The hadronic running of the electromagnetic coupling and electroweak mixing angle

Marco Cè, Antoine Gérardin, Georg von Hippel, Harvey Meyer, Kohtaroh Miura, Konstantin Ottnad, Andreas Risch, Teseo San José, Jonas Wilhelm, Hartmut Wittig

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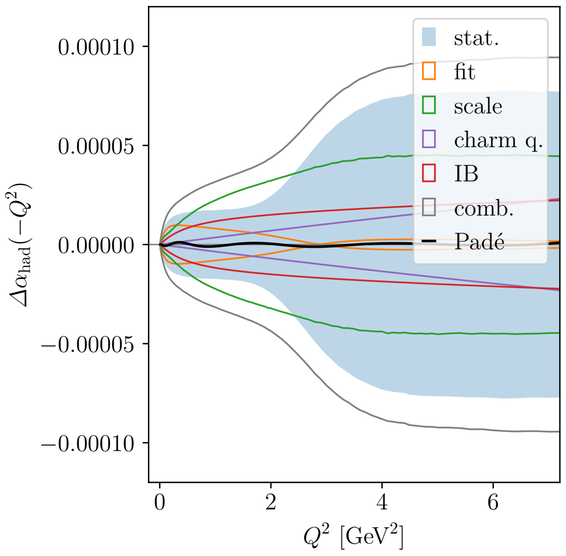
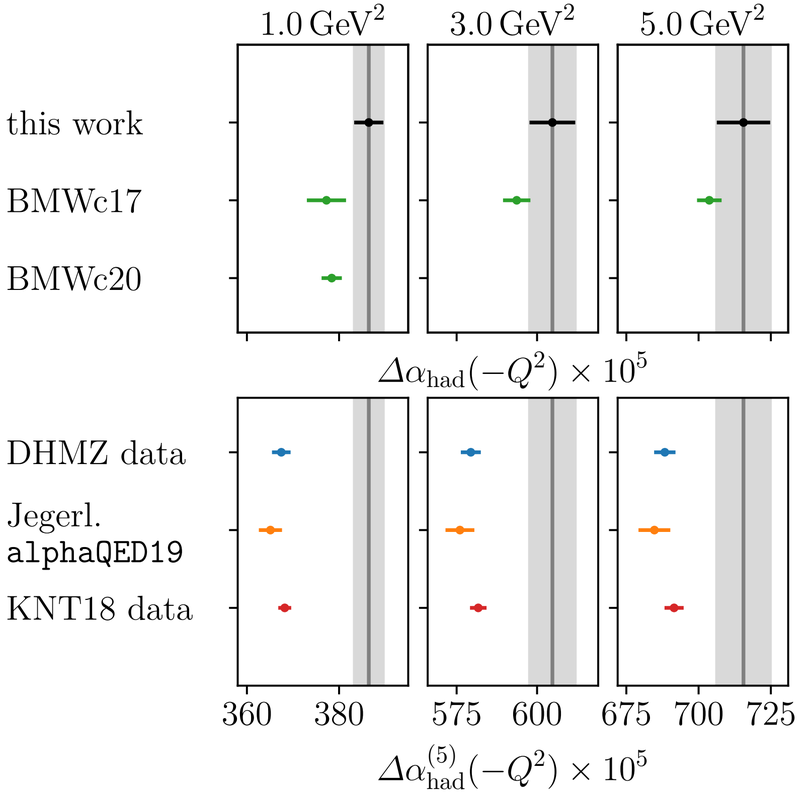
Lattice result for the hadronic running of α

[Cè et al., arXiv:2203.08676]

Starting point: Results for $\Delta\alpha_{\rm had}(-Q^2)$ for Euclidean momenta $0 \leq Q^2 \leq 7\,{\rm GeV}^2$ **[T. San José, TUE 17:10]**

Rational approximation:





- Mainz/CLS and BMWc (2017) differ by 2–3% at the level of 1–2σ
- Tension between Mainz/CLS and phenomenology by $\sim 3\sigma$ for $Q^2 \gtrsim 3\,{\rm GeV}^2$
- Tension increases to $\gtrsim 5\sigma$ for $Q^2 \lesssim 2\,{\rm GeV}^2$
  (smaller statistical error due to ansatz for continuum extrapolation)

Systematic uncertainties from fit ansatz, scale setting, charm quenching, isospin-breaking and missing bottom quark contribution (five flavour theory) included in error budget

Consistency of the Standard Model

Hadronic running at Z-pole: $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ $\rightarrow$ key quantity in global electroweak fit

$\Delta\alpha_{\rm had}$ related to hadronic vacuum polarisation contribution to muon $g-2$:

$$\Delta\alpha^{(5)}_{\rm had}(q^2) = -\frac{\alpha\, q^2}{3\pi} \fint_{m^2_{\pi^0}}^{\infty} ds\, \frac{R(s)}{s(s-q^2)}, \qquad R(s) = \frac{3s}{4\pi\,\alpha(s)}\,\sigma(\mathrm{e}^+\mathrm{e}^- \rightarrow \text{hadrons})$$

$$a_\mu^{\rm hvp} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m^2_{\pi^0}}^{\infty} ds\, \frac{R(s)\,\hat{K}(s)}{s^2}, \qquad 0.63 \lesssim \hat{K}(s) \leq 1$$

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Task: convert lattice result for $\Delta\alpha_{\rm had}^{(5)}(-Q^2)$ to an estimate of $\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$ and compare to global electroweak fit

How to evaluate $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$

Method 1: Direct dispersion relation (DR)

$$\Delta\alpha^{(5)}_{\rm had}(q^2) = -\frac{\alpha\, q^2}{3\pi} \fint_{m^2_{\pi^0}}^{\infty} ds\, \frac{R(s)}{s(s-q^2)} \qquad \text{for } q^2 = M_Z^2$$

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[Chetyrkin et al., Nucl Phys B482 (1996) 213; Eidelman et al., Phys Lett B454 (1999) 369; Jegerlehner, hep-ph/9901386, arXiv:0807.4206]

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Perturbation theory: $$\left[\Delta\alpha^{(5)}_{\rm had}(M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-M_Z^2)\right] = 0.000\,045(2)$$ *[Jegerlehner, CERN Yellow Report, 2020]*

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- $\Delta\alpha_{\rm had}(-Q_0^2)$ accessible in lattice QCD or by result from MUonE experiment
- Can check validity of perturbative QCD down to $Q_0^2 \sim 5\,{\rm GeV}^2$:

$$\left[\Delta\alpha_{\rm had}^{(5)}(-M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-Q_0^2)\right]_{\rm pQCD/Adler} \quad \text{versus} \quad \left[\Delta\alpha_{\rm had}^{(5)}(-M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-Q_0^2)\right]_{\rm DR}$$

Evaluation of $\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$

[Cè et al., arXiv:2203.08676]

Input: Lattice result for $\Delta\alpha_{\rm had}(-Q_0^2)$ for $Q_0^2 = 3 - 7\,{\rm GeV}^2$

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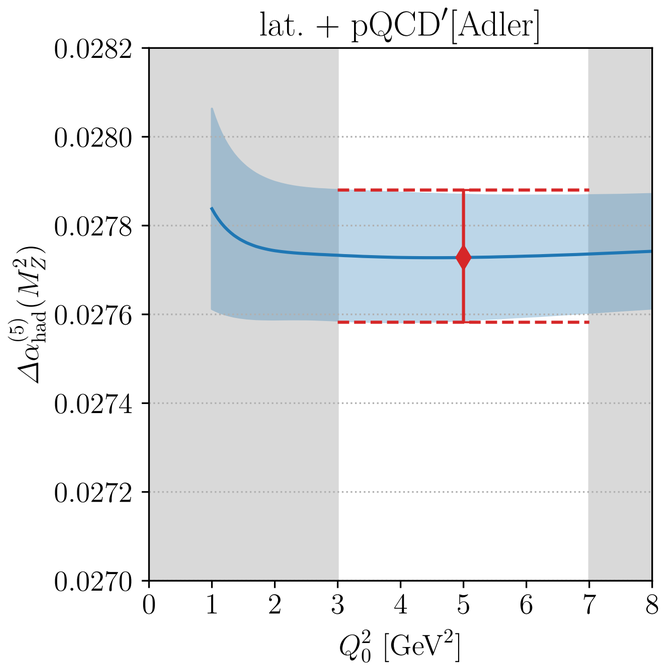

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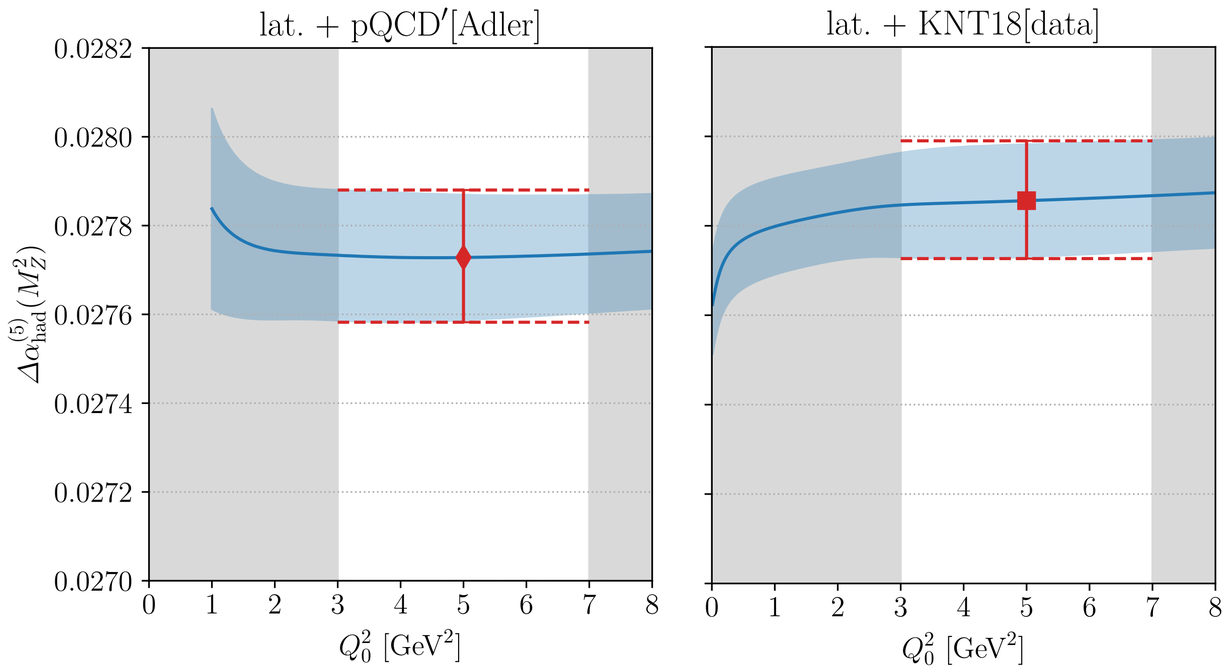

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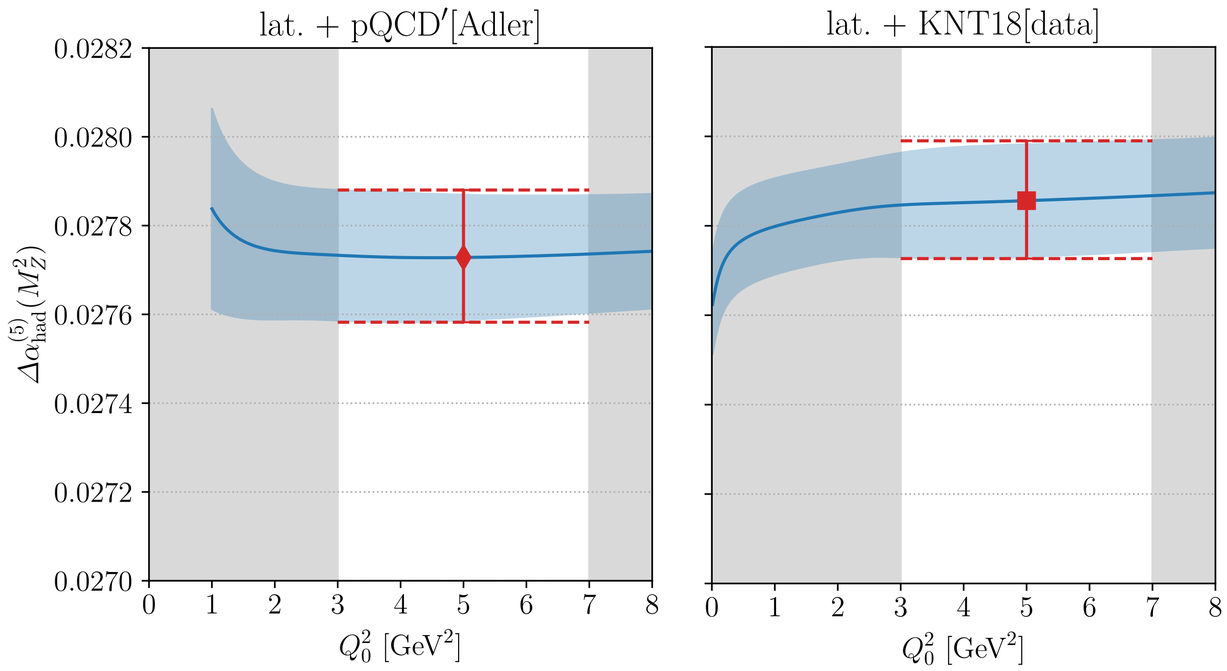
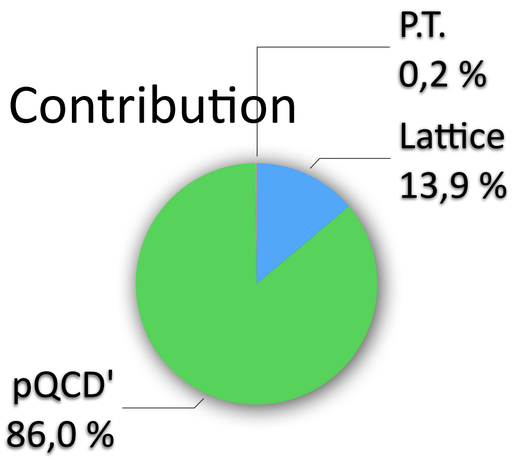


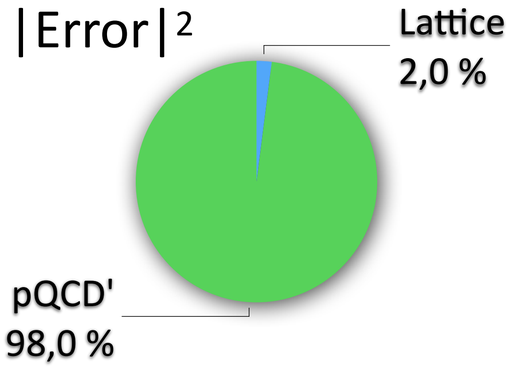
Final estimate: lattice + pQCD/Adler

$$\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = 0.027\,73(9)_{\rm lat}(2)_{\rm btm}(12)_{\rm pQCD}$$
$$= 0.027\,73(15)$$

(error contains ambiguity in the choice of Q_0^2)

Euclidean split technique: relative contributions to $\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$





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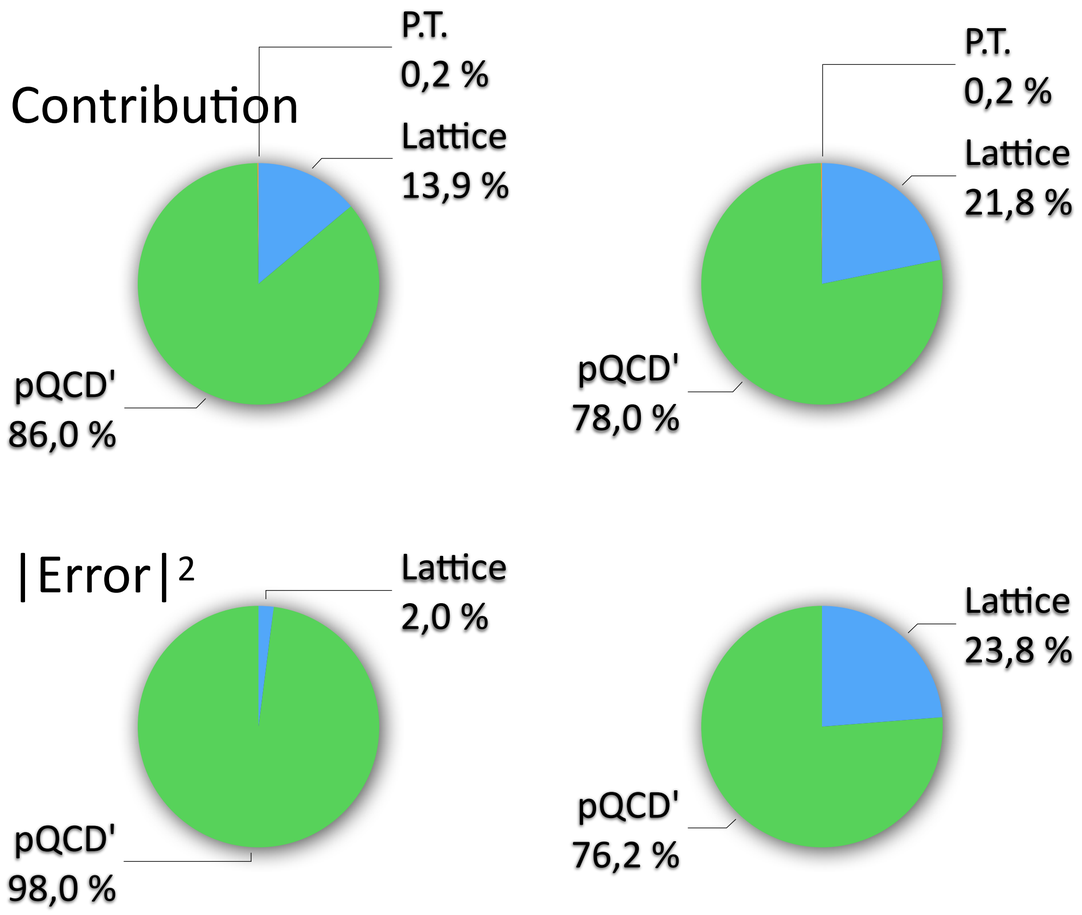

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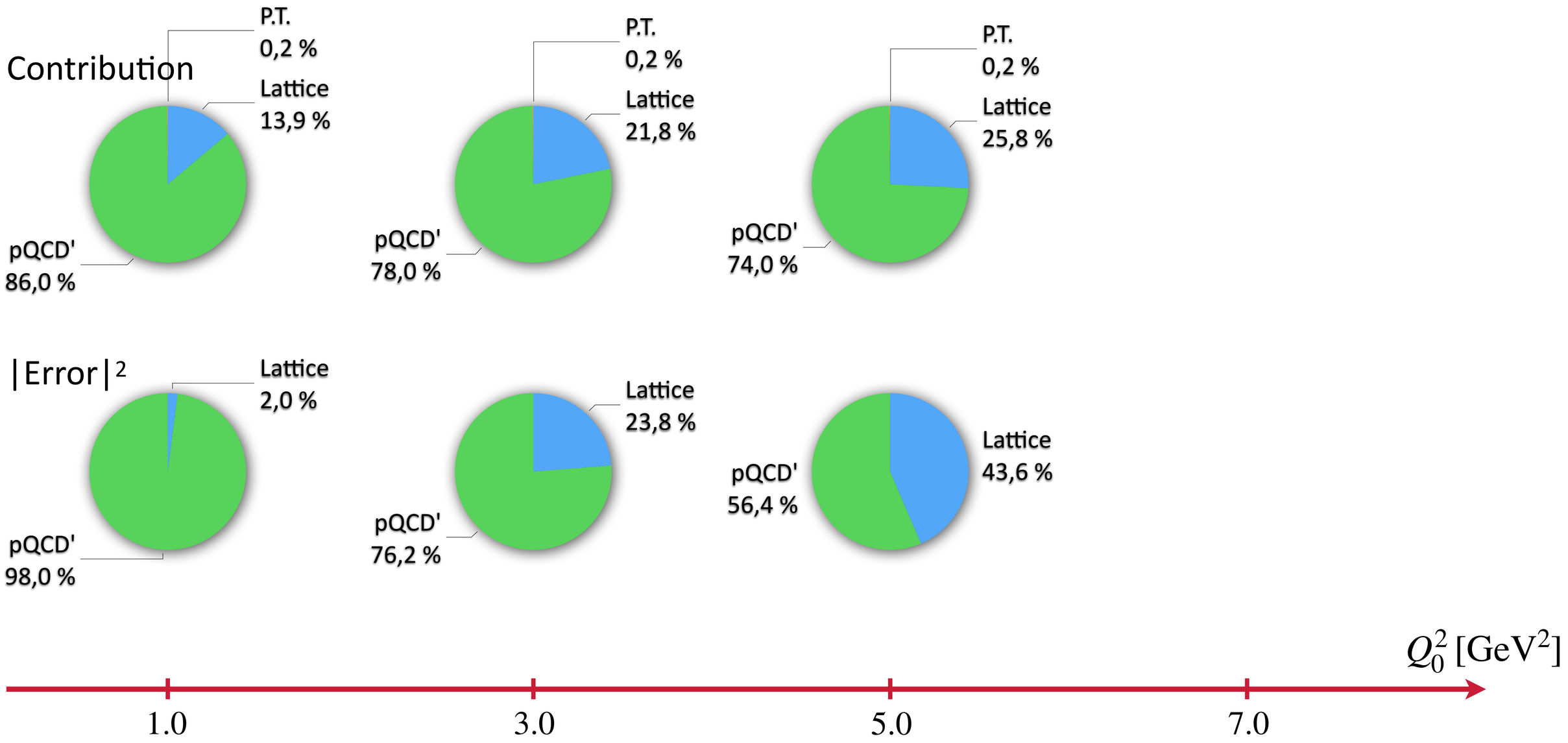

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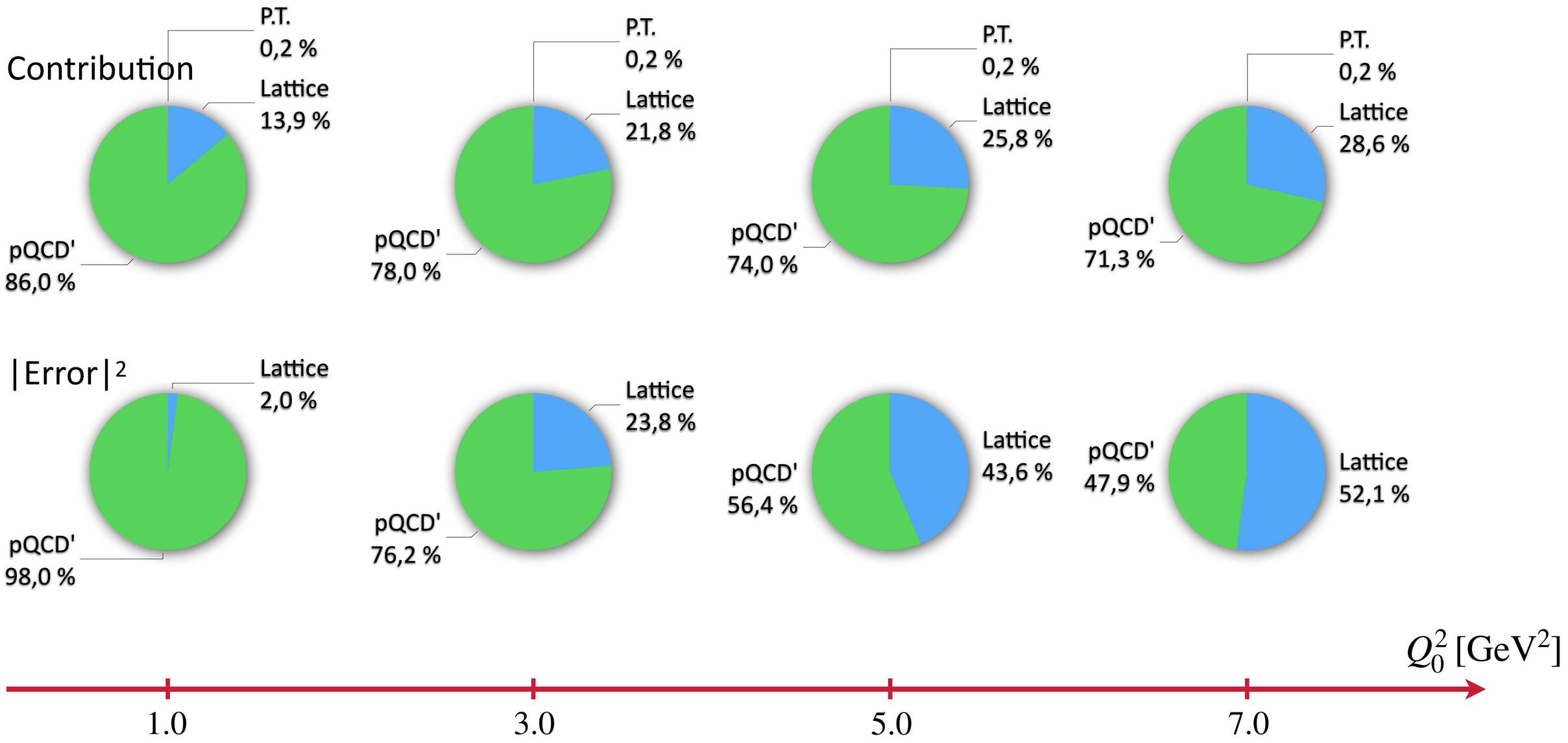

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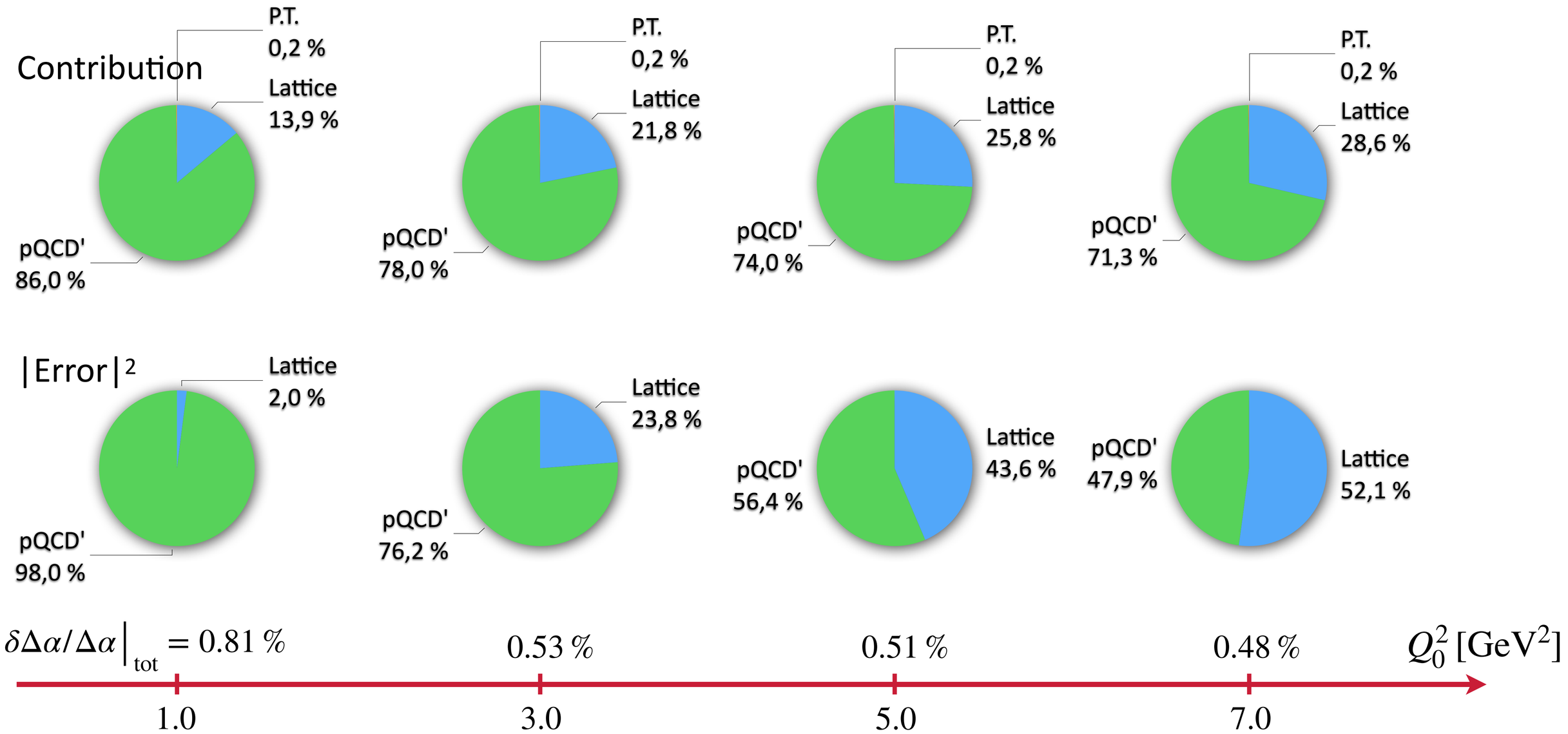

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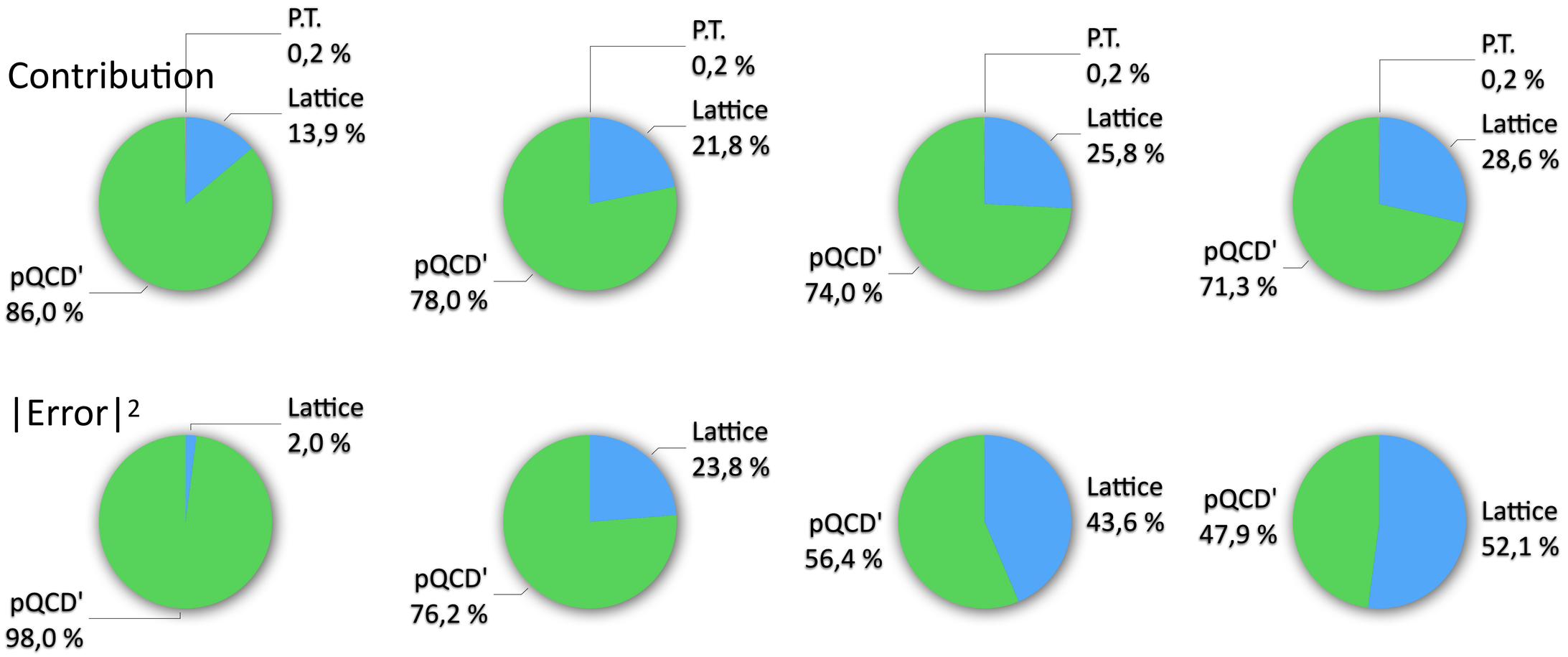


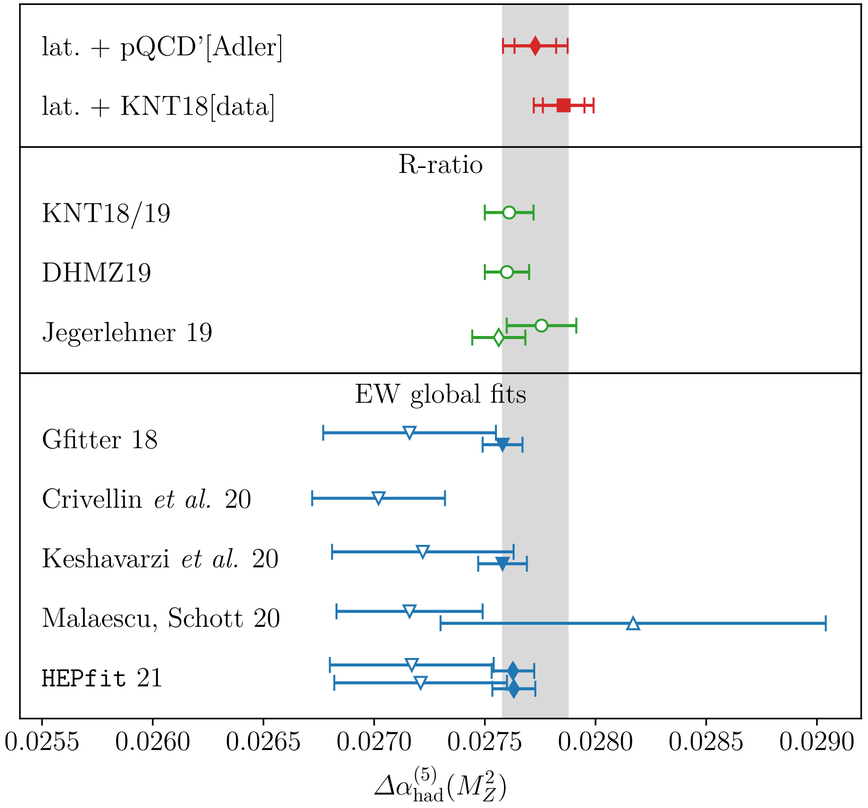
Can tune Q_0^2 to optimise the reliability

- Lattice QCD accounts for $\sim 25\,\%$ of the value of $\Delta\alpha_{\rm had}(M_Z^2)$ and for $\sim (25-50)\,\%$ of the variance

Comparison with phenomenology and electroweak fit

Mainz/CLS: $\Delta\alpha_{\rm had}(M_Z^2) = 0.027\,73(15)$

(pQCD/Adler + lattice input)



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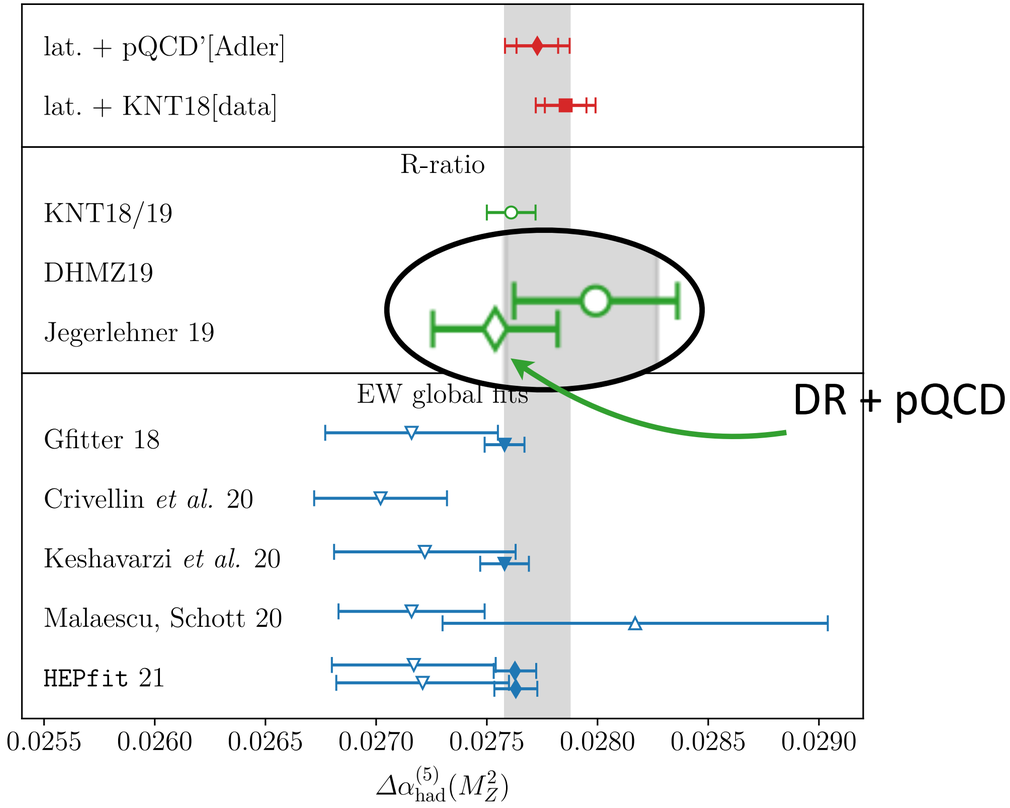

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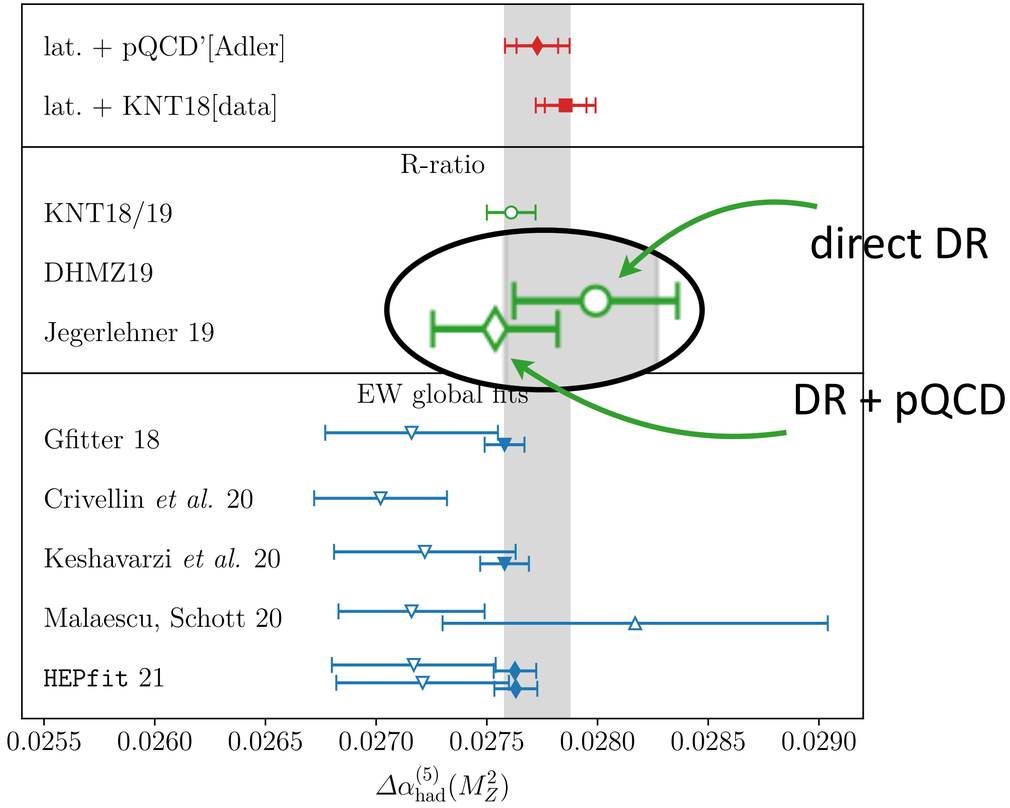

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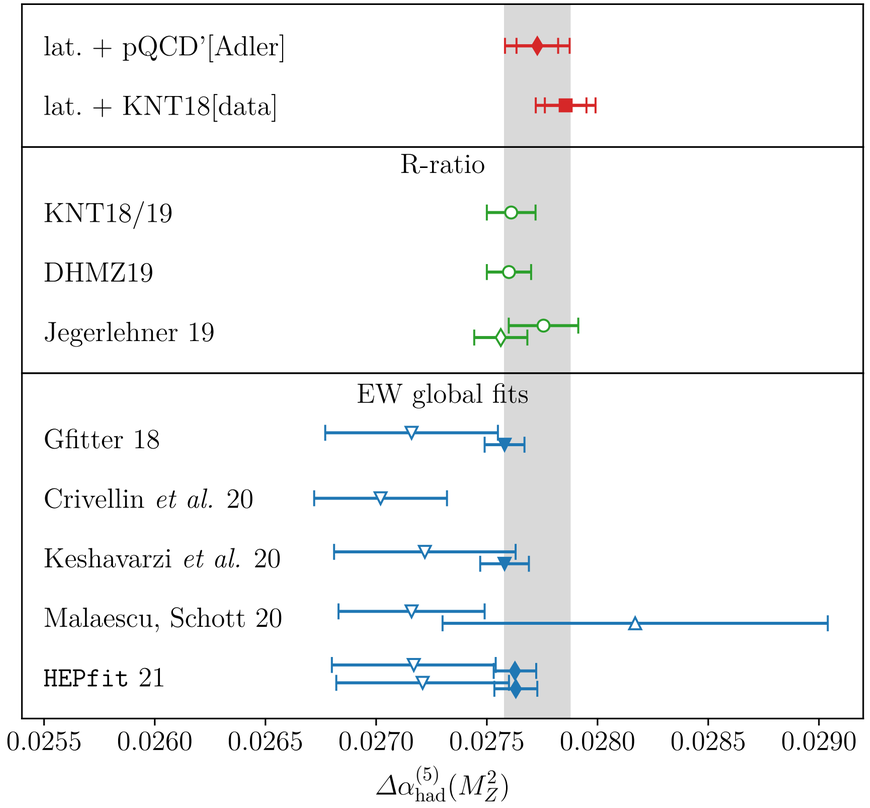

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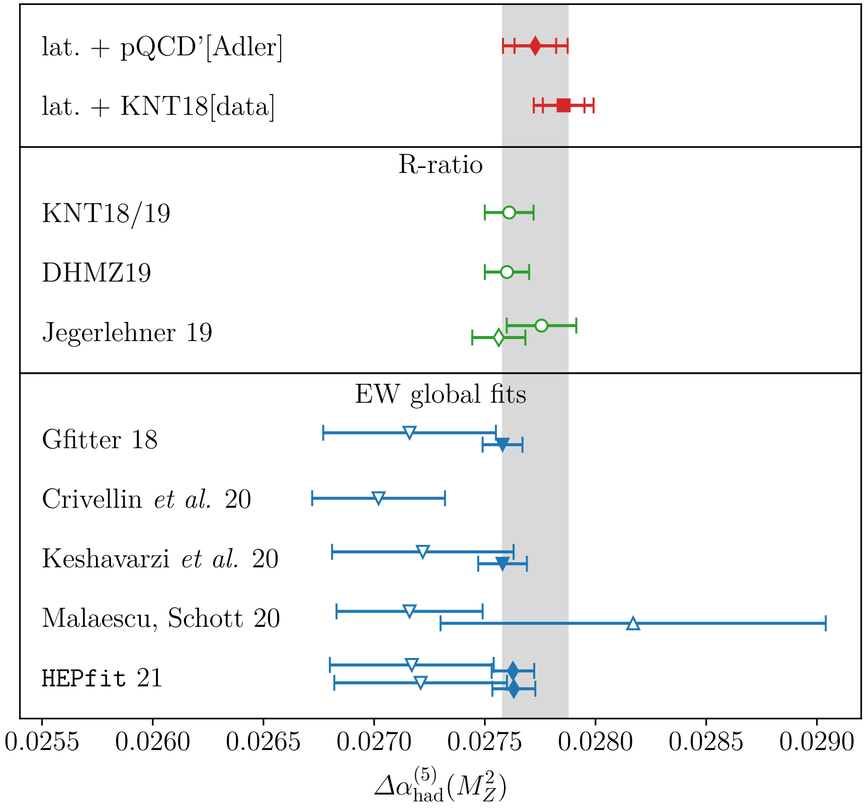
- Running from $-Q_0^2$ to $-M_Z^2$ is correlated



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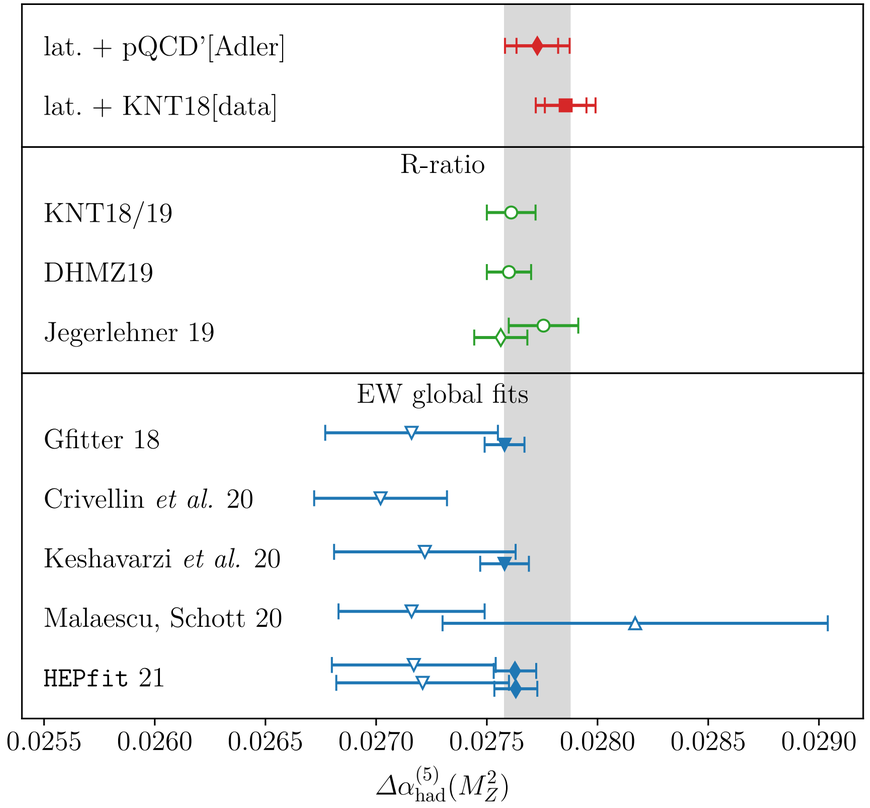

- Agreement within errors at Z-pole obscures the fact that there is a tension of $\sim 3\sigma$ for $Q_0^2 \sim (3-7)\,\mathrm{GeV}^2$
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Electroweak global fit:

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Comparison with phenomenology and electroweak fit

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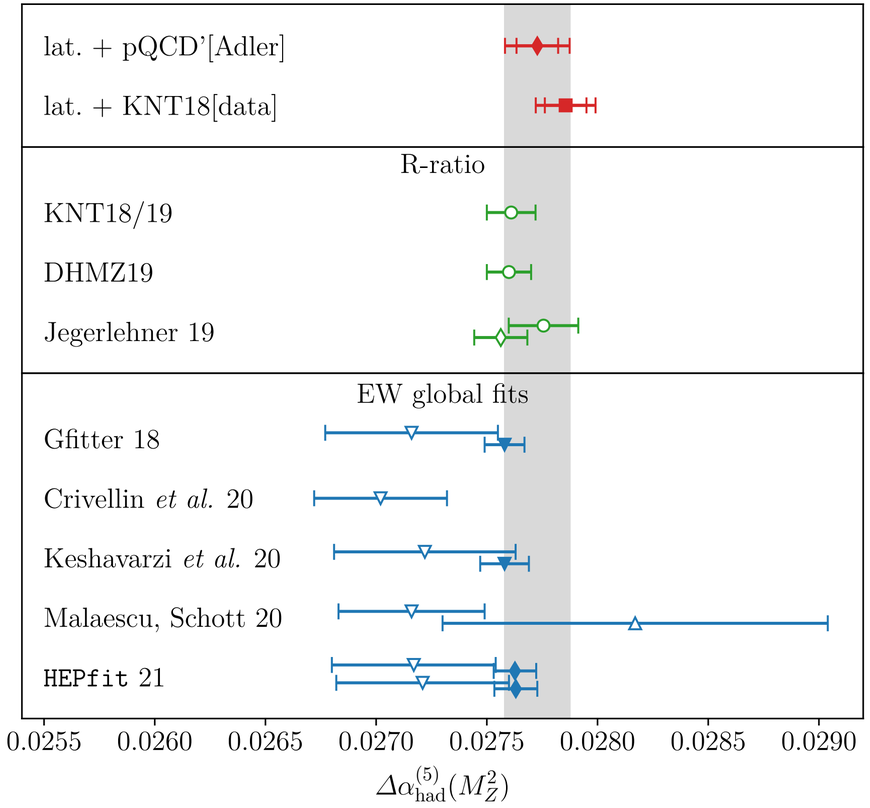

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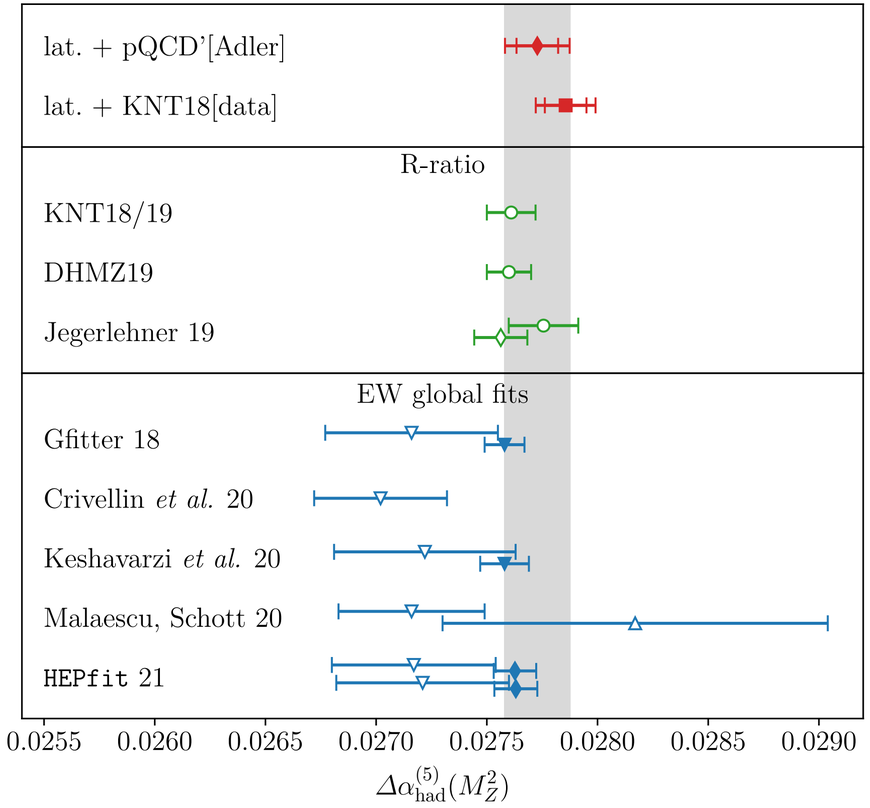

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▼ ◆ : same as ▽ but using priors for $\Delta\alpha_{\rm had}(M_Z^2)$ centred about R-ratio estimate / BMWc

Summary and Discussion

- Lattice+pQCD/Adler estimate for $\Delta\alpha_{\rm had}(M_Z^2)$ broadly agrees with global electroweak fit
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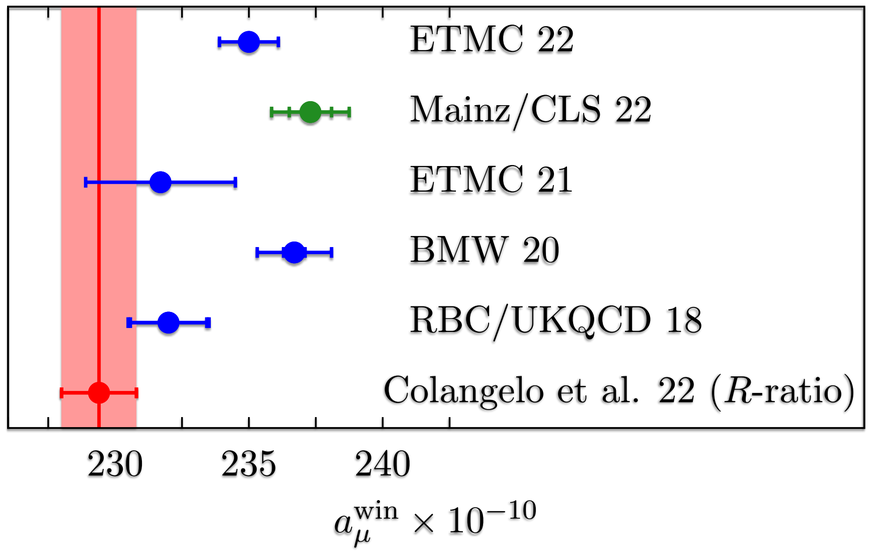

[Cè et al., arXiv:2206.06582]

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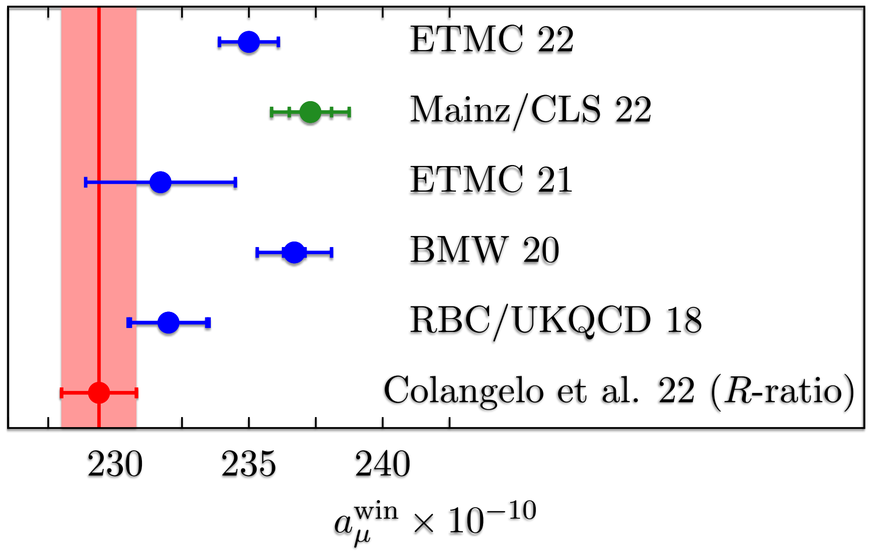
$\Rightarrow$ Standard Model can accommodate a larger value for a_μ without contradicting electroweak precision data



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