Light quark masses from a mixed action with Wilson twisted mass valence quarks

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>cls   ALPHA Collaboration

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mixed action

setup : mixed action with Wilson twisted mass (Wtm) valence quarks
on CLS $N_f = 2 + 1$ ensembles

motivation :

- alternative/complementary way to control lattice artefacts
  $\rightsquigarrow$ universality
- target :
  - light-quark sector : sea/valence matching, scale setting
    see talk by Alejandro Sáez [Wed.,14:30]
    light-quark masses
  - heavy-quark sector :
    quark masses + leptonic and semi-leptonic decays
    see talks by Alessandro Conigli [Wed.,15:40]
    & by Julien Frison [Wed.,15:00]

sea sector : $N_f = 2+1$ CLS [1411.3982, 1608.08900, 1712.04884]

- lattice action:
  - gauge action: Lüscher-Weisz gauge action (tlSym)
  - fermion action: $N_f = 2+1$ Wilson fermions with non-perturbative $c_{\rm SW}$
- open boundary conditions in time : relevant for heavy-quark physics
- chiral trajectory

$$M_q = \mathrm{diag}\,(m_{q_u}, m_{q_d}, m_{q_s})$$

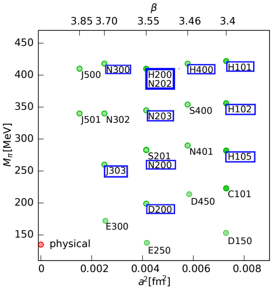
$$\mathrm{tr}\, M_q \;=\; m_{q_u} + m_{q_d} + m_{q_s} \;=\; \mathrm{const.}$$

renormalised chiral trajectory [Bruno, Korzec, Schaefer, 1608.08900]

$$\phi_4 \;\equiv\; 8t_0 \left(\tfrac{1}{2} M_\pi^2 + M_K^2 \right) \;=\; \tfrac{1}{2}\phi_2 + \phi_K \;=\; \mathrm{const} \;=\; \phi_4^{\rm phys}$$

- lattice spacings: $a \approx$ 0.087, 0.077, 0.065, 0.050, 0.037 fm

$M_\pi L \geq 3.9$



see talk by Wolfgang Soeldner [Tue.,15:40]

valence quarks: Wilson twisted mass

[ALPHA, hep-lat/0101001; Frezzotti and Rossi hep-lat/0306014, Pena et al., hep-lat/0405028]

- valence action

$$D_{\mathrm{Wtm}} = D_{\mathrm{W}}^{\mathrm{SW}} + m \pm i\gamma_5\mu$$

- maximal twist $\omega = \frac{\pi}{2}$:

$$m = \tilde{m}_{cr} \quad \rightsquigarrow \quad m_{12}^{\mathrm{val}} = 0$$

$$\mu = \{\mu_{ud}, \mu_s, \mu_c\}$$

- properties :
  - absence of $\mathcal{O}(a\mu)$ lattice artefacts at maximal twist
  - SW term: same renormalization in sea and valence, valence flavour breaking cutoff effects
  - μ acts as an infrared cutoff

- mixed action : match sea & valence quark masses (at maximal twist)

  $\rightsquigarrow$ scale setting in isoQCD with input : M_π, M_K, f_π, f_K $\qquad$ t_0 with 1.2% rel. error

  see talk by Alejandro Sáez [Wed.,14:30]

  correlations from scale setting procedure are taken into account

mixed action : lattice artefacts

[A. Bussone et al., 1812.01474]

extension of [Bhattacharya et al., hep-lat/0511014] to Wtm

- singlet and non-singlet bilinears and masses
- improvement of the twisted mass μ_j : mixed action with valence Wtm at maximal twist

$$\widehat{\mu}_j = \frac{1}{Z_{\rm P}} \mu_j \left[1 + a\overline{b}_\mu {\rm tr} M_{\rm sea}\right] + O(a^2)$$

$\overline{b}_\mu = O(g_0^4)$

- Wilson fermions : current quark mass from PCAC relation

$$\widehat{m}_{\rm ij} = \frac{Z_{\rm A}}{Z_{\rm P}} m_{\rm ij} \left[1 + a(\overline{b}_{\rm A} - \overline{b}_{\rm P}){\rm tr} M_{\rm sea} + a(\tilde{b}_{\rm A} - \tilde{b}_{\rm P}) m_{\rm ij}\right] + O(a^2)$$

$Z_{\rm P}$: non-perturbative [Schrödinger Functional (SF)]

[ALPHA, 1802.05243]

$Z_{\rm A}$: non-perturbative [chirally rotated SF]

[ALPHA, 1808.09236]

$m_{\rm ij}$ includes non-perturbative c_A

[ALPHA, 1502.04999]

$\tilde{b}_{\rm A} - \tilde{b}_{\rm P}$: non-perturbative

[ALPHA, 1906.03445]

$\overline{b}_{\rm A}$ & $\overline{b}_{\rm P} = O(g_0^4)$

mixed action : lattice artefacts twist angle

- flavours : i=1,2 $\to (u, d)$; i=3,4 $\to (s, s')$

$$\widehat{\mu}_{\rm i} = = \frac{1}{Z_{\rm P}}\,\mu_{\rm i}\,\left[1 + a\bar{b}_\mu {\rm tr} M_{\rm sea}\right] + O(a^2)$$

$$\bar{b}_\mu = O(g_0^4)$$

- Wilson twisted mass fermions : current quark mass

$$\widehat{m}_{\rm ij}^{\rm val} = \frac{Z_{\rm A}}{Z_{\rm P}}\, m_{\rm ij}^{\rm val}\,[1 + a(\bar{b}_{\rm A} - \bar{b}_{\rm P}){\rm tr}M_{\rm sea} + a(\tilde{b}_{\rm A} - \tilde{b}_{\rm P})m_{\rm ij}^{\rm val}] + O(a\mu_{\rm i}^2) + O(a^2)$$

$m_{\rm ij}^{\rm val}$ includes non-perturbative c_A [ALPHA, 1502.04999]

$\tilde{b}_{\rm A} - \tilde{b}_{\rm P}$: non-perturbative [ALPHA, 1906.03445] ; $\bar{b}_{\rm A}$ & $\bar{b}_{\rm P} = O(g_0^4)$

- deviation from maximal twist : $\theta_{\rm i}$

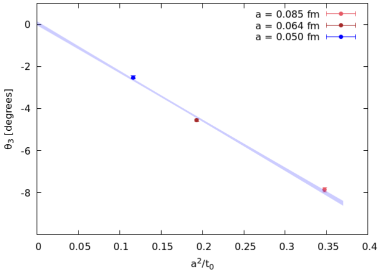
$$\tan\theta_{\rm i} = \tan\left(\frac{\pi}{2} - \omega_{\rm i}\right) = \frac{\widehat{m}_{\rm ij}^{\rm val}}{\widehat{\mu}_{\rm i}} = \frac{Z_{\rm A} m_{\rm ij}^{\rm val}}{\mu_{\rm i}}\,[1 + a(\tilde{b}_{\rm A} - \tilde{b}_{\rm P})m_{\rm ij}^{\rm val}] + O(a\mu_{\rm i}) + O(a^2)$$

tuning to maximal twist : $\theta_1 = 0$

$\rightsquigarrow$ what are the deviations θ_3

in the strange quark sector?

$\phi_2 = 0.349(03)$
$\phi_4 = 1.098(10)$



RGI quark mass : RG running $\quad N_f = 3$

non-perturbative running [Schrödinger Functional] [ALPHA, 1802.05243]

from $\mu_{\rm had} = 233(8)\,\mathrm{MeV}$ to $\mu_{\rm pt} \sim \mathrm{O}(M_W)$

RGI quark masses :

$$M_{\rm ij} \equiv \frac{1}{2}(M_{\rm i} + M_{\rm j}) = \frac{M}{\widehat{m}(\mu_{\rm had})}\ \widehat{m}_{\rm ij}(\mu_{\rm had})$$

$$\frac{M}{\widehat{m}(\mu_{\rm had})} = 0.9148(88) \qquad [1\%]$$

continuum factor: valid for Wilson and Wtm regularizations

light-quark masses: continuum-chiral extrapolations

Following [ALPHA, 1911.08025]

$$\phi_{\rm ij} \equiv \sqrt{8\,t_0}\,\widehat{m}_{\rm ij}$$

$$\phi_4 \equiv 8t_0\left(\tfrac{1}{2}M_\pi^2 + M_K^2\right) = \tfrac{1}{2}\phi_2 + \phi_K$$

SU(3) NLO χPT + $\left[O(a^2) + O(a^2\phi_2)\right]$ cutoff effects

$$\frac{\phi_{12}}{\phi_{13}} = \frac{\phi_2}{\phi_K}\left[1 + \frac{p_2}{p_1}\left(\frac{3}{2}\phi_2 - \phi_4\right) - K\left(\mathcal{L}_2 - \mathcal{L}_\eta\right)\right] + \frac{a^2}{8t_0}(2\phi_4 - 3\phi_2)\left[d_1 + d_3\phi_2\right]$$

$$\frac{\phi_{12}}{\phi_{13}} \propto \frac{2m_{ud}}{m_{ud}+m_s}$$

$$2\frac{\phi_{13}}{\phi_K} + \frac{\phi_{12}}{\phi_2} = 3p_1 + 2p_2\phi_4 + p_3K\left(\mathcal{L}_2 + \mathcal{L}_\eta\right) + \frac{a^2}{8t_0}\left[d_2 + d_4\phi_2\right]$$

$$\frac{1}{\sqrt{8t_0}}\left(2\frac{\widehat{m}_{13}}{M_K^2} + \frac{\widehat{m}_{12}}{M_\pi^2}\right)$$

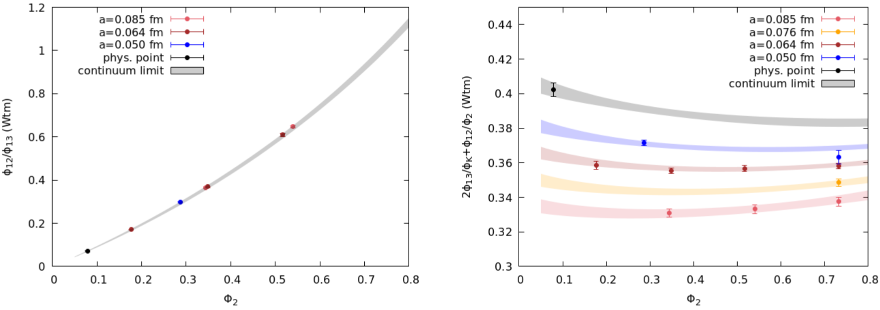
$$\mathcal{L}_2 \equiv \phi_2 \ln\phi_2\,; \quad \mathcal{L}_\eta \equiv \phi_\eta \ln\phi_\eta$$

$$K \equiv (8t_0\,16\pi^2 f_0^2)^{-1} \approx (8t_0\,16\pi^2 f_{\pi K}^2)^{-1}$$

$$f_{\pi K} = \frac{2}{3}\left(\frac{1}{2}f_\pi + f_K\right)$$

light-quark masses : Wilson twisted mass

$$\frac{\phi_{12}}{\phi_{13}} \propto \frac{2m_{ud}}{m_{ud}+m_s} \qquad \frac{1}{\sqrt{8t_0}}\left(\frac{2\widehat{m}_{13}}{M_K^2} + \frac{\widehat{m}_{12}}{M_\pi^2}\right)$$



continuum-chiral extrapolations :

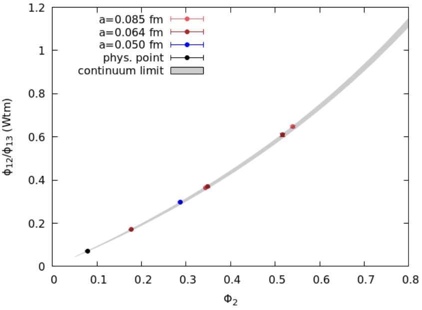
$\chi^2/\langle\chi^2\rangle = 0.9$

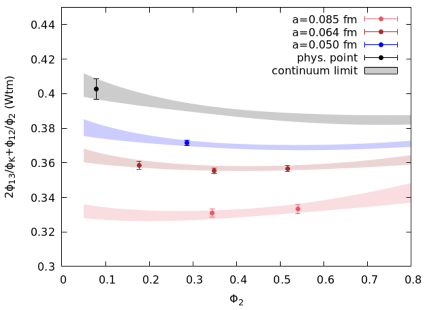
- SU(3) NLO χPT combining $\dfrac{\phi_{12}}{\phi_{13}}$ and $\dfrac{2\phi_{13}}{\phi_K} + \dfrac{\phi_{12}}{\phi_2}$
- discretization effects: $O(a^2)$, $O(\phi_2 a^2)$
- no cuts in M_π and in a

light-quark masses : cuts in mass   Wilson twisted mass

$$\frac{\phi_{12}}{\phi_{13}} \propto \frac{2m_{ud}}{m_{ud}+m_s}$$

$$\frac{1}{\sqrt{8t_0}}\left(\frac{2\widehat{m}_{13}}{M_K^2} + \frac{\widehat{m}_{12}}{M_\pi^2}\right)$$





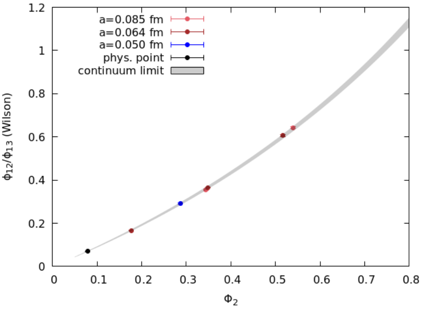
continuum-chiral extrapolations : $\chi^2/\langle\chi^2\rangle = 0.8$

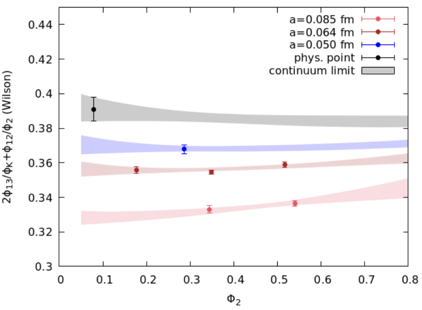
- SU(3) NLO χPT combining $\dfrac{\phi_{12}}{\phi_{13}}$ and $\dfrac{2\phi_{13}}{\phi_K} + \dfrac{\phi_{12}}{\phi_2}$
- discretization effects: $O(a^2)$, $O(\phi_2 a^2)$
- cuts in data : drop heavier pion masses

light-quark masses : cuts in mass — Wilson

$$\frac{\phi_{12}}{\phi_{13}} \propto \frac{2m_{ud}}{m_{ud}+m_s}$$

$$\frac{1}{\sqrt{8t_0}} \left(\frac{2\widehat{m}_{13}}{M_K^2} + \frac{\widehat{m}_{12}}{M_\pi^2} \right)$$





continuum-chiral extrapolations :

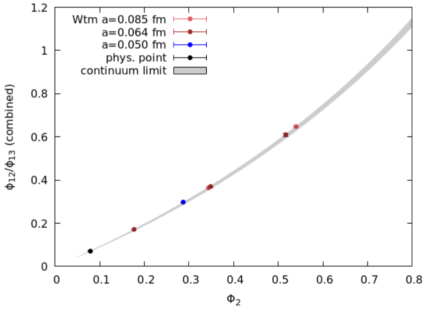
$\chi^2/\langle\chi^2\rangle = 0.7$

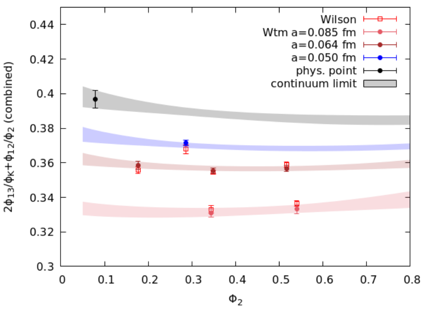
- SU(3) NLO χPT combining $\dfrac{\phi_{12}}{\phi_{13}}$ and $\dfrac{2\phi_{13}}{\phi_K} + \dfrac{\phi_{12}}{\phi_2}$
- discretization effects: $O(a^2)$, $O(\phi_2 a^2)$
- cuts in data : drop heavier pion masses

light-quark masses : cuts in mass combined Wilson and Wtm

$$\frac{\phi_{12}}{\phi_{13}} \propto \frac{2m_{ud}}{m_{ud}+m_s}$$

$$\frac{1}{\sqrt{8t_0}}\left(\frac{2\widehat{m}_{13}}{M_K^2}+\frac{\widehat{m}_{12}}{M_\pi^2}\right)$$





continuum-chiral extrapolations : $\chi^2/\langle\chi^2\rangle = 0.9$

- SU(3) NLO χPT combining $\dfrac{\phi_{12}}{\phi_{13}}$ and $\dfrac{2\phi_{13}}{\phi_K}+\dfrac{\phi_{12}}{\phi_2}$
- discretization effects: $O(a^2)$, $O(\phi_2 a^2)$
- cuts in data : drop heavier pion masses

light-quark masses: M_{ud}

Wilson twisted mass

systematic effects [ongoing] :

- mass dependence :

  SU(3) NLO χPT

- discretization effects :

  [a1] : $O(a^2)$

  [a2] : $O(a^2) + O(\phi_2 a^2)$

- data cuts :

  [c0] : no cuts $N_{\mathrm{cut}} = 0$

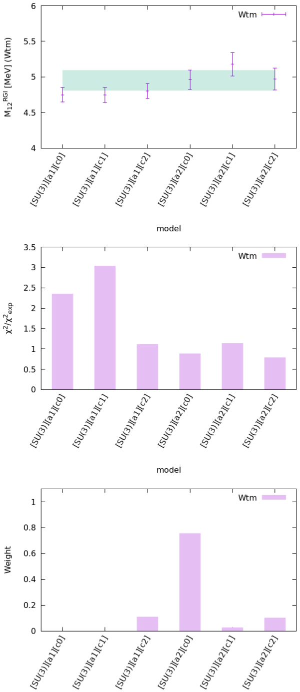
  [c1] : drop coarsest a $N_{\mathrm{cut}} = 5$

  [c2] : drop heavier pion masses $N_{\mathrm{cut}} = 4$

- model averaging : apply weights W

$$W \propto \exp\left(-\frac{1}{2}\left[\frac{\chi^2}{\langle\chi^2\rangle}N_{\mathrm{dof}} + 2N_{\mathrm{par}} + 2N_{\mathrm{cut}}\right]\right)$$

[Jay & Neil, 2008.01069]



light-quark masses: M_{ud}

Wilson

systematic effects [ongoing] :

- mass dependence :

  SU(3) NLO χPT

- discretization effects :

  [a1] : $O(a^2)$

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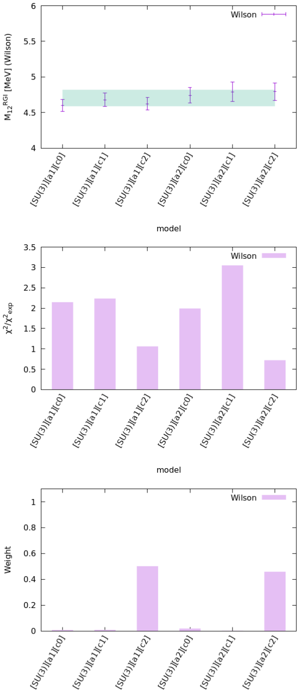

light-quark masses: M_{ud}

combined Wilson + Wtm

systematic effects [ongoing] :

- mass dependence :

  SU(3) NLO χPT

- discretization effects :

  [a1] : $O(a^2)$

  [a2] : $O(a^2) + O(\phi_2 a^2)$

- data cuts :

  [c0] : no cuts $\quad N_{\rm cut} = 0$

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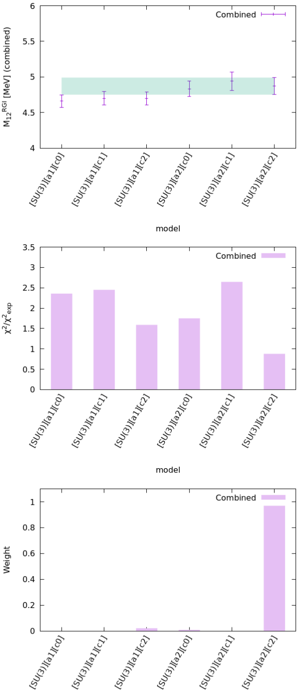
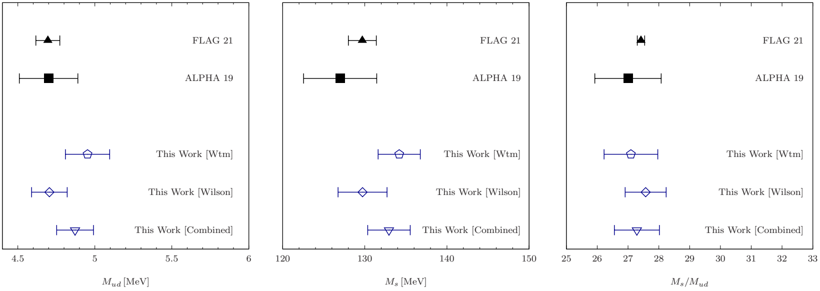


RGI light-quark masses $N_f = 2+1$ preliminary



combining Wilson + Wtm :

M_{ud} : [2.5%] rel. error — 41% of the error from $M/\widehat{m}(\mu_{\rm had})$ with [1%]

M_s : [1.9%] — 51% of the error from $M/\widehat{m}(\mu_{\rm had})$

M_s/M_{ud} : [2.7%] — sizeable contribution from scale setting error

$\rightsquigarrow$ under investigation

other new determinations based on CLS $N_f = 2+1$: see talk by Gunnar Bali [Wed.,14:00]

conclusions

- mixed action : Wilson twisted mass on Wilson fermions
- determination of quark masses : m_{ud}, m_s
- ongoing analysis of systematic effects

- ongoing extension to lighter ensembles and finer lattice spacing
- heavy-quark physics

  see talks by Alessandro Conigli [Wed.,15:40] & by Julien Frison [Thu.,15:00]