f_K and f_π from staggered QCD

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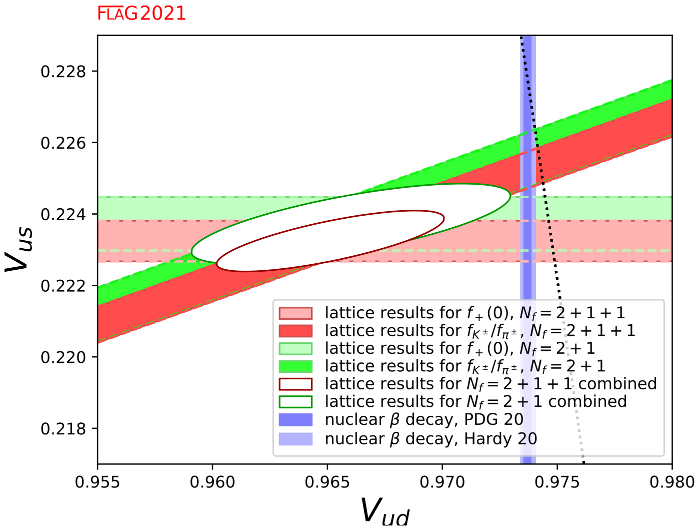
on behalf of the Budapest-Marseille-Wuppertal collaboration

Background



- Pseudoscalar decay constants provide input for CKM matrix elements
- F_K/F_π and kaon decay experiments constrain $|V_{us}|$ and $|V_{ud}|$: [FLAG '21]

$$\left|\frac{V_{us}}{V_{ud}}\right| \frac{F_{K^\pm}}{F_{\pi^\pm}} = 0.2760(4)$$

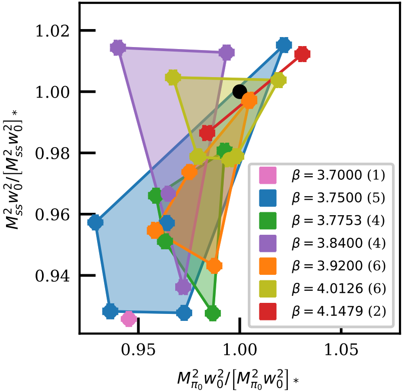
- Together with lattice $f_+(0)$:
  - Tension with β decay (blue bands)
  - $\sim 2.8\sigma$ from CKM unitarity (dotted line)

Simulations

- Tree-level Symanzik gauge action
- $N_f = 2 + 1 + 1$ staggered fermions
- stout smearing
  4 steps, $\varrho = 0.125$

β	a[fm]	$L \times T$	#conf
3.7000	0.1315	48×64	904
3.7500	0.1191	56×96	2072
3.7753	0.1116	56×84	1907
3.8400	0.0952	64×96	3139
3.9200	0.0787	80×128	4296
4.0126	0.0640	96×144	6980
4.1479	0.0483	128×192	4439

- $L \sim 6\,\mathrm{fm}, \quad T \sim 9\,\mathrm{fm}$
- M_π and M_{ss} around physical point



Scale setting

- Simulations and analysis in the isospin-symmetric limit
- Quark masses set with M_{ss} and M_{π_0}
- Use w_0-scale, defined from logarithmic derivative of gauge-action density along gradient flow:

$$W_\tau[U] = \frac{d(\tau^2 E[U,\tau])}{d\log\tau}$$

$$\langle W_{\tau=w_0^2}\rangle = 0.3$$

- Find physical w_0 by setting the scale with M_Ω and taking the continuum limit of $w_0 M_\Omega$, including isospin breaking [BMWc '20]

$$[w_0]_* = 0.17236(70)\text{fm}$$

- Similarly, take the continuum limit of M_{ss} [BMWc '20]

$$[M_{ss}]_* = 689.89(49)\text{MeV}$$

Pseudoscalar mass

- Focus on goldstone taste:

$$P_{ab} = \overline{\psi}_a \{\gamma_5 \otimes \xi_5\} \psi_b$$

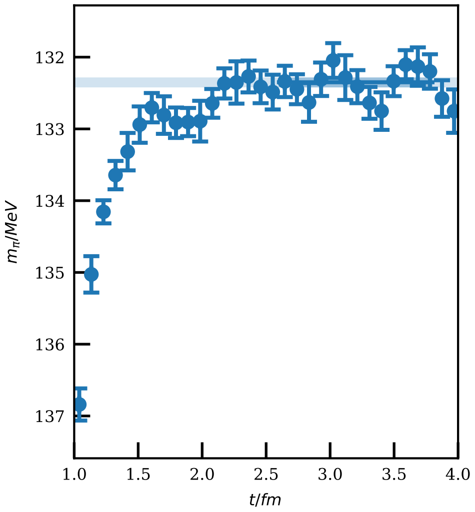
- Compute correlator

$$G(t) = \frac{1}{V_s} \sum_{\vec{x}} \langle P_{ab}(t,\vec{x})\, P^{\dagger}_{ab}(0) \rangle$$

- Extract local effective mass

$$\tilde{M} = \frac{1}{\Delta} \cosh^{-1} \frac{G(t+\Delta) + G(t-\Delta)}{2G(t)}$$

$$\Delta \approx 0.2\text{fm},\ 0.4\text{fm}$$



Decay constants

- Want to compute $f_\alpha = \langle\Omega|A_4|\alpha\rangle / m_\alpha$
- For the goldstone taste:

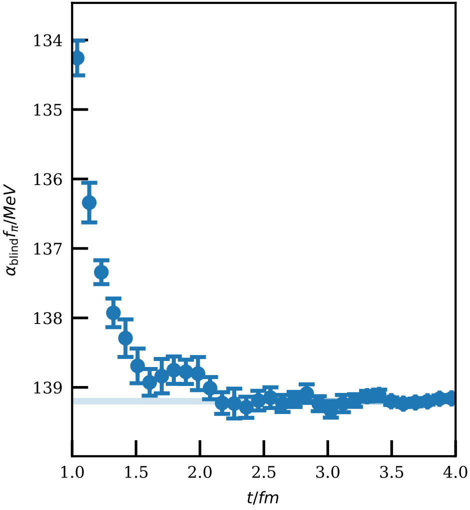
$$(m_1 + m_2)\langle\Omega|P|\alpha\rangle = \langle\Omega|\partial_4 A_4|\alpha\rangle$$

- So, for large $t << T$

$$G(t) \to \frac{M^3 f^2 \cosh\left(M[t - \frac{T}{2}]\right)}{2(m_1 + m_2)^2 e^{\frac{MT}{2}}\left(1 - e^{-MT}\right)}$$

- Define effective f

$$\tilde{f} = \sqrt{\frac{2G(t)(m_1 + m_2)^2 e^{\frac{\tilde{M}T}{2}}\left(1 - e^{-\tilde{M}T}\right)}{\tilde{M}^3 \cosh\left(\tilde{M}[t - \frac{T}{2}]\right)}}$$



Continuum limit

- Expect discretisation errors to scale like

$$A_2 \left[a^2 \alpha_s(1/a)^n\right] + A_4 \left[a^2 \alpha_s(1/a)^n\right]^2 + A_6 \left[a^2 \alpha_s(1/a)^n\right]^3 + \cdots$$

- Anomalous dimension n unknown
- To incorporate resulting uncertainty, include different values of n
- Taste-breaking of pseudoscalar masses observed to scale like $n = 3$
- Expand in powers of the taste breaking

$$\Delta_{KS} w_0^2 \underset{\sim}{\propto} a^2 \alpha_s(1/a)^3$$

- Also include even powers of lattice spacing, corresponding to $n = 0$

Global fit

- Global fit form:

$$\frac{f_K}{f_\pi}[w_0, M_{\pi_0}, M_{ss}] = A(a^2) + A'(\Delta_{KS} w_0^2) + B(a^2)\left(M_{\pi_0}^2 w_0^2 - \left[M_{\pi_0}^2 w_0^2\right]_*\right) + C(a^2)\left(M_{ss}^2 w_0^2 - \left[M_{ss}^2 w_0^2\right]_*\right)$$

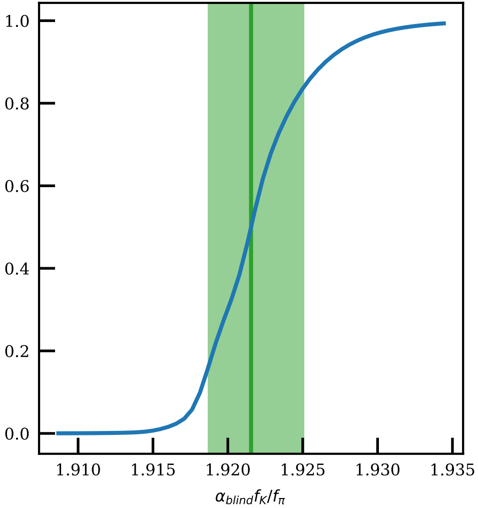
- A, B, and C are polynomials in $a^2 = [w_0]_*^2 / w_0^2$
- A' is a polynomial in Δ_{KS}
- B and C constant or linear
- A and A' up to cubic
- One of A and A' must be at least linear

Error budget

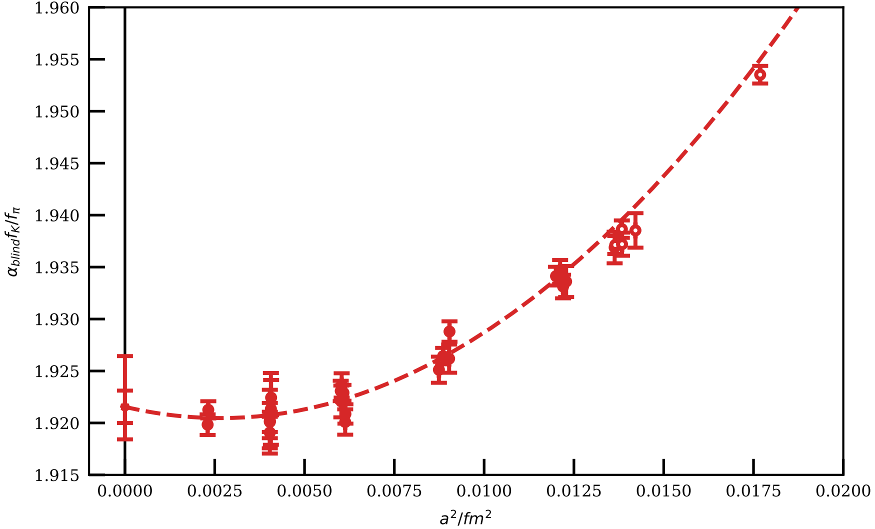
- Statistical errors from jackknife
- Perform multiple analyses with different choices made
- Combine into histogram
  - Modified AIC weight for beta cuts & polynomial order

$$\exp\left[-\frac{1}{2}\left(\chi^2 + 2n_{par} - n_{data}\right)\right]$$

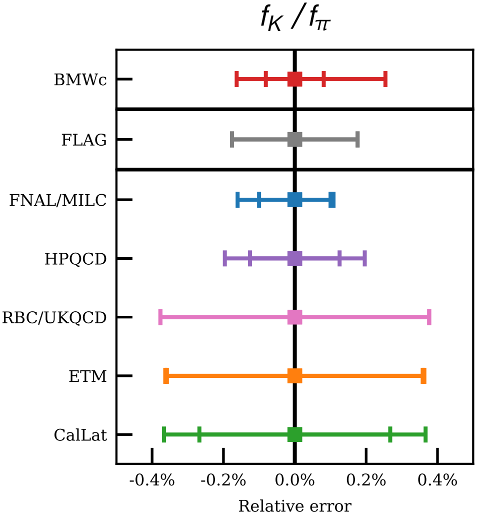
  - Flat weights for other choices
- Compute one-sigma confidence band
- Total error is half the width



Continuum extrapolation for f_K/f_π

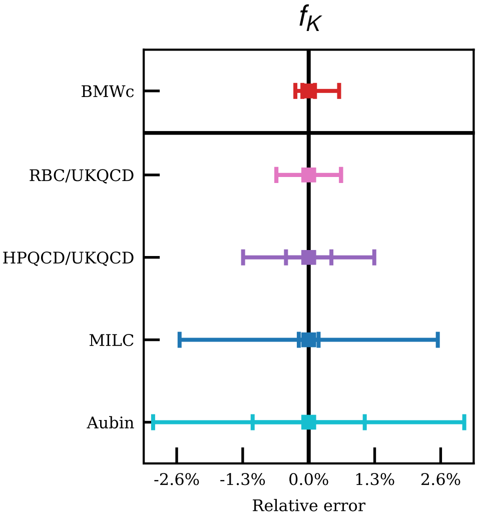


Error budget for f_K / f_π



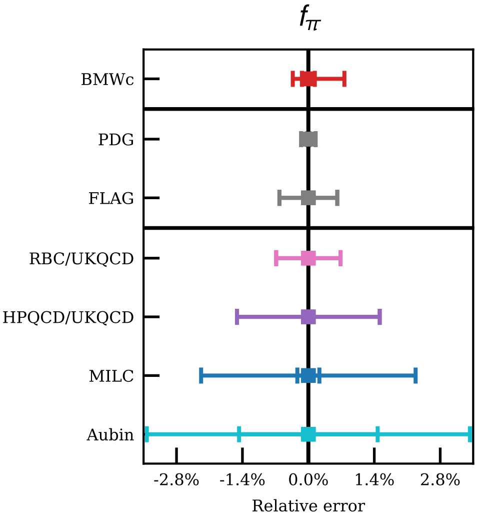
- Overall error budget 0.21%
- Dominated by the order of the A polynomial
- Closely followed by w_0
- FLAG average 0.17%
- FNAL/MILC 0.13%

Error budget for $f_K w_0$



- Overall error budget for $f_K w_0$ 0.20%
- w_0 error: 0.4%
- Gives error of 0.43% for f_K
- FLAG average 0.19% (dependent on PDG f_π)
- RBC/UKQCD 0.64%

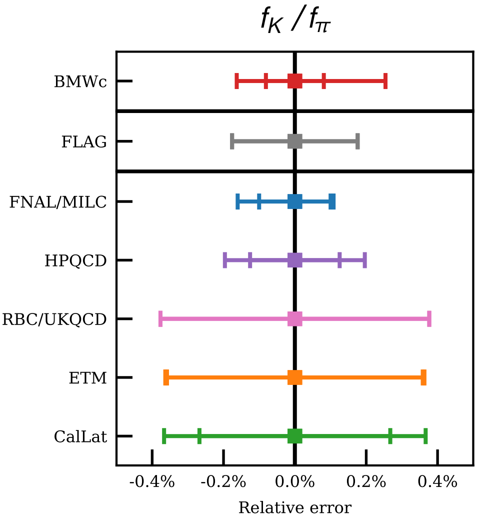
Error budget for $f_\pi w_0$



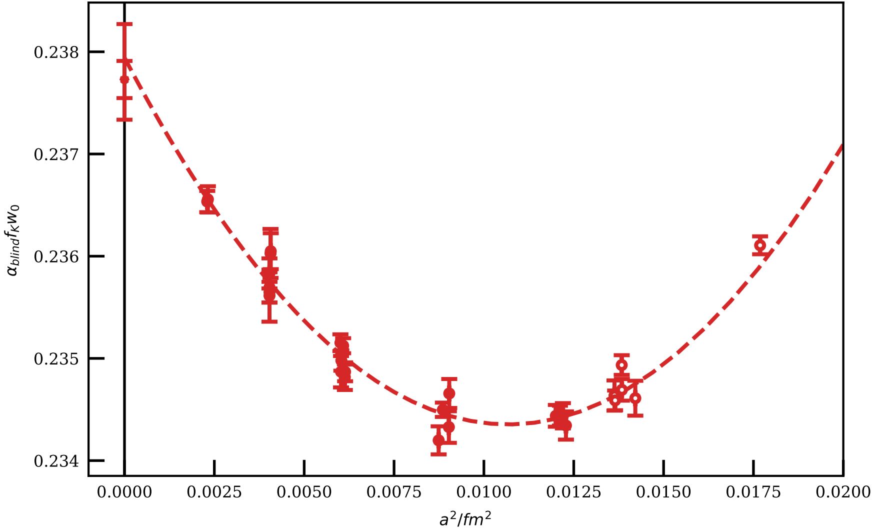
- Overall error budget for $f_\pi w_0$ 0.22%
- w_0 error: 0.4%
- Gives error of 0.55% for f_π
- PDG value 0.15% [PDG '14]
- FLAG average 0.61%
- RBC/UKQCD 0.68%
- All results in the FLAG average are $N_f = 2 + 1$

Conclusion

- Isospin-symmetric study of decay constants
- 0.21% determination of f_K/f_π
- 0.22% determination of $f_\pi w_0$
  - Gives 0.55% f_π
- 0.20% determination of $f_K w_0$
  - Gives 0.43% f_K
- Not unblinded yet



Continuum extrapolation for $f_K w_0$



Continuum extrapolation for $f_\pi w_0$