Testing universality in Lattice gauge theories

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Universality

Symanzik effective theory

- Any lattice action that we simulate S_{latt} can be described by an effective action

$$S_{\text{latt}} \overset{a\to 0}{\sim} S_{\text{cont}} + a^2 S_2 + \ldots$$

- Spectral quantities computed on the lattice have an asymptotic expansion

$$\langle O \rangle_{\text{latt}} \overset{a\to 0}{\sim} \langle O \rangle + a^2 \langle O S_2 \rangle_c + \ldots$$

Questions

- Which S_{latt} has smaller cutoff effects?
- How to investigate this? **[Husung @ Fri]**
- Is g_0 small enough? (i.e. can we claim 0.2% precision simulating $a \sim 0.05 - 0.13$ fm?)

Universality

Symanzik effective theory

- Any lattice action that we simulate $S_{\rm latt}$ can be described by an effective action

$$S_{\rm latt} \overset{a\to 0}{\sim} S_{\rm cont} + a^2 S_2 + \ldots$$

- Spectral quantities computed on the lattice have an asymptotic expansion

This talk

- Checking that different actions give the same results (after $a \to 0$ extrapolation) is becoming more and more pressing.
- Some points on checks on scaling violations
- Preliminary results on pure gauge

- How to investigate this? **[Husung @ Fri]**
- Is g_0 small enough? (i.e. can we claim 0.2% precision simulating $a \sim 0.05 - 0.13$ fm?)

Flow scales

Most natural quantities

- Numerically very precise
- Little systematic (i.e. no signal to noise)

Two natural candidates

- t_0 - like scales **[Luscher '10]**

$$t^2 \langle E(t) \rangle \Big|_{t=t_c} = \begin{cases} 0.3 & (t_c = t_0) \\ 0.5 & (t_c = t_1) \end{cases}$$

- w_0 - like scales **[BMW '10]**

$$t \frac{\mathrm{d}}{\mathrm{d}t} t^2 \langle E(t) \rangle \Big|_{t=w_c^2} = \begin{cases} 0.285 & (w_c = w_A) \\ 0.550 & (w_c = w_B) \end{cases}$$

NOTE

Weird choice for w - like scales

$$w_A^2 \approx t_0; \quad w_B^2 \approx t_1 \, .$$

Testing scaling with flow quantities: The wrong way

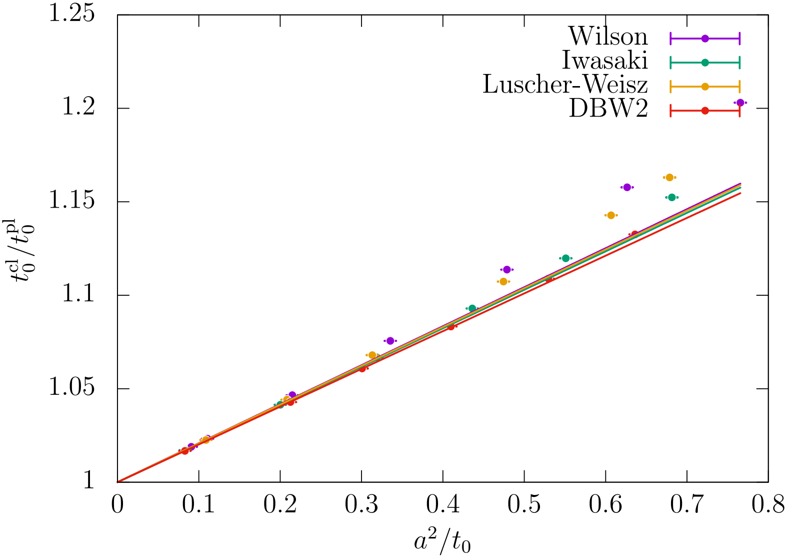
Ideal quantity: $t_0^{\mathrm{pl}}/t_0^{\mathrm{cl}}$

- We know the continuum limit

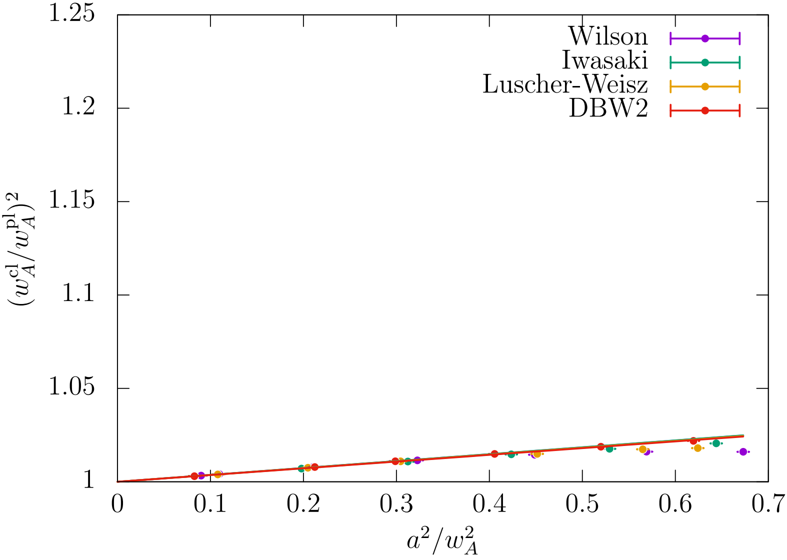
$$\lim_{a\to 0}\frac{t_0^{\mathrm{pl}}}{t_0^{\mathrm{cl}}} = 1$$

- Extremely precise (correlated numerator/denominator)
- Same game with $(w_A^{\mathrm{pl}}/w_A^{\mathrm{cl}})^2$

Testing scaling with flow quantities: The wrong way



Testing scaling with flow quantities: The wrong way



Testing scaling with flow quantities: The wrong way

Wrong conclusions

- Wilson, Iwasaki, LW, DB2 all have similar cutoff effects (see [Husung, Fri]).
- w_0 - like scales have much smaller cutoff effects
- Violations to a^2 scaling are below 8% at $a < 0.08$ fm for t_0 - like scales
- Violations to a^2 scaling are below 1% at $a < 0.08$ fm for w_0 - like scales

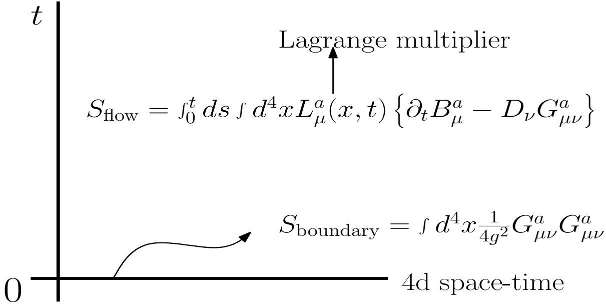
Testing scaling with flow quantities: The wrong way

Symanzik effective description for flow quantities

- $t^2\langle E(t)\rangle$ is a *non-local* observable (i.e. smeared over a distance $\sqrt{8t}$)
- Special care to interpret scaling violations of flow quantities

5d Local formulation [Lüscher, Weisz '11]

We can see the theory as a 5d local field theory [Zinn-Justin '86, Zinn-Justin, Zwanziger '88]



$$S_{\text{Total}} = S_{\text{flow}} + S_{\text{boundary}}$$

The important point

- No loops on the bulk $\Rightarrow$ "Classical theory" at $t > 0$

Symanzik effective theory for the gradient flow [A. Ramos, S. Sint '15]

Symanzik effective theory has several "parts"

$$S^{5d}_{\rm latt} \overset{a\to 0}{\sim} S^{5d}_{\rm cont} + a^2 S_{2,b} + a^2 S_{2,fl} + \dots$$

- "Usual" corrections
- Affects <u>all</u> quantities (i.e. $m_p, g-2, t_0, \dots$)
- Determined by the action that you simulate (i.e. Iwasaki/Wilson, Domain Wall/Clover)
- Affects <u>only</u> flow quantities
- Determined by *how you integrate the flow equations* (i.e. Wilson/Symanzik flow)

Symanzik expansion of a flow quantity $O \overset{a\to 0}{\sim} O_0 + a^2 O_2$

$$\langle O\rangle_{\rm latt} \overset{a\to 0}{\sim} \langle O_0\rangle + a^2\left\{\langle O_2\rangle + \langle O_0 S_{2,b}\rangle + \langle O_0 S_{2,fl}\rangle\right\}$$

Theory "classical" at $t>0$:

$$\begin{aligned} \text{Use Zeuthen flow} \quad &\Longrightarrow \quad S_{2,fl} = 0 \\ \text{Use Classically improved observables} \quad &\Longrightarrow \quad O_2 = 0 \end{aligned}$$

Non-perturbative result/all orders

Understanding $t_0^{\rm pl}/t_0^{\rm cl}$

Apply Symanzik expansion for t_0

$$t_0^{\rm pl} \overset{a\to 0}{\sim} t_0 - \frac{a^2}{D}\left\{t_0^2\langle E(t_0)S_{2,b}\rangle + t_0^2\langle E(t_0)S_{2,fl}\rangle + t_0^2\langle E_2^{\rm pl}(t_0)\rangle\right\}$$
$$t_0^{\rm cl} \overset{a\to 0}{\sim} t_0 - \frac{a^2}{D}\left\{t_0^2\langle E(t_0)S_{2,b}\rangle + t_0^2\langle E(t_0)S_{2,fl}\rangle + t_0^2\langle E_2^{\rm cl}(t_0)\rangle\right\}$$

The ratio/difference does not say anything useful

$$\frac{t_0^{\rm pl}}{t_0^{\rm cl}} \overset{t\to 0}{\sim} 1 - \frac{a^2}{D}\left\{t_0^2\langle E_2^{\rm pl}(t_0)\rangle - t_0^2\langle E_2^{\rm cl}(t_0)\rangle\right\}$$

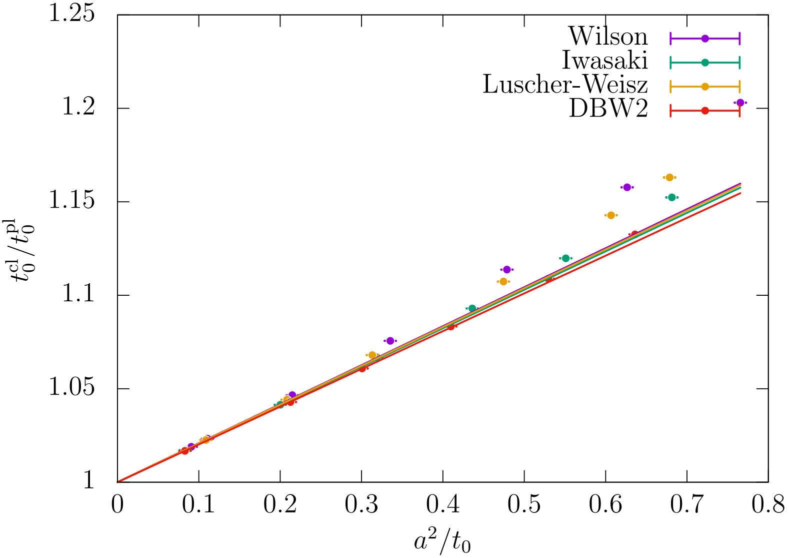
- Insensitive to $S_{2,b}$ that is the only piece that affects $m_p, g-2, \ldots$
- Only sensitive to something that can be made zero explicitly: Choose

$$E^{\rm latt}(t) = \frac{4}{3}E^{\rm pl}(t) - \frac{1}{3}E^{\rm cl}(t)$$

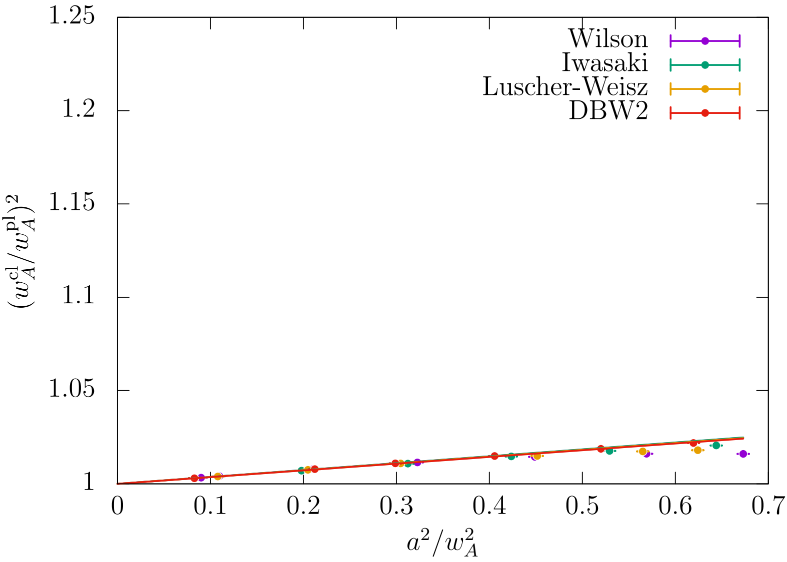
and

$$E_2^{\rm latt}(t) = 0$$

Understanding $t_0^{\rm pl}/t_0^{\rm cl}$



Understanding $t_0^{\mathrm{pl}}/t_0^{\mathrm{cl}}$



Improvement of the flow [A. Ramos, S. Sint '15]

$$\langle O \rangle_{\mathrm{latt}} \overset{a \to 0}{\sim} \langle O_0 \rangle + a^2 \left\{ \langle O_2 \rangle + \langle O_0 S_{2,b} \rangle + \langle O_0 S_{2,fl} \rangle \right\}$$

The Zeuthen flow: $S_{2,fl} = 0$

$$a^2 \frac{d}{dt} V_\mu(x,t) = -g_0^2 \left(1 + \frac{a^2}{12} D_\mu D_\mu^* \right) \frac{\delta S^{\mathrm{LW}}[V]}{\delta V_\mu(x,t)} V_\mu(x,t)$$

Classically improved observables: $O_2 = 0$

$$E^{\mathrm{latt}}(t) = \frac{4}{3} E^{\mathrm{pl}}(t) - \frac{1}{3} E^{\mathrm{cl}}(t)$$

Extra required improvement parameter: $c_b(g_0^2)$

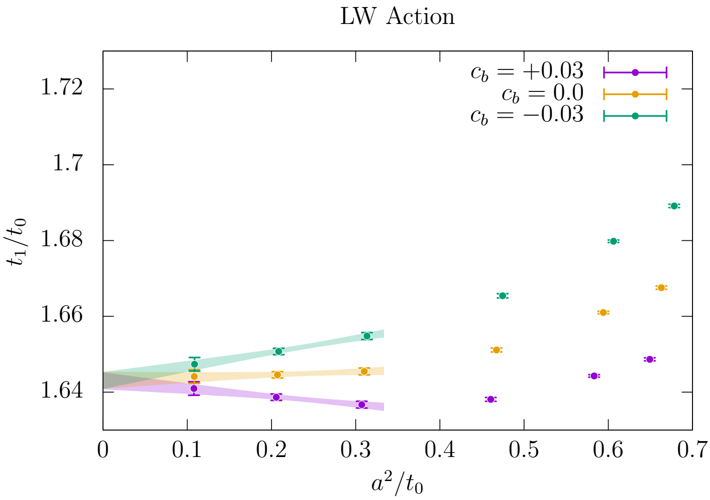
- Shift in the initial condition (similar to τ-shift [Cheng et. al. '14])

$$V_\mu(t,x)\Big|_{t=0} = \exp\{c_b g_0^2 \partial_{x,\mu} S_g[U]\} U_\mu(x)$$

- Tree-level improvement requires $c_b^{(0)}(g_0^2) = 0$. Reasonable range $|c_b| < 0.03$
- If a flow quantity is insensitive to $c_b \Longrightarrow$ "spectral" quantity

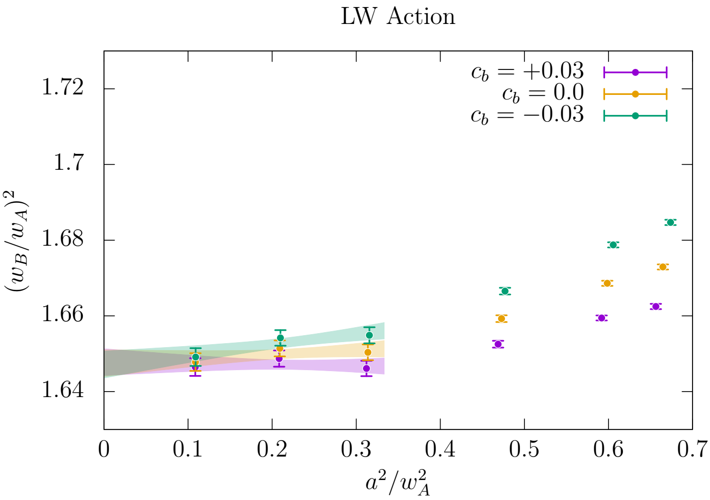
c_b dependence

$$\langle O \rangle_{\text{latt}} \overset{a \to 0}{\sim} \langle O_0 \rangle + a^2 \left\{ \cancel{\langle O_2 \rangle} + \langle O_0 S_{2,b} \rangle + \cancel{\langle O_0 S_{2,fl} \rangle} \right\}$$



c_b DEPENDENCE

$$\langle O\rangle_{\mathrm{latt}} \overset{a\to 0}{\sim} \langle O_0\rangle + a^2\left\{\cancel{\langle O_2\rangle} + \langle O_0 S_{2,b}\rangle + \cancel{\langle O_0 S_{2,fl}\rangle}\right\}$$



c_b DEPENDENCE

- t_0-like scales more sensitive to c_b than w_0-like scales

Another point of view for the c_b effect

A shift in the initial condition

$$V_\mu(t,x)\Big|_{t=0} = \exp\{c_b g_0^2 \partial_{x,\mu} S_g[U]\} U_\mu(x)$$

can be understood as a shift at some time $t > 0$

$$V'_\mu(t,x)\Big|_{t=t_s} = \exp\{c_b g_0^2 \partial_{x,\mu} S_g[V]\} V_\mu(t_s,x)$$

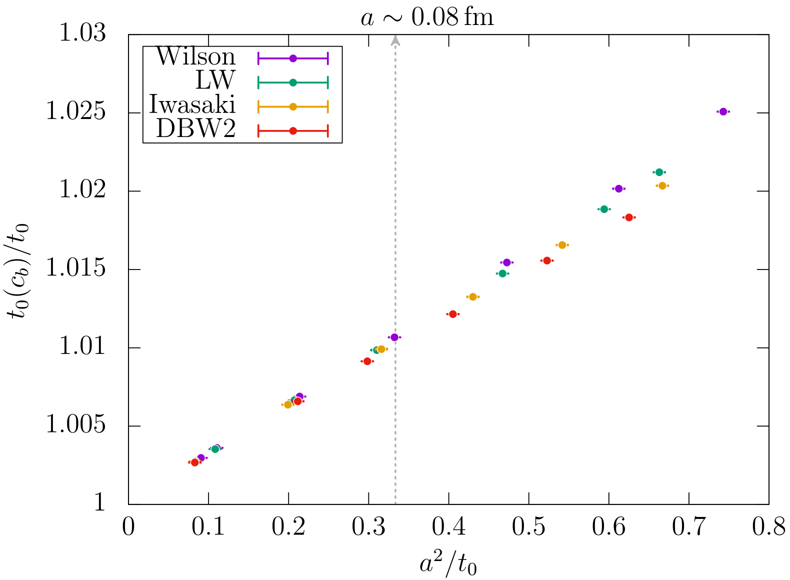
In particular if you use $t^2\langle E(t+c_b a^2)\rangle$ to determine t_0:

$$t_0(c_b) \overset{a\to 0}{\sim} t_0 - \frac{a^2}{D}\left\{ t_0^2 \langle E(t_0) S_{2,b}\rangle + c_b t_0^2 \frac{\mathrm{d}}{\mathrm{d}t}\Big|_{t_0} E(t)\right\}$$

c_b moved to positive flow time: Classical effect, pure a^2-term. Different c_b Does not give useful information

$$\frac{t_0(c_b)}{t_0} \overset{t\to 0}{\sim} 1 - \frac{a^2}{D}\left\{ c_b t_0^2 \frac{\mathrm{d}}{\mathrm{d}t}\Big|_{t_0} E(t)\right\}$$

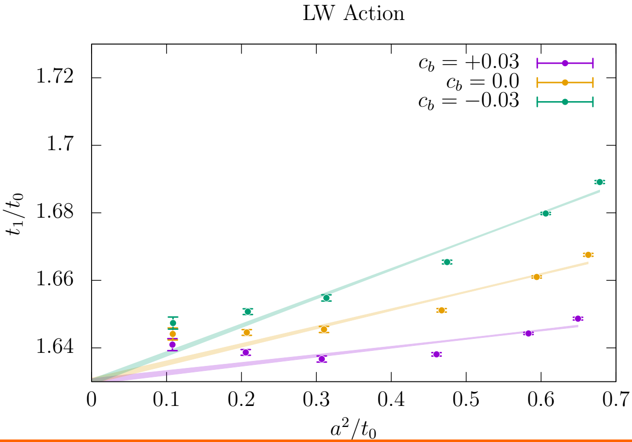
c_b dependence



Testing universality

It is difficult

- Changes in flow discretization, comparing t_0^{pl} and t_0^{cl}, comparing $t_0(c_b)$ with t_0 give no information!: Trivial "classical" a^2-effects.



Testing universality

It is difficult

- Changes in flow discretization, comparing t_0^{pl} and t_0^{cl}, comparing $t_0(c_b)$ with t_0 give no information!: Trivial “classical” a^2-effects.

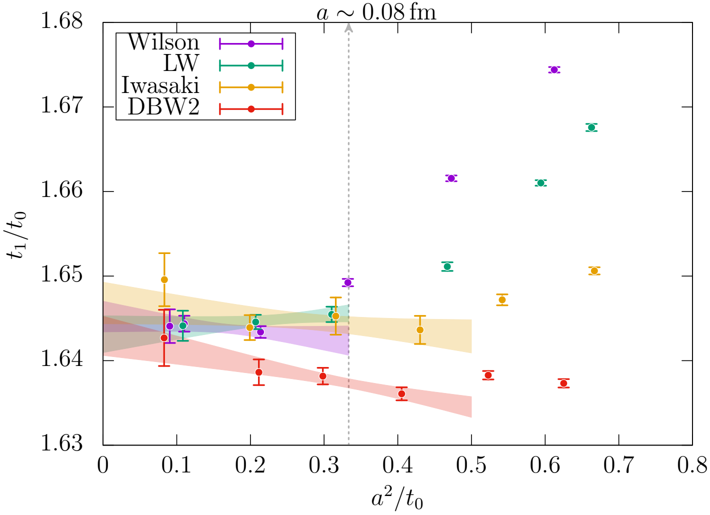
Viable strategy

$$\langle O \rangle_{\mathrm{latt}} \overset{a\to 0}{\sim} \langle O_0 \rangle + a^2 \left\{ \cancel{\langle O_2 \rangle} + \langle O_0 S_{2,b} \rangle + \cancel{\langle O_0 S_{2,fl} \rangle} \right\}$$

- Use Zeuthen flow/classically improved observables
- Use ratios t_1/t_0 or w_A^2/w_B^2
- These quantities can be considered “spectral quantities”: Probes of cutoff effects of your action

Scaling of t_1/t_0

$$\langle O\rangle_{\mathrm{latt}} \overset{a\to 0}{\sim} \langle O_0\rangle + a^2\left\{\cancel{\langle O_2\rangle} + \langle O_0 S_{2,b}\rangle + \cancel{\langle O_0 S_{2,fl}\rangle}\right\}$$



Scaling of t_1/t_0

$$\langle O\rangle_{\text{latt}} \overset{a\to 0}{\sim} \langle O_0\rangle + a^2\left\{\cancel{\langle O_2\rangle} + \langle O_0 S_{2,b}\rangle + \cancel{\langle O_0 S_{2,fl}\rangle}\right\}$$

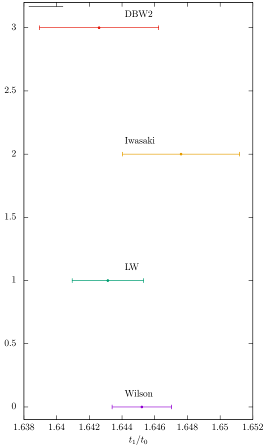
- Good agreement between at the % level
- Very far from asymptotic prediction **[Husung, Fri]**:

$$\mathcal{P}(a) = \mathcal{P}(0) + a^2 c^{\text{latt}}\left[A[\alpha(1/a)]^{\hat{\gamma}_1} + B[\alpha(1/a)]^{\hat{\gamma}_2}\right]$$

and

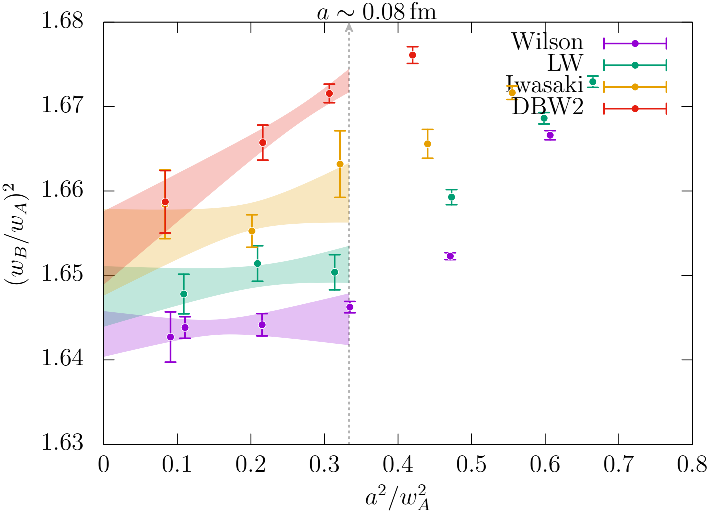
$$\frac{c^{\text{Iwasaki}}}{c^{\text{Wilson}}} = 3; \quad \frac{c^{\text{DBW2}}}{c^{\text{Wilson}}} = 16\,.$$

Is g_0^2 small enough?



Scaling of w_B/w_A

$$\langle O\rangle_{\text{latt}} \overset{a\to 0}{\sim} \langle O_0\rangle + a^2\left\{\cancel{\langle O_2\rangle} + \langle O_0 S_{2,b}\rangle + \cancel{\langle O_0 S_{2,fl}\rangle}\right\}$$



Scaling of w_B/w_A

$$\langle O\rangle_{\rm latt} \overset{a\to 0}{\sim} \langle O_0\rangle + a^2\left\{\cancel{\langle O_2\rangle} + \langle O_0 S_{2,b}\rangle + \cancel{\langle O_0 S_{2,fl}\rangle}\right\}$$

- Reasonable agreement between at the 2% level
- Far from asymptotic prediction **[Husung, Fri]**:

$$\mathcal{P}(a) = \mathcal{P}(0) + a^2 c^{\rm latt}\left[A[\alpha(1/a)]^{\hat\gamma_1} + B[\alpha(1/a)]^{\hat\gamma_2}\right]$$

and

$$\frac{c^{\rm Iwasaki}}{c^{\rm Wilson}} = 3; \qquad \frac{c^{\rm DBW2}}{c^{\rm Wilson}} = 16\,.$$

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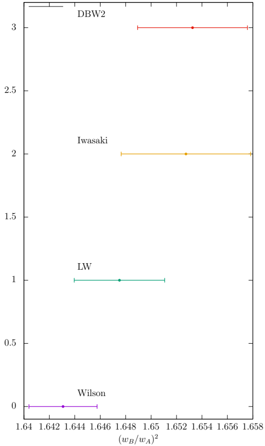

Conclusions

Testing universality/Scaling properties of an action

- Flow quantities are ideal, but classical improvement of flow/observables has to be implemented in order to avoid looking at irrelevant a^2-clasical effects:
  - Use the Zeuthen flow
  - Use $E(t) = (4/3)E^{\mathrm{pl}} - (1/3)E^{\mathrm{cl}}$
- Information taken from comparing $E^{\mathrm{pl}}, E^{\mathrm{cl}}, c_b, \ldots$ is misleading (only probes classical a^2 effects).
- Testing universality requires simulating different actions
- With classical improvement, flow scales t_0, w_0 are ideal probes, as good as spectral quantities.
- Preliminary results in pure gauge
  - Comparison of Wilson/Iwasaki/LW/DBW2
  - Reasonable agreement ($1-2\%$)
  - More precise data on the way

Is g_0^2 small enough to claim sub-percent precision?

- Numbers seem off the known asymptotic behavior. (But more data needed)