}(-t)}{f_+(0)}$$

$J_{+,0}(t)$: known function ['84 Leutwyler, Roos]





inner: stat. error; outer: total error = stat. + sys. errors in quadrature

Reasonably agree with experimental values ['10 Antonelli *et al.*]

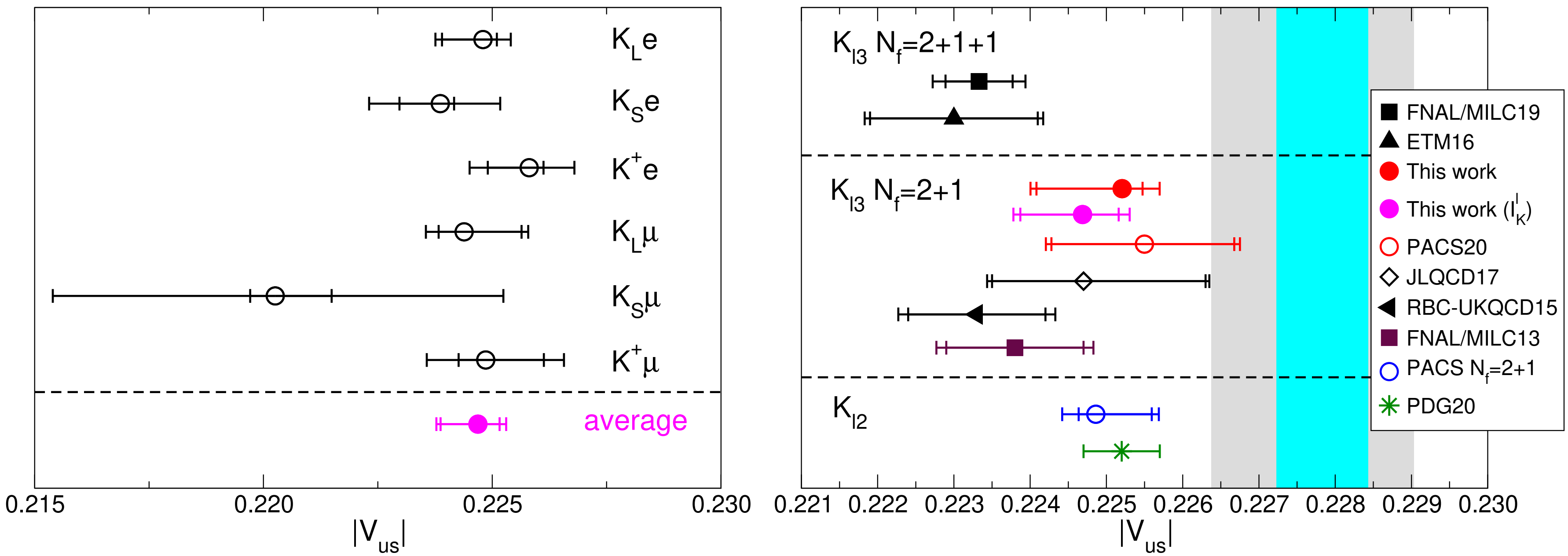
Large uncertainty from choice of $g_{+,0}^{\mathrm{cur}}$

$|V_{us}|$ using I_K^ℓ

$$|V_{us}| = \sqrt{\frac{\Gamma_{K_{\ell 3}}}{C_{K_{\ell 3}}(f_+(0))^2 I_K^\ell}}$$

Two parts calculated in lattice QCD

$\Gamma_{K_{\ell 3}}, C_{K_{\ell 3}}$ ['10 Antonelli *et al.*, '18 Seng *et al.*, '20 Seng *et al.*]



inner: lattice error, outer: total error from lattice + experimental errors

Weighted average of 6 decay processes using experimental errors

Good agreement with $|V_{us}|$ using only $f_+(0)$

Summary

Observables from q^2 dependence of K_{l3} form factors with PACS10 confs

$L \gtrsim 10$[fm] at physical point with two a [PACS:'20 + arXiv:2206.08654]

- Reasonably agree with experiment and previous lattice calculations
  - Slope and curvature
  - Phase space integral I_K^ℓ
- $|V_{us}|$ using I_K^ℓ: Good agreement with $|V_{us}|$ using $f_+(0)$

  Lattice QCD is useful to determine I_K^ℓ as well as $f_+(0)$.

Future works

- Reduce uncertainty in $a \to 0$ extrapolation

  Calculations with 3rd PACS10 configuration in smaller a

  more reliable $a \to 0$ extrapolations
- Nonperturbative estimate of systematic uncertainties

  e.g., isospin breaking

Back up

fit forms $g^{\text{cur}}_{+,0}$

fit A empirically chosen from data behavior

$$g^{\text{loc}}_{+}(q^2,a) = 0 \qquad g^{\text{loc}}_{0}(q^2,a) = d^0_{21}a^2q^2$$

$$g^{\text{con}}_{+}(q^2,a) = e_{10}a + e^{+}_{11}aq^2 \qquad g^{\text{con}}_{0}(q^2,a) = e_{10}a + e^{0}_{11}aq^2$$

fit B all errors in a^2

$$g^{\text{loc}}_{+}(q^2,a) = d_{20}a^2 + d^{+}_{21}a^2q^2 \qquad g^{\text{loc}}_{0}(q^2,a) = d_{20}a^2 + d^{0}_{21}a^2q^2$$

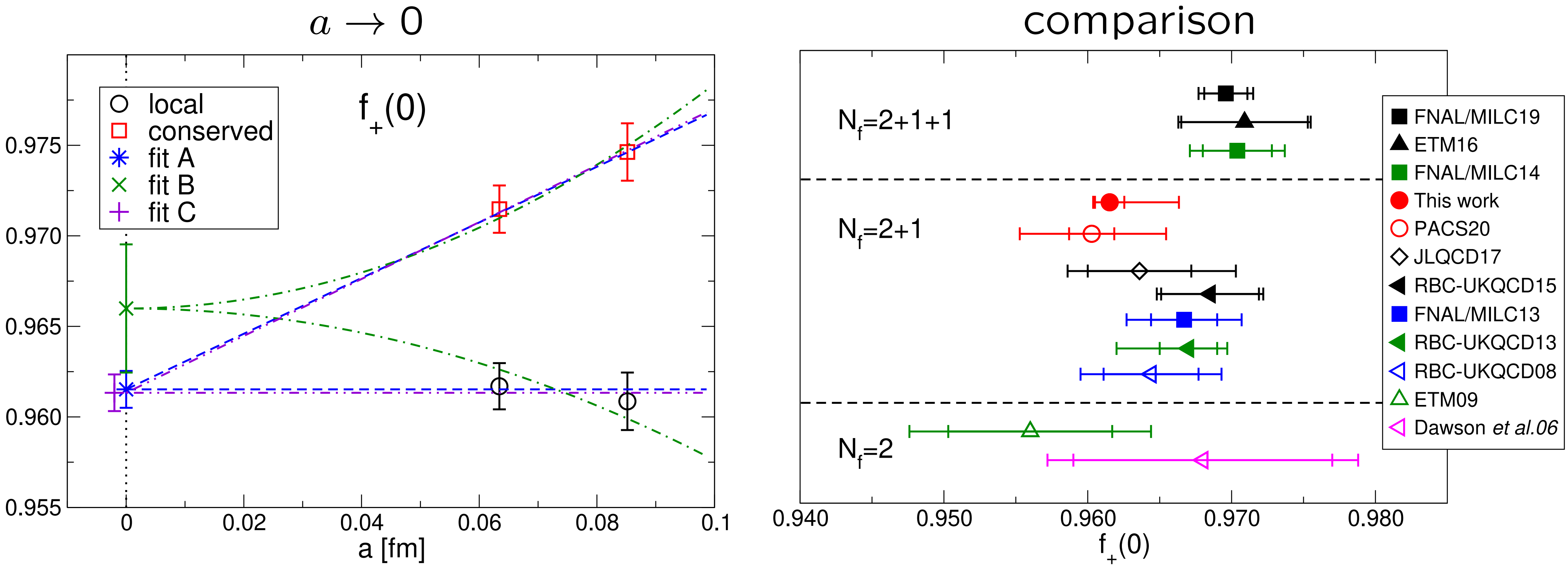
$$g^{\text{con}}_{+}(q^2,a) = e_{20}a^2 + e^{+}_{21}a^2q^2 \qquad g^{\text{con}}_{0}(q^2,a) = e_{20}a^2 + e^{0}_{21}a^2q^2$$

fit C a^2q^4 error, based on fit A

$$g^{\text{loc}}_{+}(q^2,a) = d^{+}_{22}a^2q^4 \qquad g^{\text{loc}}_{0}(q^2,a) = d^{0}_{22}a^2q^4$$

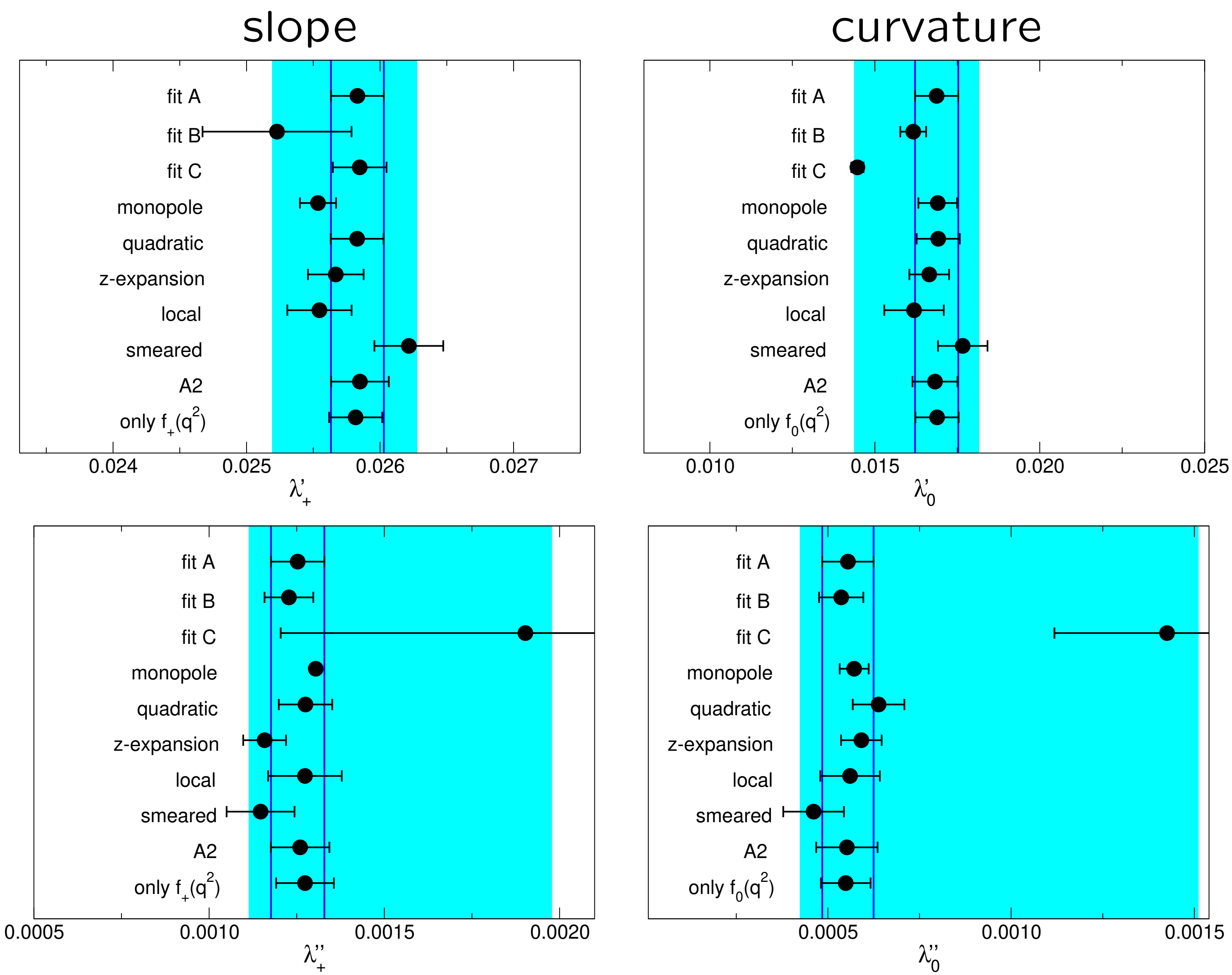
$$g^{\text{con}}_{+}(q^2,a) = e_{10}a + e^{+}_{11}aq^2 + e^{+}_{12}aq^4 \qquad g^{\text{con}}_{0}(q^2,a) = e_{10}a + e^{0}_{12}aq^4$$

Result of $f_+(0)$



$a \to 0$
comparison
local
conserved
fit A
fit B
fit C
f+(0)
0.975
0.970
0.965
0.960
0.955
0
0.02
0.04
0.06
0.08
0.1
a [fm]
Nf=2+1+1
Nf=2+1
Nf=2
FNAL/MILC19
ETM16
FNAL/MILC14
This work
PACS20
JLQCD17
RBC-UKQCD15
FNAL/MILC13
RBC-UKQCD13
RBC-UKQCD08
ETM09
Dawson et al.06
0.940
0.950
0.960
0.970
0.980
f+(0)

Stability of slope and curvature



Phase space integral I_K^ℓ

$$I_K^\ell = \int_{m_\ell^2}^{t_{\mathrm{max}}} dt\, \frac{\lambda^{3/2}}{M_K^8} \left(1 + \frac{m_\ell^2}{2t}\right) \left(1 - \frac{m_\ell^2}{t}\right)^2 \left(\overline{F}_+^2(t) + \frac{3 m_\ell^2 \Delta_{K\pi}^2}{(2t + m_\ell^2)\lambda} \overline{F}_0^2(t)\right)$$

$\ell = e, \mu$, $t_{\mathrm{max}} = (M_K - M_\pi)^2$, $\lambda = (t - (M_K + M_\pi)^2)(t - t_{\mathrm{max}})$, $\Delta_{K\pi} = M_K^2 - M_\pi^2$,
$\overline{F}_{+,0}(t) = f_{+,0}(-t)/f_+(0)$

$I_{K^0}^\ell$: $K^0 \to \pi^- \ell^+ \nu_\ell$
$M_K = m_{K^0} = 0.497611$ GeV, $M_\pi = m_{\pi^-} = 0.13957061$ GeV

$I_{K^+}^\ell$: $K^+ \to \pi^0 \ell^+ \nu_\ell$
$M_K = m_{K^+} = 0.493677$ GeV, $M_\pi = m_{\pi^0} = 0.1349770$ GeV

Calculation method R-local source with local current

Details in PACS:PRD101,9,094504(2020)

3-point function*

$$C_{V_\mu}(t,p) = \langle 0|O_K(t_{\rm sep},\mathbf{0})V_\mu(t,\mathbf{p})O_\pi^\dagger(0,\mathbf{p})|0\rangle$$
$$= \frac{Z_\pi Z_K}{Z_V}\frac{M_\mu(p)}{4E_\pi M_K}e^{-E_\pi t}e^{-M_K(t_{\rm sep}-t)} + \cdots$$

$p \equiv |\mathbf{p}| = \left|\frac{2\pi}{L}\mathbf{n}\right|$, $|\mathbf{n}| = 0$–6

with periodic boundary

2-point function* $X = \pi, K$

$$C_X(t,p) = \langle 0|O_X(t,\mathbf{p})O_X^\dagger(0,\mathbf{p})|0\rangle = \frac{Z_X^2}{2E_X}\left(e^{-E_X t} + e^{-E_X(2T-t)}\right) + \cdots$$

*Averaging ones with periodic, anti-periodic temporal boundary conditions reducing wrapping around effect in 3pt, and doubling periodicity in 2pt

$Z_V = \sqrt{Z_V^\pi Z_V^K}$ from electromagnetic form factor $F_{\pi,K}(0) = 1$

Ratio $R_\mu(t,p)$

$$R_\mu(t,p) = \frac{Z_\pi Z_K Z_V C_{V_\mu}(t,p)}{C_\pi(t,p)C_K(t_{\rm sep}-t,0)} = M_\mu(p) + \cdots$$

Constant in $R_\mu(t,p)$ corresponds to $M_\mu(p) = \langle \pi(p)|V_\mu|K(0)\rangle$

Conserved current case: $V_\mu \to \widetilde{V}_\mu$ and $Z_V = 1$

R-smear source case: $Z_\pi, Z_K \to Z_\pi(p), Z_K(0)$

Calculation method

Details in PACS:PRD101,9,094504(2020)

Constant in $R_\mu(t,p)$ corresponds to $M_\mu(p) = \langle \pi(p)|V_\mu|K(0)\rangle$

K_{l3} form factors $f_+(q^2), f_0(q^2)$

$$\langle \pi(p)|V_\mu|K(0)\rangle = (p_K + p_\pi)_\mu f_+(q^2) + (p_K - p_\pi)_\mu f_-(q^2)$$

$$f_0(q^2) = f_+(q^2) - \frac{q^2}{M_K^2 - M_\pi^2} f_-(q^2)$$

$p_K = (M_K, \mathbf{0}), p_\pi = (E_\pi, \vec{p})$
$q^2 = -(M_K - E_\pi)^2 + p^2$

Calculation of physical quantities

$R_4(t,p), R_i(t,p) \to M_4(p), M_i(p) \to f_+(q^2), f_0(q^2)$ at each q^2

except for $p = 0$, where only $f_0(q^2)$

$\to q^2$ interpolations to $q^2 = 0$ for $f_+(q^2), f_0(q^2)$

1. $f_+(0)\,(= f_0(0)) \to |V_{us}|$

$$|V_{us}|f_+(0) = 0.21654(41)$$ [Moulson:PoS(CKM2016)033(2017)]

2. slope and curvature

$$\lambda_+^{(n)} = \frac{M_{\pi^-}^{2n}}{f_+(0)} \frac{d^n f_+(0)}{d(-q^2)^n}, \quad \lambda_0^{(n)} = \frac{M_{\pi^-}^{2n}}{f_+(0)} \frac{d^n f_0(0)}{d(-q^2)^n}$$