Intermediate window observable for the hadronic vacuum polarization contribution to the muon $g-2$ from O(a) improved Wilson quarks [2206.06582, Cè et al.]

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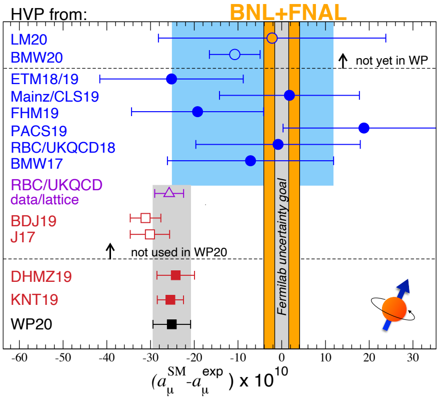


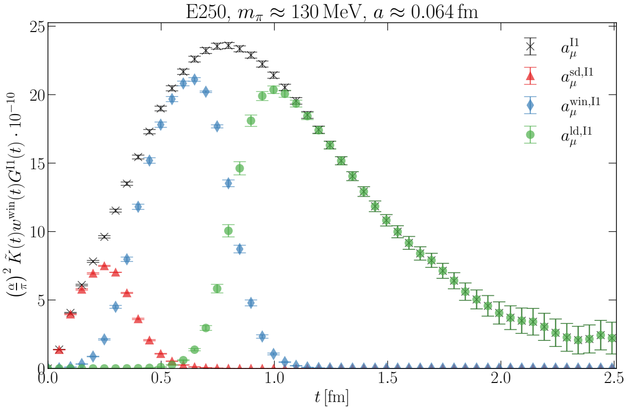
$\leftarrow$ Status for $a_\mu^{\rm hvp}$ [2203.15810, Colangelo et al.]

- Prediction in [2002.12347, BMWc] deviates **significantly** from data-driven results.
- High-precision lattice calculations needed. Major challenges:
  - Cutoff effects at short distances t
  - Exponential deterioration of signal-to-noise ratio at large t (with traditional Monte Carlo methods)

- Focus on benchmark quantities to compare among lattice collaborations: Time windows in the Time Momentum Representation [1801.07224, Blum et al.]

Time-momentum representation [1107.4388, Bernecker and Meyer]:

$$a_\mu^{\mathrm{win}} := \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt \widetilde{K}(t)\, G(t) \cdot \left[\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)\right],$$



$$G(t) = -\frac{a^3}{3} \sum_{k=1}^{3} \sum_{\vec{x}} \langle j_k^{\mathrm{em}}(t, \vec{x})\, j_k^{\mathrm{em}}(0) \rangle$$

$$\Theta(t, t', \Delta) := \tfrac{1}{2}\left(1 + \tanh[(t - t')/\Delta]\right)$$

$$t_0 = 0.4\,\mathrm{fm}, t_1 = 1.0\,\mathrm{fm}, \Delta = 0.15\,\mathrm{fm}.$$

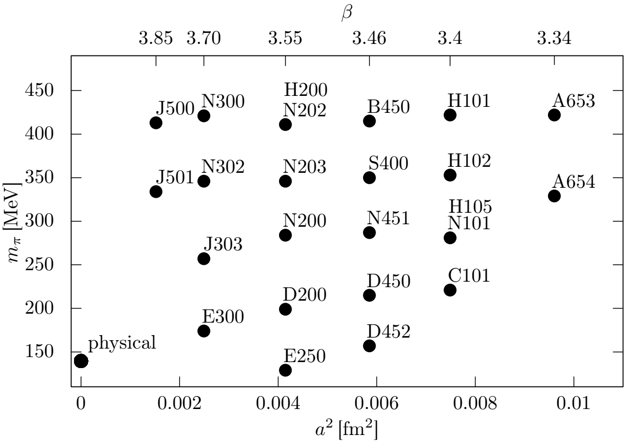
Intermediate window:

- Cutoff effects suppressed.
- No signal-to-noise problem.
- 1-3 per-mil uncertainty.

Aim for high-precision determination of a_μ^{win} – need to carefully control all sources of systematic uncertainty:

- Finite box size
- Unphysical quark masses
- Finite lattice spacing
- Iso-symmetric gauge configurations
- Missing charm and bottom quarks in the sea (insignificant ✓)

$2+1$ FLAVOR CLS ENSEMBLES



- $\mathrm{O}(a)$ improved Wilson-clover fermions.
- Open boundary conditions in temporal direction
- $a\mathrm{Tr}[M_\mathrm{q}] = 2am_\mathrm{l} + am_\mathrm{s} = \mathrm{const.}$
- Six values of $a \in [0.039, 0.099]\,\mathrm{fm}$, a factor of 6.4 in a^2.
- $m_\pi \in [129, 422]\,\mathrm{MeV}$

Scale: Either use $\sqrt{t_0^{\mathrm{phys}}} = 0.1443(15)\,\mathrm{fm}$ [2112.06696, Straßberger et al.] or express dimensionfull quantities in terms of af_π [1103.4818, Xu et al.][1904.03120, Gérardin et al.]

- Work in isospin decomposition of the electromagnetic current

$$j_\mu^{\mathrm{em}} = \tfrac{2}{3}\bar{u}\gamma_\mu u - \tfrac{1}{3}\bar{d}\gamma_\mu d - \tfrac{1}{3}\bar{s}\gamma_\mu s + \tfrac{2}{3}\bar{c}\gamma_\mu c + \ldots \quad = j_\mu^{I=1} + j_\mu^{I=0} + \tfrac{2}{3}\bar{c}\gamma_\mu c + \ldots, ,$$

Isovector: $j_\mu^{I=1} = \tfrac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)$, Isoscalar: $j_\mu^{I=0} = \tfrac{1}{6}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d - 2\bar{s}\gamma_\mu s)$

- Two discretizations of the vector current: local and conserved

$$J_\mu^{(\mathrm{L}),a}(x) = \overline{\psi}(x)\gamma_\mu \frac{\lambda^a}{2}\psi(x)\,,$$
$$J_\mu^{(\mathrm{C}),a}(x) = \frac{1}{2}\left(\overline{\psi}(x+a\hat{\mu})(1+\gamma_\mu)U_\mu^\dagger(x)\frac{\lambda^a}{2}\psi(x) - \overline{\psi}(x)(1-\gamma_\mu)U_\mu(x)\frac{\lambda^a}{2}\psi(x+a\hat{\mu})\right)\,,$$

$\mathrm{O}(a)$ IMPROVED VECTOR CURRENTS

- Improved vector currents are given by

$$J_\mu^{(\alpha),a,\mathrm{I}}(x) = J_\mu^{(\alpha),a}(x) + ac_\mathrm{V}^{(\alpha)}(g_0)\,\tilde{\partial}_\nu \Sigma_{\mu\nu}^a(x)\,, \qquad \text{with} \quad \alpha \in \mathrm{L}, \mathrm{C}$$

- Renormalization and mass-dependent improvement of local currents via

$$J_\mu^{(\mathrm{L}),3,\mathrm{R}}(x) = Z_\mathrm{V}\left[1 + 3\bar{b}_\mathrm{V} am_\mathrm{q}^\mathrm{av} + b_\mathrm{V} am_{\mathrm{q},l}\right] J_\mu^{(\mathrm{L}),3,\mathrm{I}}(x)\,,$$

$$J_\mu^{(\mathrm{L}),8,\mathrm{R}}(x) = Z_\mathrm{V}\left[1 + 3\bar{b}_\mathrm{V} am_\mathrm{q}^\mathrm{av} + \frac{b_\mathrm{V}}{3} a(m_{\mathrm{q},l} + 2m_{\mathrm{q},s})\right] J_\mu^{(\mathrm{L}),8,\mathrm{I}}(x)$$

$$+ Z_\mathrm{V}\left(\frac{1}{3} b_\mathrm{V} + f_\mathrm{V}\right)\frac{2}{\sqrt{3}} a(m_{\mathrm{q},l} - m_{\mathrm{q},s})\, J_\mu^{(\mathrm{L}),0,\mathrm{I}}(x)\,,$$

- Two independent non-perturbative determinations of $Z_\mathrm{V}, c_\mathrm{V}^L, c_\mathrm{V}^C, b_\mathrm{V}, \bar{b}_\mathrm{V}$:

Set 1: Large-volume, CLS ensembles [1811.08209, Gérardin et al.]

Set 2: Small volume, Schrödinger functional [2010.09539, ALPHA],[1805.07401, Fritzsch]

differ by higher order cutoff effects. f_V is of $\mathrm{O}(g_0^6)$ and unknown.

Finite-size effects

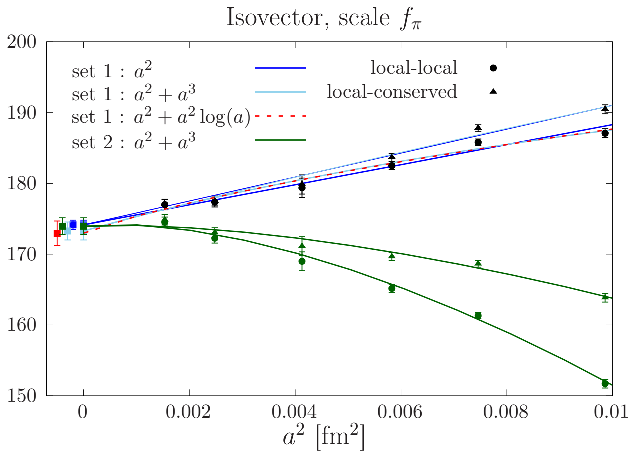
- Finite-size corrections applied to the isovector correlator.
- Correction for $t < \frac{(m_\pi L/4)^2}{m_\pi}$: Hansen-Patella method [1904.10010][2004.03935]
  - Expansion in the pion winding number.
  - Using monopole parametrization of the electromagnetic pion form factor.
- Large distances: MLL [1105.1892, Meyer] [hep-lat/0003023, Lellouch and Lüscher]:
  - Compute difference between finite and infinite-volume isovector correlator
  - Based on the time-like pion form factor.
  - Applied at large Euclidean distances $\rightarrow$ less relevant for a_μ^{win}.
- This is the only correction applied to the lattice data!
  Of similar size as statistical uncertainty for a_μ^{win}.

- Separate extrapolations of isovector, isocalar and charm contributions.
- General fit ansatz, not possible to resolve all parameters at once:

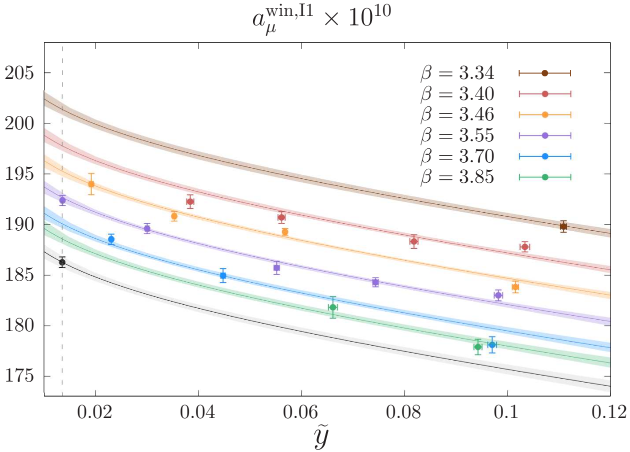
$$
\begin{aligned}
a_\mu^{\mathrm{win,f}}(X_a, X_\pi, X_K) = a_\mu^{\mathrm{win,f}}(0, X_\pi^{\mathrm{exp}}, X_K^{\mathrm{exp}}) \\
+ \beta_2 X_a^2 + \beta_3 X_a^3 + \delta X_a^2 X_\pi + \epsilon X_a^2 \log X_a \\
+ \gamma_1 (X_\pi - X_\pi^{\mathrm{exp}}) + \gamma_2 (f(X_\pi) - f(X_\pi^{\mathrm{exp}})) \\
+ \gamma_0 \left(X_K - X_K^{\mathrm{phys}}\right)
\end{aligned}
$$

where $\quad X_a \sim a\,, \qquad X_\pi \sim m_\pi^2\,, \qquad X_K \sim m_K^2 + \frac{1}{2} m_\pi^2$

- Light quark mass effects: $f(X_\pi) \in \{0;\ \log(X_\pi);\ X_\pi^2;\ 1/X_\pi;\ X_\pi \log(X_\pi)\}$
- Dedicated computation of priors for γ_0 (Backup).
- Final result and uncertainties from model average.

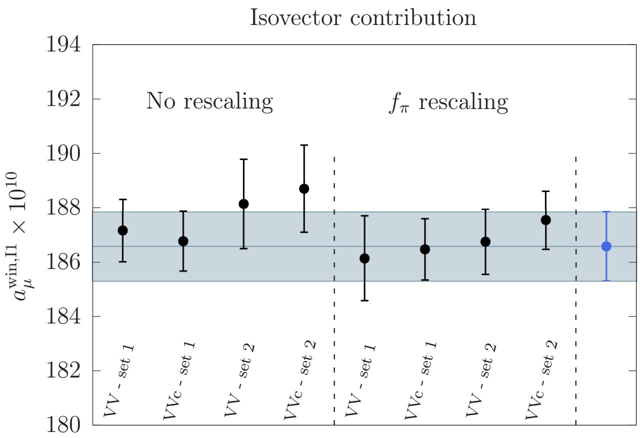


- Two sets of equally valid improvement coefficients.
- No cutoff effects of $\mathrm{O}(a^3)$ resolved for Set 1.
- Independent extrapolations compatible in the continuum $\rightarrow$ strong cross-check of our extrapolations.
- No sign of modification $a^2 \rightarrow (\alpha_{\mathrm{s}}(1/a^2))^{\hat{\Gamma}} a^2$ [1912.08498, Husung et al.] Nikolai Husung's talk

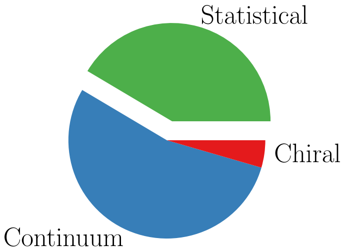


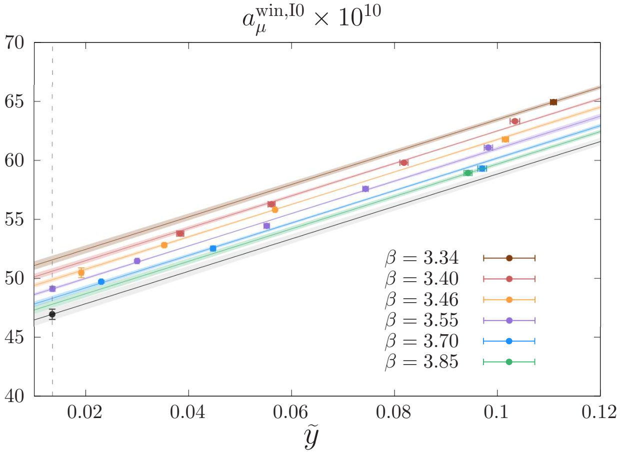
- f_π rescaling, local-local current and Set 1.
- Curvature in $\tilde{y} = \frac{m_\pi^2}{8\pi f_\pi^2}$ is needed to describe the data.
- Variation in the chiral extrapolation does not change the result significantly.
- Singular fit ansatz favored.

$\longrightarrow$ Martin Hoferichter's talk

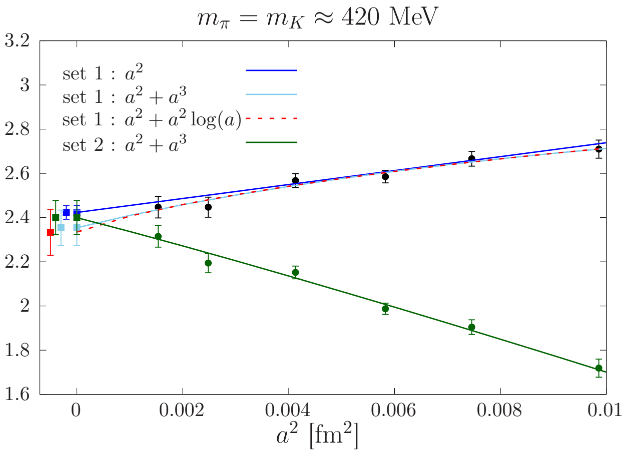


- Eight different combinations of discretization and improvement procedures.
- Model averages in each category
- Final result by combining L and C of Set 1.





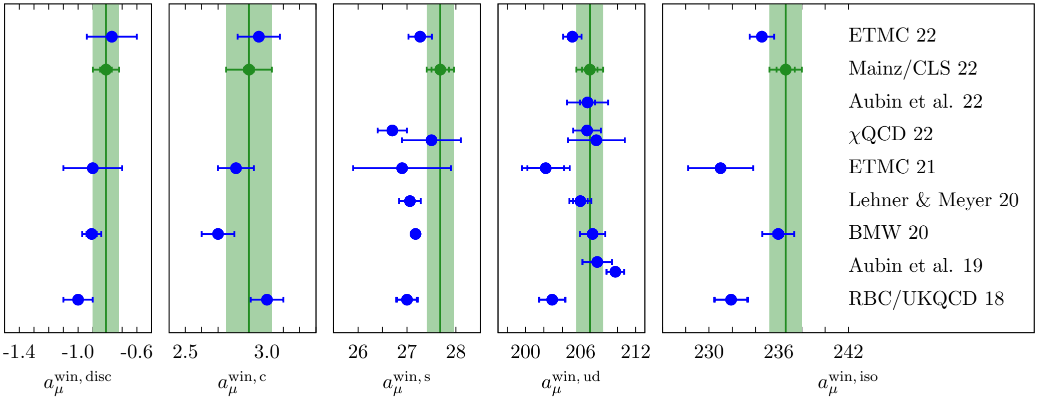
- f_π rescaling, local-local current and Set 1.
- Choose non-singular fit ansatz, $f(X_\pi) \in \{0;\ X_\pi^2;\ X_\pi \log(X_\pi)\}$
- Charm contribution not included at this stage.



- Charm quark included in partially-quenched setup.
- Effect of missing charm loops estimated to be $< 0.02\%$ for a_μ^win

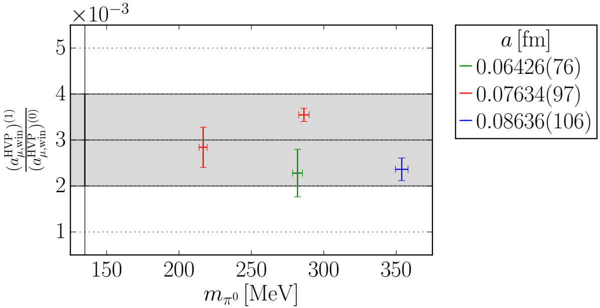
  $\rightarrow$ negligible thanks to separation of scales.
- Mass-dependent renormalization scheme.
- Show only local-conserved current, large cutoff effects for local-local.

Comparison with lattice results for $a_\mu^{\mathrm{win,iso}}$



$$a_\mu^{\mathrm{win,iso}} = a_\mu^{\mathrm{win,I1}} + a_\mu^{\mathrm{win,I0}} + a_\mu^{\mathrm{win,c}} = (236.60 \pm 0.79_{\mathrm{stat}} \pm 1.13_{\mathrm{syst}} \pm 0.05_{\mathrm{Q}}) \times 10^{-10}$$

- Tension with EMTC 21 and RBC/UKQCD 18 estimates for $a_\mu^{\mathrm{win,iso}}$ mainly from light quark contribution. EMTC 22 and BMW 20 consistent with our result.

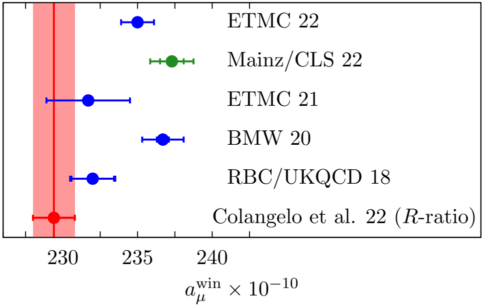


- QED$_\mathrm{L}$-action [0804.2044, Hayakawa and Uno] for IR regularisation, Coulomb gauge.
- Reweighting based on perturbative expansion [1303.4896, de Divitiis et al.] in $\Delta\varepsilon = \varepsilon - \varepsilon^{(0)} = (\Delta m_\mathrm{u}, \Delta m_\mathrm{d}, \Delta m_\mathrm{s}, \Delta\beta = 0, e^2)$

- Ongoing effort [2112.00878, Risch and Wittig]: four ensembles included, so far.
- IB in scale setting [2112.08262, Segner et al.] and QED-FV effects to be considered.
- Uncertainty on relative correction $0.3(1)\%$ doubled in final result for a_μ^{win}.

- Isospin-breaking correction $+(0.70 \pm 0.47) \times 10^{-10}$ included:

$$a_\mu^{\rm win} = (237.30 \pm 0.79_{\rm stat} \pm 1.13_{\rm syst} \pm 0.05_{\rm Q} \pm 0.47_{\rm IB}) \times 10^{-10}$$



- 3.9σ tension with data-driven estimate in [2205.12963, Colangelo et al.].
- Genuine difference between lattice and data-driven results?

- We observe tension with data-driven estimates for a_μ^{win}.
- Systematic effects from continuum extrapolation seem to be under control:
  - Non-perturbative $\mathrm{O}(a)$ improvement
  - 6 resolutions $< 0.1\,\mathrm{fm}$ with $a_{\mathrm{max}}^2/a_{\mathrm{min}}^2 > 6$.
  - Two discretizations of the vector current, two sets of improvement procedures.
  - So-far no sign of logarithmic corrections to a^2 scaling.
- Uncertainties from chiral extrapolation and finite-volume correction are subleading.

Outlook

- Investigation of other windows might help to clarify the situation.
- Short-distance window:
  - Cutoff effects from short-distance singularities need proper treatment [0807.1120, Della Morte et al.][2106.15293, Cè et al.] Rainer Sommer's talk.
  - Systematic uncertainties will dominate and need to be properly estimated.
- Sub-percent precision on a_μ^{hvp} needs reduction of our statistical uncertainties.
- Application of variance reduction techniques at physical pion masses in progress: Low Mode Averaging and information from π-π scattering analyses (see Srijit Paul's talk).

The chiral trajectory

- The light and strange quark mass dependence is parameterized via

$$X_\pi = \tilde{y} = \frac{m_\pi^2}{8\pi f_\pi^2}, \qquad X_K = y_{K\pi} = \frac{m_K^2 + \frac{1}{2}m_\pi^2}{8\pi f_{K\pi}^2}, \text{ where } f_{K\pi} = \frac{2}{3}(f_K + \frac{1}{2}f_\pi)$$

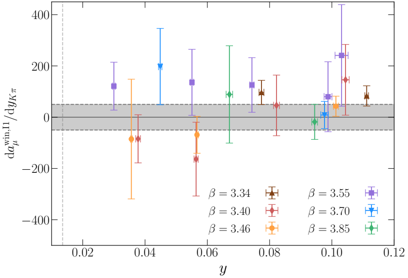
  when using f_π rescaling.

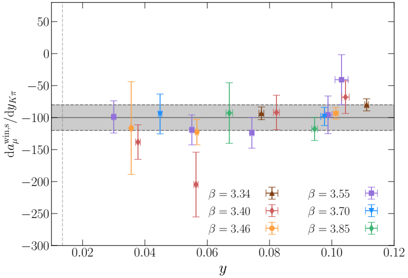
- $2am_\mathrm{l} + am_\mathrm{s} = \mathrm{const}$ on our ensembles. This implies $X_K \sim \mathrm{const}$ up to $\mathrm{O}(a)$ and NLO χPT effects.

- Correct for small deviation $\Delta X_K = X_K^\mathrm{phys} - X_K$ in global fit.

- No independent variation of the Kaon mass, fit parameter γ_0 not stable.

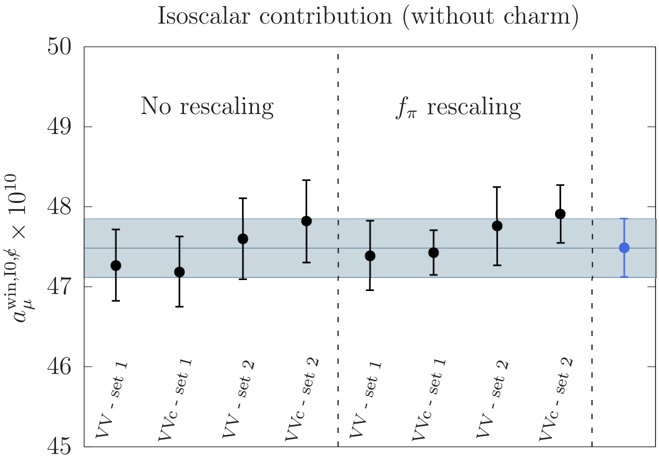
Correcting the mistuning of the chiral trajectory





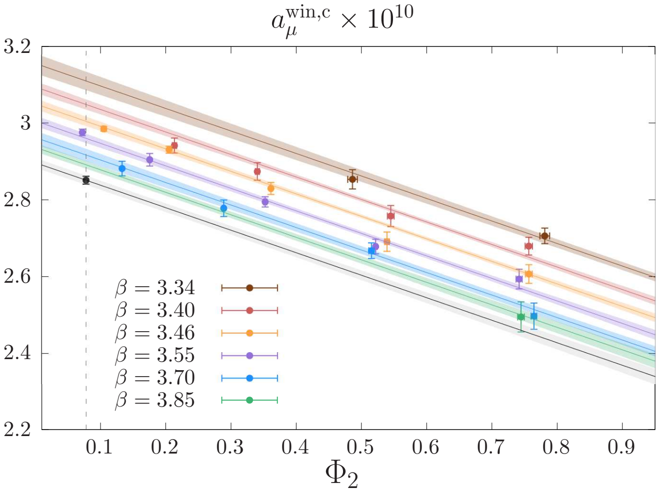
- Explicit computation of $\gamma_0 = \frac{\mathrm{d}\langle a_\mu^{\mathrm{win}}\rangle}{\mathrm{d}X_K}$.
- Based on mass-derivatives $\frac{\mathrm{d}\langle\mathcal{O}\rangle}{\mathrm{d}m_{\mathrm{q},i}}$ and a first-order Taylor expansion [1608.08900, Bruno et al.].
- No cutoff or quark mass effects resolved.
- Use results as priors for fit parameter γ_0.
  - No significant strange quark mass dependence for $a_\mu^{\mathrm{win,I1}}$.
  - Negative contribution for $a_\mu^{\mathrm{win,s}}$.
- Results confirmed by pheno estimates.

Model averages: Isoscalar contribution



- Eight different combinations of discretization and improvement procedures.
- Model averages in each category, including systematic uncertainty from choice of fit model.
- Final result by combining L and C of Set 1.

Chiral extrapolation of charm contribution



- Charm mass set via D_s.
- m_s not constant on our ensembles.
- Extrapolation in $\Phi_2 = 8t_0 m_\pi^2$
- Effect of missing charm loops estimated to be $< 0.02\%$ for a_μ^{win}

  $\rightarrow$ negligible thanks to separation of scales.