Leading strange and charm contribution to HVP with C* boundary conditions

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1. Motivation

The anomalous magnetic moment of the muon is one of the most promising observables for new physics searches. Currently, the SM's prediction [1] exhibits a 4.2σ discrepancy with the experiments [2]. Our research aims to evaluate the hadronic contribution to the muon g-2 on lattice, with a subpercent precision, where the isospin breaking effects become important. One approach to deal with them is to use C* boundary conditions in space to simulate QCD+QED in a finite volume. In this preliminary work we focus on the leading HVP from strange and charm quarks in QCD simulations with C* bcs.

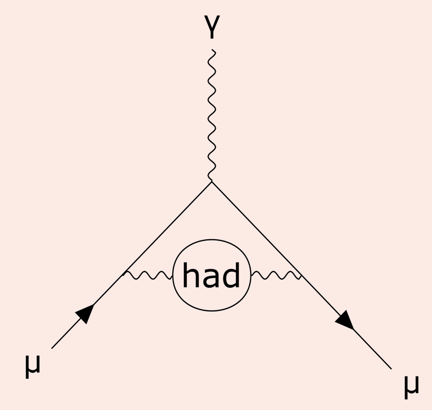
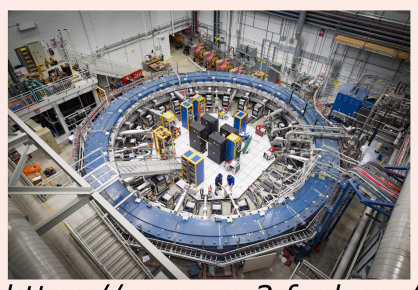
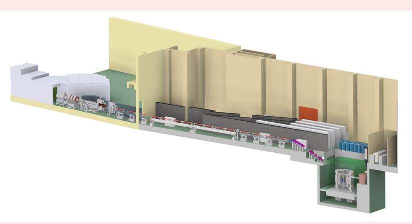


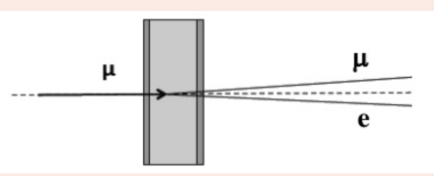
Fig.1: HVP contribution to g-2.



https://muon-g-2.fnal.gov/



https://g-2.kek.jp/overview/



MUonE : R.Pillato ICHEP22

Fig.2: Current and upcoming experiments for g-2.

2. Lattice set-up

We use the configurations generated by the RC* collaboration (openQ*D code: https://gitlab.com/rcstar/openQxD)

- Lüscher-Weisz action for the SU(3) field
- N_f=3+1 *O(a)*-improved Wilson fermions
- periodic bcs in time and C* bcs in all three spatial directions
- two lattice volumes: $64\text{x}32^3$ and $80\text{x}48^3$

ensemble	β	$\kappa_{u,d,s}$	κ_c	$c_{SW,SU(3)}$	N_{confs}
A400a00b324	3.24	0.134407	0.12784	2.18859	200
B400a00b324	3.24	0.134407	0.12784	2.18859	108

Table 1: Parameters of our ensemles [5].

- the ensembles have $m_\pi \sim$ 400 MeV and lattice spacing a= 0.0539(3) fm[1].

The observable is the two-point function of the electromagnetic current:

- two lattice discretizations for the EM vector current

$$V^{loc}_\mu(x) = \bar\psi(x)\gamma_\mu\psi(x)$$

$$V^{ps}_\mu(x) = \tfrac{1}{2}[\bar\psi(x+\hat\mu)\,(1+\gamma_\mu)\,U^\dagger_\mu(x)\psi(x) + -\bar\psi(x)\,(1-\gamma_\mu)\,U_\mu(x)\psi(x+\hat\mu)]$$

- time-momentum representation [3]

$$G(x_0) = \tfrac{1}{3}\sum_{k=1}^{3}\sum_{\vec{x}\in V^3}\langle V_k(x)V_k(0)\rangle$$

- in this representation we take the convolution of G with the QED kernel

$$a^{HVP}_\mu = \left(\tfrac{\alpha}{\pi}\right)^2\int_0^\infty \mathrm{d}t\, G(t)\tilde{K}(t;m_\mu)$$

- to get the connected part of the correlator

$$G(x_0) = -\tfrac{1}{3}\sum_k\sum_{\vec{x}} q^2\,\mathrm{tr}\left[\gamma_k D^{-1}(x|0)\gamma_k D^{-1}(0|x)\right]$$

[1]Updated measurements of the lattice spacing on both the ensemble will be presented in poster session B: "Tuning of QCD+QED simulations with C* boundary conditions" by A. Cotellucci.

3. C* boundary conditions

The novelty of the approach is the use of C* boundary conditions in space, which allow for a gauge-invariant and local formulation of the QED without the zero modes in a finite volume.

(QCD+QED simulations: see J. Lücke's talk in "Hadron spectroscopy and interactions VII", Friday 4.40 pm; and poster by A. Cotellucci in Poster session B).

- double volume: physical+mirror lattice
- orbifold construction
- fields' constraints:

$$\psi_f(x+L_k\hat{e}_k) = C^{-1}\bar\psi^T(x)$$
$$\bar\psi_f(x+L_k\hat{e}_k) = -\psi^T(x)C$$
$$U_\nu(x+L_k\hat{e}_k) = U^*_\nu(x)$$

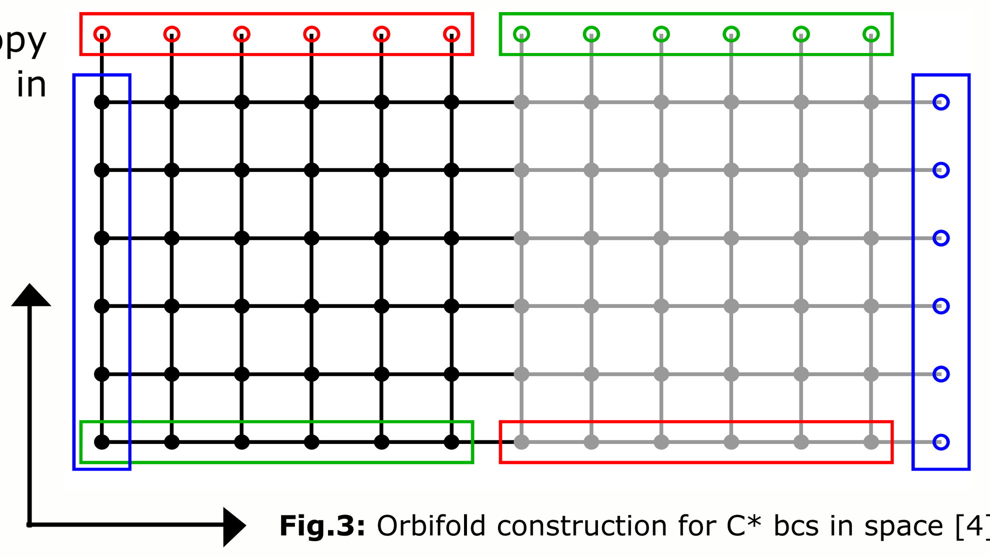


Fig.3: Orbifold construction for C* bcs in space [4].

4. Results

The renormalization constant is evaluated from the ratio between the correlators

$$R(t) = \frac{\langle V^{ps}_k(x)V^{loc}_k(0)\rangle}{\langle V^{loc}_k(x)V^{loc}_k(0)\rangle}$$

Ensemble	$Z_V^{(m_s)}$	$Z_V^{(m_c)}$
A400	0.6713(7)	0.6065(2)
B400	0.6707(5)	0.6071(3)

Table 2: Renormalization constants.

We integrate the lattice data up to a cut-off x_0^{cut} and extrapolate with the single exponential $Ae^{-am_V x_0}$ (for now)

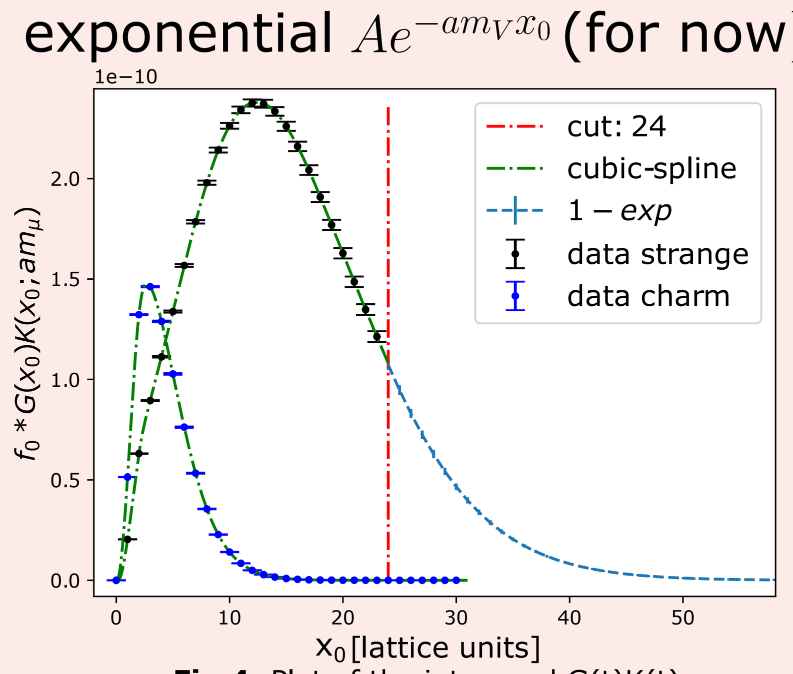


Fig.4: Plot of the integrand G(t)K(t).

Ensemble,type	a^s_μ	a^c_μ
A400a00b324,loc-loc	46.7(7)	7.83(8)
B400a00b324,loc-loc	48.5(7)	7.81(9)
A400a00b324, ps-loc	46.2(7)	6.18(7)
B400a00b324, ps-loc	48.0(7)	6.16(7)

Table 3: Results on the two ensembles

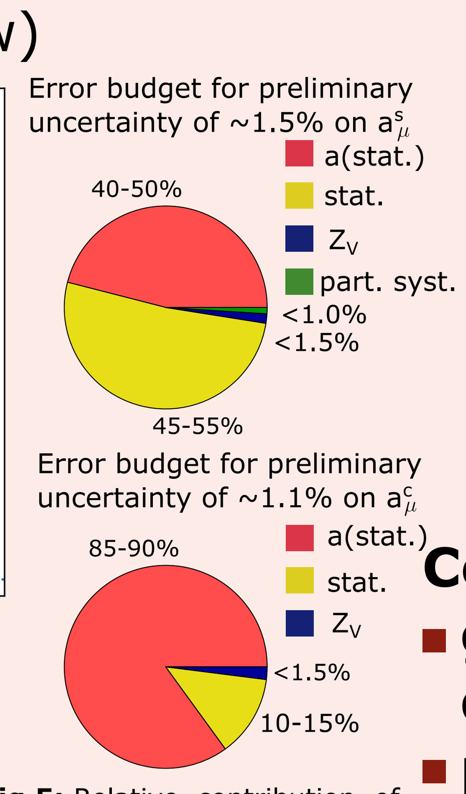


Fig.5: Relative contribution of the accounted uncertainties for a^s_μ (top pannel) and a^c_μ (bottom panel).
Unaccounted uncertainties: phys. pion mass, scale setting, continuum extrapolation.

The valence quarks' masses are tuned based on the physical vector mesons

$$m_\phi = 1019.461 \pm 0.016 \text{ MeV}$$
$$m_{J/\psi} = 3096.900 \pm 0.006 \text{ MeV}$$

(https://pdg.lbl.gov/)

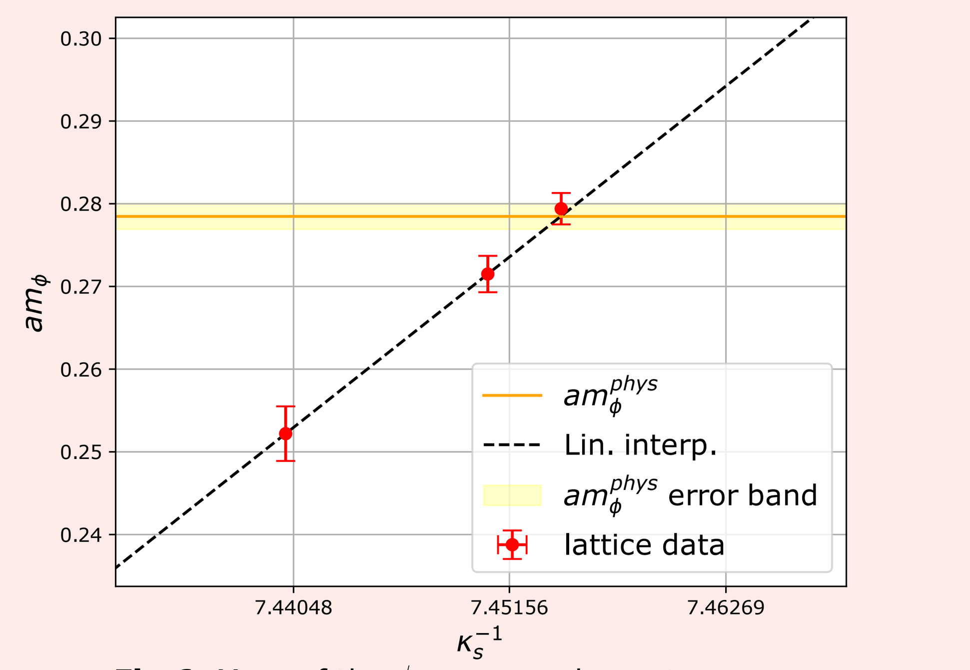


Fig.6: Mass of the ϕ meson vs bare strange mass.

Conclusions at unphysical m_π

- good statistical precision for heavy quark HVP
- precise scale setting needed
- finite-volume effects under control

(FVE with C* bcs: see talk by S. Martins in "QCD in searches for physics beyond the SM III",Thurs. 10.20 am)

5. Signal-to-noise

- comparison of the noise for the three types of correlator

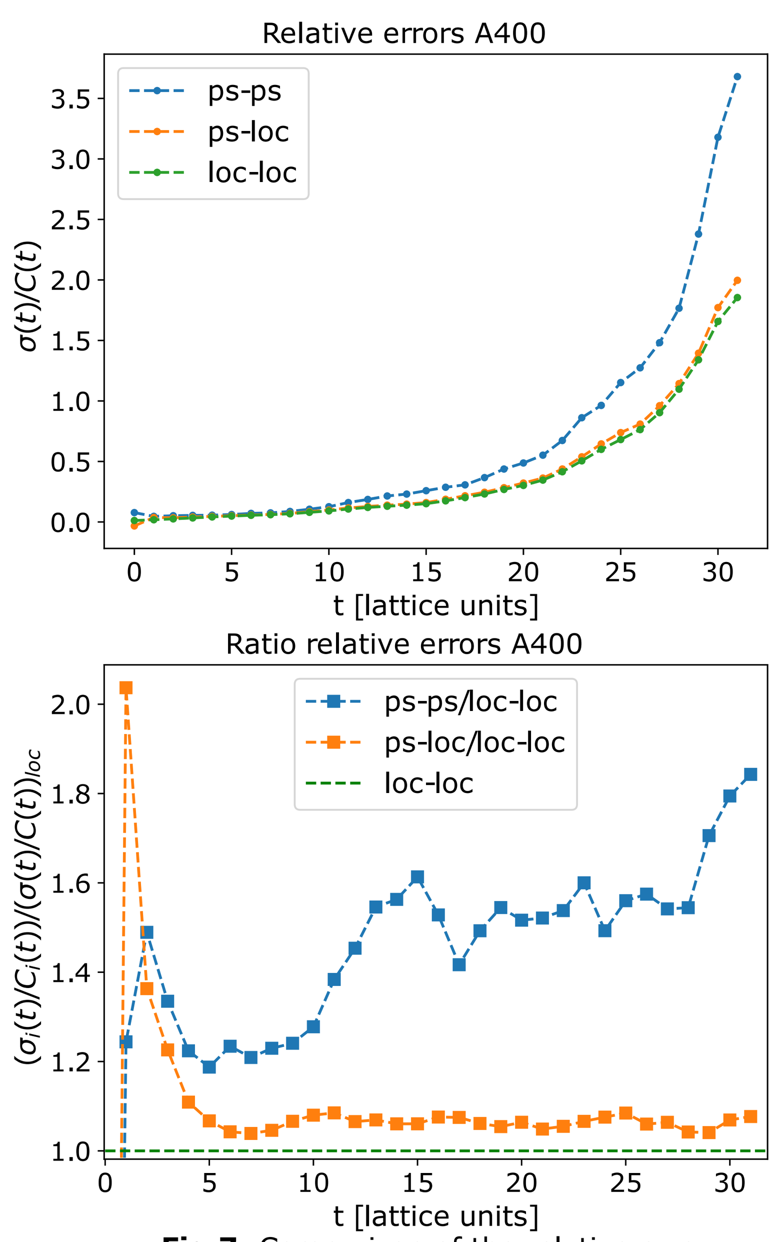


Fig.7: Comparison of the relative errors.

Statistics = 60 confs x 10 point sources

- ps-loc/loc-loc ~1.1
- ps-ps/loc-loc ~1.5
- CPU time: ps-ps= loc-loc x 4

Next steps

- On QCD ensembles:
  - leading light contribution to HVP
  - disconnected quark diagrams
  - isospin breaking effects
  - include reweighting factors in the analysis (to correct for the choice of rational approximation)
- Measurement of a^{HVP}_μ on the QCD +QED dynamical ensembles

(see talks by A. Altherr and R. Gruber in the session "QCD in searches for physics beyond the SM III",Thurs. 9.20 am)

References

[1] T. Aoyama et al., Phys. Rep., vol. 887, p.1-166, 2020.
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