Progress on the exploratory calculation of the rare Hyperon decay $\Sigma^+ \to p\ell^+\ell^-$

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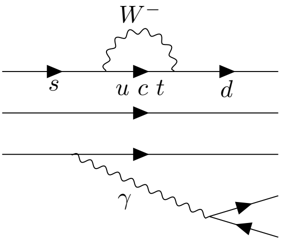
Introduction

Rare Hyperon Decay

- $\Sigma^+ \to p\ell^+\ell^-$ is an $s \to d$ FCNC process, suppressed in the SM
- Potentially sensitive to new physics
  [Geng et al. 10.1007/JHEP02(2022)178]
- Baryonic equivalent of the rare kaon decay $K \to \pi\ell^+\ell^-$
  (see Ryan Hill's talk [Thur. 11:50])

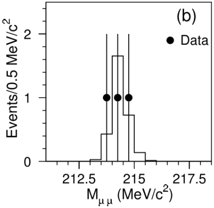


Related Information

- Finite volume corrections to $\Sigma^+ \to p\ell^+\ell^-$ covered in Felix Erben's talk [Thur. 12:30]
- For details on theoretical work presented here and Felix's talk, see
  [F. Erben et al. arXiv:2208.XXXXX]

Motivation: Experiment

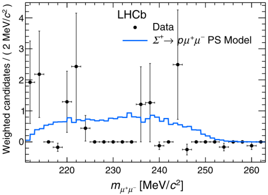
Hyper CP (2005)



- Observed 3 events
- $\mathcal{B}(\Sigma^+ \to p\mu^+\mu^-)_{HCP} = 8.6^{+6.6}_{-5.4} \pm 5.5 \times 10^{-8}$
- Originally thought this could be signal for a new particle of mass $\simeq 214 MeV$

[HyperCP 10.1103/PhysRevLett.94.021801]

LHCb (2018)



- $\mathcal{O}(10)$ events
- $\mathcal{B}(\Sigma^+ \to p\mu^+\mu^-)_{LHCb} = 2.2^{+1.8}_{-1.3} \times 10^{-8}$
- No evidence for new particle
- Interest in improving measurement with the latest data, including first measurement of angular observables and $\mathcal{B}(\Sigma^+ \to pe^+e^-)$

[LHCb 10.1103/PhysRevLett.120.221803]

[LHCb 10.1007/JHEP05(2019)048]

Motivation: Theory

- SM prediction of the 4 hadronic form factors (a,b,c,d) come from combination of sources
- Real parts from experimental input and vector meson dominance
- Imaginary parts from unitarity cuts and various types of Baryon Chiral Perturbation Theory
- Dominated by the long distance contribution

$$\Sigma^+ \to p\gamma^* \to p\mu^+\mu^-$$

$$1.6 \times 10^{-8} < \mathcal{B}(\Sigma^+ \to p\mu^+\mu^-) < 8.9 \times 10^{-8}$$
$$-1.4 \times 10^{-5} < A_{FB}(\Sigma^+ \to p\mu^+\mu^-) < 0.6 \times 10^{-5}$$

- Large ranges mainly coming from partially unconstrained real parts of a and b
- Sensitivity to new physics limited by this uncertainty

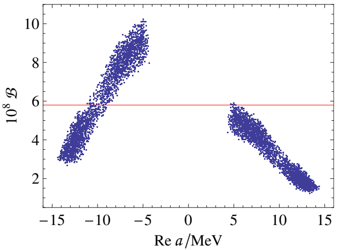
[X. He 10.1103/PhysRevD.72.074003]

[X. He et al. 10.1007/JHEP10(2018)040]

[Geng et al. 10.1007/JHEP02(2022)178]

Motivation: Lattice

- In near term lattice can complement existing methodology by constraining *Re a* and *Re b* to reduce range of $\mathcal{B}$



[X. He et al. 10.1007/JHEP10(2018)040]

- In the further future go to a fully lattice determination of $\mathcal{B}$ including imaginary parts (see Felix's talk [Thur. 12:30])

Minkowski Amplitude

- Long distance part $\Sigma^+ \to p\gamma^*$ given by amplitude

$$\mathcal{A}_\mu = \int d^4x \, \langle p(\boldsymbol{p})| \, T[\mathcal{H}_W(x) J_\mu(0)] \left|\Sigma^+(\boldsymbol{k})\right\rangle$$

- J_μ: Electromagnetic current
- H_w: $s \to d$ effective weak Hamiltonian

$$\mathcal{H}_W = \frac{G_f}{\sqrt{2}} V_{us} V_{ud}^* \left[C_1(Q_1^u - Q_1^c) + C_2(Q_2^u - Q_2^c) + ...\right]$$

with 4-quark operators

$$Q_1^q = (\bar{d}\gamma^{L\mu} s)(\bar{q}\gamma_\mu^L q) \qquad\qquad Q_2^q = (\bar{d}\gamma^{L\mu} q)(\bar{q}\gamma_\mu^L s)$$

Minkowski Amplitude

- Form factor decomposition

$$\mathcal{A}_\mu = \bar{u}_p(\boldsymbol{p}) \left[i\sigma_{\mu\nu} q^\nu (a + b\gamma_5) + (q^2\gamma_\mu - q_\mu \not{q})(c + d\gamma_5) \right] u_\Sigma(\boldsymbol{k})$$
$$q = k - p$$

- Spectral representation

$$\mathcal{A}_\mu = i \int d\omega \left(\frac{\rho_\mu(\omega)}{E_\Sigma(\boldsymbol{k}) - \omega + i\epsilon} - \frac{\sigma_\mu(\omega)}{\omega - E_p(\boldsymbol{p}) - i\epsilon} \right)$$

- In finite volume spectral functions have the form

$$\rho_\mu(\omega)_L = \sum_\alpha \frac{\delta(\omega - E_\alpha(\boldsymbol{k}))}{2E_\alpha(\boldsymbol{k})} \langle p(\boldsymbol{p})| J_\mu |E_\alpha(\boldsymbol{k})\rangle \langle E_\alpha(\boldsymbol{k})| \mathcal{H}_W |\Sigma(\boldsymbol{k})\rangle$$
$$\sigma_\mu(\omega)_L = \sum_\beta \frac{\delta(\omega - E_\beta(\boldsymbol{p}))}{2E_\beta(\boldsymbol{p})} \langle p(\boldsymbol{p})| \mathcal{H}_W |E_\beta(\boldsymbol{p})\rangle \langle E_\beta(\boldsymbol{p})| J_\mu |\Sigma(\boldsymbol{k})\rangle$$

Euclidean Correlators

- 4-point correlation function

$$\Gamma^{(4)}_{\mu}(t_p, t_H, t_\Sigma) = \int d^3\boldsymbol{x} \ \langle \psi_p(t_p, \boldsymbol{p}) \ \mathcal{H}_W(t_H, \boldsymbol{x}) J_\mu(0) \ \bar{\psi}_\Sigma(t_\Sigma, \boldsymbol{k}) \rangle$$

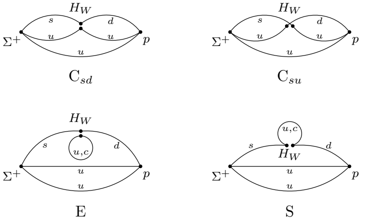
- Amputate external propagation and overlap factors

$$\begin{aligned}
\hat{\Gamma}^{(4)}_{\mu}(t_H) &= \frac{4 E_\Sigma E_p}{Z_p Z_\Sigma^*} e^{+E_p t_p} e^{-E_\Sigma t_\Sigma} \ \Gamma^{(4)}_{\mu} \\
t_H < 0: \quad &= \int_0^\infty d\omega \ \rho_\mu(\omega)_L \ e^{-(E_\Sigma - \omega) t_H} \\
t_H > 0: \quad &= \int_0^\infty d\omega \ \sigma_\mu(\omega)_L \ e^{-(\omega - E_p) t_H}
\end{aligned}$$

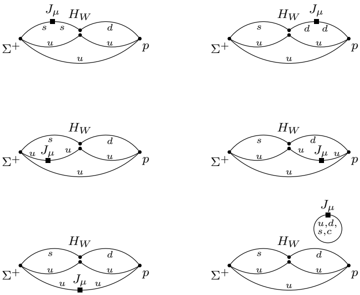
- Assuming ground state dominance of external states

Euclidean Correlators

- 4 diagrams for 3-point functions of H_w



- 24 diagrams for 4-point functions



etc

Integrated Correlator

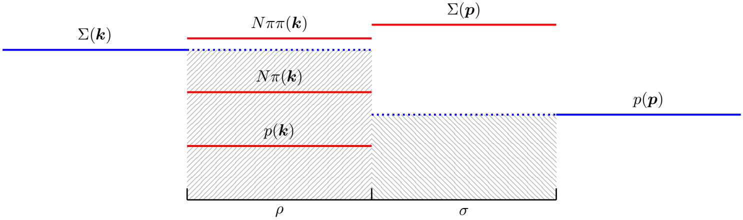
- Integrate amputated 4-point function within window $[-T_a, T_b]$

$$
\begin{aligned}
I_\mu^{(4)}(T_a, T_b) &= -i \int_{-T_a}^{T_b} dt_H \; \hat{\Gamma}_\mu^{(4)}(t_H) \\
&= i \int_0^\infty d\omega \left[\rho_\mu(\omega)_L \; \frac{1 - e^{(E_\Sigma - \omega) T_a}}{E_\Sigma - \omega} - \sigma_\mu(\omega)_L \; \frac{1 - e^{-(\omega - E_p) T_b}}{\omega - E_p} \right] \\
&= \mathcal{A}_\mu + T_a, T_b \text{ exp terms} + \text{FV corrections}
\end{aligned}
$$

- FV corrections will be covered in Felix's talk [Thur. 12:30]
- Remove T_b exp terms by taking $T_b \to \infty$
- $T_a \to \infty$ limit blows up for region of ρ_μ spectrum with $\omega < E_\Sigma$

Growing Exponentials

$$I_\mu^{(4)}(T_a, T_b) = i \int_0^\infty d\omega \left[\rho_\mu(\omega)_L \frac{1 - e^{(E_\Sigma - \omega)T_a}}{E_\Sigma - \omega} - \sigma_\mu(\omega)_L \frac{1 - e^{-(\omega - E_p)T_b}}{\omega - E_p} \right]$$



Example physical point intermediate spectra with $\boldsymbol{k} = \mathbf{0}$ and $|\boldsymbol{p}| = 400$ MeV

- Must remove growing exponentials before taking the $T_a \to \infty$ limit
- In practice might also have to remove slowly decaying exponentials to improve convergence as $T_a, \ T_b \to \infty$

Single Proton removal: Explicit subtraction

- Can construct the problematic exponentials and explicitly remove them from the integrated 4-point function
- Requires energies and matrix elements from 3-point functions of H_w and J_μ
- Example: single proton intermediate state

$$\frac{1}{2E_p(\boldsymbol{k})} \langle p(\boldsymbol{p})| J_\mu |p(\boldsymbol{k})\rangle \langle p(\boldsymbol{k})| \mathcal{H}_W |\Sigma(\boldsymbol{k})\rangle \frac{e^{-(E_\Sigma(\boldsymbol{k})-E_p(\boldsymbol{k}))T_a}}{E_\Sigma(\boldsymbol{k})-E_p(\boldsymbol{k})}$$
$$\sim \hat{\Gamma}^{(3)}_{J_\mu} \cdot \hat{\Gamma}^{(3)}_{H_W} \frac{e^{-(E_\Sigma(\boldsymbol{k})-E_p(\boldsymbol{k}))T_a}}{E_\Sigma(\boldsymbol{k})-E_p(\boldsymbol{k})}$$

- Can be applied to all intermediate states

Single Proton removal: (Pseudo)scalar shift

- Can also remove a single growing exponential via a (pseudo)scalar operator shift to $\mathcal{H}_w$. Amplitude invariant due to chiral Ward identities [RBC UKQCD 10.1103/PhysRevD.88.014508]

$$\mathcal{H}_W' = \mathcal{H}_W - c_S \bar{d}s - c_P \bar{d}\gamma_5 s \quad \Rightarrow \quad \mathcal{A}' = \mathcal{A}$$

- Choose c_S and c_P such that a single matrix element vanishes

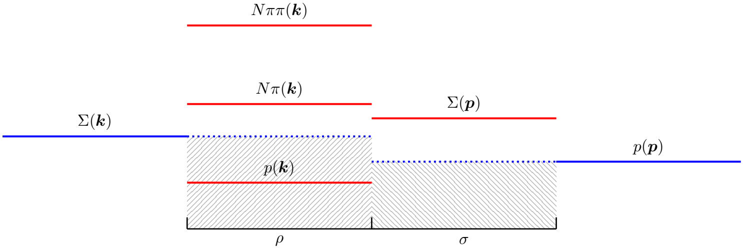
$$\langle p(\boldsymbol{k})| \mathcal{H}_W' |\Sigma(\boldsymbol{k})\rangle = \bar{u}_p \left[(a_H - c_S a_S) + (b_H - c_P b_P)\gamma_5\right] u_\Sigma = 0$$
$$\therefore \quad c_S = \frac{a_H}{a_S} \quad \text{and} \quad c_P = \frac{b_H}{b_P}$$

- In general, need both operators to remove intermediate state (unlike in the rare Kaon decay) [RBC UKQCD 10.1103/PhysRevD.92.094512]
- Σ at rest ($\boldsymbol{k} = \mathbf{0}$) is a special kinematic point as it removes need for pseudoscalar shift since

$$\bar{u}_p^r(\mathbf{0})\gamma_5 u_\Sigma^s(\mathbf{0}) = 0$$

Exploratory Calculation: Ensemble details

- Iwasaki gauge action $\beta = 2.13$ [RBC-UKQCD 10.1103/PhysRevD.83.074508]
- 2+1f Shamir domain-wall fermions
- $a \simeq 0.1fm \simeq (1730\ MeV)^{-1}$
- $24^3 \times 64\ (\times 16)_{L_s}$
- $m_\pi \simeq 340\ MeV$ and $m_K \simeq 580\ MeV$
- $m_N \simeq 1150\ MeV$ and $m_\Sigma \simeq 1350\ MeV$



Exploratory Calculation: Measurement Details

- At present only measuring C_{sd} and C_{su} diagrams (Non-Eye)
- Gauge fixed Gaussian sources
- Source-Sink sampling (x4x4)
- Time translation (x32)
- Kinematics $\boldsymbol{k} = \mathbf{0}$, $\boldsymbol{p} = \frac{2\pi}{L}(1, 0, 0)$
- Grid/Hadrons

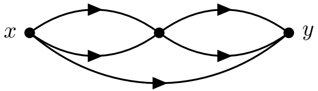
GRID

[github.com/paboyle/Grid]

Hadrons

[github.com/aportelli/Hadrons]

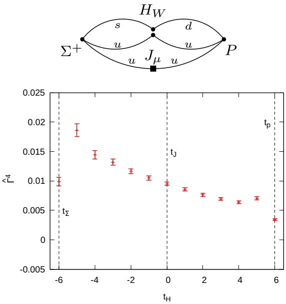
Source-Sink sampling

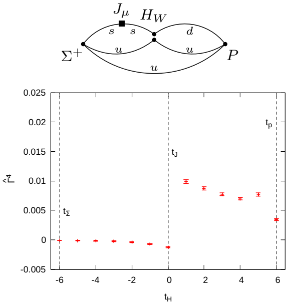


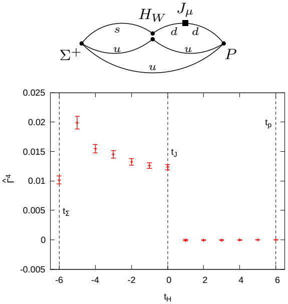
[Y. Li et al. 10.1103/PhysRevD.103.014514]

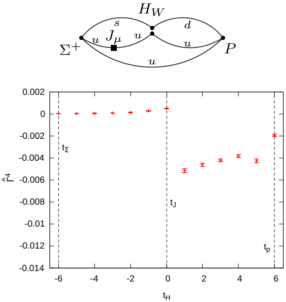
- Positions x and y fixed in solves
- Full volume sum for momentum projection requires $\sim 14,000$ solves
- Approximate with sum over N random position samples
- Could get up to $1/N$ error scaling

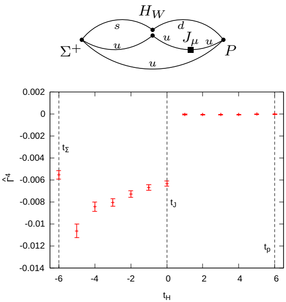
Exploratory Calculation: Preliminary Data



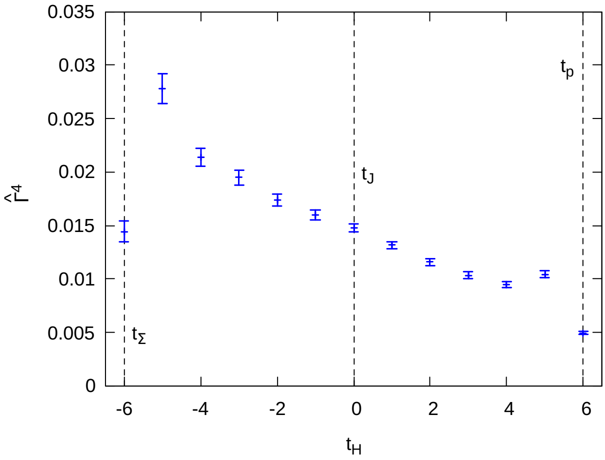








Exploratory Calculation: Preliminary Data



- Connected contribution to the amputated 4-point function
- Need to integrate and remove single proton intermediate state via both methods

Conclusion/Outlook

Conclusion

- $\Sigma^+ \to p\ell^+\ell^-$ is a process of interest to the community
- We have theoretical framework to extract the amplitude
  [F. Erben et al. arXiv:2208.XXXXX]
- Working towards an exploratory computation with heavy pion mass

Outlook

- Need to complete data collection (including Eye-type diagrams) and full analysis to be performed
- Move towards a physical point calculation where finite volume effects are important (see Felix's talk [Thur. 12:30])

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