Isospin Breaking corrections in vector-vector correlators for $(g-2)_\mu$ and τ decays

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for the RBC/UKQCD Collaboration

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Window fever - I

Hadronic Vacuum Polarization (HVP) contribution to a_μ

Time-momentum representation [Bernecker, Meyer, '11]

$$G^\gamma(t) = \frac{1}{3} \sum_k \int d\boldsymbol{x} \, \langle j_k^\gamma(x) j_k^\gamma(0) \rangle \quad \rightarrow \quad a_\mu = 4\alpha^2 \sum_t w_t G^\gamma(t)$$

Windows in Euclidean time [RBC/UKQCD '18]

$$a_\mu^W = 4\alpha^2 \sum_t w_t \, G^\gamma(t) \, [\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)]$$
$$t_0 = 0.4 \text{ fm} \quad t_1 = 1.0 \text{ fm} \quad \Delta = 0.15 \text{ fm}$$

allow for in-depth cross-checks

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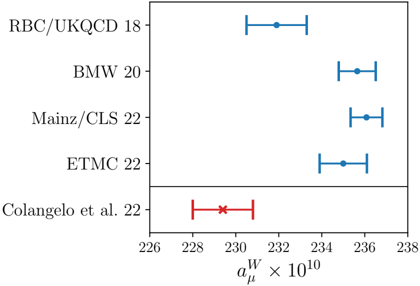
Window fever - II

Status of intermediate window ($0.4 - 1.0$ fm, $\Delta = 0.15$ fm)



Several lattice collaborations agree

Updated results (RBC/UKQCD, FNAL/HPQCD ...) soon [e.g. Lehner on Fri]

Data-driven approach [Colangelo et al. '22]

$$a_\mu^W \times 10^{10} = 229.4(1.4)\ [\text{total}]$$
$$138.3(1.2)\ [\pi\pi]$$

$\pi\pi$ is 60% of mean of a_μ^W

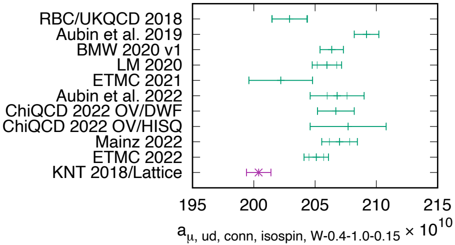
$\pi\pi$ is $> 80\%$ of error of a_μ^W

Add one player to the game:

τ data

Window fever - II

Status of intermediate window ($0.4 - 1.0$ fm, $\Delta = 0.15$ fm)



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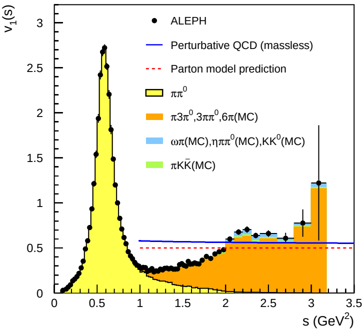
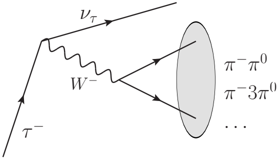
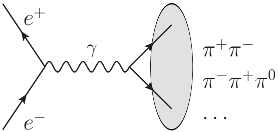
Motivations for τ





$V - A$ current

Final states $I = 1$ charged



EM current

Final states $I = 0, 1$ neutral

τ data can improve $a_\mu[\pi\pi]$

$\rightarrow$ 72% of total Hadronic LO

$\rightarrow$ competitive precision on a_μ^W

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Isospin Corrections

Restriction to $e^+e^- \to \pi^+\pi^-$ and $\tau^- \to \pi^-\pi^0\,\nu_\tau$

$$v_0(s) = \frac{s}{4\pi\alpha^2}\sigma_{\pi^+\pi^-(\gamma)}(s)$$

$$v_-(s) = \frac{m_\tau^2}{6|V_{ud}|^2}\frac{\mathcal{B}_{\pi\pi^0}}{\mathcal{B}_e}\frac{1}{N_{\pi\pi^0}}\frac{dN_{\pi\pi^0}}{ds}\left(1-\frac{s}{m_\tau^2}\right)^{-1}\left(1+\frac{2s}{m_\tau^2}\right)^{-1}\frac{1}{S_{\mathrm{EW}}}$$

Isospin correction $v_0 = R_{\mathrm{IB}} v_-$ $\quad R_{\mathrm{IB}} = \frac{\mathrm{FSR}}{G_{\mathrm{EM}}}\frac{\beta_0^3|F_\pi^0|^2}{\beta_-^3|F_\pi^-|^2}$ [Alemani et al. '98]

0. S_{EW} electro-weak radiative correct. [Marciano, Sirlin '88][Braaten, Li '90]

1. Final State Radiation of $\pi^+\pi^-$ system [Schwinger '89][Drees, Hikasa '90]

2. G_{EM} (long distance) radiative corrections in τ decays
Chiral Resonance Theory [Cirigliano et al. '01, '02]
Meson Dominance [Flores-Talpa et al. '06, '07]

3. Phase Space ($\beta_{0,-}$) due to $(m_{\pi^\pm} - m_{\pi^0})$

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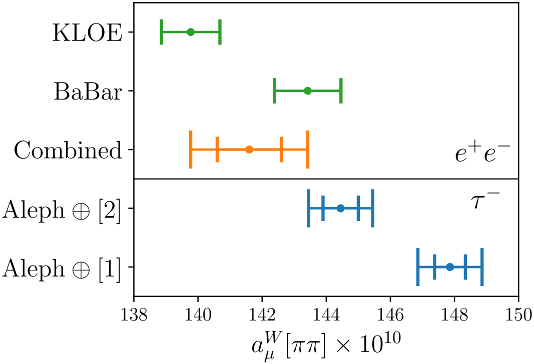
Window fever - III

(my) PRELIMINARY analysis of exp.data

syst. errs require further investigation/understanding



Isospin-breaking:

[1]: w/o $\rho\gamma$ mixing

[Davier et al.]

[Jegerlehner, Szafron]

[2]: w/ $\rho\gamma$ mixing

[Jegerlehner, Szafron]

What is $\rho\gamma$? too much to say, too little time to explain everything...

From the $(g-2)$ White Paper

" ... it appears that, at the required precision to match the e^+e^- data, the present understanding of the IB corrections to τ data is unfortunately not yet at a level allowing their use for the HVP dispersion integrals. "

"The ratio $|F_0(s)/F_-(s)|^2$ is the most difficult to estimate reliably, since a number of different IB effects may contribute."

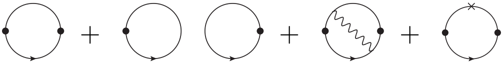
Contribution to a_μ

Time-momentum representation [Bernecker, Meyer, '11]

$$G^\gamma(t) = \frac{1}{3}\sum_k \int d\boldsymbol{x}\ \langle j_k^\gamma(x) j_k^\gamma(0)\rangle \quad \rightarrow \quad a_\mu = 4\alpha^2 \sum_t w_t G^\gamma(t)$$

Isospin decomposition of u, d current

$$j_\mu^\gamma = \frac{i}{6}\left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d\right) + \frac{i}{2}\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right) = j_\mu^{(0)} + j_\mu^{(1)}$$

$G_{00}^\gamma \leftarrow \langle j_k^{(0)}(x) j_k^{(0)}(0)\rangle =$  $\ldots$

$G_{01}^\gamma \leftarrow \langle j_k^{(0)}(x) j_k^{(1)}(0)\rangle =$  $\ldots$

$G_{11}^\gamma \leftarrow \langle j_k^{(1)}(x) j_k^{(1)}(0)\rangle =$  $\ldots$

Decompose $a_\mu = a_\mu^{(0,0)} + a_\mu^{(0,1)} + a_\mu^{(1,1)}$

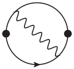
Neutral vs Charged

$$\frac{i}{2}\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right), \begin{bmatrix} I=1 \\ I_3 = 0 \end{bmatrix} \quad \rightarrow \quad j_\mu^{(1,-)} = \frac{i}{\sqrt{2}}\left(\bar{u}\gamma_\mu d\right), \begin{bmatrix} I=1 \\ I_3 = -1 \end{bmatrix}$$

Isospin 1 charged correlator $G_{11}^W = \frac{1}{3}\sum_k \int d\boldsymbol{x}\, \langle j_k^{(1,+)}(x) j_k^{(1,-)}(0)\rangle$

$$\delta G^{(1,1)} \equiv G_{11}^\gamma - G_{11}^W$$

[MB et al.' Latt18]

$$= Z_V^4 (4\pi\alpha) \frac{(Q_\mathrm{u} - Q_\mathrm{d})^4}{4} \Bigg[$$

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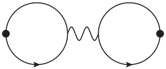
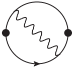
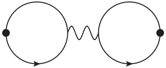
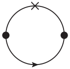

$\Bigg]$

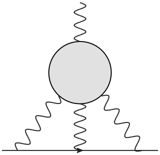
$$G_{01}^\gamma = Z_V^4 \frac{(Q_u^2 - Q_d^2)^2}{2} (4\pi\alpha) \Bigg[$$

$+ 2\times$

$+$

$+ \dots \Bigg]$

$$+ Z_V^2 \frac{Q_u^2 - Q_d^2}{2} (m_u - m_d) \Bigg[2\times$$

$+ \dots \Bigg]$

$\dots$ = subleading diagrams

Synergy



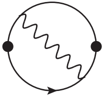
from QCD we need a 4-point function $f(x, y, z, t)$:

known kernel with details of photons and muon line

1 pair of point sources (x, y), sum over z, t exact at sink

stochastic sampling over (x, y) (based on $|x - y|$)

Successfull strategy: x10 error reduction [RBC '16]



from QCD we need a 4-point function $f(x, y, z, t)$:

$(g-2)_\mu$ kernel + photon propagator

Similar problem $\rightarrow$ re-use HLbL point sources!

The RBC & UKQCD collaborations

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BNL and BNL/RBRC
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Peter Boyle (Edinburgh)
Taku Izubuchi
Chulwoo Jung
Christopher Kelly
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Nobuyuki Matsumoto
Shigemi Ohta (KEK)
Amarjit Soni
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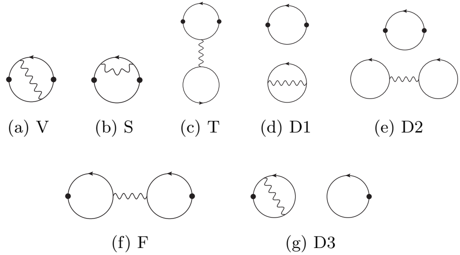
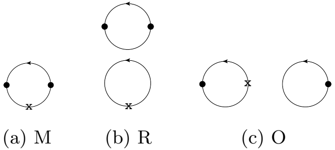


[Blum et. al. '18]



Done:

leading diagrams on coarse 24^3 ensemble $a^{-1} \simeq 1$ GeV

$V, S, F, M, D3, O$ (analysis to be finalized soon)

On-going:

cross-checks for data generation and analysis

$\rightarrow$ calculation on finer ensemble 48I

calculations of subleading diagrams

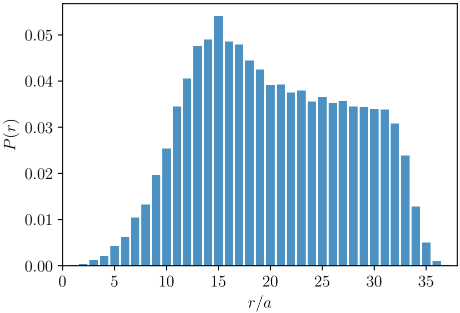
Sampling strategy

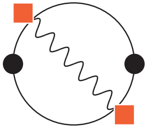
Propagators on disk from HLbL project [Phys.Rev.Lett. 118 (2017)]

$$\tilde{V}_\Gamma(x_0, z_0, r) = \sum_{\boldsymbol{x},\boldsymbol{z}} \mathrm{tr}\Big[\Gamma D^{-1}(x,0)\gamma_\nu D^{-1}(0,z)\Gamma D^{-1}(z,r)\gamma^\nu D^{-1}(r,x)\Big]$$

$$V_\Gamma(|x_0 - z_0|) = \sum_r \Delta(r)\tilde{V}_\Gamma(x_0, z_0, r)$$

$O(10^3)$ points $\to$ $O(10^6)$ pairs





contract photon offline
$\to$ study QED_L vs QED_∞

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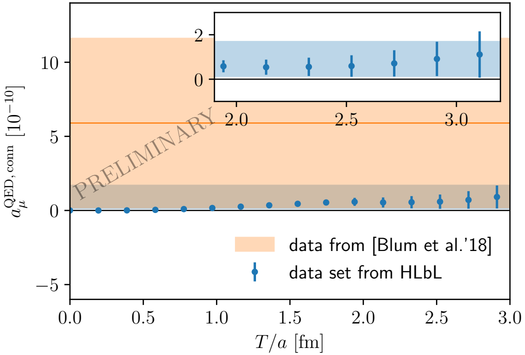
QED valence connected

Stat. improvements from data of HLbL project [Phys.Rev.Lett. 118 (2017)]

contribution of diagrams V, S to a_μ



Coarse ensemble 32ID

$\sim 3 \cdot 10^3$ point pairs

$O(10)$ configurations

preliminary (rough) analysis

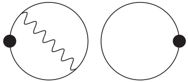
plain sum up to 3 fm

$\times 4$ reduction in stat. error

only stat. error showed

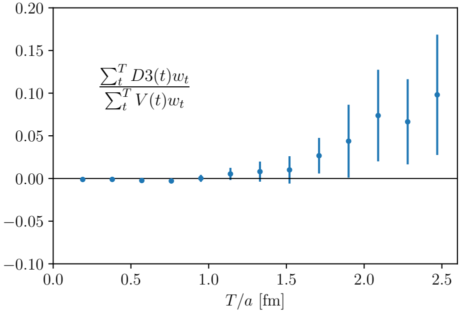
expected QED conn. error $\leq 3 \times 10^{-10} \rightarrow$ matches target

QED VALENCE DISCONNECTED



Preliminary (run2)
Point sources at exchanged photon vertices

Coarse lattice $a \simeq 0.2$ fm



Observe suppression relative to V

matches target accuracy

not yet explored full statistics (running)

Conclusions

These are exciting times for $(g-2)_\mu$:

- $<$1% goal reached by BMWc, to be expected from other collabs
- windows powerful intermediate tool to validate full calculation
- QED+SIB crucial to reach target uncertainty

As a bi-product we get $\Delta a_\mu[\tau]$ for τ data:

1. first lattice calculation of $\Delta a_\mu[\tau]$ almost complete
   study energy cut at τ mass (e.g. Backus-Gilbert method)
2. comparing with experiment requires
   - re-evaluation of radiative corrections [in collab. w/ Cirigliano]
   - lattice fully inclusive: understand role higher channels [private exchange Maltman, Golterman et al.]
3. tests/checks previous calculations [Jegelehner, Szafron][Davier et al.]

Thanks for your attention

Backup slides

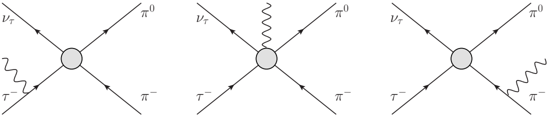
Long distance QED - I

At low energies relevant degrees of freedom are mesons

- Chiral Perturbation Theory [Cirigliano et al. '01, '02]
- Meson dominance model [Flores-Talpa et al. '06, '07]

Corrections casted in one function $v_-(s) \to v_-(s) G_{\mathrm{EM}}(s)$

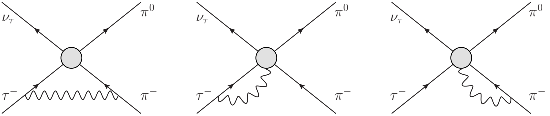
Real photon corrections



Real + virtual

$\to$ IR divergences cancel

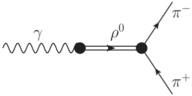
Virtual photon corrections



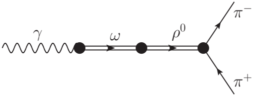
Pion form factors

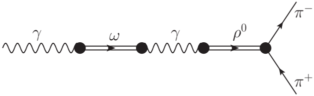
[Gounaris, Sakurai '68]
[Kühn, Santamaria '90]

$$F_\pi^0(s) \propto \frac{m_\rho^2}{D_\rho(s)}$$

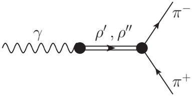


$$\times \left[1 + \color{red}{\delta_{\rho\omega}} \frac{s}{D_\omega(s)} \right]$$

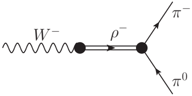




$$+ \frac{m_X^2}{D_X(s)} \qquad X = \rho' \, , \rho''$$



$$F_\pi^-(s) \propto \frac{m_{\rho^-}^2}{D_{\rho^-}(s)} + (\rho' \, , \rho'')$$



Sources of IB breaking in phenomenological models

$m_{\rho^0} \neq m_{\rho^\pm}$, $\Gamma_{\rho^0} \neq \Gamma_{\rho^\pm}$, $m_{\pi^0} \neq m_{\pi^\pm}$

$\rho - \omega$ mixing $\delta_{\rho\omega} \simeq O(m_\mathrm{u} - m_\mathrm{d}) + O(e^2)$

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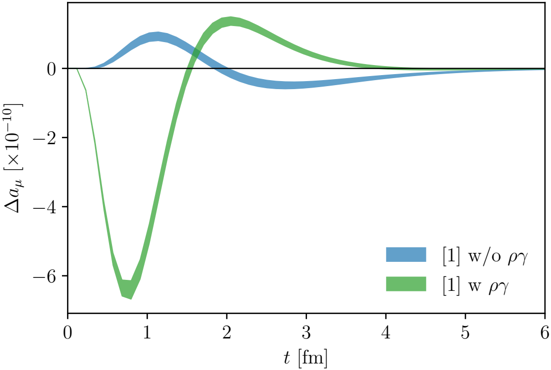
Comparison in Euclidean time more natural for Lattice

[1] = [Jegelehner, Szafron '17]

modified $\rho\gamma$ coupling
large negative Δa_μ

very important to examine integrand



From $(g-2)$ White Paper
" .. an increasing effect above the ρ peak that appears uncomfortably large." $\rightarrow$ translates into negative big dip below 1 fm

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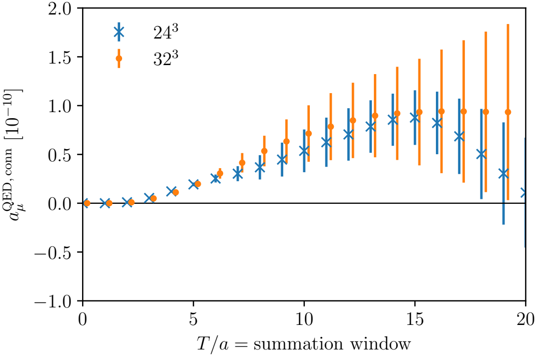
Finite volume errors



$$a_\mu^{\mathrm{QED,conn}} = V + 2S$$

FV study at coarse
$a^{-1} \sim 1$ GeV

Finite volume errors

empirical observation: diagrams may have largish FV errors

cancellation of FV effects in physical combinations

similar observation in ChPT, e.g. [Bijnens, Portelli '19]

Strong isospin breaking

Accurate determination from multiple valence calculations

- independent determination from point sources only 8k / 1M
- on-going check if full 1M can be competitive