Gradient flow scale setting with tree-level improvement

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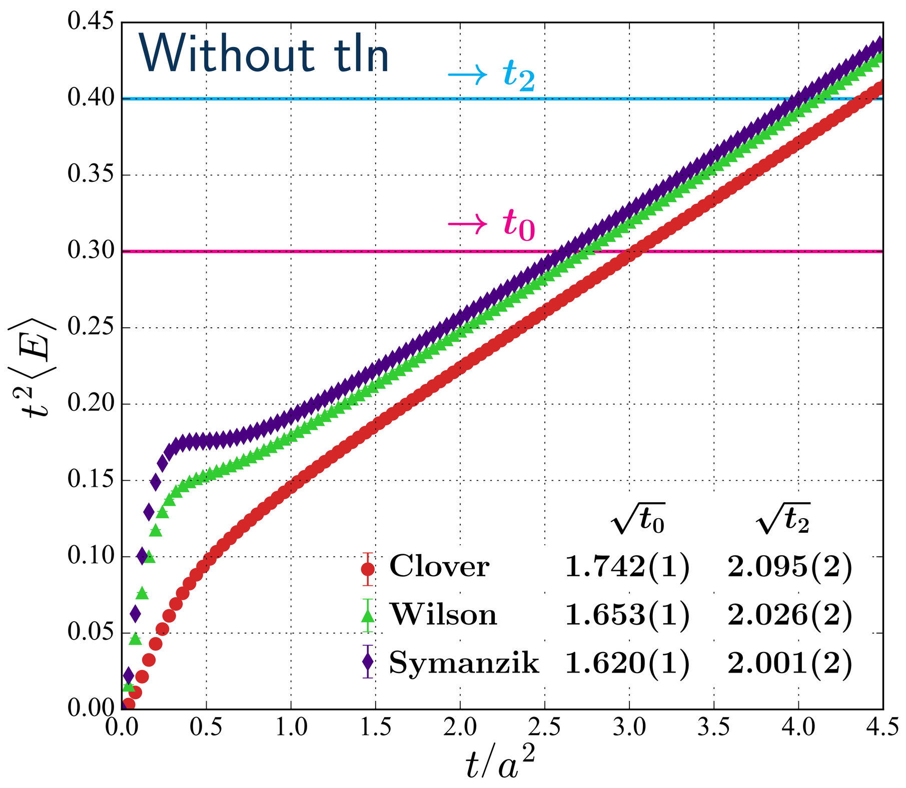
INTRODUCTION

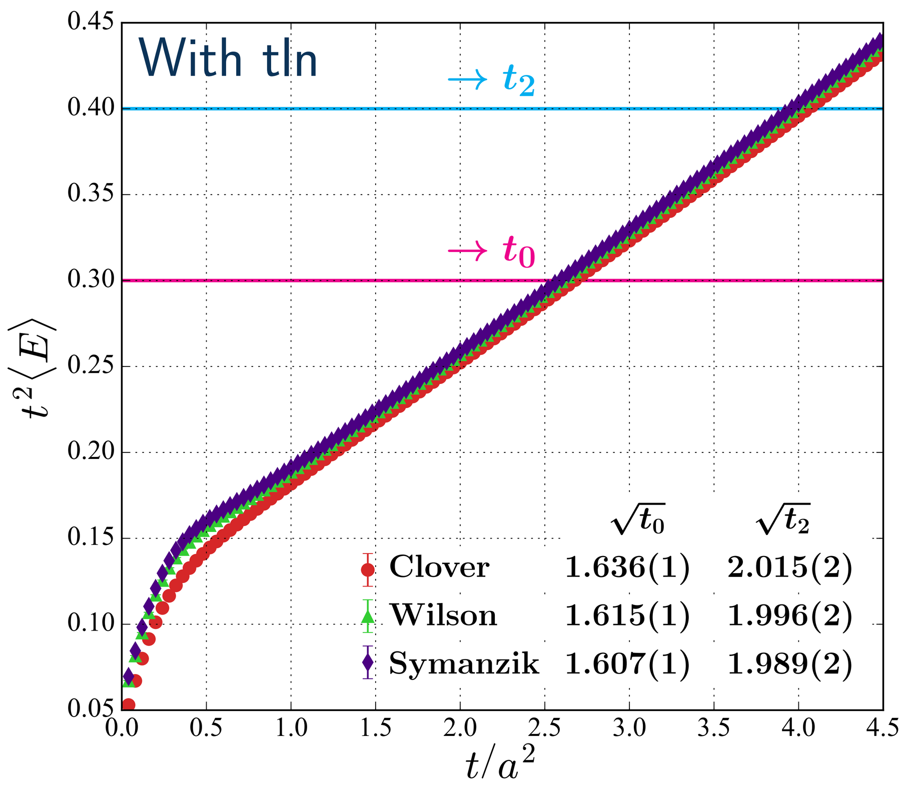
- Precise determination of the lattice scale is crucial for all physics
- Gradient flow scales are easy to calculate and precise, however, $\sqrt{t_0}$ [1] and w_0 [2] can exhibit large cutoff effects
- Study discretization errors of $\sqrt{t_0}$ and w_0
  - RBC-UKQCD's 2+1 flavor Shamir domain-wall fermion and Iwasaki gauge field ensembles [3-7]; Wilson gradient flow measurements [1]
  - Three operators: Clover (C), Wilson (W), and Symanzik (S) with and without tree-level normalization (tln) corrections [8]

Ensembles

	β	L/a	T/a	am_l	am_s^{sea}	am_{res}
C1	2.13	24	64	0.005	0.040	0.003154(15)
C2	2.13	24	64	0.010	0.040	0.003154(15)
M1	2.25	32	64	0.004	0.030	0.0006697(34)
M2	2.25	32	64	0.006	0.030	0.0006697(34)
M3	2.25	32	64	0.008	0.030	0.0006697(34)
F1	2.31	48	96	0.002144	0.02144	0.0009679(21)
X1	2.37	32	64	0.0047	0.0186	0.0006296(58)

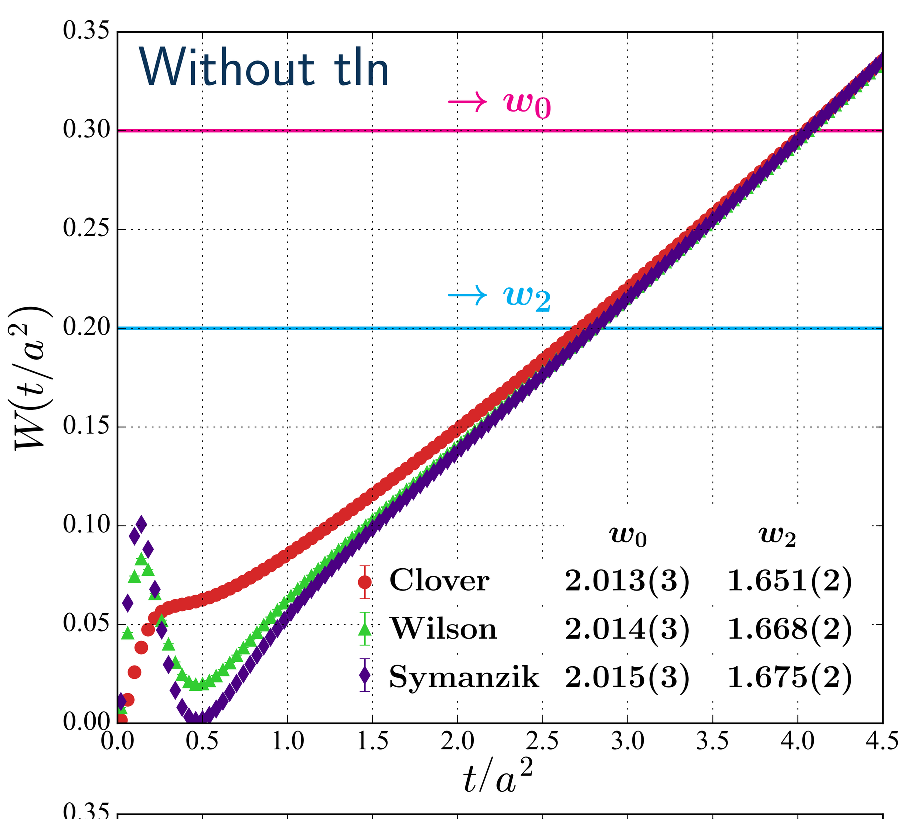
Scale $\sqrt{t_0}$ on M1

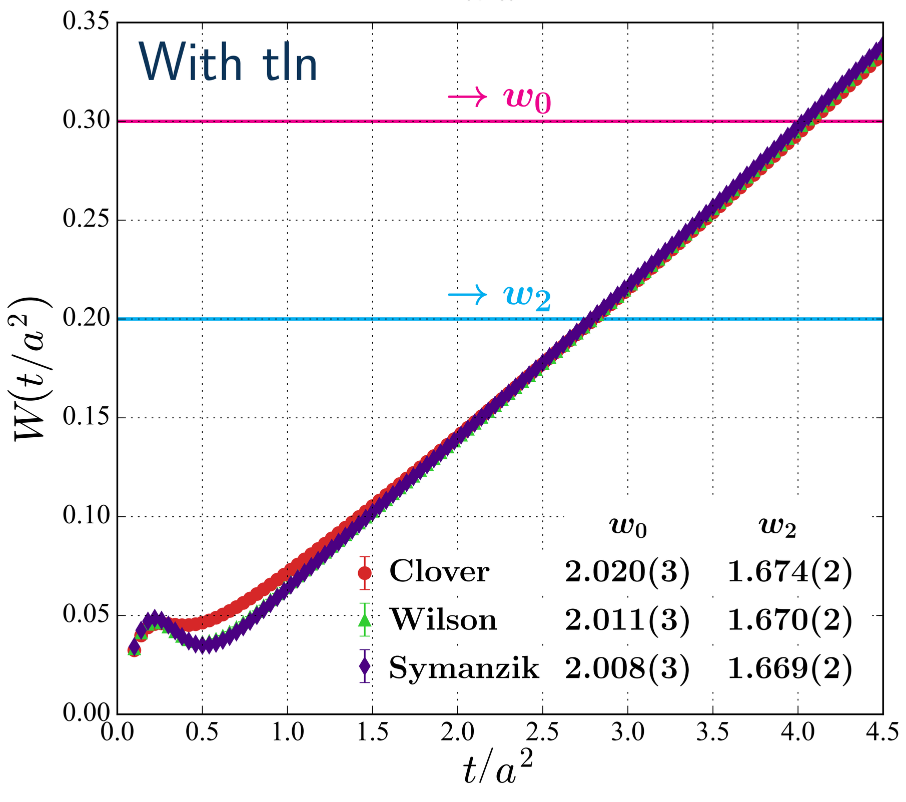




- Tln reduces cutoff effects between different operators

Scale w_0 on M1

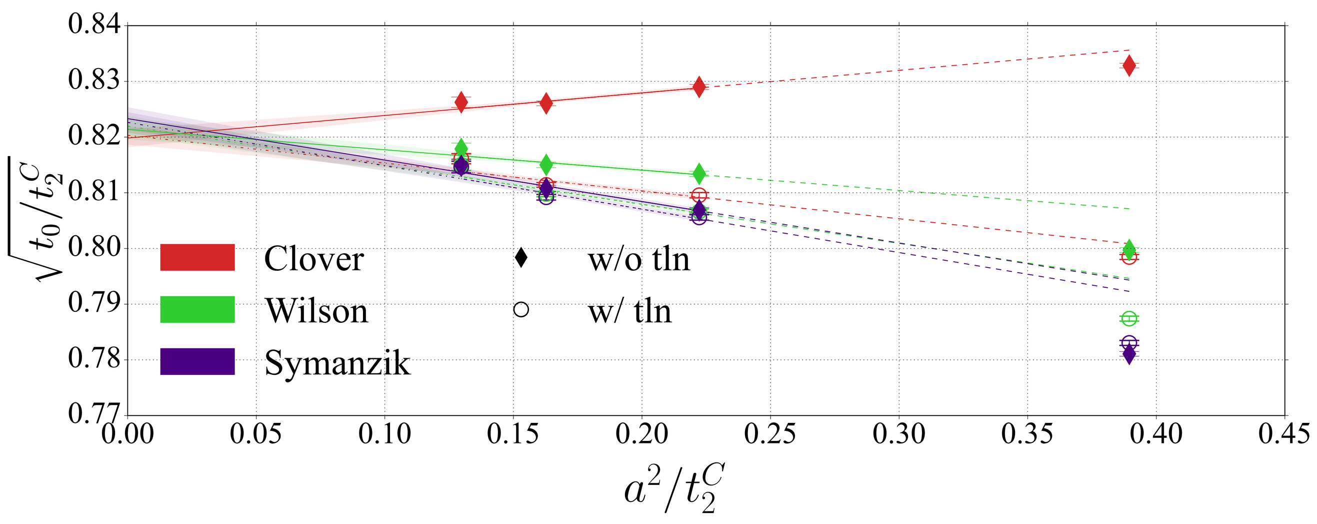




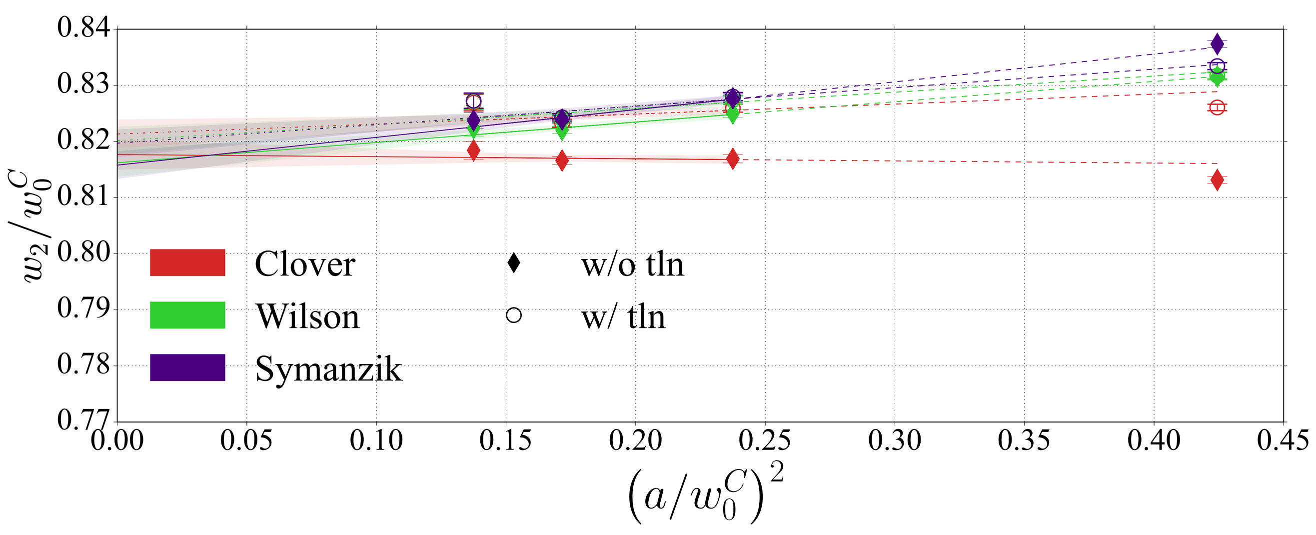
- Different operators for w_0 show little cutoff effects

Identifying cutoff effects

- Operator with small/no cutoff effects should predict the same relative scale, independent of where the scale is defined
- Ratios of scales should be independent of the lattice spacing

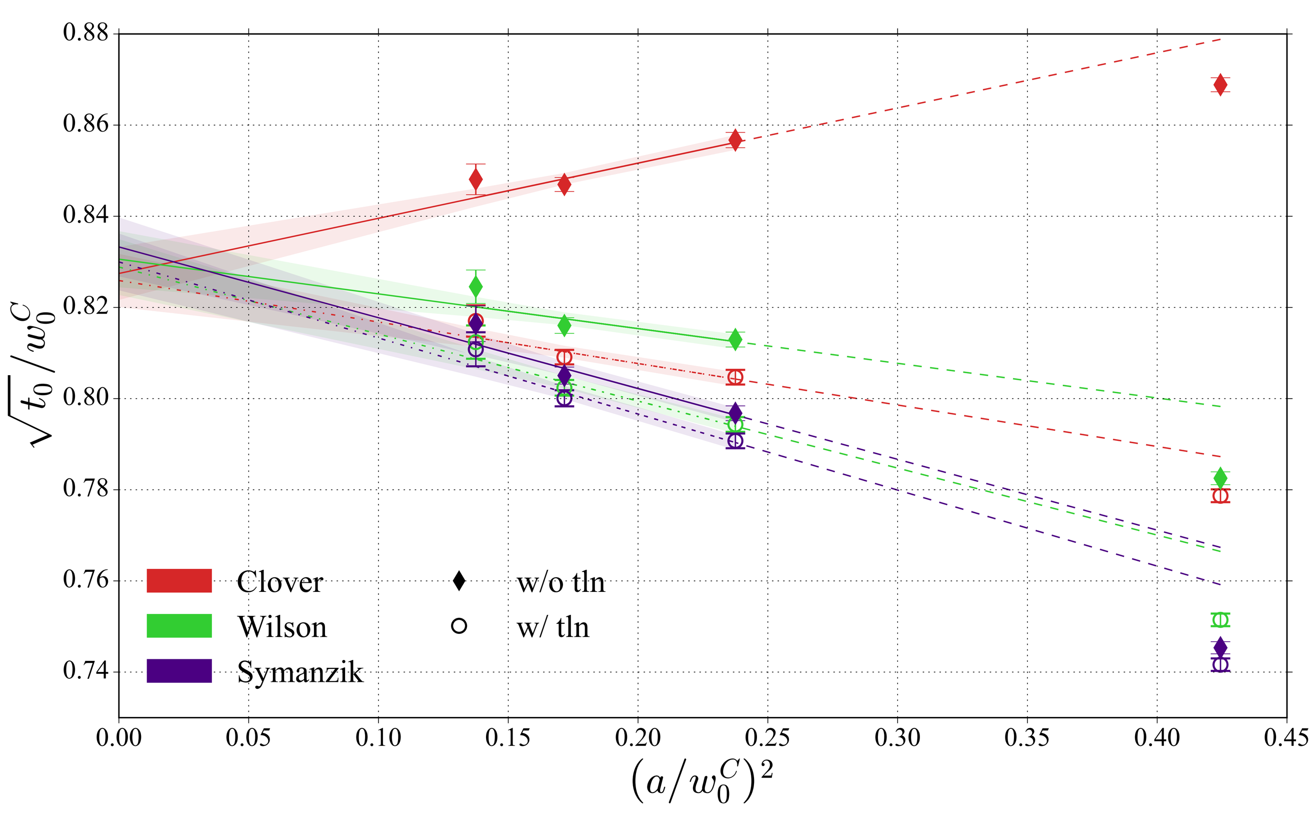


- $\sqrt{t_0/t_2^C}$ vs. a^2/t_2^C suggests Clover operator $< 2\%$ cutoff effects
- Symanzik and Wilson operators have about 5% cutoff effects



- w_2/w_0^C suggests all w scales have similar cutoff effects $< 2\%$
- Optimal operator likely close to Clover, no tln

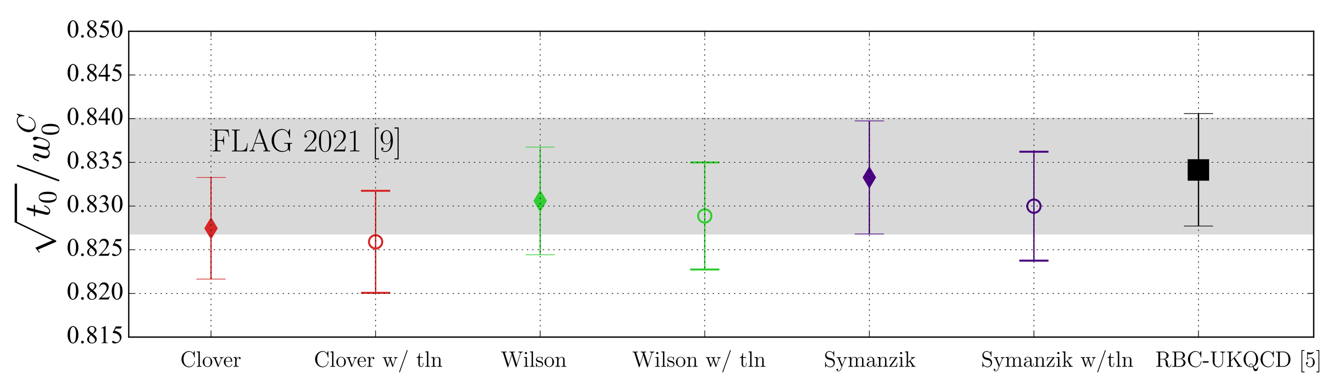
Continuum limit of chirally-extrapolated data



SUMMARY

Some gradient flow scales show large discretization effects

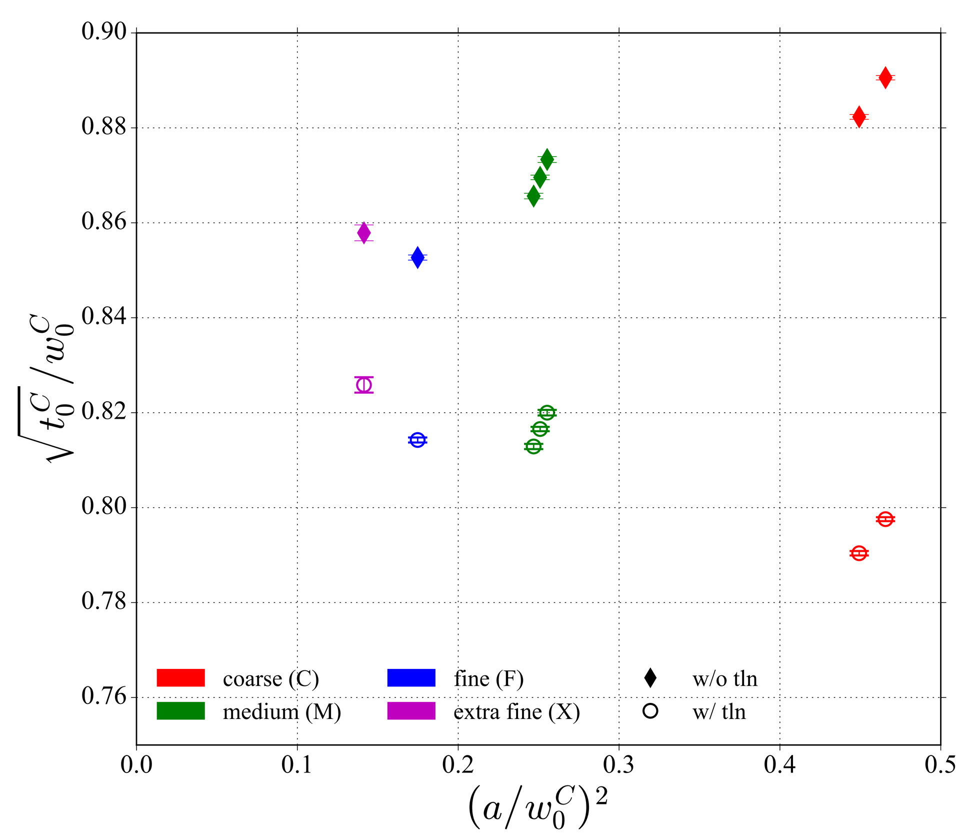
- For Iwasaki gauge with Wilson flow w scales are preferred
- Discretization effects between different operators are subdominant
- Tree-level normalization reduces spread between different operators, but does not lead to an overall improvement
- Consistent continuum limit when extrapolating three finest ensembles, coarsest ensembles need higher order corrections



Tree-level Normalization

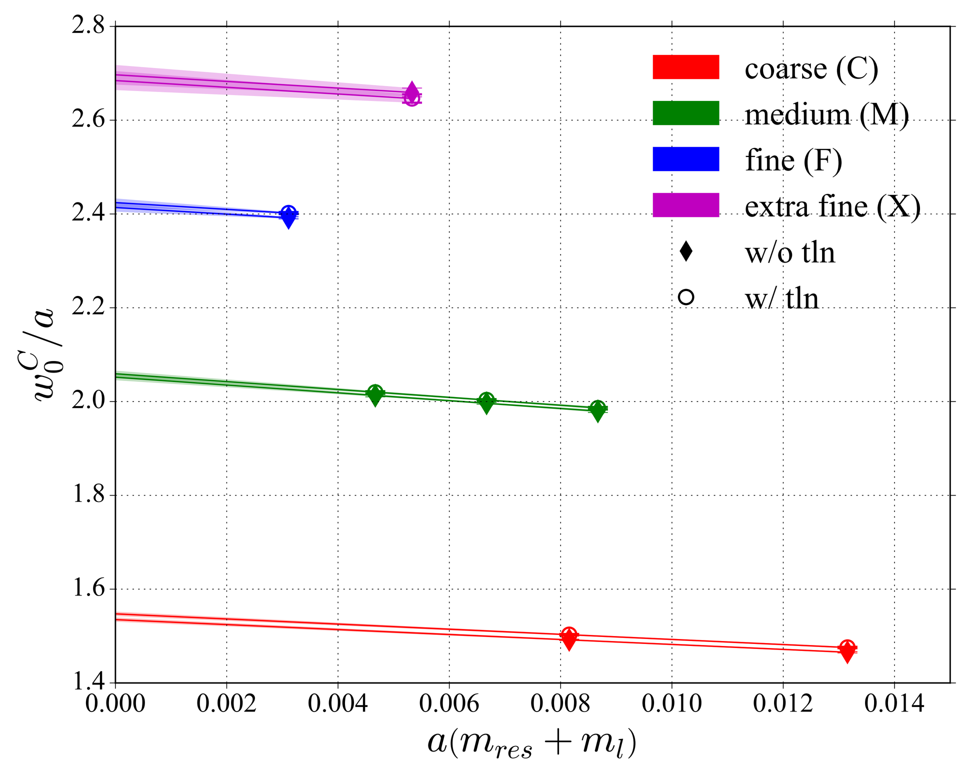
- Removes cutoff effects at tree-level for given gauge action, gradient flow, and operator
- Replace $t^2\langle E(t)\rangle$ with $t^2\langle E(t)\rangle/C(t,L,T)$
- Determine $C(t,L,T)$ perturbatively [8]
- Reduces cutoff effects between operators for $\sqrt{t_0}$
- Notable improvement for determinations of the step-scaling β function for SU(3) with $N_f = 4$, 6, 8, 10, 12 [10-12]
- Does not improve strong coupling loop-corrections

Comparison of $\sqrt{t_0^C}/w_0^C$

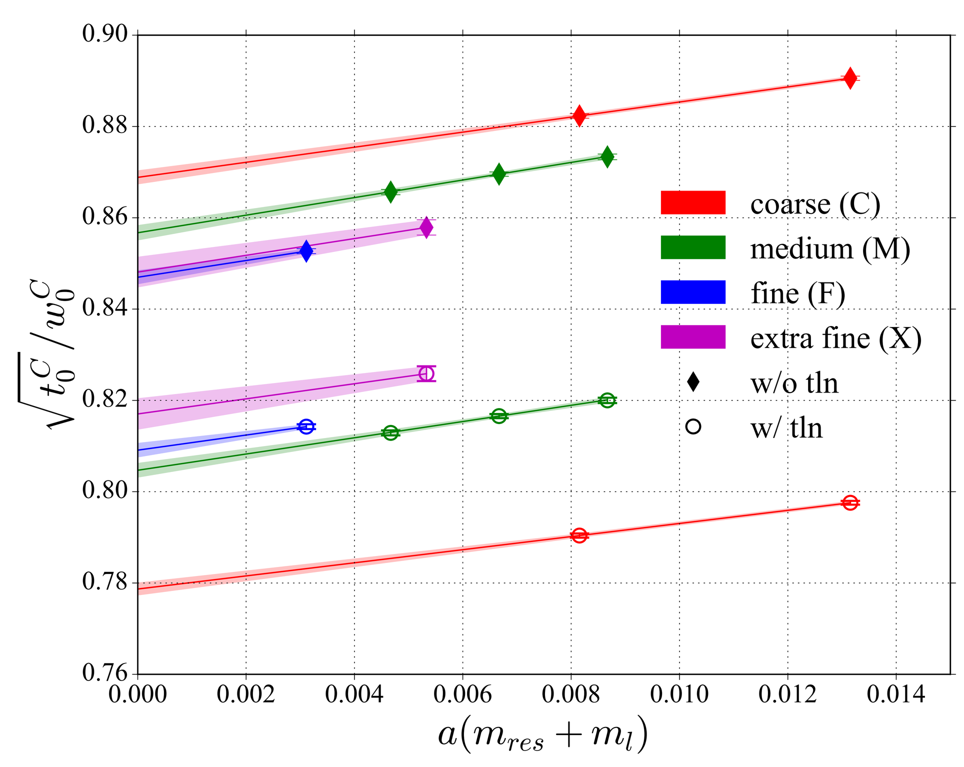


- Clearly resolved quark mass dependence

Chiral extrapolation of w_0^C



Chiral extrapolation of $\sqrt{t_0^C}/w_0^C$



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