

RG- *running*
of the tensor
operator / *in a*
 χSF
setup
for $N_f = 3$
QCD |

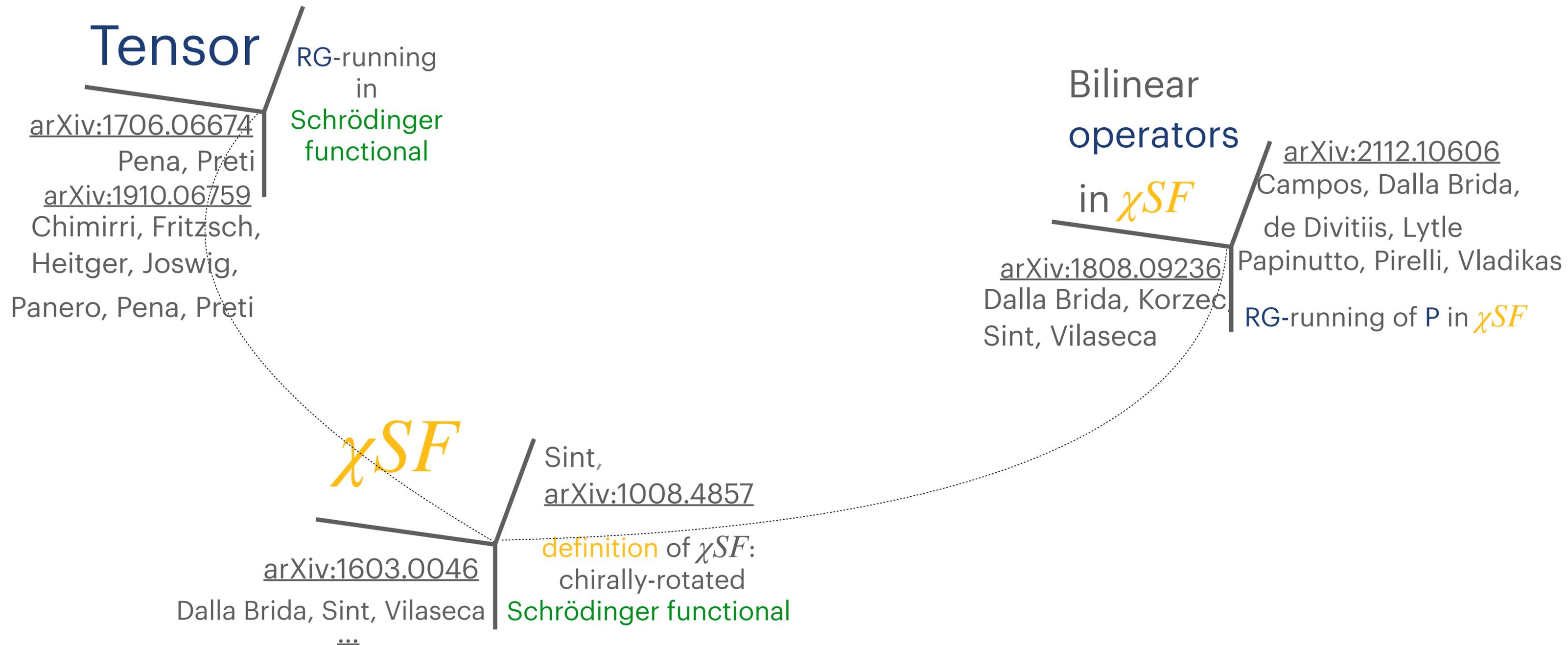
ALPHA
Collaboration

LATTICE
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8TH–13TH
BONN
GERMANY

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Previous results and goal



RG-running of the tensor operator for $N_f = 3$ QCD in a χSF setup

RG- running

in a **mass-independent** renormalisation scheme

we can define the **RGE** for the renormalised operator $\bar{O}(\mu) = Z_O(\mu)O$ and the coupling $\bar{g}(\mu)$

$$\mu \frac{\partial \bar{O}}{\partial \mu} = \gamma_O(\bar{g}(\mu)) \bar{O}(\mu)$$

$$\mu \frac{\partial \bar{g}}{\partial \mu} = \beta(\bar{g}(\mu))$$

with

$$\gamma_O(\bar{g}) \underset{\bar{g} \rightarrow 0}{\sim} -\bar{g}^2 (\gamma_0 + \gamma_1 \bar{g}^2 + \gamma_2 \bar{g}^4 + O(\bar{g}^6))$$

$$\beta(\bar{g}) \underset{\bar{g} \rightarrow 0}{\sim} -\bar{g}^3 (b_0 + b_1 \bar{g}^2 + b_2 \bar{g}^4 + O(\bar{g}^6))$$

RG-running of the tensor operator **for Nf = 3 QCD in a χ_{SF} setup**

from which we obtain the **RG**-invariants

$$\Lambda_{QCD} = \mu [b_0 \bar{g}^2(\mu)]^{-\frac{b_1}{2b_0^2}} e^{-\frac{1}{2b_0 \bar{g}(\mu)^2}} e^{-\int_0^{\bar{g}(\mu)} dg \left(\frac{1}{\beta(g)} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g} \right)}$$

$$O^{RGI} = \bar{O}(\mu) \left[\frac{\bar{g}^2(\mu)}{4\pi} \right]^{-\frac{\gamma_0}{2b_0}} e^{-\int_0^{\bar{g}(\mu)} dg \left(\frac{\gamma_X(g)}{\beta(g)} - \frac{\gamma_0}{b_0 g} \right)}$$

We can factorise the running in many evolutions between two scales:

$$O^{RGI} = \frac{O^{RGI}}{\bar{O}(\mu_1)} \frac{\bar{O}(\mu_1)}{\bar{O}(\mu_2)} \cdots \frac{\bar{O}(\mu_n)}{\bar{O}(\mu_{had})} \bar{O}(\mu_{had})$$

from which we naturally define

$$\sigma_O(s, u) = \frac{\bar{O}(\mu_2)}{\bar{O}(\mu_1)} \quad \text{with} \quad s = \frac{\mu_1}{\mu_2}$$

RG-running of the tensor operator for **Nf = 3 QCD** in a χSF setup

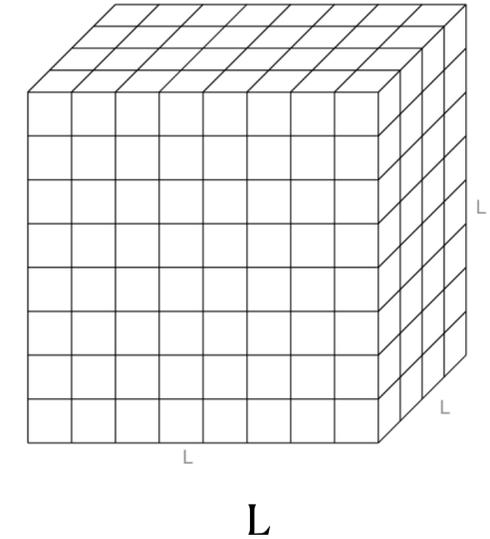
while for the coupling

$$u = \bar{g}^2(\mu) \quad \sigma(s, u) = \bar{g}^2(\mu/s)$$

Now, with $s=2$, from the **RGE**

$$2 = \exp - \int_{\sqrt{u}}^{\sqrt{\sigma(u)}} dg \frac{1}{\beta(g)}$$

$$\sigma_O(u) = \exp \int_{\sqrt{u}}^{\sqrt{\sigma(u)}} dg \frac{\gamma_O(g)}{\beta(g)}$$



On the lattice, the **renormalisation scale** can be identified as $\mu = 1/L$ and

$$u = \bar{g}^2(L) \rightarrow \Sigma(u, a/L) = \bar{g}^2(2L)$$

$$\sigma(u) = \lim_{a \rightarrow 0} \Sigma(u, a/L)$$

$$\sigma_O(u) = \lim_{a \rightarrow 0} \Sigma_O(u, a/L)$$

RG-running of the tensor operator **for Nf = 3 QCD in a χSF setup**

Tensor operator

Flavour non-singlet tensor current

$$O \rightarrow T_{\mu\nu}^{f_1 f_2}(x) = i\bar{\psi}_{f_1}(x)\sigma_{\mu\nu}\frac{1}{2}\psi_{f_2}(x), \quad \text{with } \sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$$

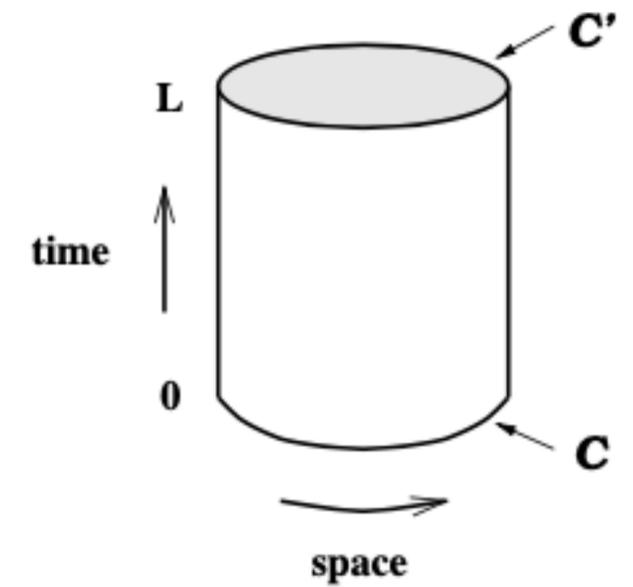
- **phenomenological interest for effective Hamiltonian amplitudes (rare heavy meson decays, neutron beta decays, BSM ...)**

$$A = \langle f | H_{eff} | i \rangle = C_W(\mu) \langle f | \bar{O}(\mu) | i \rangle$$

$$O \sim (\bar{l}\sigma_{\mu\nu}e)(\bar{q}\sigma_{\mu\nu}u), \quad G_{\mu\nu}(\bar{q}_i\sigma_{\mu\nu}q_j)$$

Simulation details

- $N_f = 3$ massless QCD, Wilson-clover fermions
- Gauge configurations of [arXiv:1802.05243](#), [arXiv:2112.10606](#)
- Schrödinger Functional (SF) boundary conditions. $\theta = 0.5$
- χSF valence fermions
- Scheme switching at $\mu_0/2 \sim 2 \text{ GeV}$. Same renormalisation conditions for O on both sides!



SF boundary conditions

$$\begin{array}{c}
 \begin{array}{ccccccc}
 \leftarrow & \mu_1^{SF} & & \mu_0/2 & & & \mu_{had} \\
 & \sim 64 \text{ GeV} & & \sim 2 \text{ GeV} & & & \sim 200 \text{ MeV}
 \end{array} \\
 T^{RGI} = \frac{T^{RGI}}{\bar{T}(\mu_1^{SF})} \frac{\bar{T}(\mu_1^{SF})}{\bar{T}(\mu_2^{SF})} \cdots \frac{\bar{T}(\mu_k^{SF})}{\bar{T}(\mu_0/2)} \frac{\bar{T}(\mu_0/2)}{\bar{T}(\mu_1^{GF})} \frac{\bar{T}(\mu_2^{GF})}{\bar{T}(\mu_3^{GF})} \cdots \frac{\bar{T}(\mu_n^{GF})}{\bar{T}(\mu_{had})} \bar{T}(\mu_{had})
 \end{array}$$

- SF coupling
- Gradient Flow coupling
- plaquette gauge action
- Lüscher-Weisz gauge action

RG-running of the tensor operator for $N_f = 3$ QCD in a χSF setup

$$\chi SF \quad \xleftarrow{\substack{\psi = R(\frac{\pi}{2})\psi' \quad \bar{\psi} = \bar{\psi}'R(\frac{\pi}{2}) \\ \text{with } R(\alpha) = \exp(i\frac{\alpha}{2}\gamma_5\tau^3)}} \quad SF$$

- **Automatic** $O(a)$ -**improvement** for the operators

- $\langle T_{\mu\nu}^I \rangle = \langle T_{\mu\nu} \rangle + O(a^2)$
- $\langle T_{\mu\nu}^I \rangle = \langle T_{\mu\nu} \rangle + ac_T(g_0^2) \langle (\tilde{\partial}_\mu V_\nu - \tilde{\partial}_\nu V_\mu) \rangle$

after renormalisation and for $a \rightarrow 0$

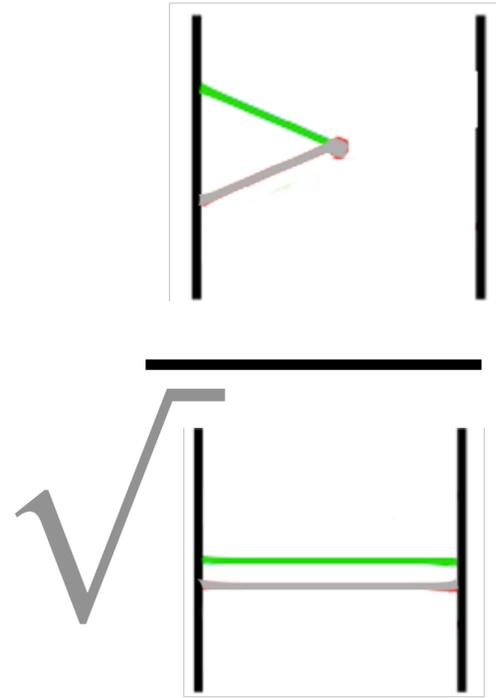
$$l_T^{ud} = l_{\tilde{T}}^{uu} = k_T$$

RG-running of the tensor operator for Nf = 3 QCD in a χSF setup

Renormalisation conditions

χSF

SF

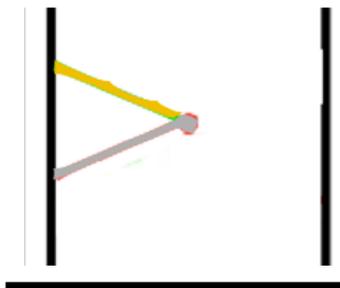


$$\mathbf{1.} \quad Z_T^{ud}(g_0^2, L/a) \frac{l_T^{ud}(T/2)}{\sqrt{g_1^{ud}}} = \left(\frac{l_T^{ud}(T/2)}{\sqrt{g_1^{ud}}} \right)^{g_0^2=0, a/L \neq 0}$$

$$\mathbf{2.} \quad Z_T^{ud}(g_0^2, L/a) \frac{l_T^{ud}(T/2)}{\sqrt{l_1^{ud}}} = \left(\frac{l_T^{ud}(T/2)}{\sqrt{l_1^{ud}}} \right)^{g_0^2=0, a/L \neq 0}$$

$$\mathbf{1.} \quad Z_T(g_0^2, L/a) \frac{k_T(T/2)}{\sqrt{f_1}} = \left(\frac{k_T(T/2)}{\sqrt{f_1}} \right)^{g_0^2=0, a/L \neq 0}$$

$$\mathbf{2.} \quad Z_T(g_0^2, L/a) \frac{k_T(T/2)}{\sqrt{k_1}} = \left(\frac{k_T(T/2)}{\sqrt{k_1}} \right)^{g_0^2=0, a/L \neq 0}$$



$$\mathbf{3.} \quad Z_T^{ud}(g_0^2, L/a) \frac{l_T^{ud}(T/2)}{-ig_{\tilde{V}}^{ud}} = \left(\frac{l_T^{ud}(T/2)}{-ig_{\tilde{V}}^{ud}} \right)^{g_0^2=0, a/L \neq 0}$$

$$\mathbf{4.} \quad Z_T^{ud}(g_0^2, L/a) \frac{l_T^{ud}(T/2)}{l_{\tilde{V}}^{uu'}} = \left(\frac{l_T^{ud}(T/2)}{l_{\tilde{V}}^{uu'}} \right)^{g_0^2=0, a/L \neq 0}$$

since \tilde{V} is the conserved lattice vector current and $Z_{\tilde{V}}^{f_1 f_2} = 1$

RG-running of the tensor operator for $N_f = 3$ QCD in a χSF setup

4 renormalisation schemes

$$Z_T^{ud}(g_0^2, L/a) l_T^{ud}(T/2) \frac{1}{(g_1^{ud})^\alpha (l_1^{ud})^\beta (-ig_{\tilde{V}}^{ud})^\gamma (l_{\tilde{V}}^{uu'})^\delta} = \left[l_T^{ud}(T/2) \frac{1}{(g_1^{ud})^\alpha (l_1^{ud})^\beta (-ig_{\tilde{V}}^{ud})^\gamma (l_{\tilde{V}}^{uu'})^\delta} \right]^{(0, a/L)}$$

requiring $(\alpha + \beta + 0.5 \gamma + 0.5 \delta) = 1$ and

$$\bar{O}' = \chi_O(\bar{g}) \bar{O} \rightarrow \gamma_O^{(1)'} = \gamma_O^{(1)} + 2b_0 \chi_O^{(1)}$$

$$\chi_O^{(1)} = r_0 \text{ extracted from } Z_T^{(1)} \sim \sum_{n=0}^{n_{max}} [r_n + s_n \log(\frac{L}{a})] (\frac{a}{L})^n$$

$$\gamma_1 = \left\{ \begin{array}{ll} 0.0062755(11) & \alpha\text{-scheme} \\ 0.0057956(11) & \beta\text{-scheme} \\ -0.0007746(11) & \gamma\text{-scheme} \\ 0.0032320(11) & \delta\text{-scheme} \end{array} \right.$$

RG-running of the tensor operator for $N_f = 3$ QCD in a χSF setup

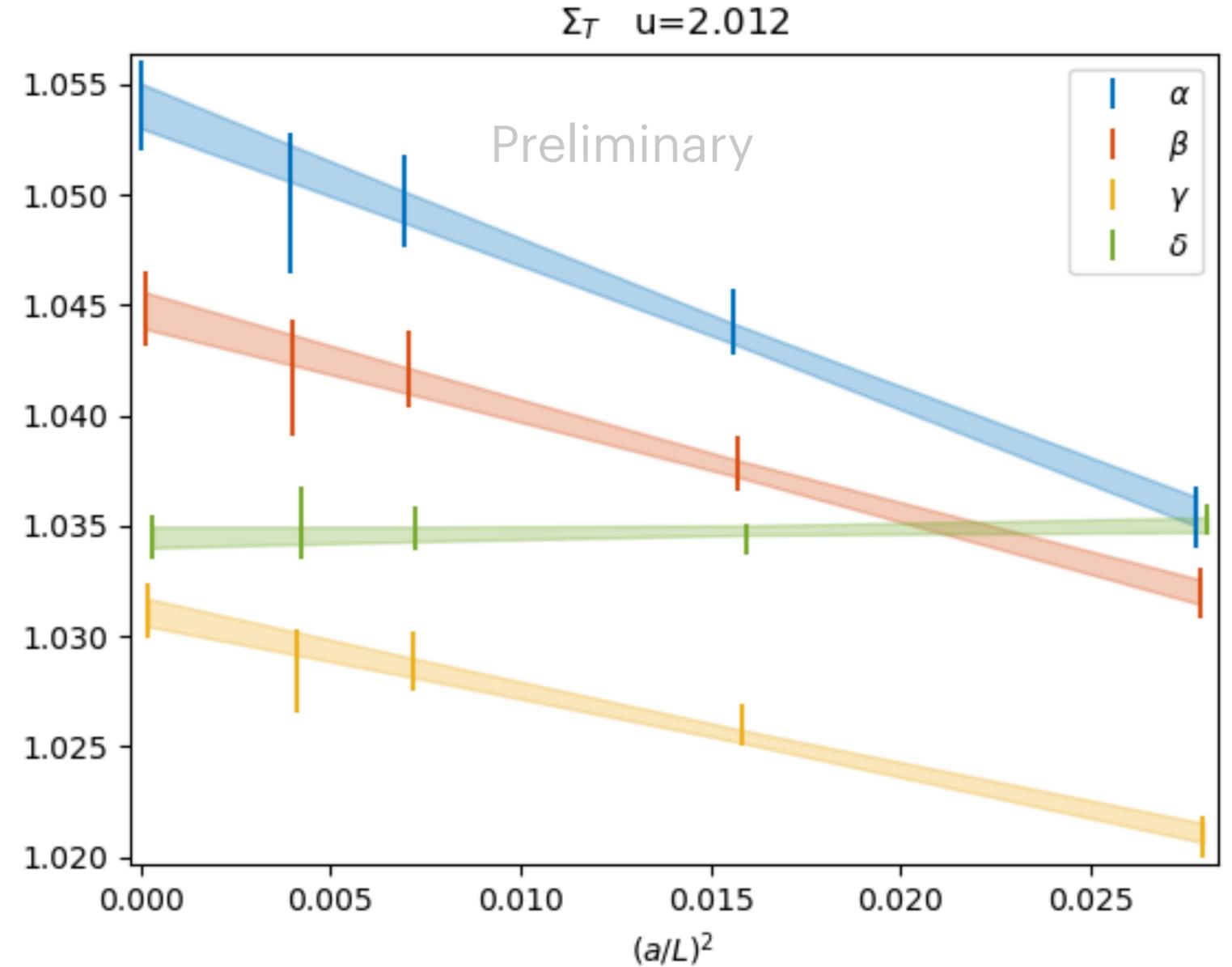
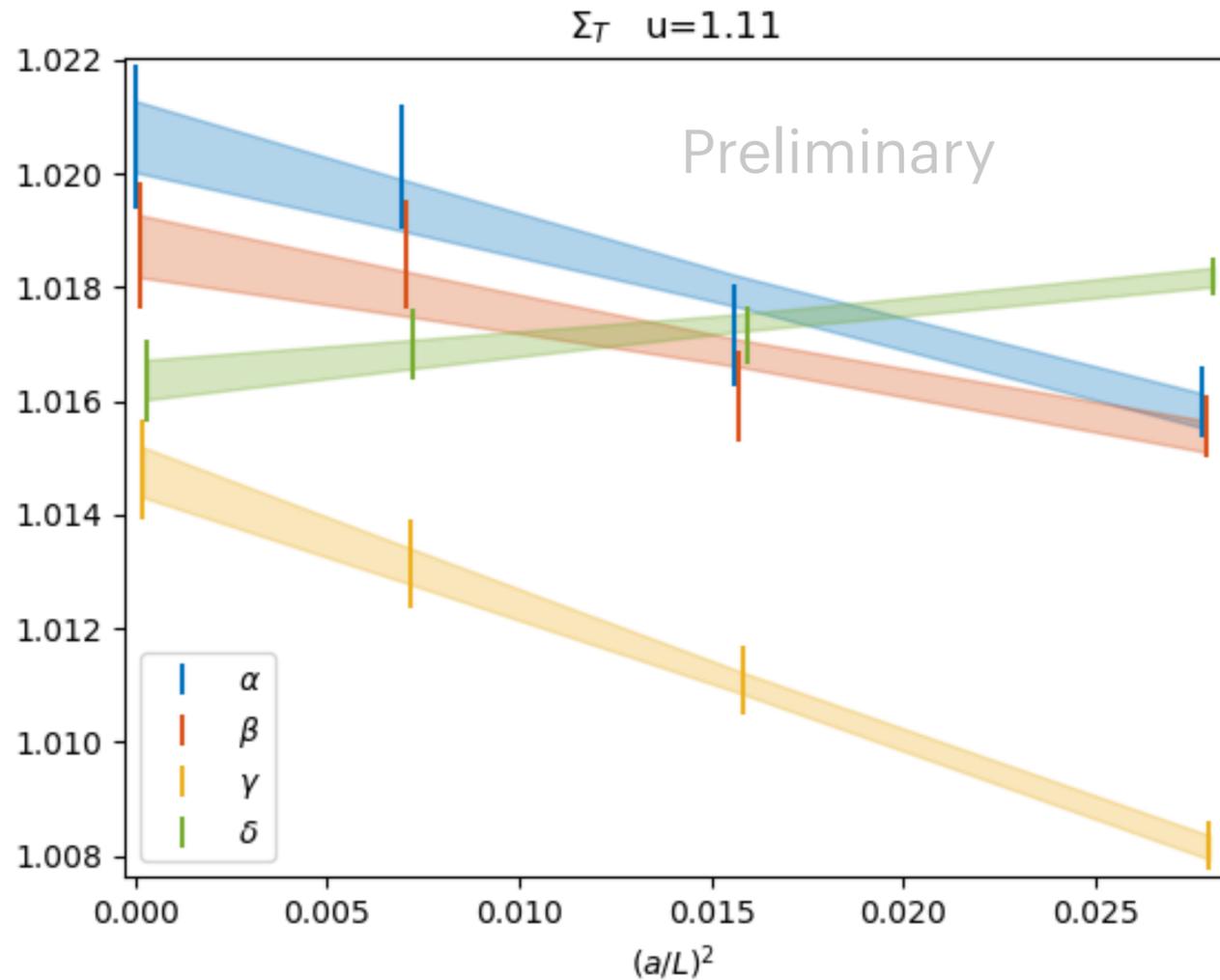
Running of the 4 schemes at high energies

The analysis is in **progress!!**



u by u fits: at fixed u

$$\Sigma_T(u, a/L) = \sigma_T(u) + \rho(u) \left(\frac{a}{L}\right)^2$$



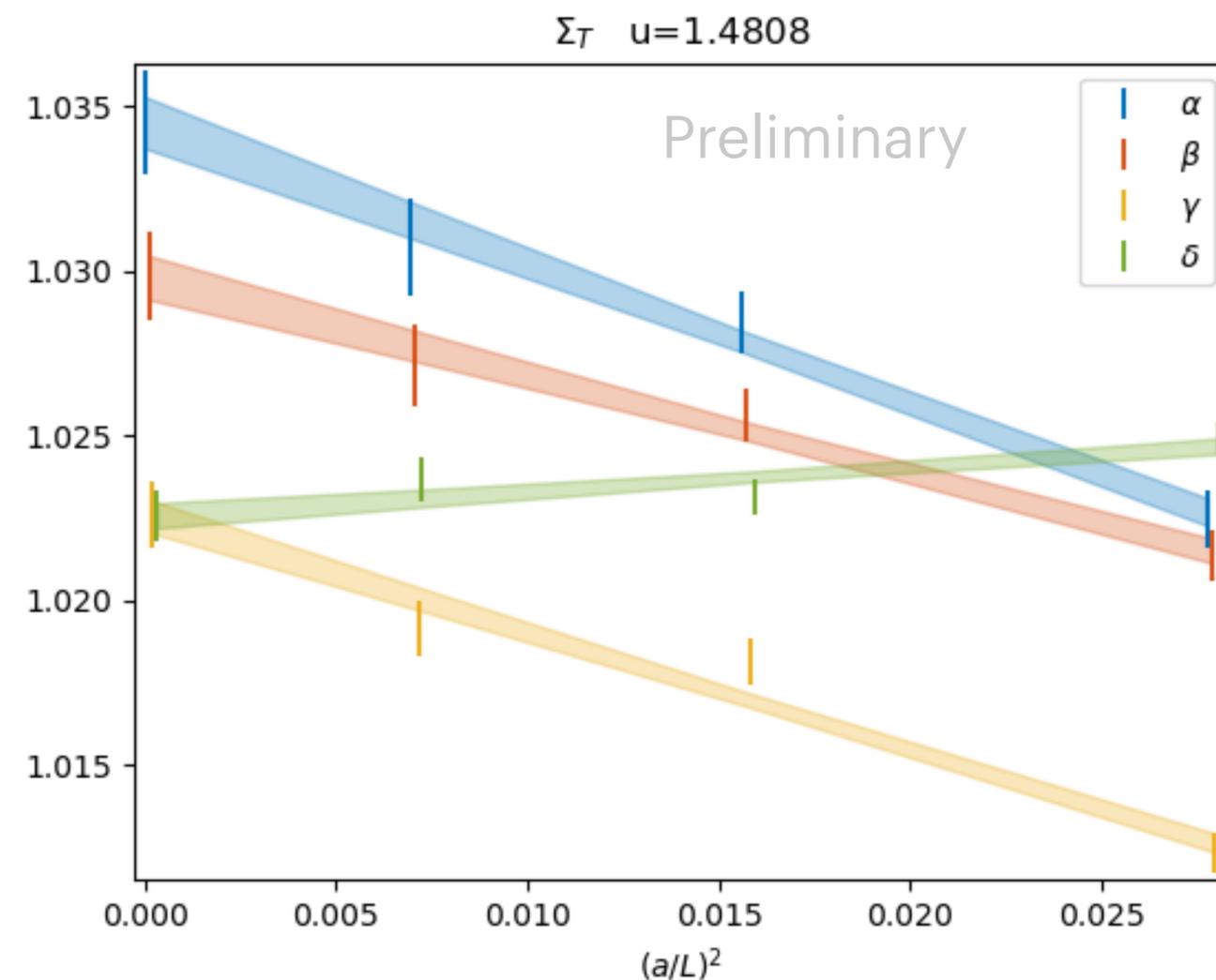
RG-running of the tensor operator for $N_f = 3$ QCD in a χSF setup

SF results: in progress



σ_T tends to have smaller errors for δ -scheme

u	σ_T^α	σ_T^β	σ_T^γ	σ_T^δ
1.11	1.0206(13)	1.0187(11)	1.0148(09)	1.0164(07)
1.1844	1.0249(13)	1.0221(11)	1.0166(10)	1.0173(08)
1.2656	1.0243(15)	1.0220(14)	1.0190(12)	1.0177(09)
1.3627	1.0310(19)	1.0270(16)	1.0209(13)	1.0199(10)
1.4808	1.0345(16)	1.0298(13)	1.0226(10)	1.0226(08)
1.6173	1.0348(23)	1.0297(19)	1.0252(15)	1.0250(12)
1.7943	1.0434(24)	1.0371(21)	1.0297(16)	1.0274(12)
2.012	1.0540(20)	1.0448(17)	1.0311(13)	1.0345(10)



RG-running of the tensor operator for $N_f = 3$ QCD in a χ SF setup

SF results: in progress



fit $\sigma_T(u)$: polynomial in u

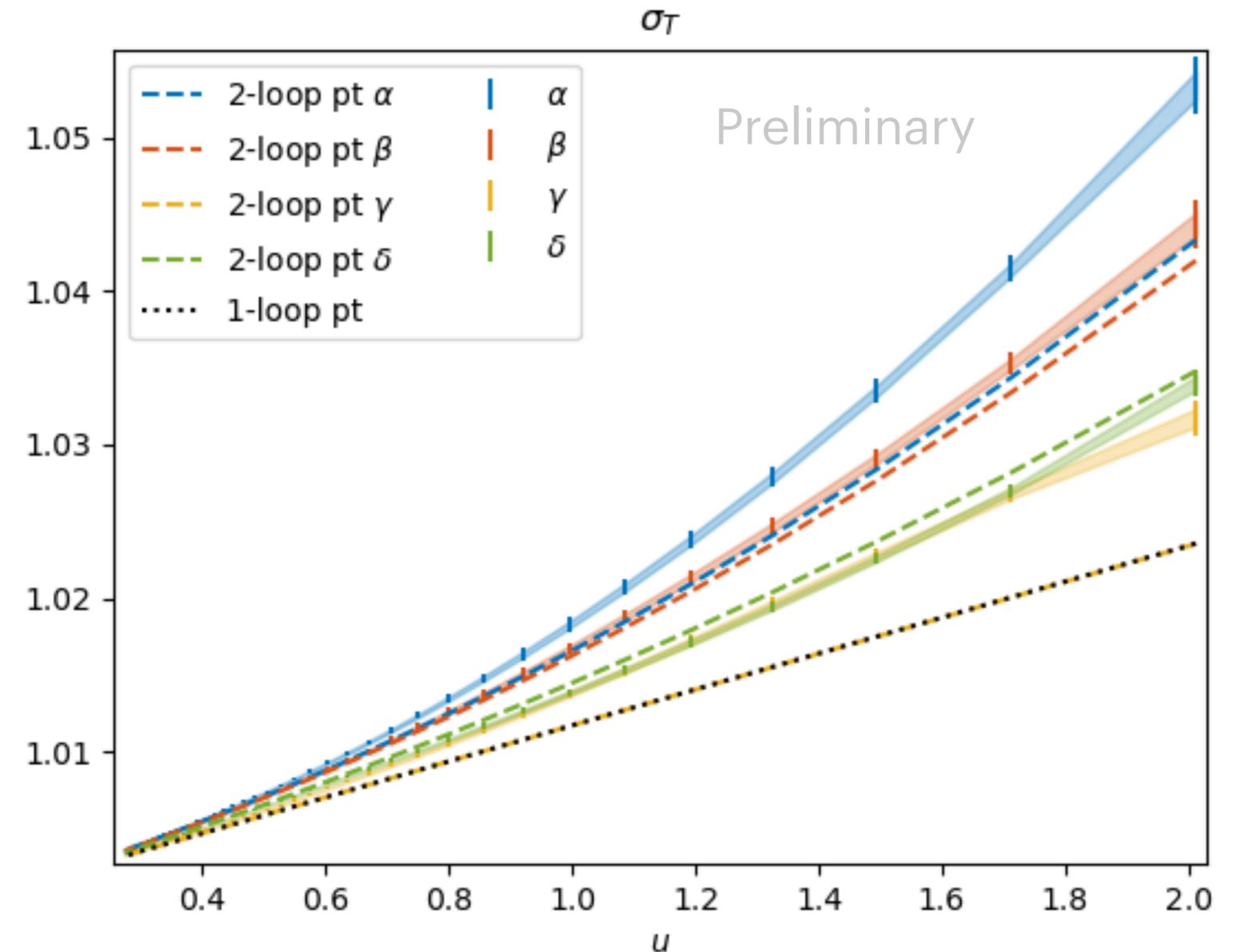
$$\sigma_T(u) = \left(1 + \rho_1 u + \rho_2 u^2 + \dots + \rho_{n_s} u^{n_s} \right)$$

with

$$\rho_1 = \gamma_0 \log(2),$$

$$\rho_2 = \gamma_1 \log(2) + (0.5\gamma_0^2 + b_0\gamma_0)\log(2)^2$$

- **good agreement with PT at high energies**
- **data of β -scheme, δ -scheme** tend to agree with 2-loop also at the lowest energies of SF range



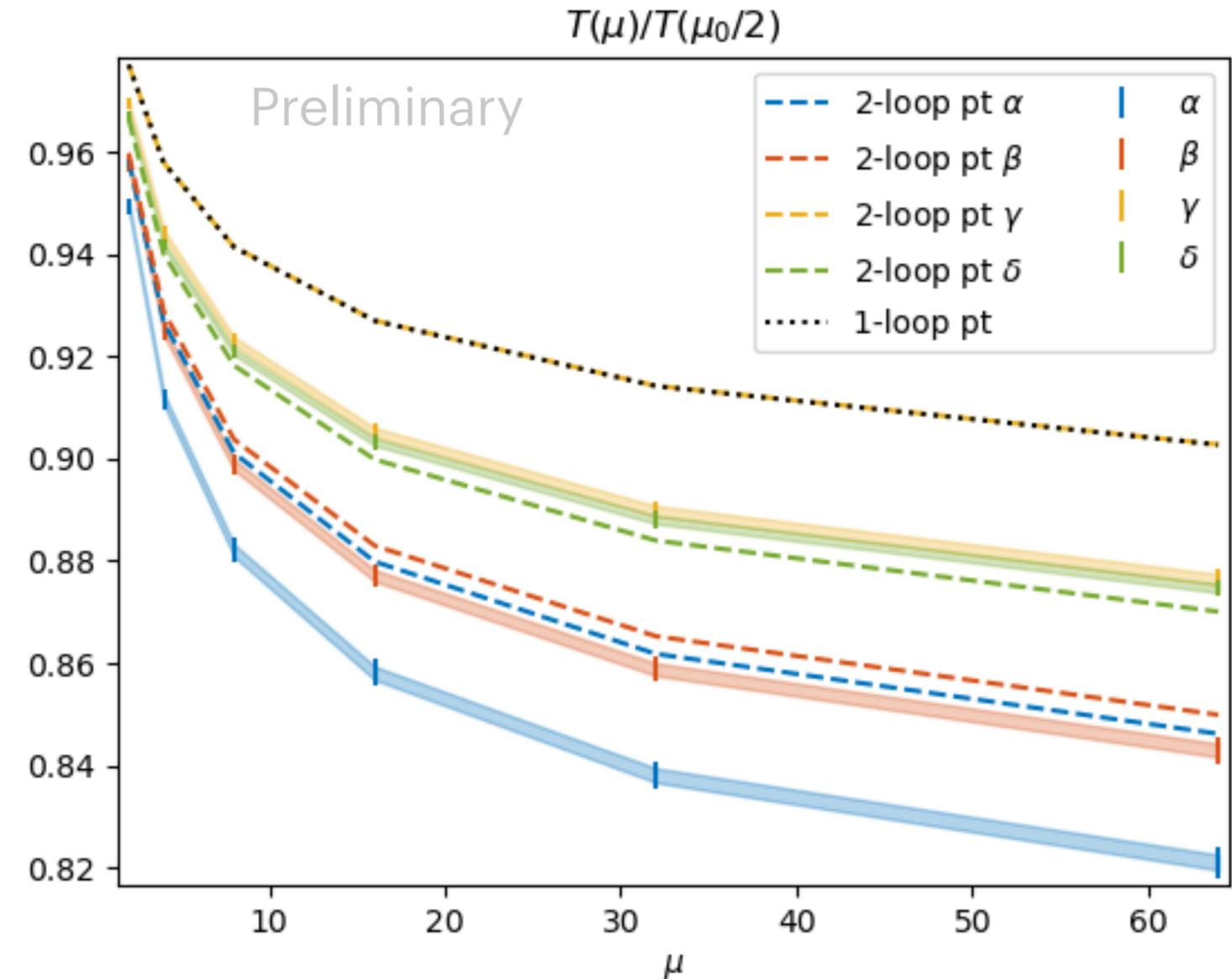
RG-running of the tensor operator for $N_f = 3$ QCD in a χ SF setup

SF results: in progress



$$\frac{\bar{T}(2^k \mu_0)}{\bar{T}(\mu_0/2)} = \prod_{n=0}^k \sigma_T^{-1}(u_n)$$

u	$\frac{\bar{T}(2^k \mu_0)^\alpha}{\bar{T}(\mu_0/2)}$	$\frac{\bar{T}(2^k \mu_0)^\beta}{\bar{T}(\mu_0/2)}$	$\frac{\bar{T}(2^k \mu_0)^\gamma}{\bar{T}(\mu_0/2)}$	$\frac{\bar{T}(2^k \mu_0)^\delta}{\bar{T}(\mu_0/2)}$
2.012	0.9493(17)	0.9575(14)	0.9692(11)	0.9671(08)
1.7126	0.9116(22)	0.9249(18)	0.9439(14)	0.9417(11)
1.4939	0.8820(24)	0.8988(21)	0.9229(16)	0.9209(13)
1.3264	0.8581(26)	0.8772(23)	0.9051(18)	0.9033(14)
1.1936	0.8381(28)	0.8588(24)	0.8898(20)	0.8881(16)
1.0856	0.8211(30)	0.8430(26)	0.8764(22)	0.8747(17)



smaller errors for δ -scheme

RG-running of the tensor operator for $N_f = 3$ QCD in a χ SF setup

SF results: in progress

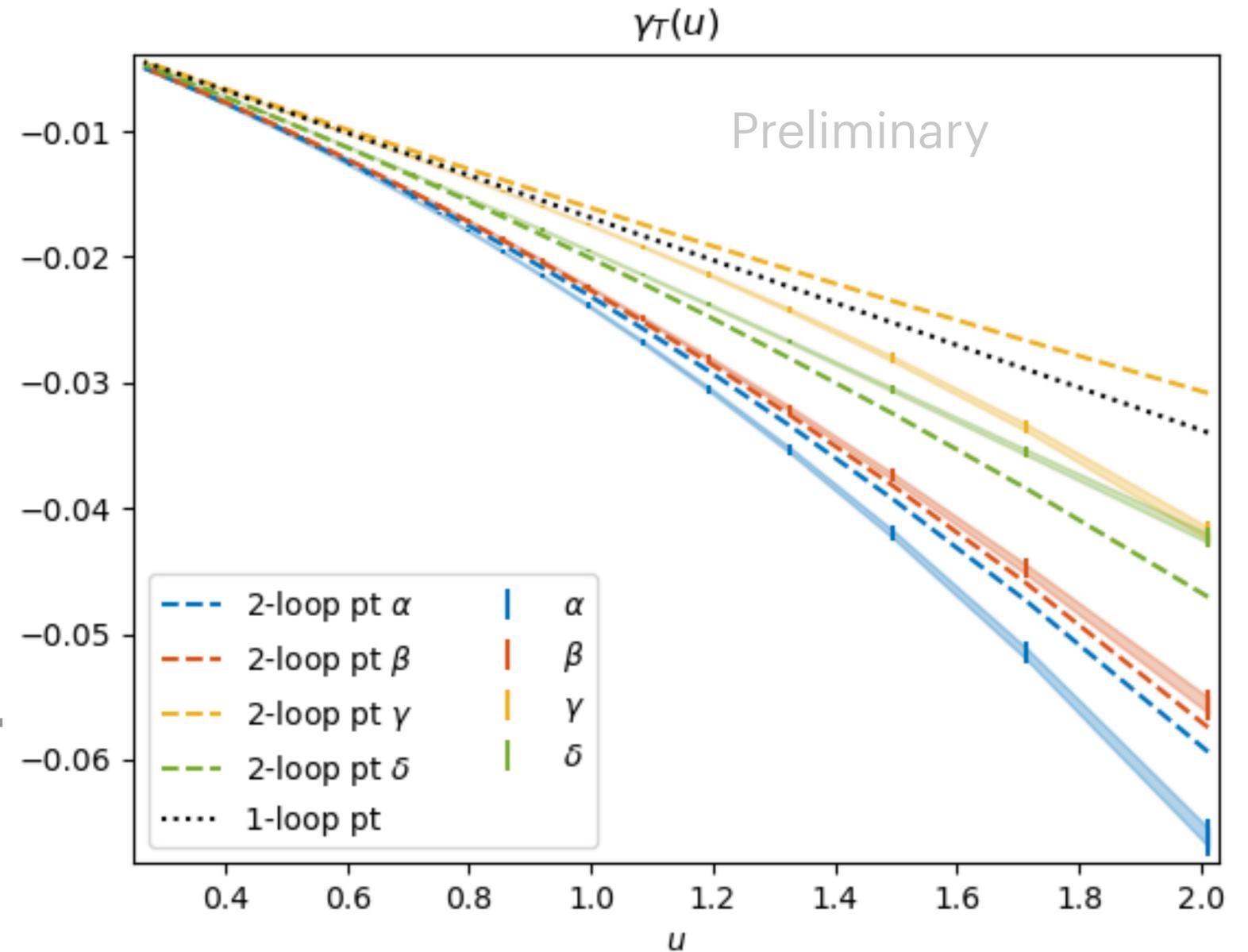


fit $\sigma_T(u)$:

$$\sigma_T(u) = \exp \int_{\sqrt{u}}^{\sqrt{\sigma(u)}} dx \frac{\gamma_T(x)}{\beta(x)}$$

with $\gamma_T(x) = -x^2 \sum_{n=0}^{n_t} \gamma_n x^{2n}$

- good agreement with PT at high energies
- data of **β -scheme**, **δ -scheme** tend to agree with 2-loop also at the lowest energies of SF range



RG-running of the tensor operator for $N_f = 3$ QCD in a χ SF setup

Summary and conclusions

- **Testing 4 different renormalisation schemes with χSF valence fermions is useful**
- From u by u fits: **δ -scheme** is promising.
- **less noisy Σ_T^δ , easily extrapolated to σ_T . Smaller error for $\bar{T}(2^k \mu_0) / \bar{T}(\mu_0/2)$ too**
- **Good agreement for $\sigma_T(u), \gamma_T(u)$ with PT at high energies. For β -scheme, δ -scheme we tend to have good agreement also at the lowest energies of SF range**

Next (always in the 4 schemes):

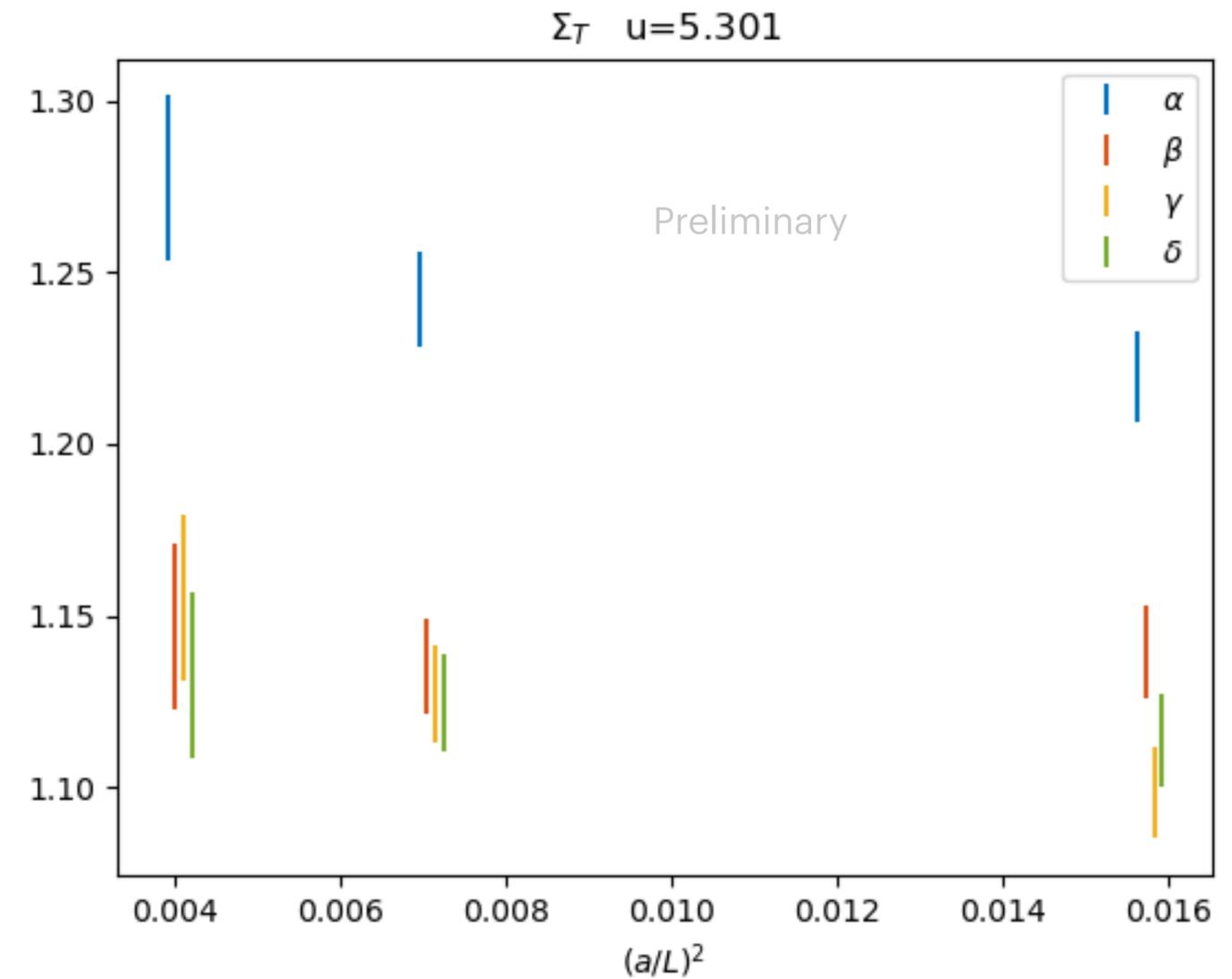
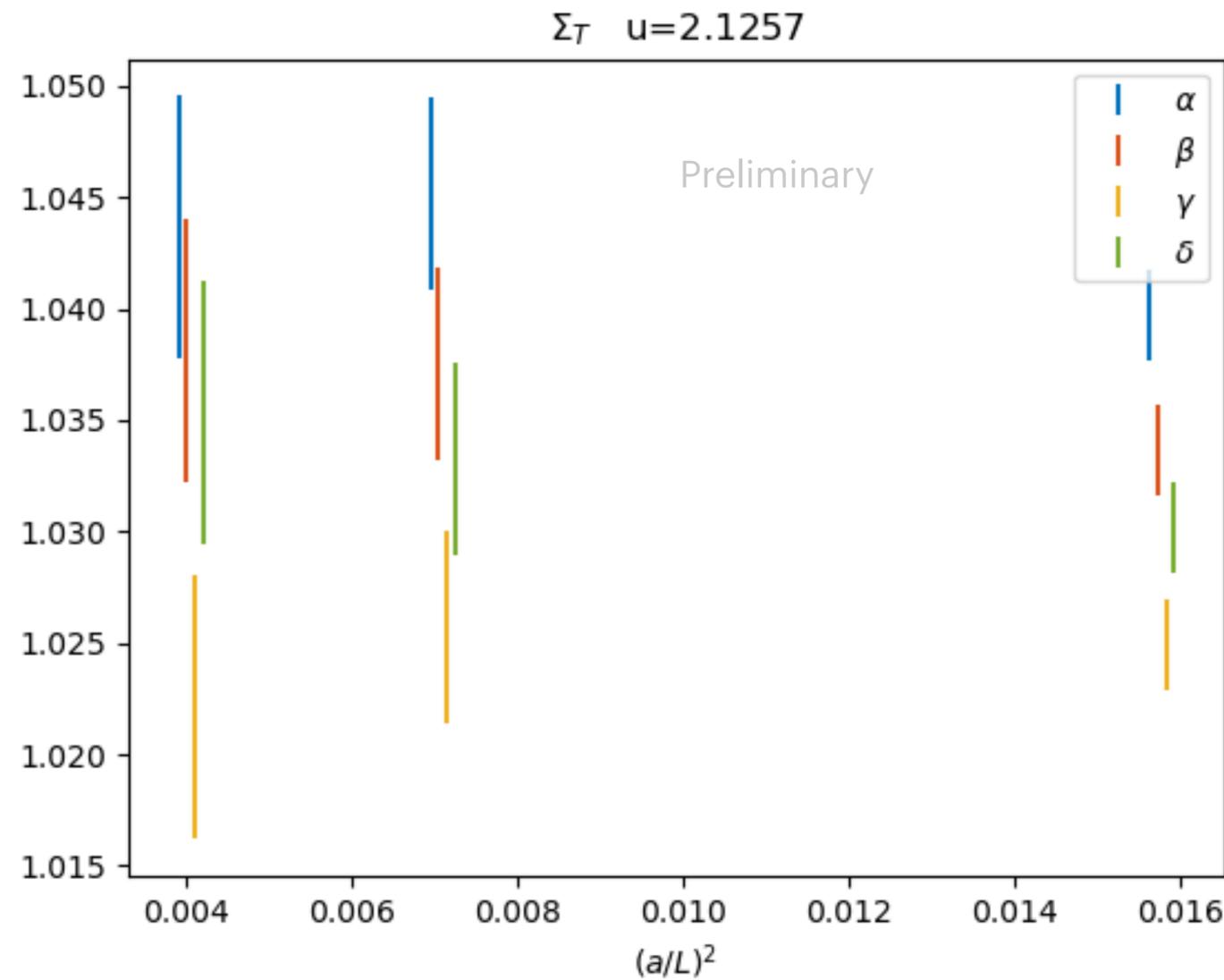
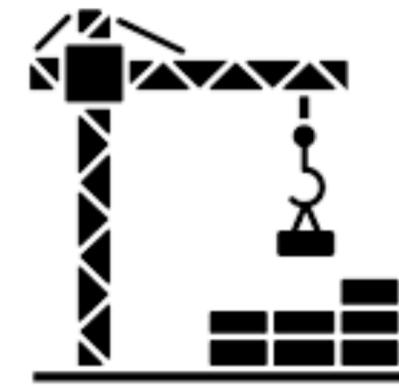
- **Complete the analysis at SF scales**
- **Complete the analysis at GF scales**
- **Compute the hadronic matching at μ_{had} , where $\mu_{had} \sim 0.2 MeV$**
- **Compute the final renormalisation factors $T^{RGI} / \bar{T}(\mu_{had})$**

Thanks for the
attention!

RG-running of the tensor operator for $N_f = 3$ QCD in a χSF setup

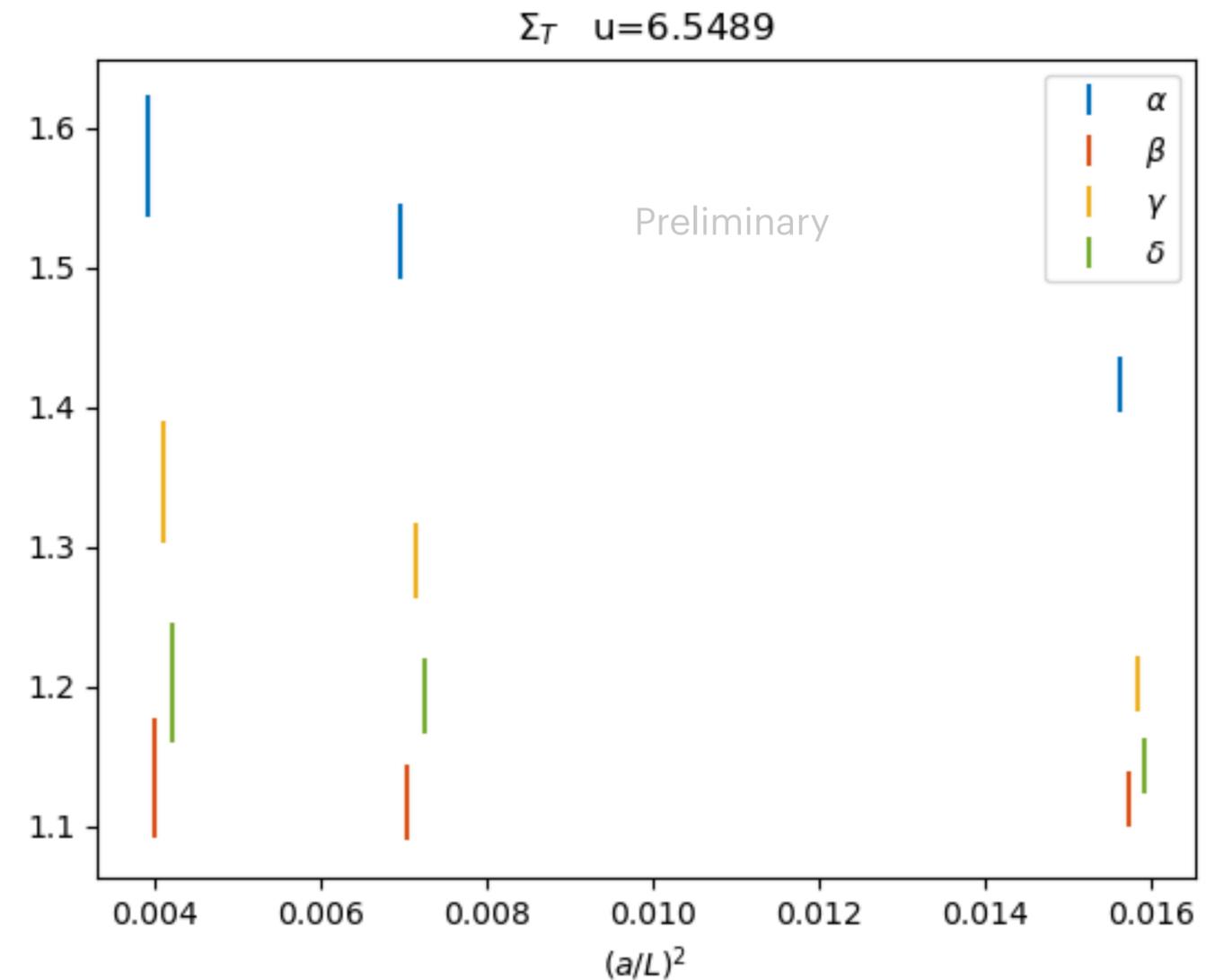
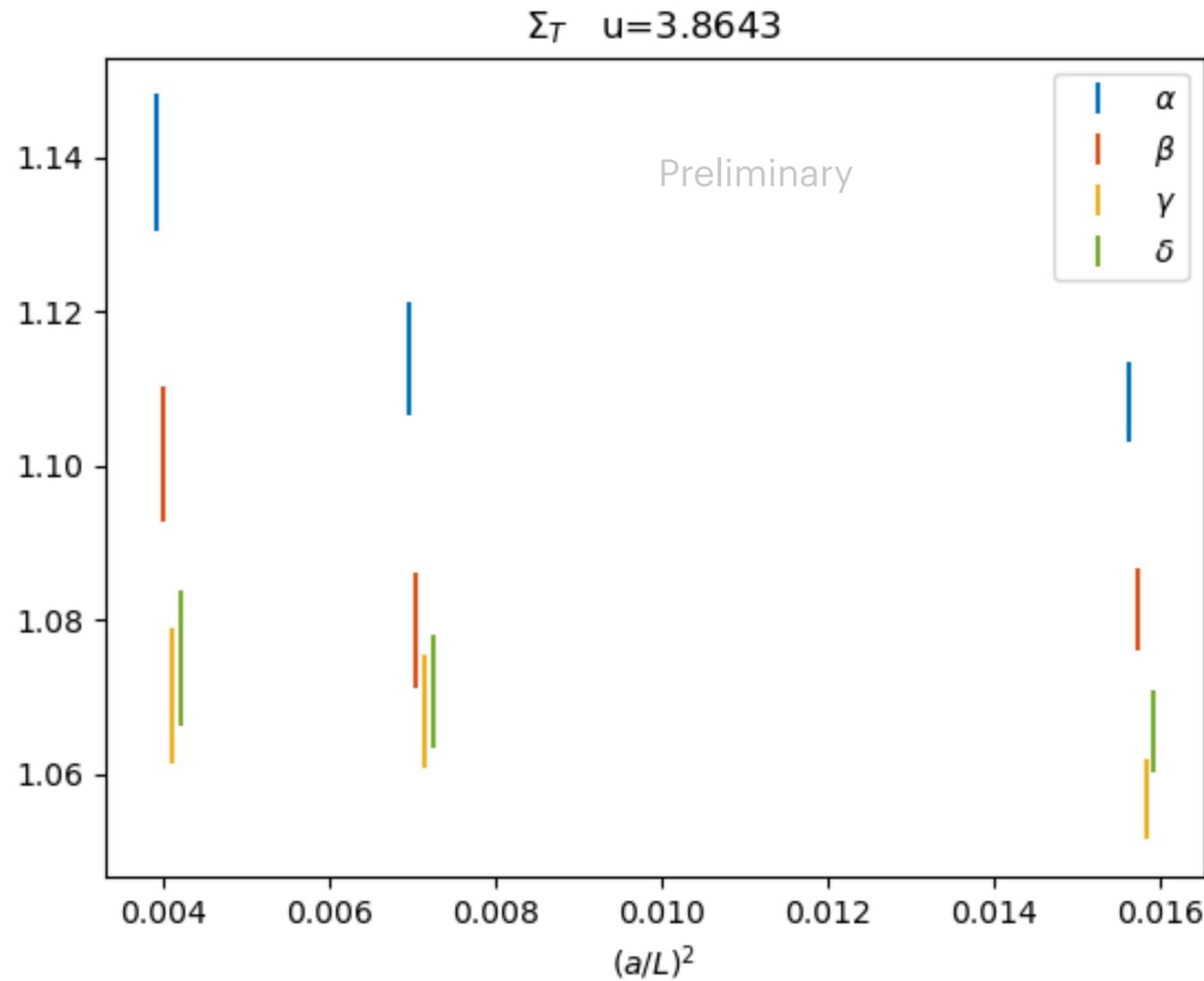
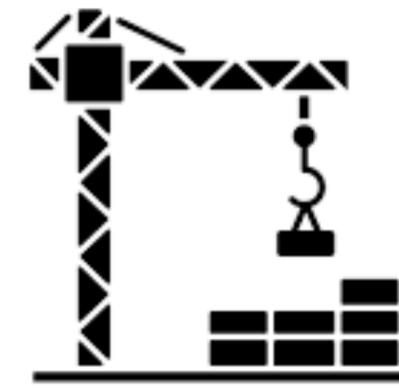
Backup slides

Just a quick glance to low energies



RG-running of the tensor operator for $N_f = 3$ QCD in a χSF setup

Just a quick glance to low energies



RG-running of the tensor operator for $N_f = 3$ QCD in a χSF setup