

Nucleon PDFs at the physical point from lattice QCD using NNLO matching

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Outline

- Motivations and formalism
- Lattice setup
- Analysis of two-point and three-point functions
- Model-independent extraction of lowest moments
- Model-dependent fits

All results are preliminary!

Motivations

- Remove pion-mass systematic by working at the physical point
- Extract nucleon PDFs using various methods
 - Leading-twist OPE
 - Model-dependent fits
 - x-space matching with Hybrid renormalization
- When does short-distance factorization break down?
- Check perturbative uncertainty by including NNLO matching
- Can other states be used to cancel renormalizations and/or higher twist effects?

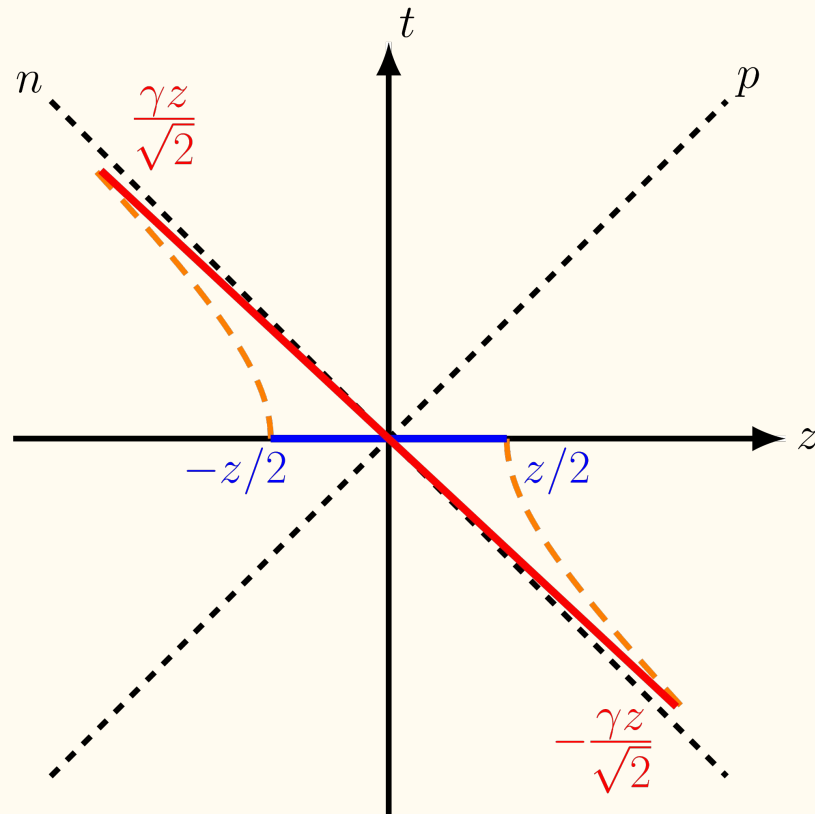
Light-cone PDFs from lattice QCD

- Cannot calculate matrix elements separated along the light cone in lattice QCD
- Instead, calculate equal-time spatially-separated matrix element of highly-boosted hadron

$$h^B(z, P_z) = \langle N; P_z | \bar{\psi}(z) \Gamma W(z, 0) \psi(0) | N; P_z \rangle$$

- Can be matched to light-cone PDF through Large-momentum Effective Theory or short-distance factorization

Theoretical Framework:
[V. Braun, D. Müller '07]
[X. Ji '13]
[A. Radyushkin '17]



[X. Ji et al., Rev. Mod. Phys. **93**, 035005, arXiv: 2004.03543]

Correlation functions

Use standard nucleon operator: $N_\alpha^{(s)}(x, t) = \varepsilon_{abc} u_{a\alpha}^{(s)}(x, t) (u_b^{(s)}(x, t)^T C \gamma_5 d_c^{(s)}(x, t))$

For two-point functions: $C^{2\text{pt}}(\vec{p}, t_{\text{sep}}; \vec{x}, t_0) = \sum_{\vec{y}} e^{-i\vec{p} \cdot (\vec{y} - \vec{x})} \mathcal{P}_{\alpha\beta}^{2\text{pt}} \langle N_\alpha(\vec{y}, t_{\text{sep}} + t_0) \bar{N}_\beta(\vec{x}, t_0) \rangle$

And three-point functions:

$$C^{3\text{pt}}(\vec{p}_f, \vec{q}, t_{\text{sep}}, t_{\text{ins}}; \vec{x}, t_0) = \sum_{\vec{y}, \vec{z}} e^{-i\vec{p}_f \cdot (\vec{y} - \vec{x})} e^{-i\vec{q} \cdot (\vec{x} - \vec{z})} \mathcal{P}_{\alpha\beta}^{3\text{pt}} \langle N_\alpha(\vec{y}, t_{\text{sep}} + t_0) \mathcal{O}^\Gamma(\vec{z}, \hat{\mathcal{L}}, t_{\text{ins}} + t_0) \bar{N}_\beta(\vec{x}, t_0) \rangle$$

$$\mathcal{O}^\Gamma(\vec{z}, \hat{\mathcal{L}}, t_{\text{ins}} + t_0) = \bar{q}(\vec{z}, t_{\text{ins}} + t_0) \Gamma \tau_3 W(\vec{z}, t_{\text{ins}} + t_0; \vec{z} + \hat{\mathcal{L}}, t_{\text{ins}} + t_0) q(\vec{z} + \hat{\mathcal{L}}, t_{\text{ins}} + t_0)$$

For unpolarized distribution: $\mathcal{P}^{2\text{pt}} = \mathcal{P}^{3\text{pt}} = \frac{1}{2}(1 + \gamma_t)$, $\Gamma = \gamma_t, \gamma_z$

Smeared-smeared (SS) and smeared-point (SP) two-point correlators  **No mixing**

Only smeared-smeared three-point correlators

Calculation setup

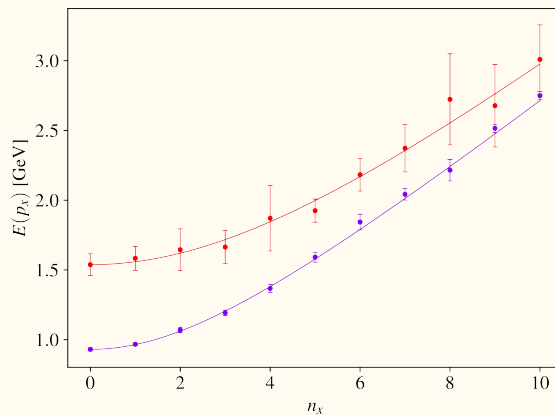
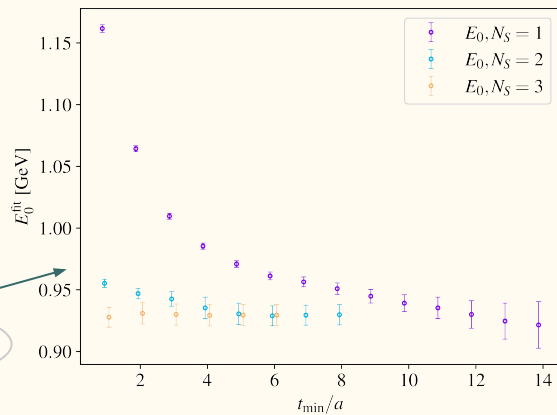
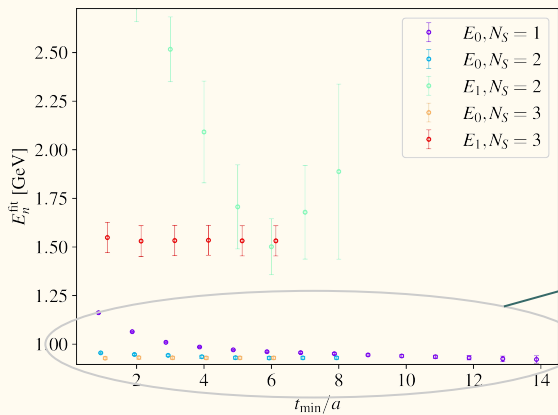
- Mixed fermion action
 - Sea quark action: $N_f = 2+1$ HISQ with physical quark masses, $L^3 \times T = 64^3 \times 64$, $a = 0.076$ fm
 - Valence quark action: $N_f = 2 + 1$ Wilson-Clover with physical quark masses, 1-HYP smeared gauge links

- Calculations done with Qlua, which utilizes the multigrid solver in QUDA
- Use momentum smearing for quarks to achieve better overlap with boosted hadrons
- Included four momentum projections to $P_x^{(f)}$ at the sink for three-point functions

$P_x^{(f)}$	k_x	t_{sep}	N_{samp}
0	0	6	16
0	0	8,10	32
0	0	12	64
1	0	6,8,10,12	32
4	2	6	32
4	2	8,10,12	128
6	3	6	20
6	3	8	100
6	3	10,12	140

Analysis of two-point functions

- Fit two-point functions to $C_N^{2\text{pt}}(\vec{p}, t_{\text{sep}}) = C_0 e^{-E_0 t_{\text{sep}}} \left[1 + \sum_{i=1}^{N-1} R_i \prod_{j=1}^i e^{-\Delta_{j,j-1} t_{\text{sep}}} \right]$
- Use SP and SS correlators to help control excited states
- All energies below largest energy are priored
- Three states required to fit full t_{sep}



Three-point function analysis

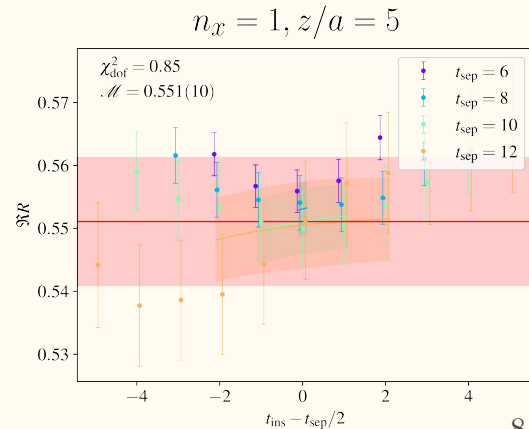
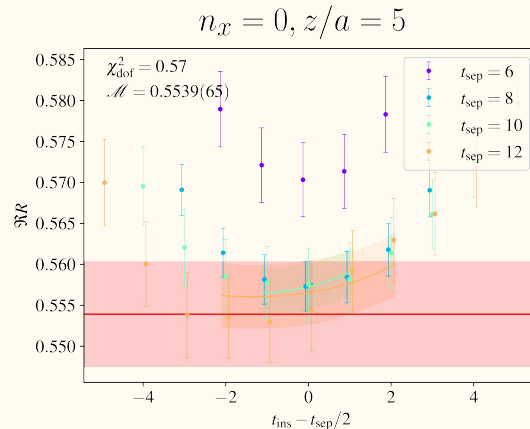
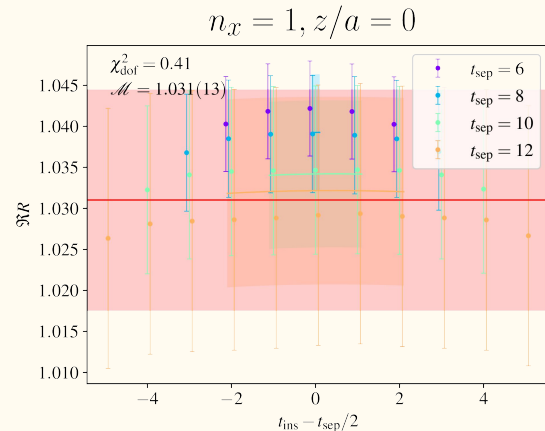
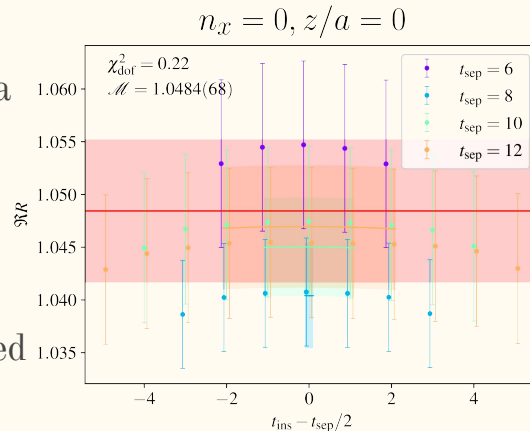
- Fit ratio of three-point to two-point data

$$R(\vec{p}_f, t_{\text{ins}}, t_{\text{sep}}) = \frac{C^{\text{3pt}}(\vec{p}_f, \vec{q} = 0, t_{\text{ins}}, t_{\text{sep}})}{C^{\text{2pt}}(\vec{p}_f, t_{\text{sep}})}$$

- Two-state fits to three-point ratio priored with ‘effective’ energy gap and amplitudes from two-state fits to two-point SS correlators

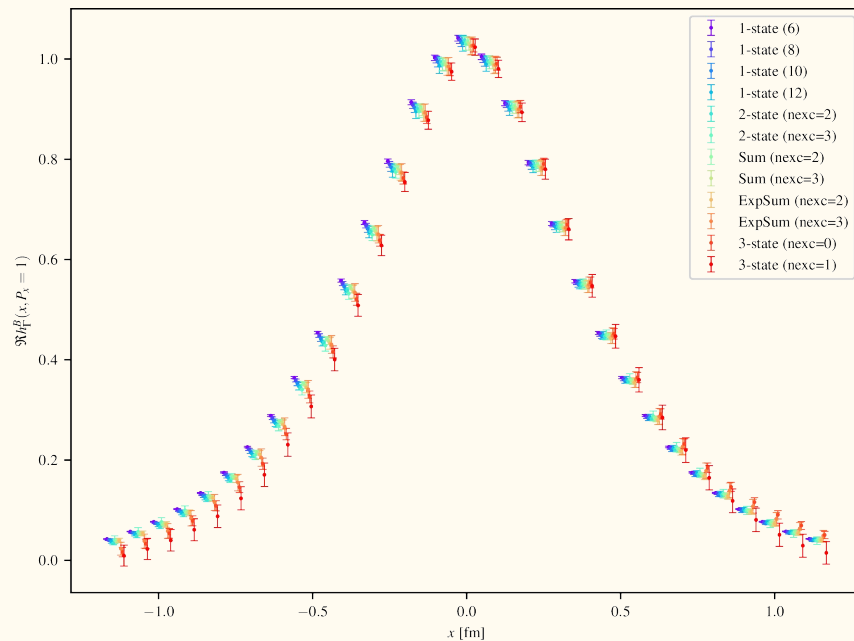
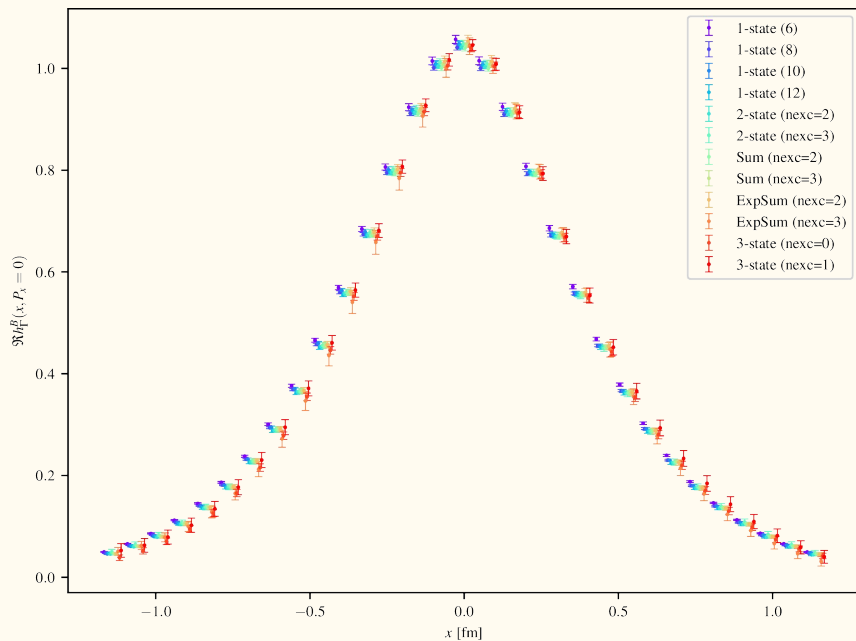
- Reasonable agreement between two-state and other fit strategies, like summation fits

$$R_{\text{sum}}(\vec{p}_f, t_{\text{sep}}) = \sum_{t_{\text{ins}}=n_{\text{exc}}a}^{t_{\text{sep}}-n_{\text{exc}}a} R(\vec{p}_f, t_{\text{ins}}, t_{\text{sep}})$$



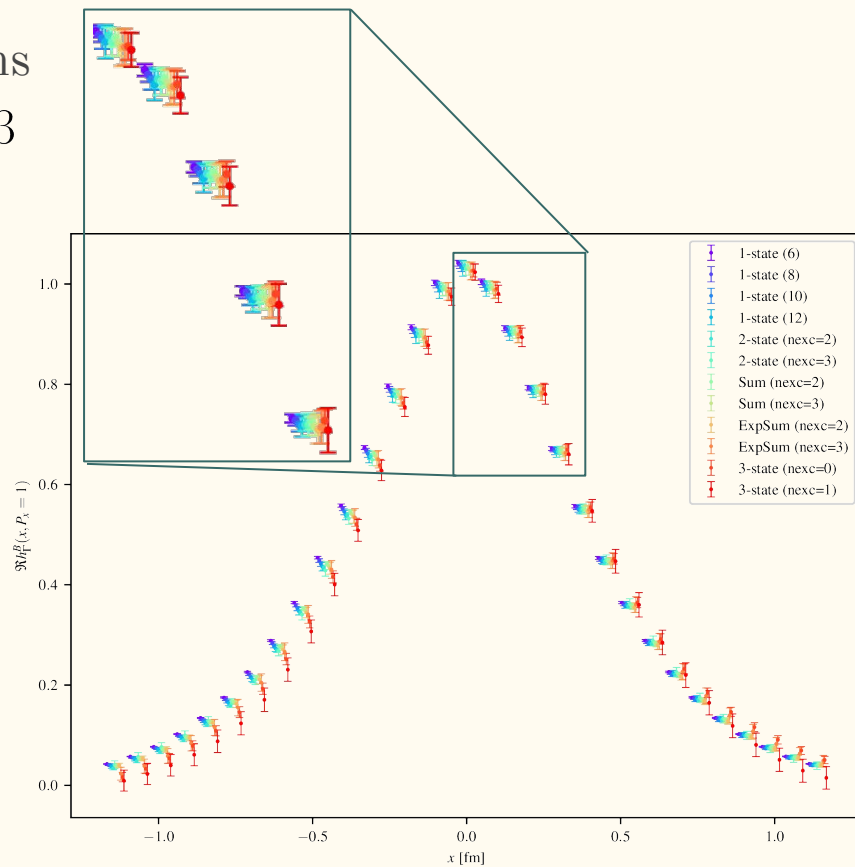
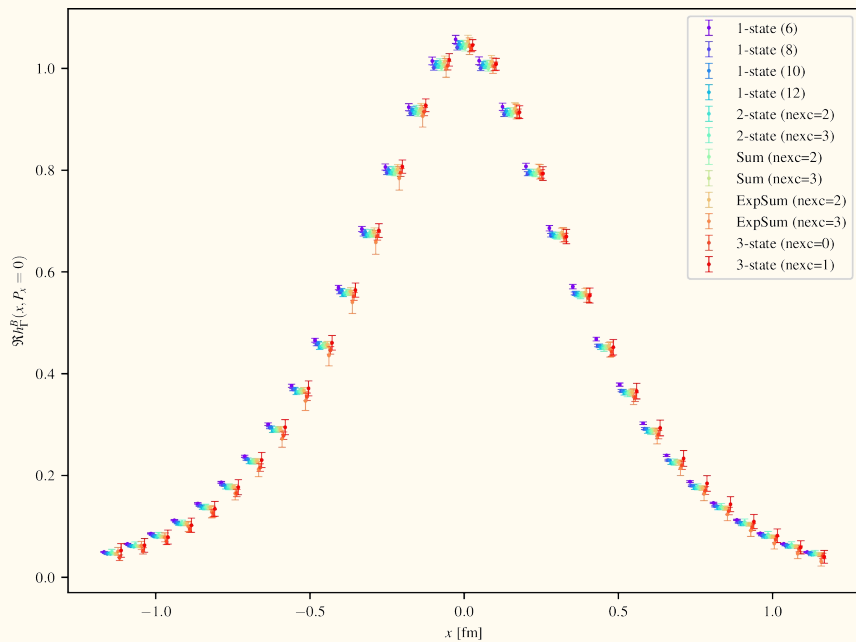
Comparison of fit strategies

- Good agreement across various fit forms
- Preferred fit is two-state with $n_{\text{exc}} = 3$



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Ratio-scheme renormalization

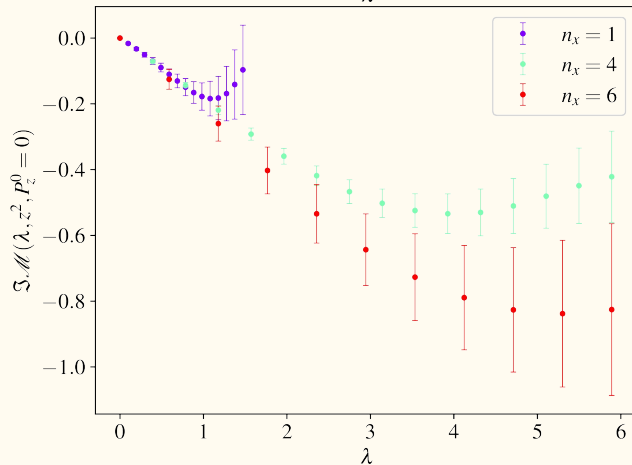
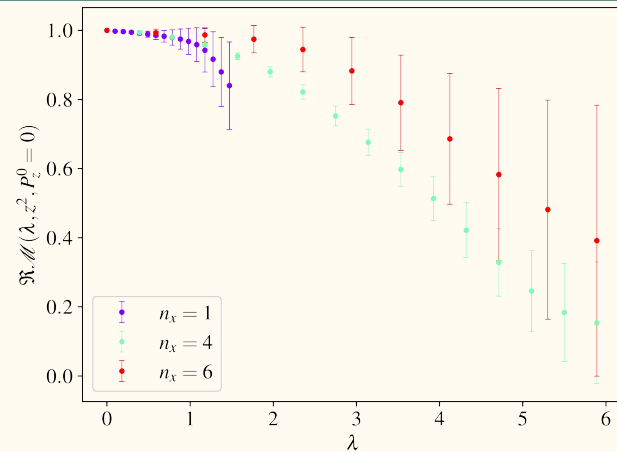
- The operator $\mathcal{O}_\Gamma(z)$ is multiplicatively renormalizable

$$h_\Gamma^B(z, P_z, a) = e^{-\delta m(a)|z|} Z_O(a) h_\Gamma^R(z, P_z, \mu)$$

- Can form renormalization-group invariants with the double ratio ($z = 0$ for exact normalization)

$$\mathcal{M}(\lambda, z^2; P_z^0, a) = \frac{h^B(z, P_z, a)}{h^B(z, P_z^0, a)} \bigg/ \frac{h^B(0, P_z, a)}{h^B(0, P_z^0, a)}, \quad \lambda \equiv z P_z$$

- Consider $\mathcal{M}(\lambda, z^2; P_z^0 = 0, a)$, referred to as the reduced Ioffe Time Distribution (rITD)
- rITD can be perturbatively matched to light-cone ITD $Q(\lambda, \mu^2)$

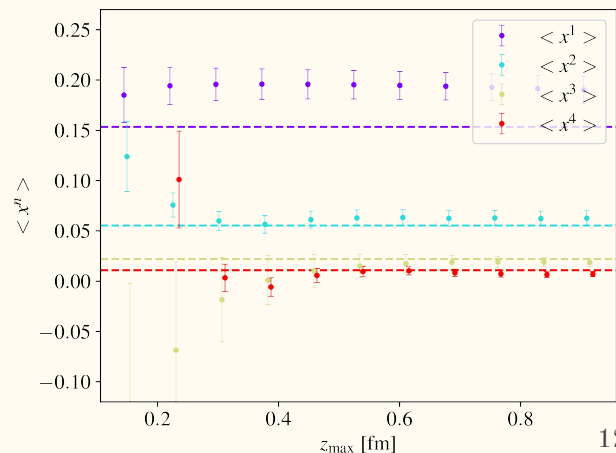
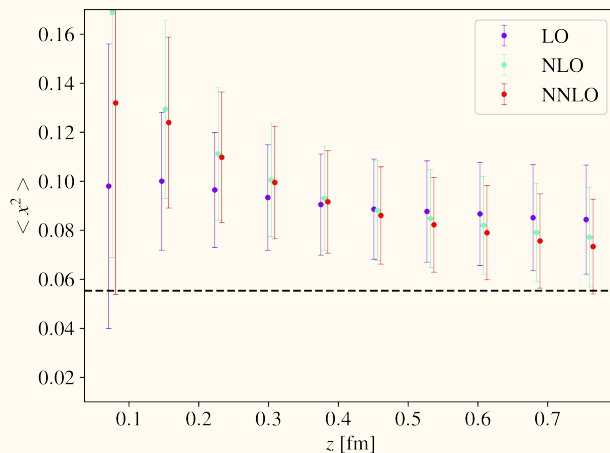
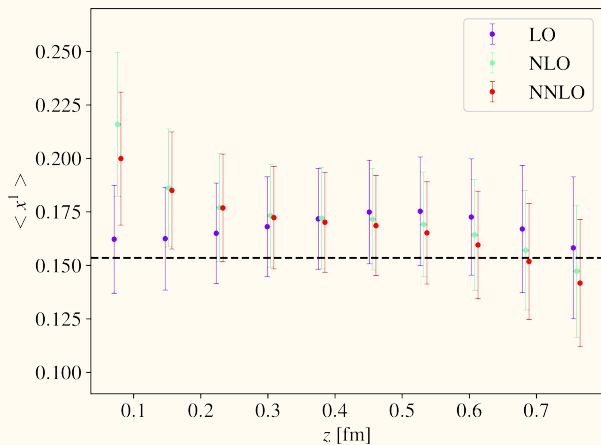


Lowest moments from leading-twist OPE

The lowest few moments can be extracted from the rITD by fits to

$$\mathcal{M}(\lambda, z^2; \lambda^0 \equiv zP_z^0) = \frac{\sum_{n=0} c_n(\mu^2 z^2) \frac{(-i\lambda)^n}{n!} \langle x^n \rangle(\mu) + \mathcal{O}(\Lambda_{\text{QCD}}^2 z^2)}{\sum_{n=0} c_n(\mu^2 z^2) \frac{(-i\lambda^0)^n}{n!} \langle x^n \rangle(\mu) + \mathcal{O}(\Lambda_{\text{QCD}}^2 z^2)}$$

where $c_n(\mu^2 z^2) \equiv C_n(\mu^2 z^2)/C_0(\mu^2 z^2)$, and $C_n(\mu^2 z^2)$ are Wilson coefficients, which have been computed up to next-to-next-leading-order (NNLO)



Model dependent fits

- Model the PDF $q_{\text{model}}(x)$ and evaluate the moments $\langle x^n \rangle_{\text{model}} = \int_0^1 dx x^n q_{\text{model}}(x)$
- Substitute $\langle x^n \rangle_{\text{model}}$ into leading-twist OPE to obtain $\mathcal{M}_{\text{model}}(\lambda, z^2; \lambda^0)$
- Fit by minimizing,
$$\chi^2 = \sum_{P_z > P_z^0}^{P_z^{\text{max}}} \sum_{z_{\text{min}}}^{z_{\text{max}}} \frac{(\mathcal{M}(\lambda, z^2; \lambda^0) - \mathcal{M}_{\text{model}}(\lambda, z^2; \lambda^0))^2}{\sigma^2(z, P_z, P_z^0)}$$
- Real and Imaginary part of rITD related to

$$q^-(x) \equiv q^u(x) - q^d(x) - (q^{\bar{u}}(x) - q^{\bar{d}}(x)), \quad q^+(x) \equiv q^u(x) - q^d(x) + (q^{\bar{u}}(x) - q^{\bar{d}}(x)), \quad x \in [0, 1]$$

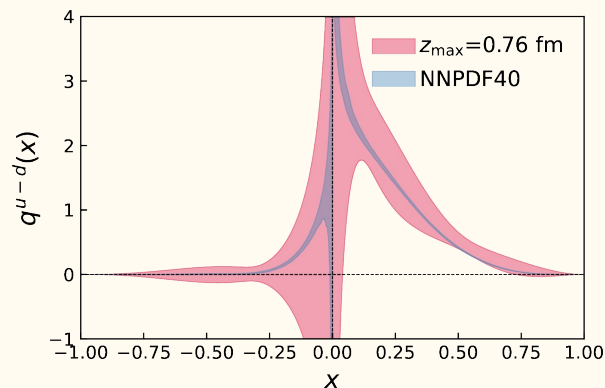
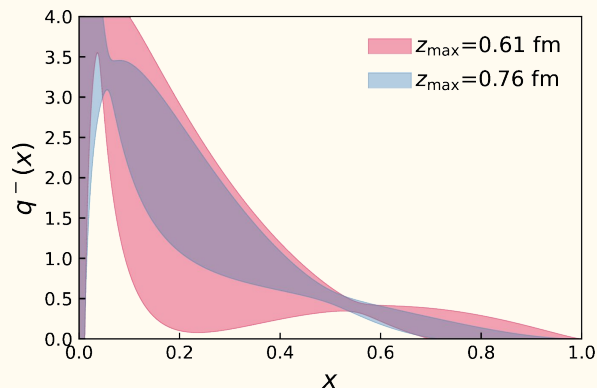
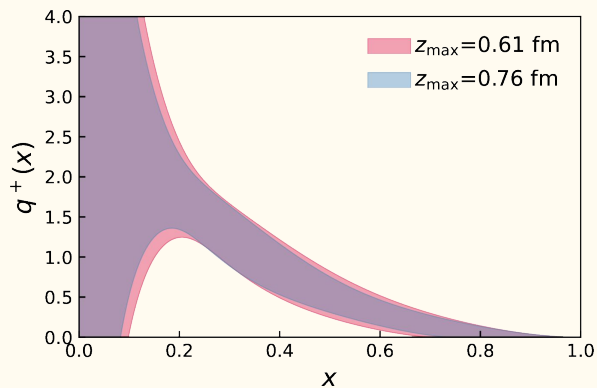
respectively, and can be expressed via a simple model

$$q^-(x; \alpha, \beta) = \frac{\Gamma(2 + \alpha + \beta)}{\Gamma(1 + \alpha)\Gamma(2 + \beta)} x^\alpha (1 - x)^\beta, \quad q^+(x; \alpha, \beta, A) = A x^\alpha (1 - x)^\beta$$

Isvector PDF from model fits

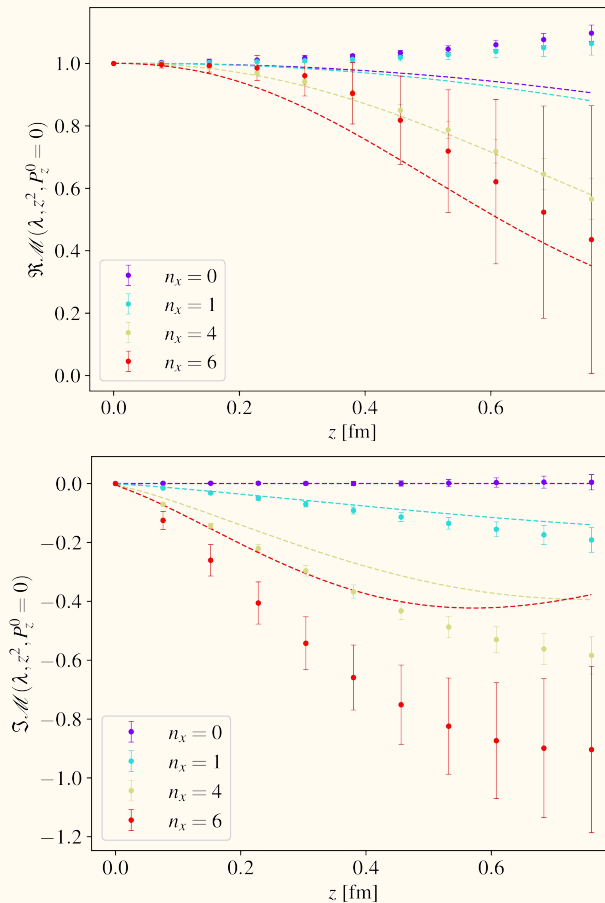
- Include all P_z and $z \in [2a, z_{\max}]$ for fit
- Use $q^f(-x) = -q^{\bar{f}}(x)$ to form isovector PDF from

$$q^{u-d}(x) = \begin{cases} \frac{q^-(x) + q^+(x)}{2}, & x > 0 \\ \frac{q^-(-x) - q^+(-x)}{2}, & x < 0 \end{cases}$$



Ratio with pion matrix elements?

- Renormalization factors should be independent of the hadron state used
- The ratio of the proton matrix elements to the pion matrix elements is in much worse agreement with NNPDF40
- Higher twist effects?



Conclusions and Outlooks

- Conclusions

- Excited-state contamination at the physical point can be controlled
- Leading-twist OPE can describe the proton ratio data for $z \sim 0.8$ fm
 - First four moments extracted
 - $\langle x \rangle$ is above result from NNPDF40
- Model dependent fits agree with NNPDF40 (with larger errors)
- Moments from leading-twist OPE in agreement with model fits
- Using other states does not seem to work

- Future work/Outlooks

- x-space matching with Hybrid renormalization
- More statistics and source-sink separations would be helpful
- Helicity and transversity distributions