The light quark masses from $N_f = 2+1$ CLS ensembles

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RQCD Collaboration

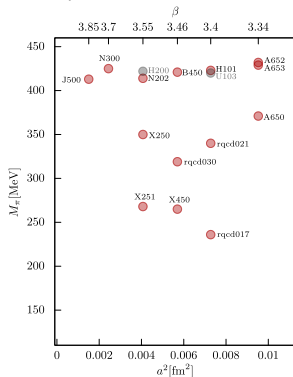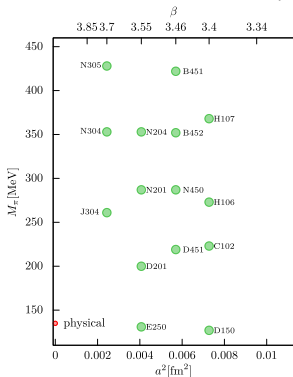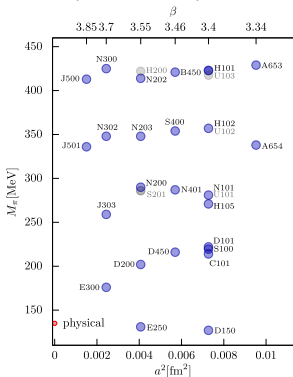Lattice 2022, Bonn   August 10, 2022

CLS ensembles: M_π vs a^2

$N_f = 2 + 1$ flavours of non-perturbatively $\mathcal{O}(a)$ improved Wilson fermions on tree level Symanzik improved glue.

High statistics: typically 6000 – 8000 MDUs, 1000 – 2000 configurations.

Aim to control all main sources of systematics (a, m_q and V). Six lattice spacings:
$a = (0.1 \searrow 0.039)$ fm, $LM_\pi \gtrsim 4$ + smaller volumes, $M_\pi = (420 \searrow 130)$ MeV.
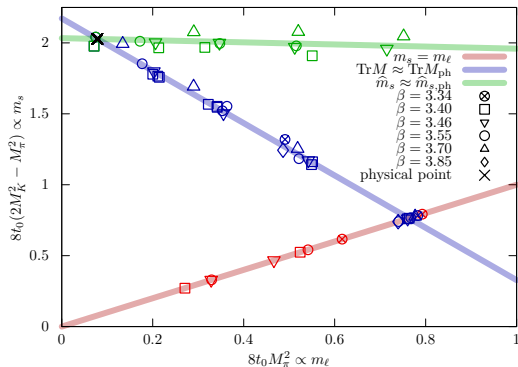

$$2m_\ell + m_s = \text{const.} \qquad m_s = \text{const.} \qquad m_\ell = m_s$$

$a < 0.06$ fm: open-boundary (ob) conditions in time.

$a > 0.06$ fm: mixture of ensembles with periodic and ob conditions.

CLS ensembles: m_ℓ-m_s plane

Three trajectories: can correct for mis-tuning of the trajectories. Good control over the quark mass dependence.

$2m_\ell + m_s = \text{const.}$: investigate SU(3) flavour breaking (flavour averaged quantities roughly constant), approach to the physical point involves $M_K \uparrow$ as $M_\pi \downarrow$.

Motivation

▶ Another independent prediction of light quark masses useful to assess systematics of existing results.

▶ Study the impact of different renormalization procedures (small lattice spacings necessary).

▶ Important to set a precise baseline to study isospin breaking effects.

▶ Map out $m_q(M_\pi^2, M_K^2)$ and $F_{\pi,K}(M_\pi^2, M_K^2)$ to determine SU(3) LECs (much less well-known than SU(2) LECs).
LO LECs F_0 and B_0: [RQCD: S. Weishäupl et al.,2201.05591].

▶ σ-terms $\sigma_{\pi N} = \sigma_{uN} + \sigma_{dN}$, σ_{sN} are defined as derivatives wrt quark masses:

$$\sigma_{uN} = m_u \left.\frac{\partial m_N}{\partial m_u}\right|_{m_d, m_s \text{ fixed}} = \frac{1}{2} M_\pi^2 \left.\frac{\partial m_N}{\partial M_\pi^2}\right|_{2M_K^2 - M_\pi^2 \text{ fixed}} \cdot \underbrace{\left[1 + \mathcal{O}\left(M^2\right)\right]}_{???}.$$

Direct determination
$\longrightarrow$ yesterday's 18:10 Hadron Structure talk by Pia Petrak.

$\mathcal{O}(a)$ Symanzik improvement

Improvement of the action (subtracting $aS^{(1)}$ from both sides):

$$S_{\text{lattice}}^{(0)} = S_{\text{continuum}} + \underbrace{aS^{(1)} + a^2 S^{(2)} + \dots}_{\text{unwanted ``physics'' at scales} \sim 1/a}$$

▶ Clover counter term $\propto i c_{SW} a \bar{q} \sigma_{\mu\nu} F_{\mu\nu} q$. c_{SW} is known non-perturbatively.

▶ Improvement of the quark masses in the lattice action (i.e. of $1/\kappa_q$).
  Not relevant since we use the axial Ward identity (AWI) masses.

▶ Improvement of the coupling $g^2 \mapsto g^2(1 + a b_g \text{Tr} \, M / N_f)$.
  Improved coupling changes if $\text{Tr} \, M = m_s + 2m_\ell \neq$ const.

The effect of b_g cancels from dimensionless combinations at each fixed
$\{\beta, \, \kappa_\ell, \, \kappa_s\}$. Therefore, we extrapolate $\sqrt{8t_0}\, \widehat{m}_q$ as functions of $8t_0 M^2$.

Improvement of the pseudoscalar and axial currents $P^{uq} = \bar{u}\gamma_5 q$ and
$A_0^{uq} = \bar{u}\gamma_0\gamma_5 q$, where $q \in \{d, s\}$:

$$P^{uq} \mapsto P^{uq} + a \left(b_P m_{12} + \tilde{b}_P \text{Tr} \, M \right) P^{uq}, \quad m_{uq} = (m_u + m_q)/2$$

$$A_0^{uq} \mapsto A_0^{uq} + a \left[c_A \partial_0 P^{uq} + \left(b_A m_{uq} + 3\tilde{b}_A \text{Tr} \, M \right) A_0^{uq} \right].$$

c_A, $b_A - b_P$ and $\tilde{b}_A - \tilde{b}_P$ are known non-perturbatively and used
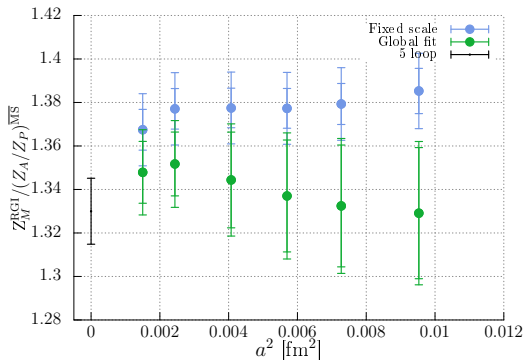(also taking their uncertainties into account).

Renormalization of the AWI quark masses

$$\widehat{m}_u + \widehat{m}_q = \boxed{\frac{Z_A}{Z_P(\mu)}} \frac{\partial_0 \langle \Omega | A_0^{uq} | P^+ \rangle}{\langle \Omega | P^{uq} | P^+ \rangle}, \quad P^+ = \pi^+, K^+ \quad \text{for} \quad q = d, s.$$

Renormalized masses computed in RGI- (no μ) and $\overline{\text{MS}}$-scheme at $\mu = 2\,\text{GeV}$.
Below: $N_f = 3$ ratios of

$Z_M^{\text{RGI}} = Z_A/Z_P^{\text{RGI}}$ from finite box step scaling function [ALPHA,1802.05243] and
$Z_A/Z_P^{\overline{\text{MS}}}(2\,\text{GeV})$ [RQCD,2012.06284], converted at 3-loops RI'-SMOM $\to \overline{\text{MS}}$.

Perturbative ratio:
$Z_m^{\text{RGI}}/Z_m^{\overline{\text{MS}}}(2\,\text{GeV}) = 1.330(14)(7)$
(5 loops)

error 1: uncertainty of $\Lambda^{\overline{\text{MS}}, N_f=3}$

error 2: $|(\text{5-loop}) - (\text{4-loop})|$

$\to \overline{\text{MS}}$ quark masses at $\mu = 2\,\text{GeV}$
with an error below 1% impossi-
ble! PT uncertainty alone is 0.5%.

Definition of the physical point

We use isoQCD: $m_\ell = (m_u + m_d)/2$. Isospin-breaking Q[C+E]D effects will affect m_ℓ only quadratically in the symmetry breaking parameters.

We define the physical point in isoQCD as

- $M_\pi = 134.8(3)\,\text{MeV}$ (see, e.g., FLAG 2016),
- $M_K = 494.2(3)\,\text{MeV}$ (see, e.g., FLAG 2016),
- $m_\Xi = 1316.9(3)\,\text{MeV}$, assuming one linear QCD and one linear QED isospin breaking term across the whole baryon octet and no QED contribution for neutral particles (Dashen theorem).
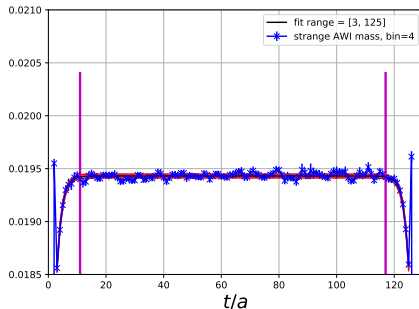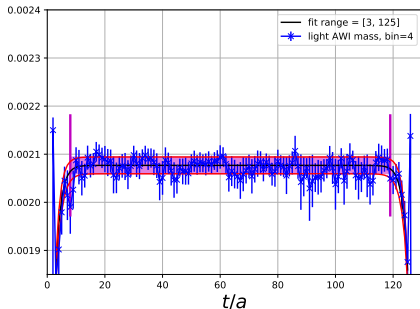 The first assumption yields the Coleman-Glashow theorem
  $0 \approx (m_p - m_n) + (m_{\Xi_0} - m_{\Xi^-}) - (m_{\Sigma^+} - m_{\Sigma^-}) = 0.06(23)\,\text{MeV}$.

We use $\sqrt{8t_{0,\text{ph}}} = 0.4098^{(20)}_{(25)}\text{fm}$ to set the scale. This is determined from the product $\sqrt{8t_0}\,m_\Xi$.
$\longrightarrow$ yesterday's 16:30 Hadron Spectroscopy talk by Sara Collins.

Then $8t_0 M_\pi^2 = 0.07835(88)$ and $(2M_K^2 - M_\pi^2)/M_\pi^2 = 25.88(12)$ define to the physical point.

Extracting the AWI masses

Example: ensemble D450 ($a \approx 0.075\,\text{fm}$, $M_\pi \approx 216\,\text{MeV}$).

"strange" is for the K-π combination $m_s = (m_s + m_u) - (m_u + m_d)/2$.

Also shown are 2-exp fits to determine the fit ranges and the resulting fit ranges.

Analysis details $\longrightarrow$ yesterday's poster (Poster B) by Wolfgang Soeldner.

Continuum, finite volume, quark mass extrapolation

$$\overline{M}^2 = \frac{1}{3}(2M_K^2 + M_\pi^2) \propto \overline{m} = \frac{1}{3}(2m_\ell + m_s), \quad \delta M^2 = 2(M_K^2 - M_\pi^2) \propto \delta m = m_s - m_\ell$$

Rescale all masses by $\sqrt{8t_0}$: $q \in \{\ell, s\}$

$$\overline{\mathbb{M}} = \sqrt{8t_0}\,\overline{M}, \quad \delta\mathbb{M} = \sqrt{8t_0}\,\delta M, \quad \mathbb{m}_q = \sqrt{8t_0}\,m_q, \quad \mathbb{a} = \frac{a}{\sqrt{8t_0^*}},$$

Extrapolation performed using the fit form

$$\mathbb{m}_q(\mathbb{M}_\pi, \mathbb{M}_K, \mathbb{a}) = \mathbb{m}_q(\mathbb{M}_\pi, \mathbb{M}_K, 0) \cdot \left[1 + \mathbb{a}^2\left(c + \bar{c}\,\overline{\mathbb{M}}^2 + \delta c_q\,\delta\mathbb{M}^2\right)\right].$$

Simultaneous fit to m_ℓ and m_s with **all correlations taken into account.**

Discretization effects: 4 parameters: c, $\bar{c}$, δc_ℓ, δc_s.

Natural choice for $\mathbb{m}_q(\mathbb{M}_\pi, \mathbb{M}_K, 0)$ is to use SU(3) ChPT.

We also considered Taylor expansions around the line of equal quark masses.

Continuum quark mass dependence

SU(3) ChPT at NLO

(finite volume corrections for M_π and M_K are small but included):

$$2B_0 m_\ell = M_\pi^2 \left\{ 1 - \mu_\pi^2 + \frac{1}{3}\mu_{\eta_8}^2 - \frac{8}{F_0^2} \left[M_\pi^2 L_{85} + \left(2M_K^2 + M_\pi^2 \right) L_{64} \right] \right\},$$

$$2B_0 m_s = \left(2M_K^2 - M_\pi^2 \right) \left[1 - \frac{8}{F_0^2} \left(2M_K^2 + M_\pi^2 \right) L_{64} \right] + M_\pi^2 \mu_\pi^2$$

$$- \frac{1}{3} \left(4M_K^2 + M_\pi^2 \right) \mu_{\eta_8}^2 - \frac{8}{F_0^2} \left(2M_K^4 - M_\pi^4 \right) L_{85},$$

where

$$\mu_P^2 = \frac{1}{2} \frac{M_P^2}{(4\pi F_0)^2} \ln \left(\frac{M_P^2}{\mu^2} \right)$$

and

$$L_{85}(\mu) = 2L_8(\mu) - L_5(\mu), \quad L_{64}(\mu) = 2L_6(\mu) - L_4(\mu).$$

Usually, $\mu = 770\,\text{MeV}$, but this choice only affects the NLO LECs $L_j(\mu)$.

4 parameters: F_0, B_0, L_{85}, L_{64}.
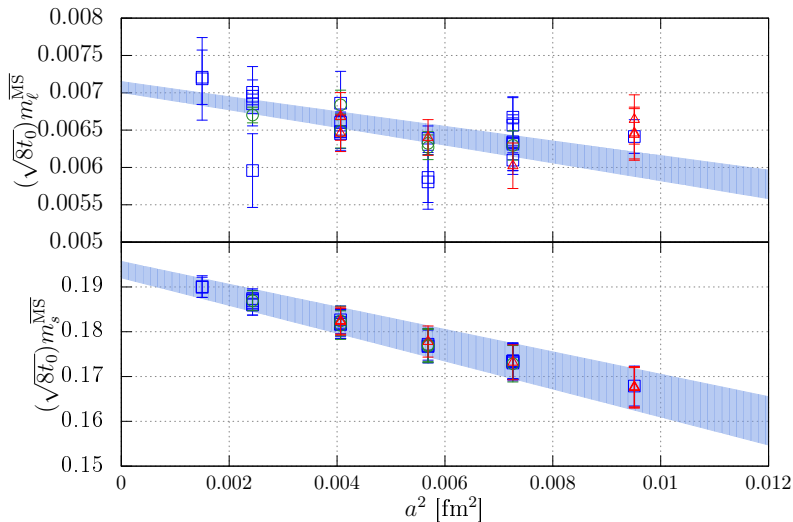
Quark mass dependence **PRELIMINARY**
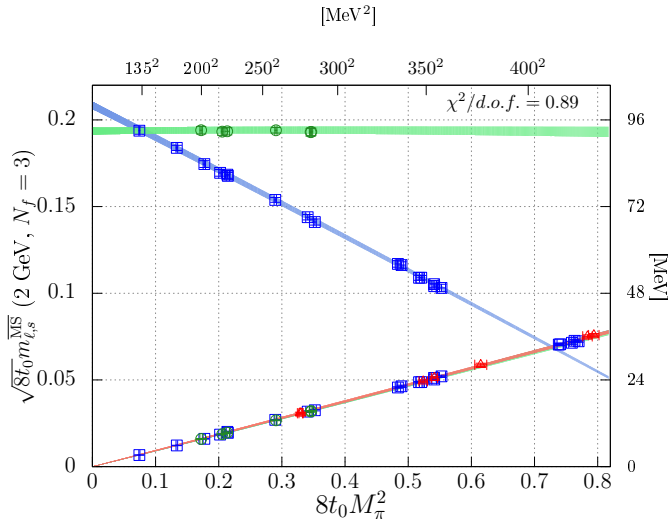
$(Z_A/Z_P$ from global fit to RI'-SMOM results.)

For the light quark mass the two physical point trajectories almost coincide with the $m_s = m_\ell$ trajectory (red). (Data moved to the continuum limit trajectories.)
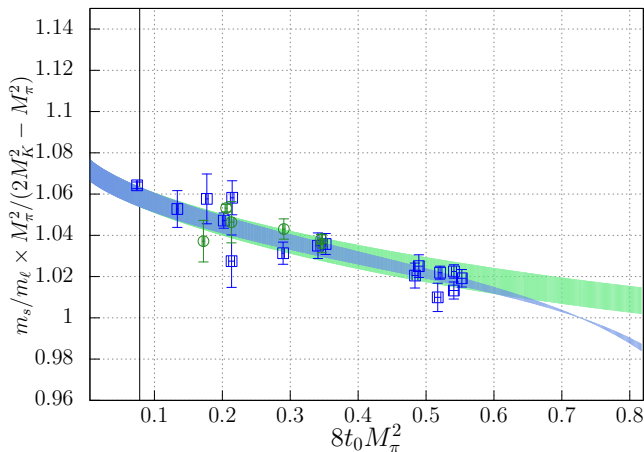
The continuum limit

Data are moved to the physical quark mass point according to the fit.

Violation of the GMOR relation for m_s/m_ℓ

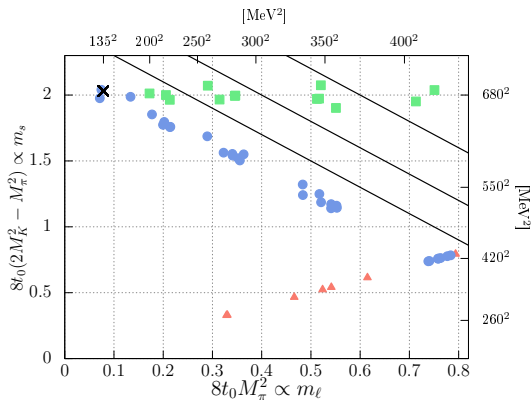$$\frac{m_s}{m_\ell} = \frac{2M_K^2 - M_\pi^2}{M_\pi^2} \cdot \boxed{\left[1 + \mathcal{O}(M^2)\right]}$$

PRELIMINARY

Assess the systematics of the fits: cuts on the data

Pseudoscalar masses:
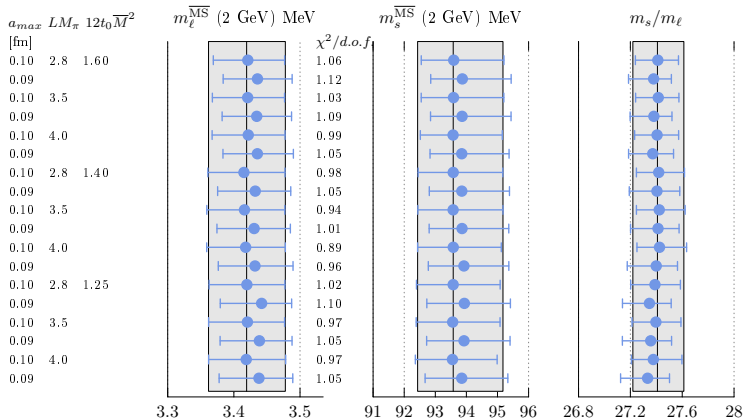include $\overline{M}^2 < (498\,\text{MeV})^2$ and impose further cuts $\overline{M}^2 < (466\,\text{MeV})^2$, $\overline{M}^2 < (440\,\text{MeV})^2$.

Finite V: $L > 2.3$ fm always. Use all data and cuts $LM_\pi > 3.5$, $LM_\pi > 4.0$.

Finite a: use all data and cut $a < 0.09$ fm.

Dependence on the data cuts
PRELIMINARY
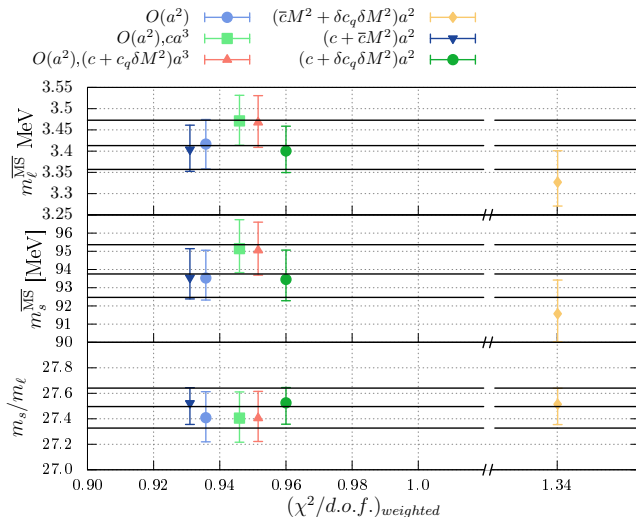

Correlated χ^2, 8 fit parameters, $43 \leq$ d.o.f. ≤ 65.

Grey bands are AIC averages with weights $\propto \exp\left[-\frac{1}{2}\left(\chi^2 - d.o.f.\right)\right]$.

(# of data points is varied, # of fit parameters is fixed!)

Results shown are for the $N_f = 3$ theory.

Future: m_s/m_ℓ from a separate fit to cancel out Z_A/Z_P and $a(\tilde{b}_A - \tilde{b}_P)\,\mathrm{tr}\,M$.

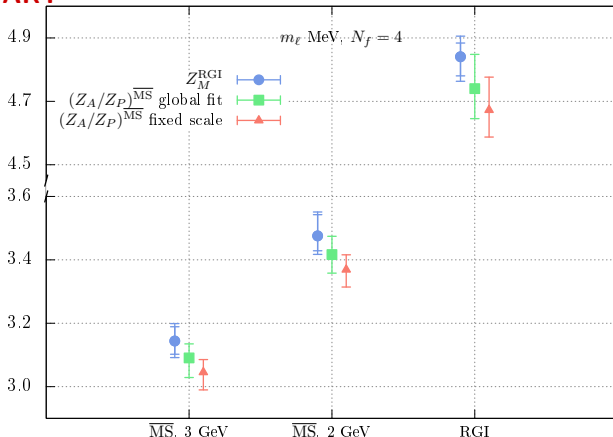# Vary the parametrization of the discretization effects

PRELIMINARY

Variation shown for fit with lowest $\chi^2/d.o.f.$, i.e. $LM_\pi > 4$, $\overline{M}^2 < (466\,\text{MeV})^2$.
($N_f = 3$ theory at $\mu = 2\,\text{GeV}$, using the RI'-SMOM global fit.)

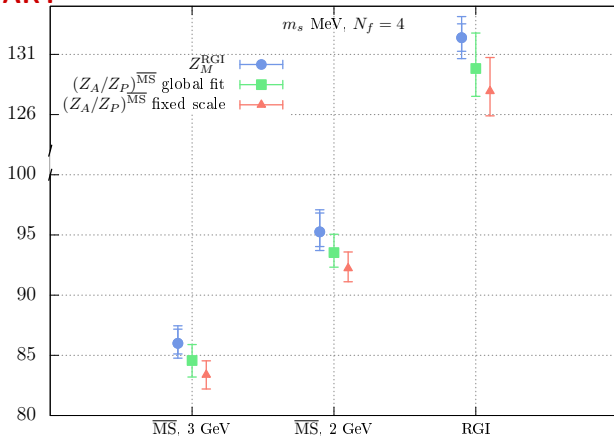# Different renormalization schemes/scales for m_ℓ ($N_f = 4$)

PRELIMINARY

Using 5-loop (β-, γ-function) running and 4-loop matching at the charm quark mass threshold.

At present, the renormalization is the dominant source of uncertainty, followed by the uncertainty of the scale $\sqrt{8t_0} = 0.4098^{(20)}_{(25)}$fm.

Different renormalization schemes/scales for m_s ($N_f = 4$)

Concentrate on $[Z_A/Z_P(2\,\text{GeV})]^{\overline{\text{MS}}}$ from the "global" RI'-SMOM fit.
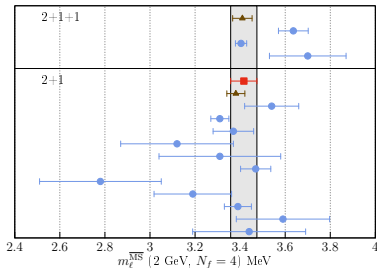
Comparison with literature values

RQCD 22: $Z_A/Z_P^{\overline{\text{MS}}, N_f=4}(2\,\text{GeV})$ determined via global fit to RI'-SMOM data.
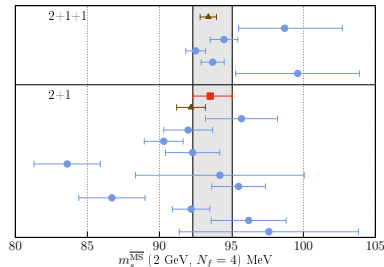
PRELIMINARY

Comparison of m_s/m_ℓ with the literature

PRELIMINARY
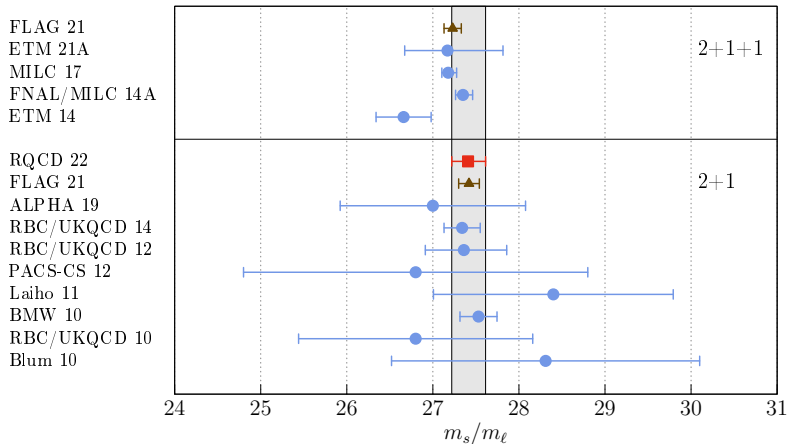
In QCD this ratio is scheme- and scale-independent.

In perturbation theory it does not depend on N_f either.

Our precision can probably be somewhat reduced by a direct fit to this ratio.

Summary and outlook

Summary

- ▶ We computed the light and strange quark masses in isoQCD, including all sources of error.

- ▶ As there is significant dependence on a^2, the continuum limit is of particular importance.

- ▶ The precision is limited by the accuracy of the renormalization to the $\overline{\text{MS}}$ (or the RGI) scheme, which was carried out very carefully.

- ▶ Nevertheless, the results agree well with FLAG averages.

Outlook

- ▶ Joint SU(3) ChPT analysis of the masses and the pseudoscalar decay constants to determine the (N)LO LECs too.
  First step in [RQCD: S. Weishäupl et al.,2201.05591].

- ▶ Reduction of error on m_s/m_ℓ through a direct fit to the ratio.

- ▶ The accuracy of the renormalization should ideally be improved upon since this is the limiting factor on the precision (also for heavy quark masses).

- ▶ The inclusion of QCD and QED isospin breaking effects would be interesting.