

Inclusive semileptonic B -decays from lattice QCD

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Based on “*Lattice QCD study of inclusive semileptonic decays of heavy mesons*” [hep-lat] 2203.11762, JHEP 2022, 83 (2022)

Collaborators:

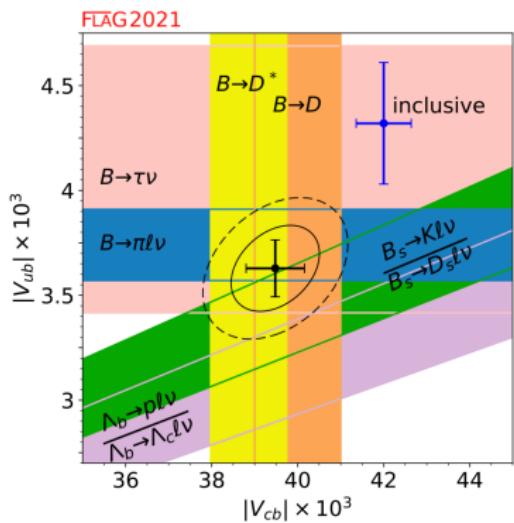
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V_{cb} problem

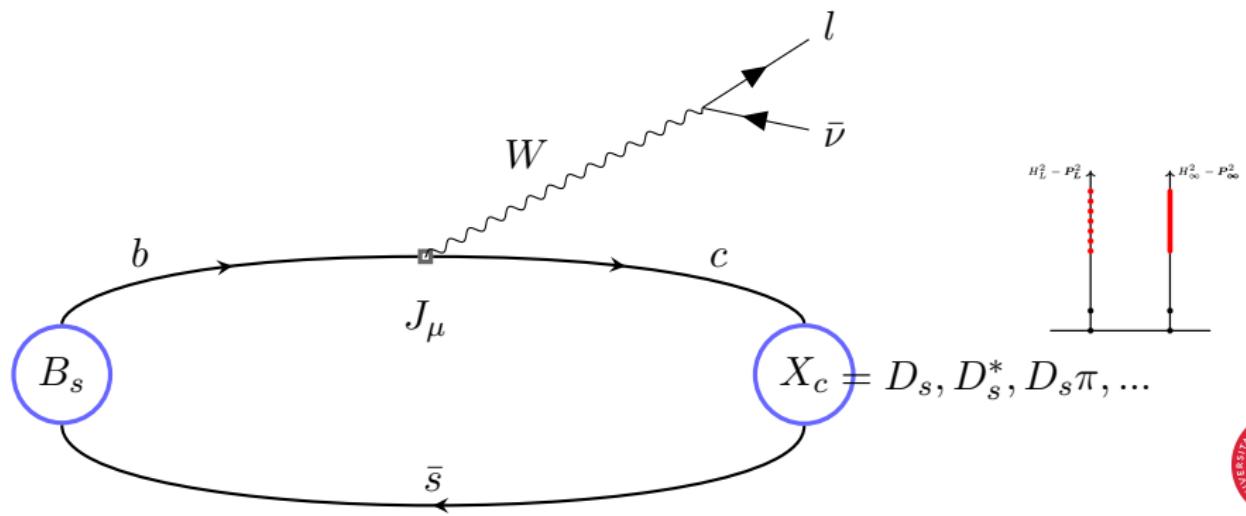
A persistent tension

The tension between the **exclusive** and **inclusive** determination of the CKM parameter V_{cb} is $\sim 3\sigma$



Inclusive $B_s \rightarrow X_c l \bar{\nu}$

$$J_\mu = (V - A)_\mu = \bar{c} \gamma_\mu (1 - \gamma_5) b$$



Decay rate

we start by considering the differential decay rate:

$$\frac{d\Gamma}{dq^2 dq^0 dE_l} = \frac{G_F^2 |V_{cb}|^2}{8\pi^2} L_{\mu\nu} W^{\mu\nu}$$

with:

$$W_{\mu\nu}(w, \mathbf{q}) = \frac{(2\pi)^3}{2m_{B_s}} \langle \bar{B}_s(\mathbf{0}) | J_\mu^\dagger(0) \delta(\hat{H} - w) \delta^3(\hat{P} + \mathbf{q}) J_\nu | \bar{B}_s(\mathbf{0}) \rangle$$

Integrating over E_l :

$$\frac{24\pi^3}{|\mathbf{q}| G_F^2 |V_{cb}|^2} \frac{d\Gamma}{d\mathbf{q}^2} = \sum_{l=0}^2 (\sqrt{\mathbf{q}^2})^{2-l} Z^{(l)}(\mathbf{q}^2)$$



Introducing the kernel

$$\Theta^{(l)}(w_{max} - w) = (w_{max} - w)^l \theta(w_{max} - w)$$

We can write

$$Z^{(l)}(\mathbf{q}^2) = \int_0^\infty dw \Theta^{(l)}(w_{max} - w) T^{(l)}(w, \mathbf{q}^2)$$

with:

$$T^{(0)} = W^{00} + \sum_{i,j=1}^3 \hat{n}_i \hat{n}_j W^{ij} - \sum_{i=1}^3 \hat{n}_i (W^{0i} + W^{i0})$$



Smooth function

On the lattice we have to introduce the lattice spacing a :

$$G^{(l)}(a\tau; \mathbf{q}) = \int_0^\infty dw \mathcal{T}_L^{(l)}(w, \mathbf{q}) e^{-aw\tau}$$

If a function is infinitely differentiable (smooth) then we can approximate it numerically:

$$f(w) = \sum_{\tau}^{\infty} g_{\tau} e^{-aw\tau}$$

So that:

$$\sum_{\tau}^{\infty} g_{\tau} G^{(l)}(a\tau; \mathbf{q}) = \int_0^\infty dw \mathcal{T}_L^{(l)}(w, \mathbf{q}) f(w)$$



Smeared kernel

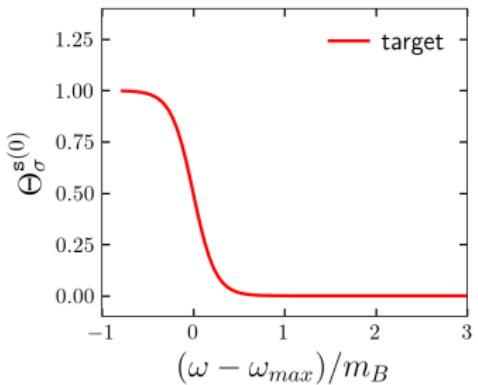
Problem

The θ -function is definitely NOT smooth

Solution

We need to introduce a *smeared* kernel θ_σ

$$\begin{aligned}\Theta_\sigma^{(l)}(w_{max} - w) &= (w_{max} - w)^l \theta_\sigma(w_{max} - w) \\ &= m_{B_s}^l \sum_{\tau=1}^{\infty} g_\tau^{(l)}(w_{max}, \sigma) e^{-aw\tau}\end{aligned}$$



Smeared spectral density

$$\sum_{\tau}^{\infty} g_{\tau}(w_{max}, \sigma) G^{(l)}(a\tau; \mathbf{q}) = \int_0^{\infty} dw T_L^{(l)}(w, \mathbf{q}) \Theta_{\sigma}^{(l)}(w_{max} - w)$$

$$Z^{(l)}(\mathbf{q}^2) = \int_0^{\infty} dw \Theta^{(l)}(w_{max} - w) T^{(l)}(w, \mathbf{q}^2)$$

$$\begin{aligned} Z_{\sigma}^{(l)}(\mathbf{q}^2) &= \int_0^{\infty} dw \Theta_{\sigma}^{(l)}(w_{max} - w) T_L^{(l)}(w, \mathbf{q}^2) \\ &= \sum_{\tau}^{\infty} g_{\tau}(w_{max}, \sigma) G^{(l)}(a\tau; \mathbf{q}) \end{aligned}$$



HLT algorithm

The details can be found in:

[Hansen, Lupo, Tantalo '19, Phys. Rev. D 99, 094508]

[hep-lat/1903.06476], see also J. Bulava plenary talk (Tue 9:20) and A. De Santis parallel talk (Thu 10:00)

we want to reconstruct:

$$\Theta_{\sigma}^{(l)}(w_{max} - w) = \sum_{\tau=1}^{\tau_{max}} g_{\tau} e^{-aw\tau}$$

$$W_{\lambda}[g] = (1 - \lambda) \frac{A[g]}{A[0]} + \lambda B[g]$$

$$\left. \frac{\partial W_{\lambda}[g]}{\partial g_{\tau}} \right|_{g_{\tau}=g_{\tau}^{\lambda}} = 0$$



$A[g^\lambda]$ is the reconstruction bias

$$A[g] = a \int_{E_0}^{\infty} dw \left\{ \Theta_\sigma^{(l)} - \sum_{\tau=1}^{\tau_{max}} g_\tau e^{-aw\tau} \right\}^2$$

$B[g^\lambda]$ is the statistical variance

$$B[g] = \sum_{\tau, \tau'=1}^{\tau_{max}} g_\tau g_{\tau'} \frac{Cov[G^{(l)}(a\tau), G^{(l)}(a\tau')]}{[G^{(l)}(0)]^2}$$

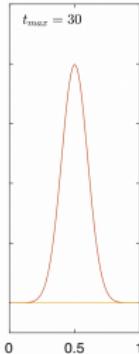
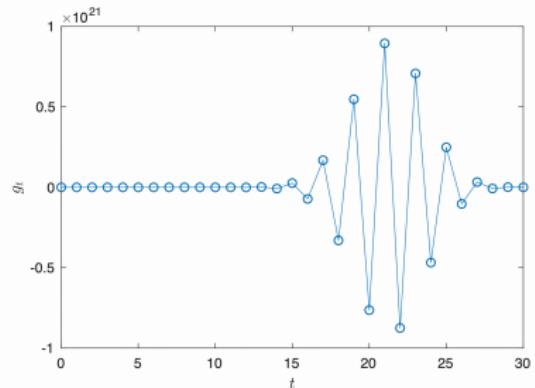


g_τ^λ defines the approximation of the kernel

$$\Theta_\sigma^\lambda = \sum_{\tau=1}^{\tau_{max}} g_\tau^\lambda e^{-aw\tau}$$

the best choice for λ can be found via:

$$\left. \frac{\partial W(\lambda)}{\partial \lambda} \right|_{\lambda=\lambda_*} = 0$$



Smeared kernel reconstruction

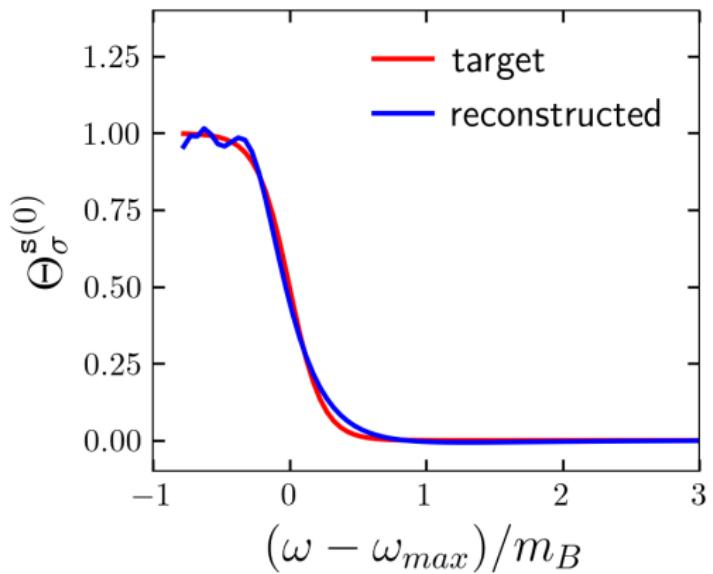


Figure: $\lambda = \lambda_*$



Smeared Kernel λ

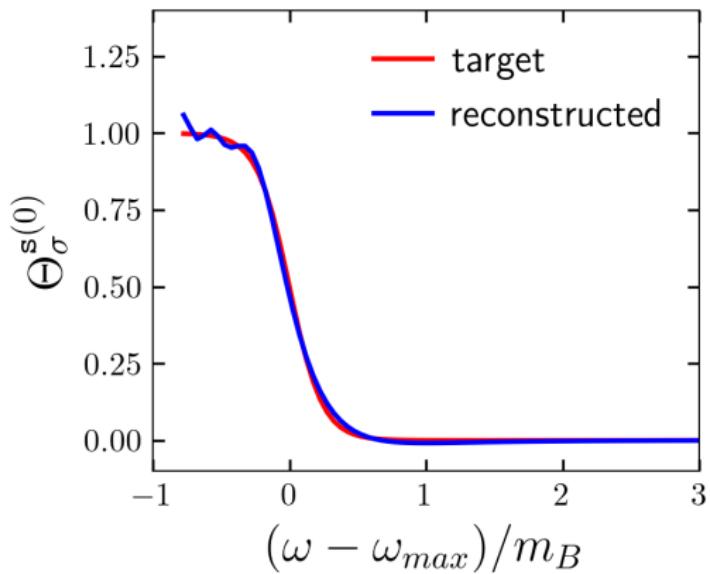


Figure: $\lambda < \lambda_*$



ETMC ensemble

B55.32 ensemble

Uses the Twisted Mass QCD action with $N_f = 2 + 1 + 1$ sea quarks.

$L^3 \times T$	N_{cfg}	a (fm)
$32^3 \times 64$	150	0.0815(30)
m_{ud}^{phys}		m_s^{phys}
3.70(17)	99.6(4.3)	m_c^{phys}
		m_π
375(13) (MeV)		

Unphysically light B_s mass

Use of $m_c \simeq m_c^{phys}$ and $m_b = 2m_c$ which gives a meson mass
 $M_{B_s} = 3.08(11)$ GeV

JLQCD ensemble

M-ud3-sa ensemble

Uses the DWF QCD action with $N_f = 2 + 1$ sea quarks.

$L^3 \times T \times L_s$	N_{cfg}	$a \text{ (fm)}$
$48^3 \times 96 \times 8$	42	0.055
am_{ud}	am_s	m_π
0.0042	0.0025	$300(1) \text{ (MeV)}$

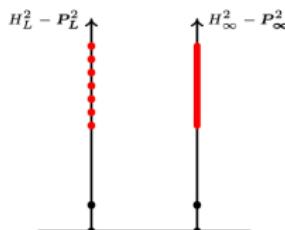
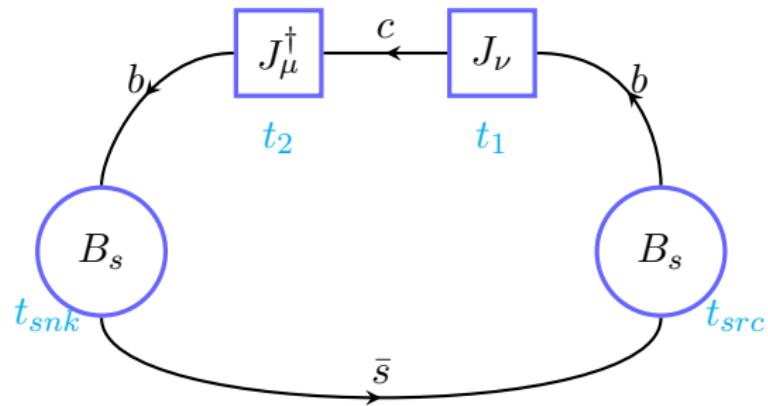
Unphysically light B_s mass

Use of $m_c = m_c^{phys}$ and $m_b = 2.44m_c$ which gives a meson mass
 $M_{B_s} \simeq 3.45 \text{ GeV}$

Euclidean 4-point Correlator

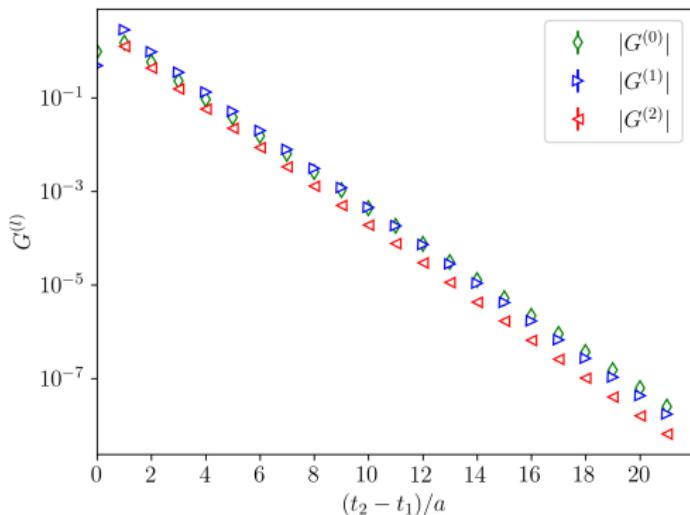
[Gambino, Hashimoto'20, Phys. Rev. Lett. 125, 032001, hep-lat/2005.13730]

$$C_{\mu\nu}(t_{snk}, t_2, t_1, t_{src}; \mathbf{q}) = \int d^3x e^{i\mathbf{q}\cdot\mathbf{x}} T\langle 0 | \tilde{\phi}_{B_s}(\mathbf{0}, t_{snk}) J_\mu^\dagger(\mathbf{x}, t_2) J_\nu(\mathbf{0}, t_1) \tilde{\phi}_{B_s}^\dagger(\mathbf{0}, t_{src}) | 0 \rangle$$



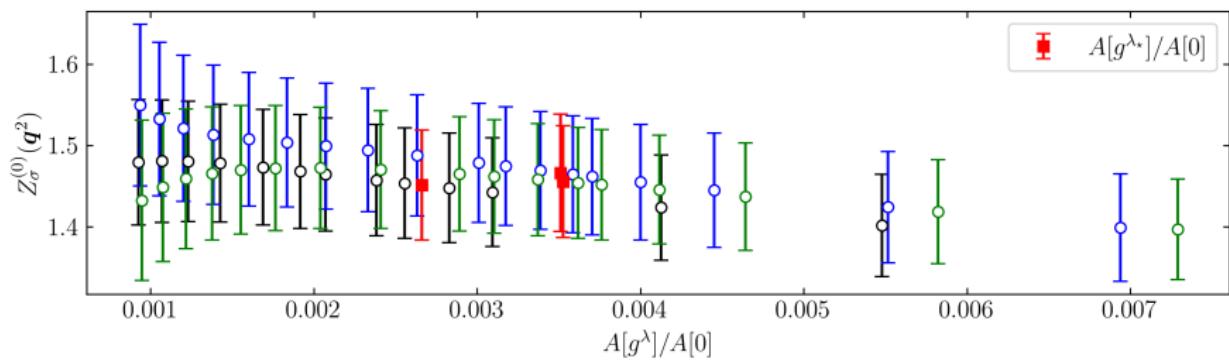
G correlators

$$G^{(l)}(t_2 - t_1; \mathbf{q}) = M_{\mu\nu}(t_2 - t_1; \mathbf{q}) = Z_{B_s} \lim_{\substack{t_{\text{snk}} \rightarrow +\infty \\ t_{\text{src}} \rightarrow -\infty}} \frac{C_{\mu\nu}(t_{\text{snk}}, t_2, t_1, t_{\text{src}}; \mathbf{q})}{C(t_{\text{snk}} - t_2)C(t_1 - t_{\text{src}})}$$



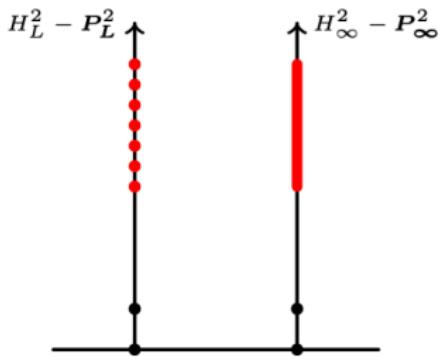
Smeared Spectral Density

Now we can apply the smeared kernel to $Z_\sigma^{(l)}(\mathbf{q})$ which is obtained by applying the coefficients g_τ^λ to the correlator $G^{(l)}(t, \mathbf{q}^2)$ and perform the integral over the allowed phase space

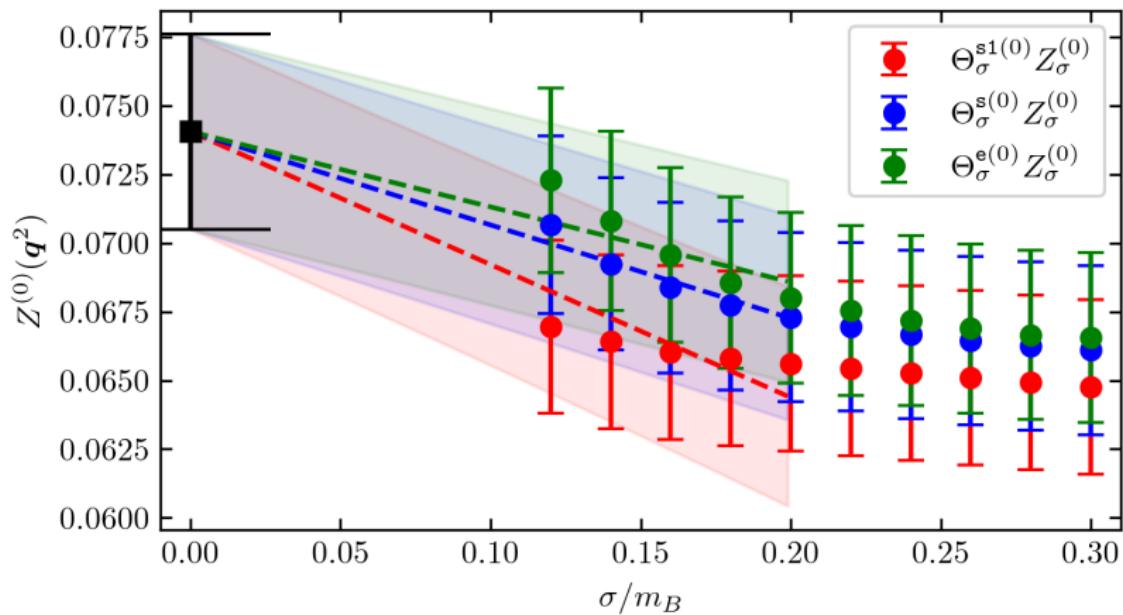


$\sigma \rightarrow 0$ limit

$$\begin{aligned}
 Z^{(l)}(\mathbf{q}^2) &= \lim_{\sigma \rightarrow 0} \left(\lim_{V \rightarrow \infty} \right) \int_0^\infty dw \mathbf{T}_L^{(l)}(w, \mathbf{q}^2) \Theta_\sigma^{(l)}(w_{max} - w) \\
 &= \lim_{\sigma \rightarrow 0} \left(\lim_{V \rightarrow \infty} \right) m_{B_s}^l \sum_{\tau} g_\tau^{(l)}(w_{max}, \sigma) G^{(l)}(a\tau, \mathbf{q})
 \end{aligned}$$



Extrapolation to $\sigma = 0$



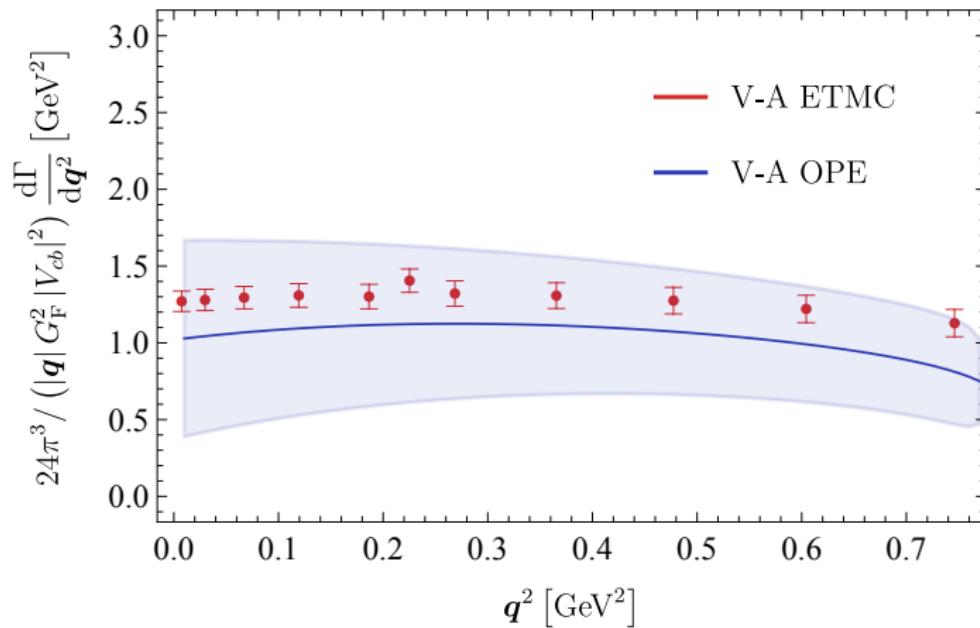
Differential decay rate

$$\frac{24\pi^3}{|\mathbf{q}|G_F^2|V_{cb}|^2} \frac{d\Gamma}{d\mathbf{q}^2} = \sum_{l=0}^2 (\sqrt{\mathbf{q}^2})^{2-l} Z^{(l)}(\mathbf{q}^2)$$



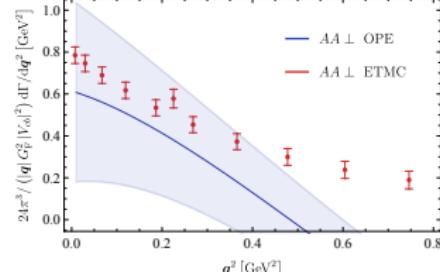
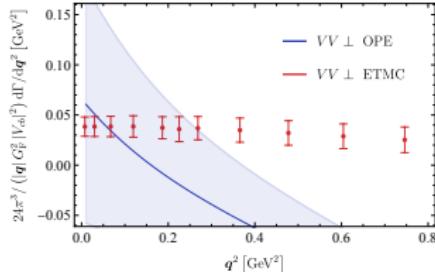
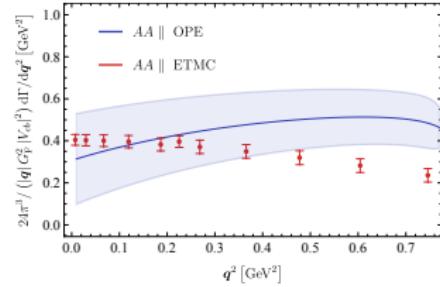
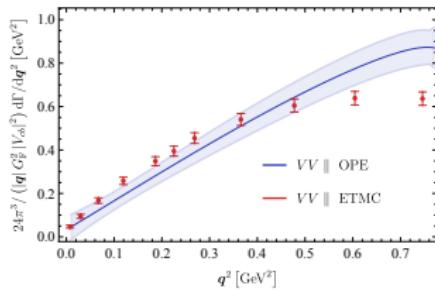
Differential decay rate comparison

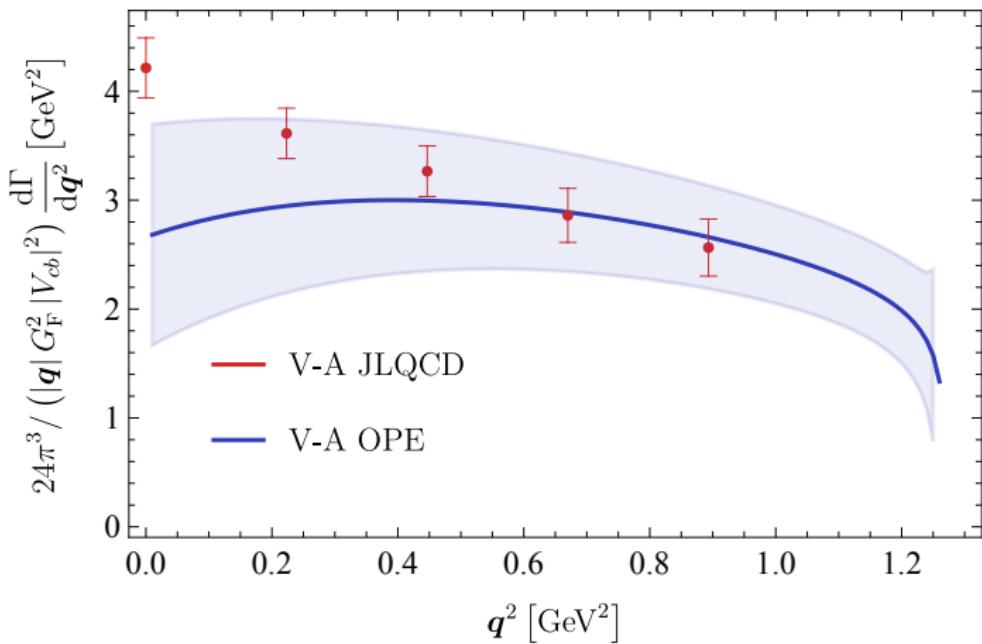
After we take the $\sigma \rightarrow 0$ limit, we can confront our results with the analytic results from the OPE

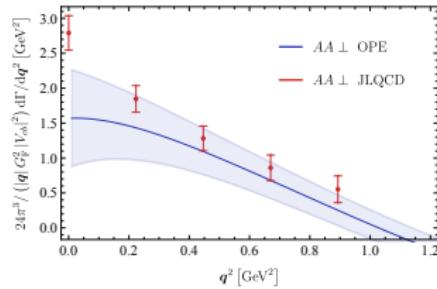
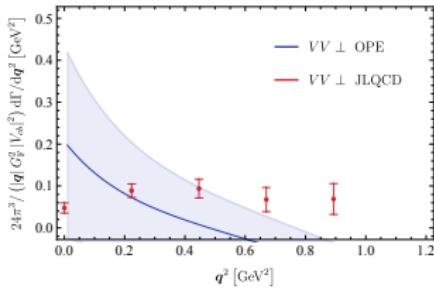
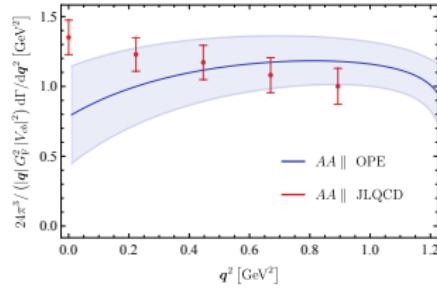
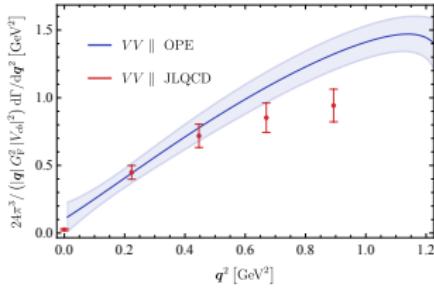


Channel decomposition

One can decompose the contribution to the differential decay rate into V_{\parallel} , V_{\perp} , A_{\parallel} , A_{\perp}







Final Results

The last thing to do is to perform the integration over q^2

	ETMC	OPE
$\Gamma/ V_{cb}^2 \times 10^{13}$ (GeV)	0.987(60)	1.20(46)
$\langle E_\ell \rangle$ (GeV)	0.491(15)	0.441(43)
$\langle E_\ell^2 \rangle$ (GeV ²)	0.263(16)	0.207(49)
$\langle E_\ell^2 \rangle - \langle E_\ell \rangle^2$ (GeV ²)	0.022(16)	0.020(8)
$\langle M_X^2 \rangle$ (GeV ²)	3.77(9)	4.32(56)

Table: Total width and moments in the ETMC case.



	JLQCD	OPE
$\Gamma/ V_{cb}^2 \times 10^{13}$ (GeV)	4.46(21)	5.7(9)
$\langle E_\ell \rangle$ (GeV)	0.650(40)	0.626(36)
$\langle M_X^2 \rangle$ (GeV 2)	3.75(31)	4.22(30)

Table: Total width and moments in the JLQCD case.



Summary & future prospects

- Calculated the inclusive decay rate of semileptonic B_s decays
- Comparison with OPE promising, need full lattice calculation
- Perform the analysis with several values of lattice spacing, volumes and b -quark masses in order to perform the extrapolations: $a \rightarrow 0$, $V \rightarrow \infty$, $m_b \rightarrow m_b^{phys}$
- Apply this method to other inclusive semileptonic decays for which we have good experimental results for Branching ratios, such as those involving the $D_{(s)}$ -meson
- Extend this method to semileptonic decays needed to estimate V_{ub} such as $B \rightarrow X_u l \nu$



BACKUP SLIDES



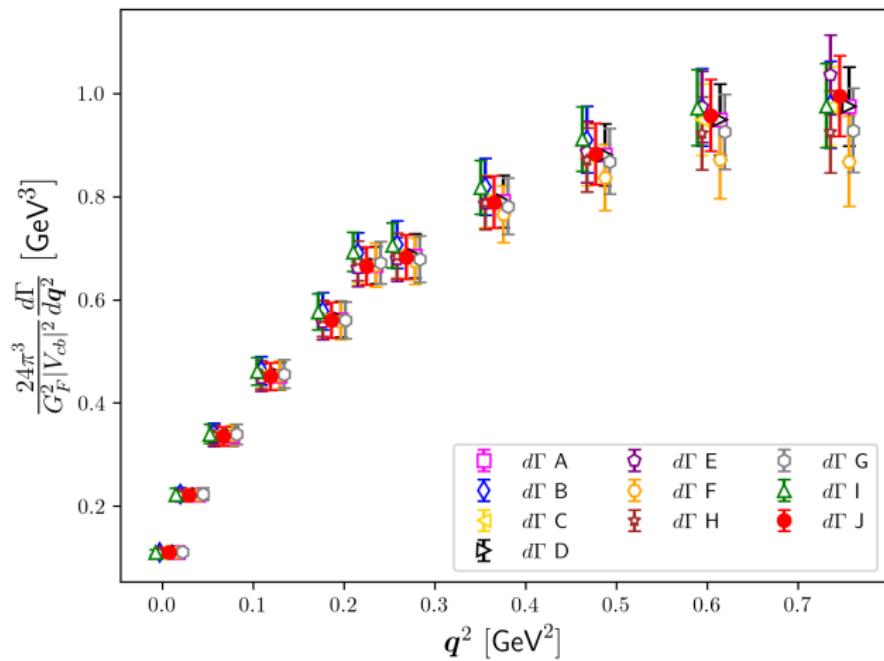
Tensor Decomposition

According to Lorentz invariance and time-reversal symmetry, the Hadronic Tensor can be decomposed as follows

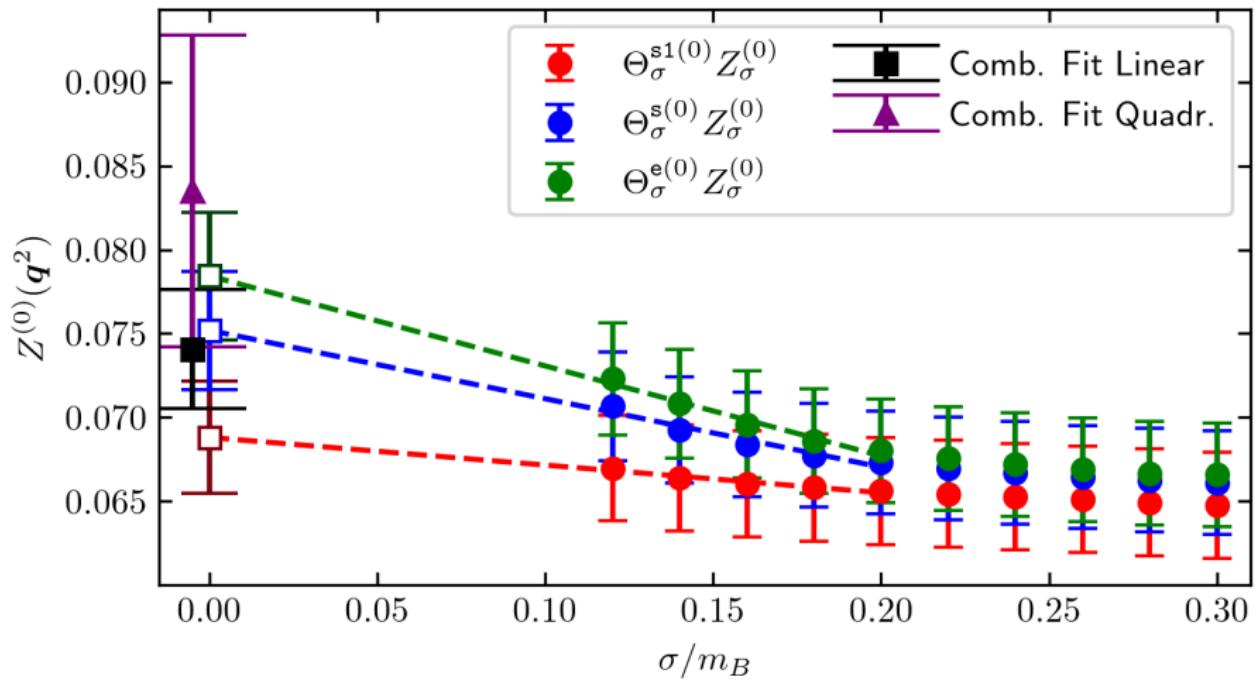
$$\begin{aligned} W^{\mu\nu}(p, q) = & -g^{\mu\nu}W_1(w, \mathbf{q}^2) + \frac{p^\mu p^\nu}{m_{B_s}^2}W_2(w, \mathbf{q}^2) - \frac{i\varepsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta}{m_{B_s}^2}W_3(w, \mathbf{q}^2) \\ & + \frac{q^\mu q^\nu}{m_{B_s}^2}W_4(w, \mathbf{q}^2) + \frac{p^\mu q^\nu + p^\nu q^\mu}{m_{B_s}^2}W_5(w, \mathbf{q}^2) \end{aligned}$$



Systematics



Extrapolation



Three kernels

$$\theta_\sigma^{\text{s}}(x) = \frac{1}{1 + e^{-\frac{x}{\sigma}}} , \quad \theta_\sigma^{\text{s1}}(x) = \frac{1}{1 + e^{-\sinh\left(\frac{x}{r^{\text{s1}}\sigma}\right)}} ,$$

$$\theta_\sigma^{\text{e}}(x) = \frac{1 + \text{erf}\left(\frac{x}{r^{\text{e}}\sigma}\right)}{2} ,$$



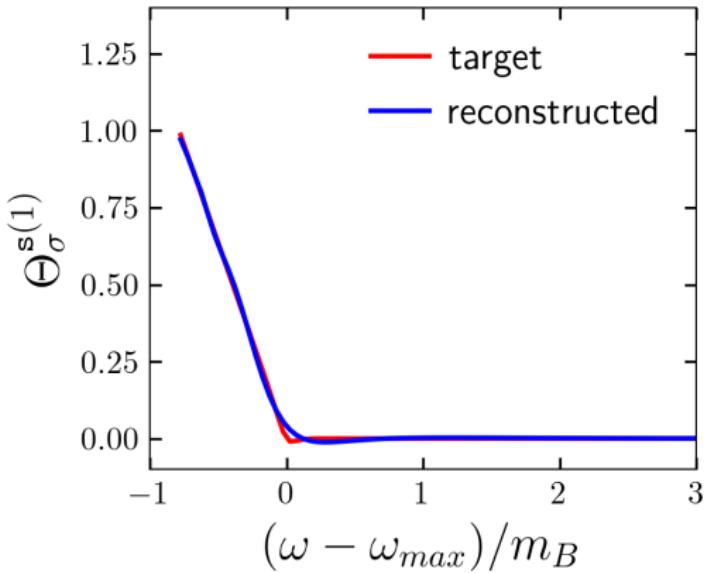


Figure: $\lambda = \lambda_*$



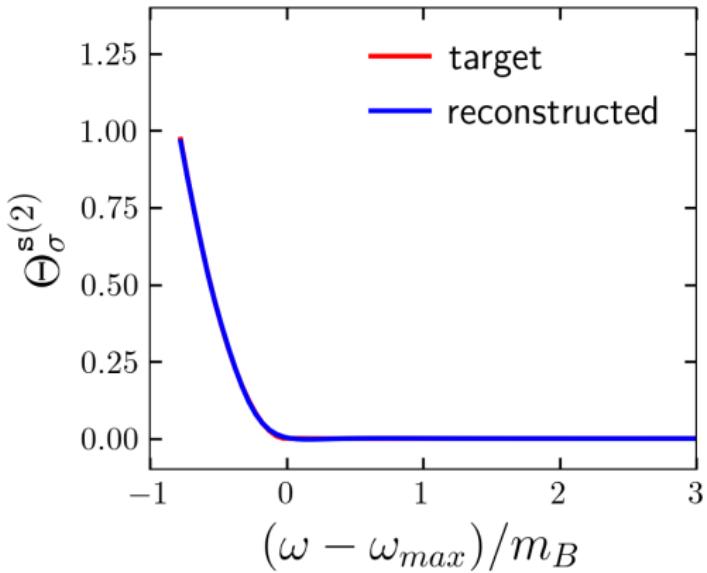


Figure: $\lambda = \lambda_*$



Modified Sigmoid

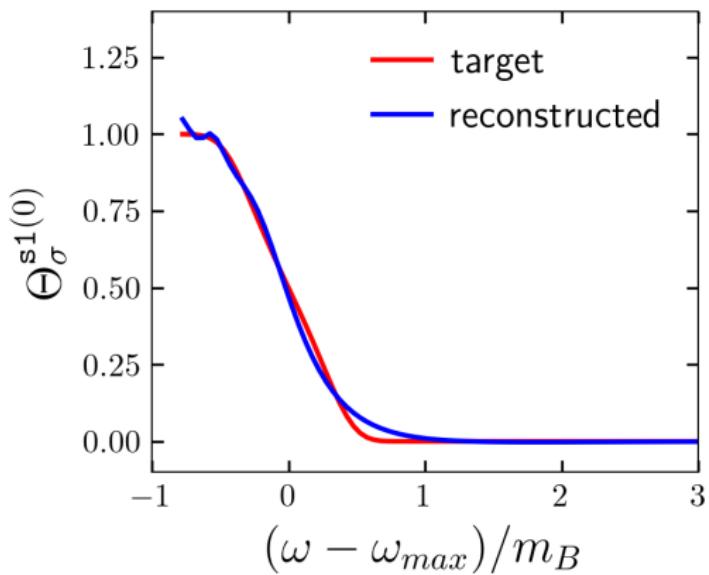


Figure: $\lambda = \lambda_*$



Error Function

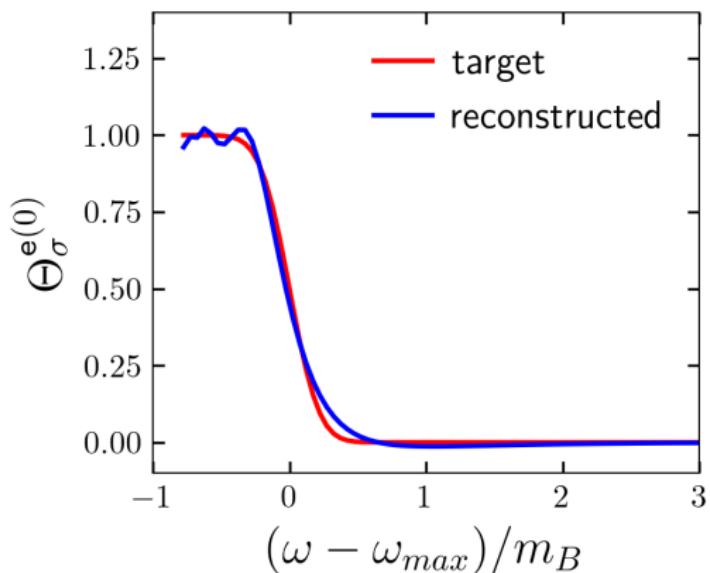


Figure: $\lambda = \lambda_*$



Lepton Energy Moment

We also looked at the Lepton Energy Moment defined as

$$L_1(\mathbf{q}^2) = \frac{\int dq_0 dE_l E_l \left[\frac{d\Gamma}{d\mathbf{q}^2 dq_0 dE_l} \right]}{\int dq_0 dE_l \left[\frac{d\Gamma}{d\mathbf{q}^2 dq_0 dE_l} \right]}$$

