Neutron Electric Dipole Moment from the θ Term with Overlap Fermion

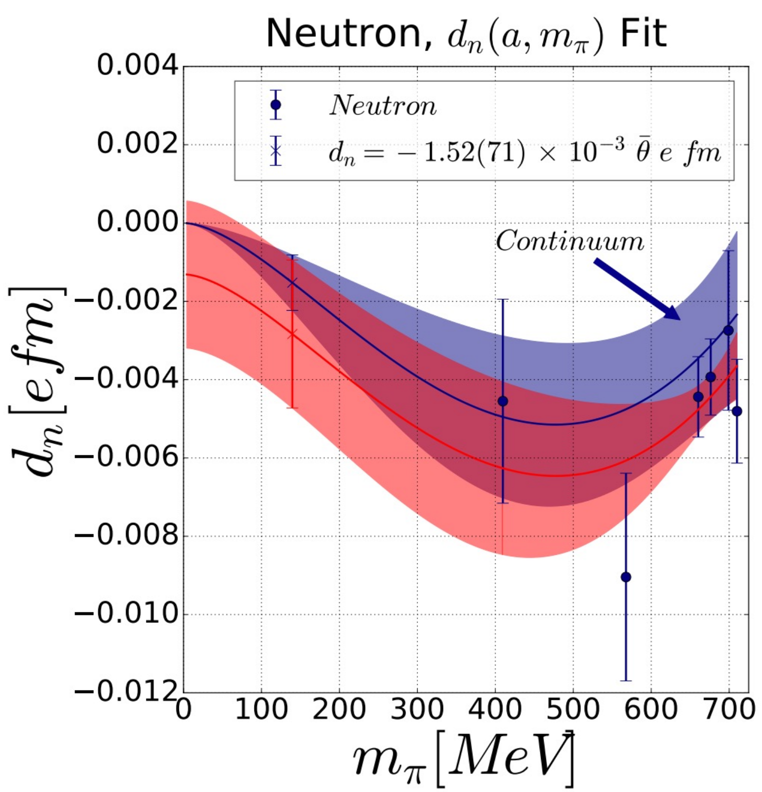
- CP violation operators and lattice methodology
- Cluster decomposition error reduction (CDER)
- Topological charge and topological susceptibility with overlap operator
- Results
- Valence and sea quark dependence – partial quenching

χ QCD Collaboration

-- Jian Liang

Lattice, Bonn, Aug. 9, 2022

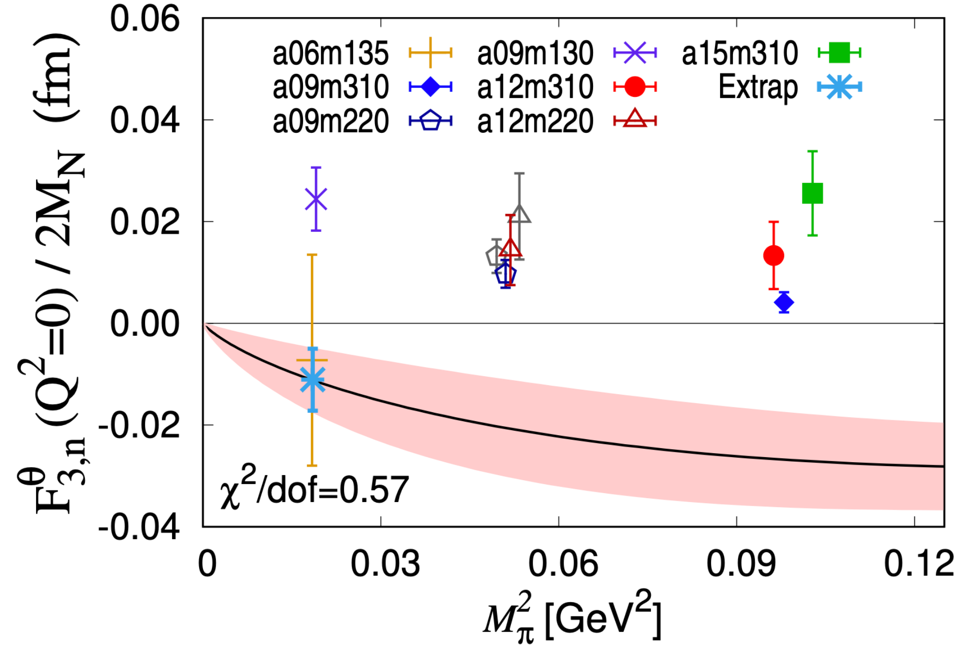
Recent results (theta term)



J. Dragos et al., *PRC* 103, 015202 (2021)

A. Shindler, Eur.Phys.J.A 57, 128 (2021)

Chiral extrapolation with heavy pion masses and with non-chiral fermion



T. Bhattacharya et. al., PRD 103, 114507 (2021)

$$d_n = -0.003(7)(20)\overline{\Theta}\ e \cdot \mathrm{fm}$$
$$d_p = 0.024(10)(30)\overline{\Theta}\ e \cdot \mathrm{fm}$$

Overlap Fermion

$$D_{ov} = 1 + \gamma_5 \frac{H_W}{\sqrt{H_w^\dagger H_W}}$$

- Chiral symmetry – Ginsparg-Wilson relation

$$\{\gamma_5, D_{ov}\} = aD_{ov}\gamma_5 D_{ov}$$

$$\{\gamma_5, D_c\} = 0, \quad D_c = D_{ov}/(1 - 1/2D_{ov})$$

Effective propagator: $1/(D_c + m)$

- Topology – Atiya-Singer theorem: $n_- - n_+ = Q = -\frac{1}{2}Tr(\gamma_5 D_{ov})$

local charge $q(x) = -\frac{1}{2}Tr(\gamma_5 D_{ov}(x,x))$

- Anomalous Ward identity -- $\partial_\mu A_\mu^0 = i2mP - i2N_f q(x)$

P. Hasenfrata et al., NP B643, 280 (2002)

- Quark spin crisis – J. Liang et al., PRD 98, 074505 (2018)

- EDM →0 when m → 0 -- D. Guadagnoli et al., JHEP 0304, 19 (2003)

Cluster Decomposition Principle

H. Araki, K. Hepp, and D. Ruelle, Helv. Phys. Acta 35, 164 (1962);
S. Weinberg, Quantum Theory of Fields,Vol 1, pp. 169

$$\left|\langle 0 | O_1(x) O_2(y) | 0 \rangle\right|_s \leq A r^{-3/2} e^{-Mr}, \quad \mathrm{r = x\text{-}y\ (space\ like)},$$

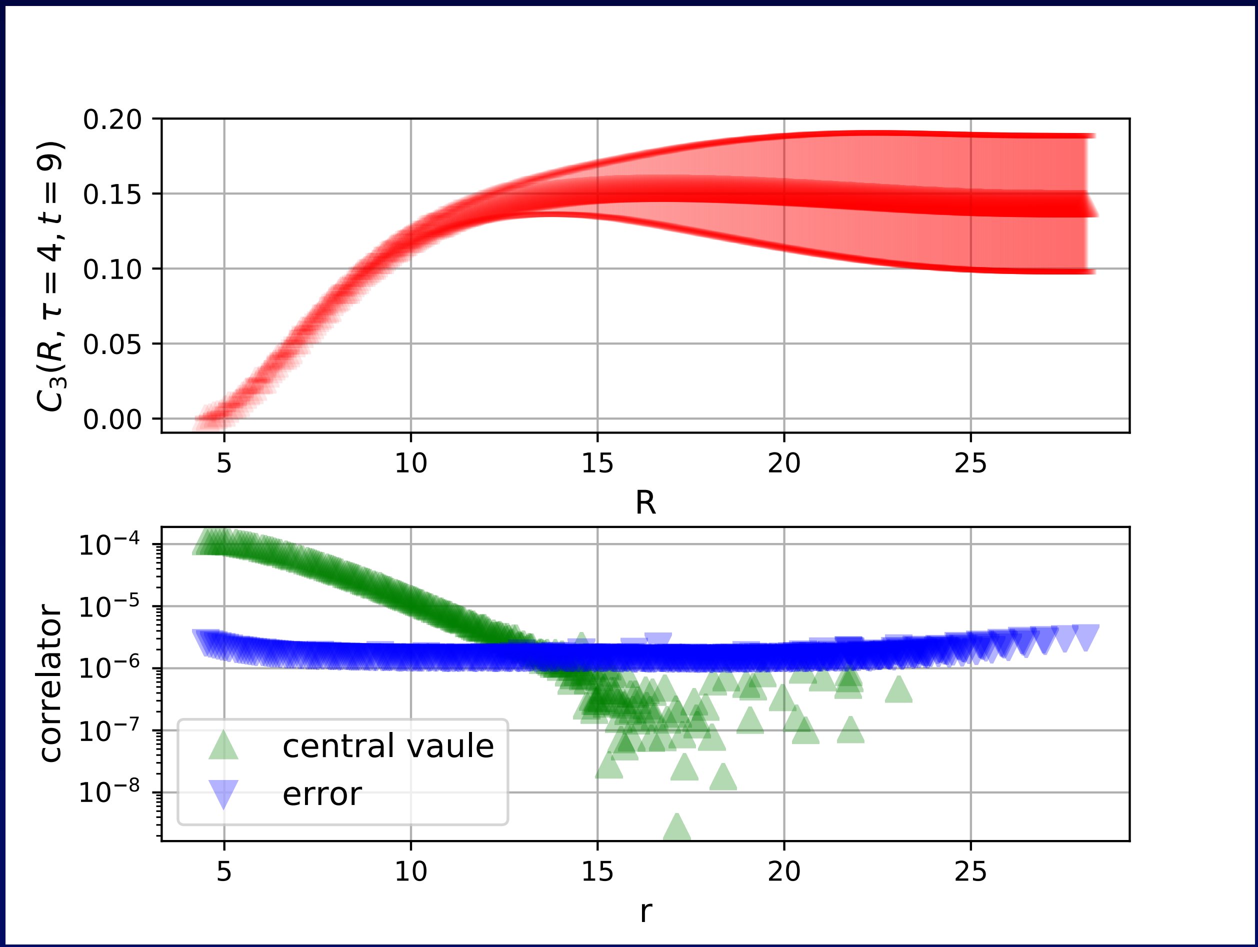
O_1 and O_2 are color-singlet operators,

Asympotic behavior of a boson propagator $K_1(r)/r$.

This point-to-point relation should hold for color-singlet operators separated by large Euclidean distance.

Strangeness in the Nucleon

$$C_3(R,\tau,t) = \left\langle \sum_{\vec{x}} \sum_{r<R} O_N(\vec{x},t) S(\vec{x}+\vec{r}\,',\tau) \bar{O}_N(\text{grid},0) \right\rangle, \quad \text{r} = \sqrt{(\vec{r}_x - \vec{x})^2 + (\tau - t)^2}$$



32^3 x 64 (32ID) RBC (4.6 fm, m_π=170 MeV)

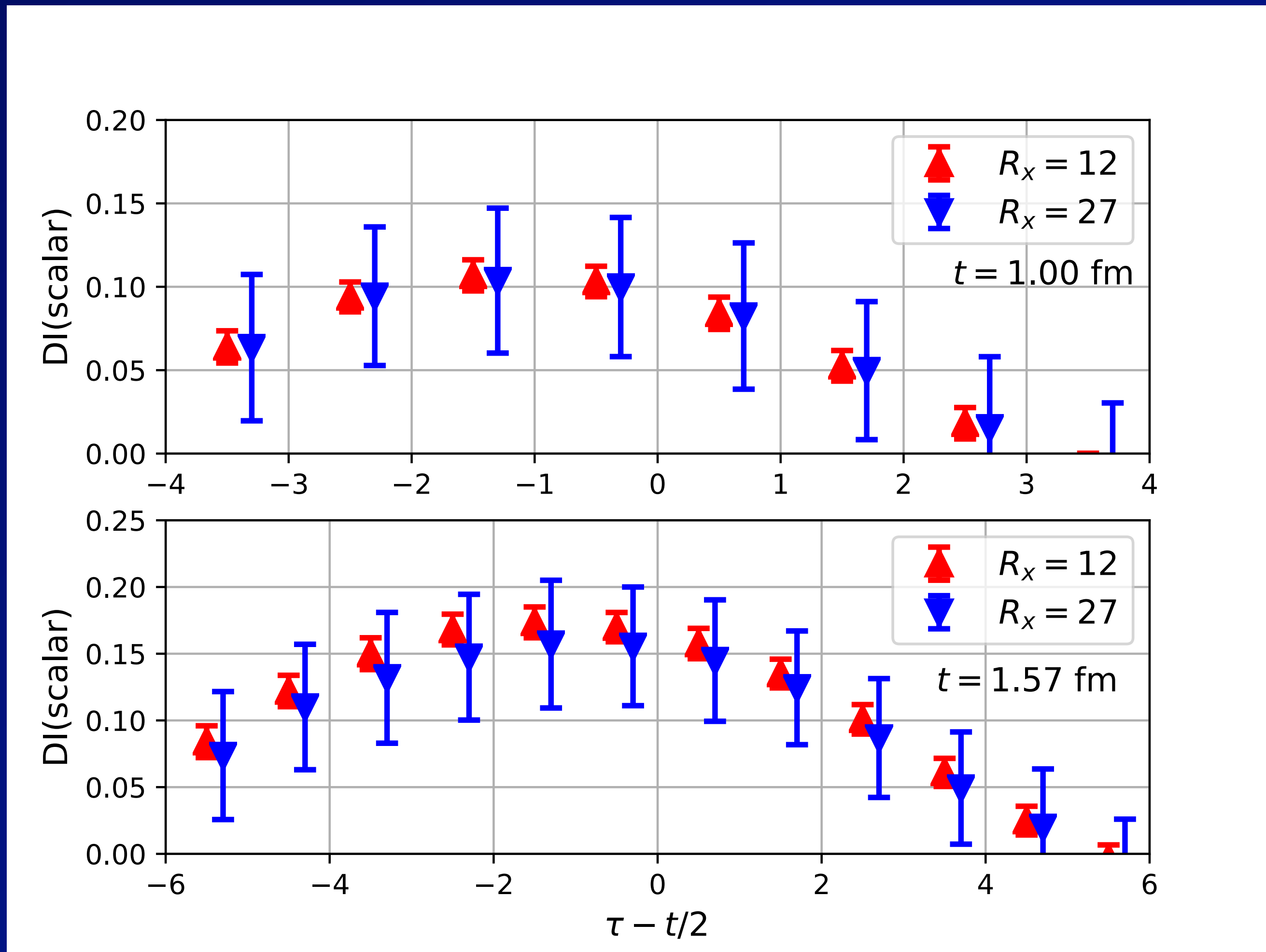
Disconnected Insertions

Cluster Decomposition Error Reduction (CDER)
Liu, **Liang** and Yang, PRD97:034507 (2018)

- Variance of disconnected insertion
- Vacuum insertion

$$\mathrm{Var}(R,t) = \frac{1}{V^2}\sum_{\vec{x}}\left(\left\langle \sum_{r_1<R} O_1(\vec{r}_1{}',t)\sum_{r_2<R} O_1^\dagger(\vec{x}+\vec{r}_2{}',t)\right\rangle \times \left\langle O_2(0,0)O_2^\dagger(\vec{x},0)\right\rangle\right) + \ldots$$

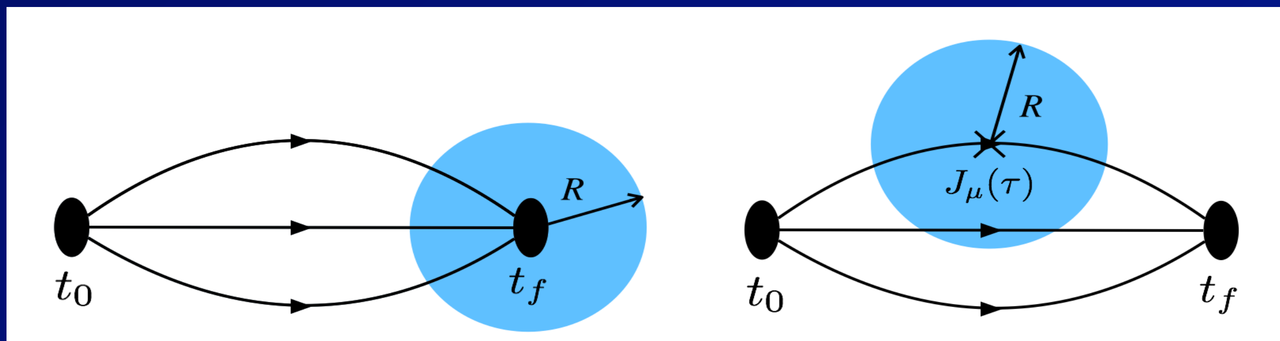
- $\mathrm{Var}(R_{max},t) = 1$, but $\mathrm{Var}(R_s,t) = V_s/V$.
- Gains singal to noise ratio: $S/N(R_s,t)/S/N(L,t) = (V/V_s)^{1/2}$
- Fast Fourier transform to calculate the truncated sum in relative coordinates.



$R_x = 12$
$R_x = 27$
$t = 1.00$ fm
DI(scalar)
0.20
0.15
0.10
0.05
0.00
−4
−3
−2
−1
0
1
2
3
4
$R_x = 12$
$R_x = 27$
$t = 1.57$ fm
DI(scalar)
0.25
0.20
0.15
0.10
0.05
0.00
−6
−4
−2
0
2
4
6
$\tau - t/2$

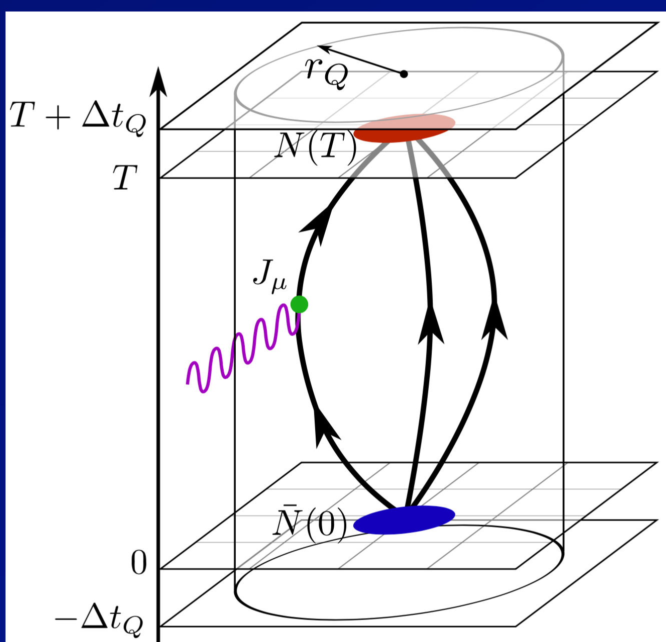
Error reduction

The cluster decomposition error reduction (CDER): Liu, **Liang** and Yang, PRD97:034507 (2018)



FFT -- V log V

Cylinder shape



Truncation in *t*-direction

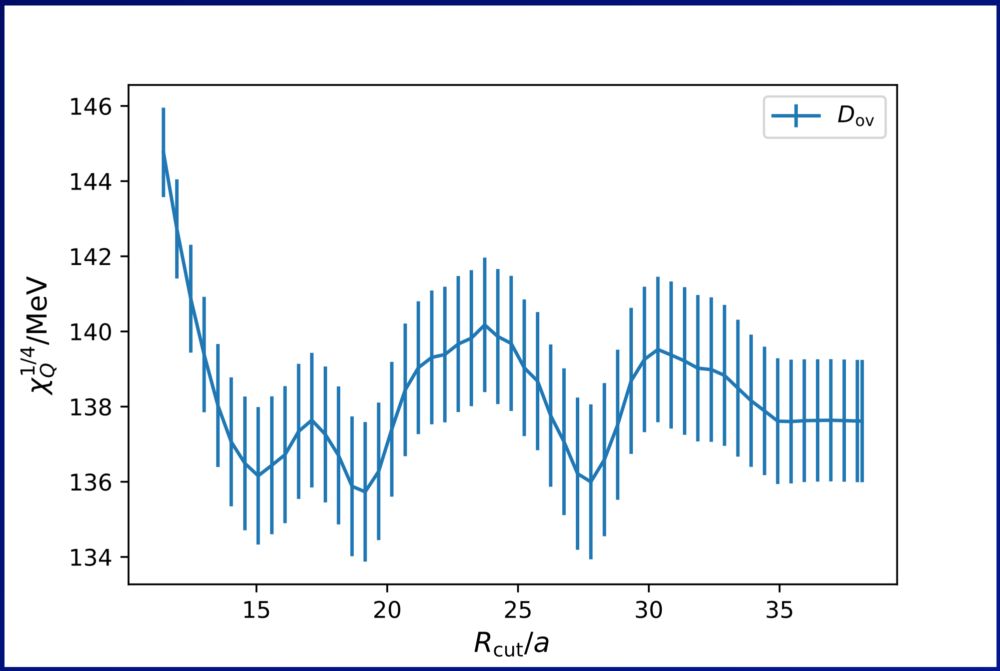
$$\overline{G}_3^{(\bar{Q})}(\boldsymbol{p}', t, \boldsymbol{q}, \tau, \Pi, \gamma_\mu, t_f, t_s)$$

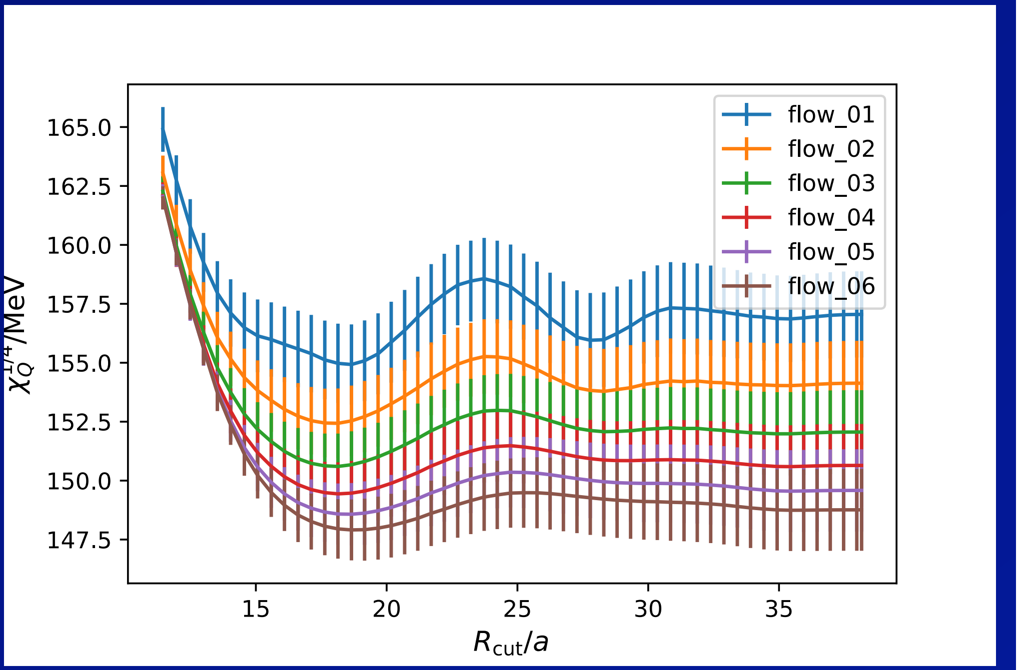
$$= a \sum_{\frac{\tau_Q}{a}=0}^{t_s/a} \Big[\Delta_3^{(\bar{Q})}(\boldsymbol{p}', t, \boldsymbol{q}, \tau, \tau_Q, \Pi, \gamma_\mu, t_f) + \Delta_3^{(\bar{Q})}(\boldsymbol{p}', t, \boldsymbol{q}, \tau, T-\tau_Q, \Pi, \gamma_\mu, t_f) \Big]$$

J. Dragos et al., arXiv:1902.03254

T. Izubuchi, H. Ohki and S. Syritsyn, arXiv:2004.10449

Topological susceptibility





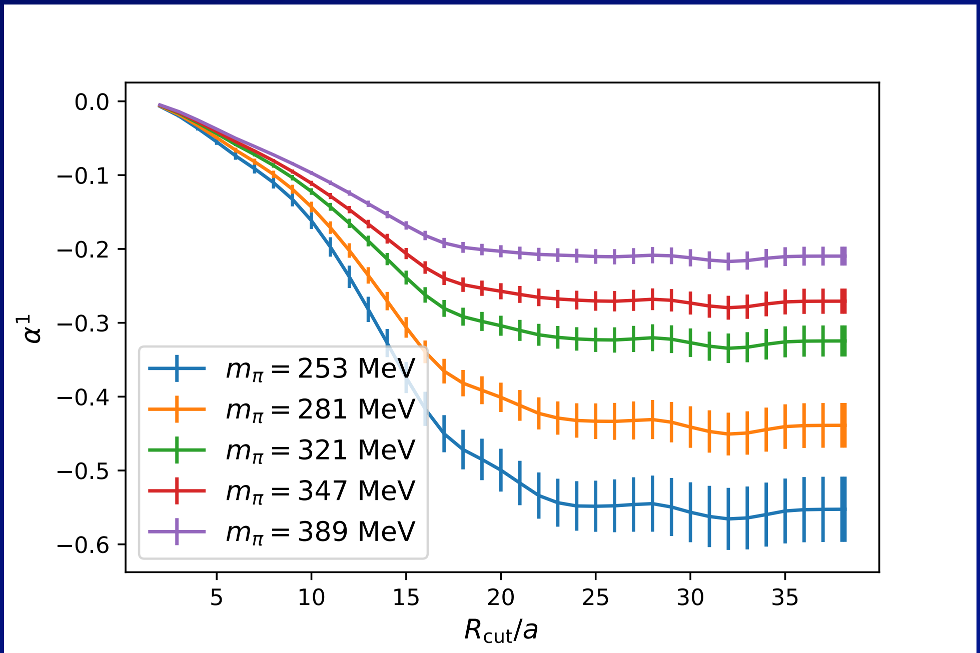
Overlap:137.6(1.6)

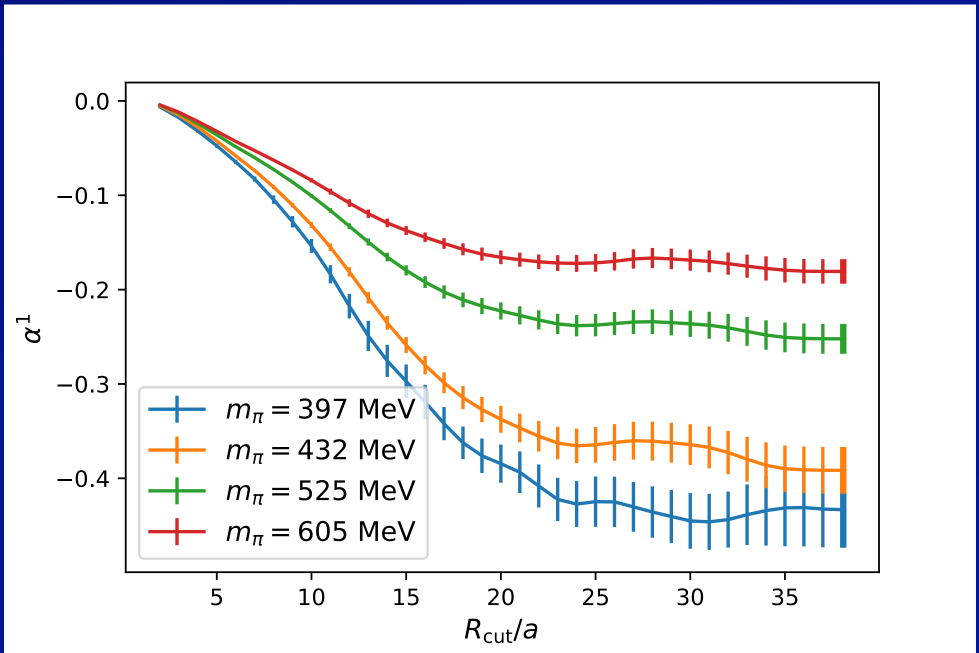
Flow_01: 157.1(1.8)
Flow_02: 154.1(1.8)
Flow_03: 152.1(1.8)
Flow_04: 150.6(1.8)
Flow_05: 149.6(1.7)
Flow_06: 148.8(1.7)

817 confs

For the flow time people usually use ($t_f = 4\sim5a^2$), although they do not drift beyond error, the values tend to be smaller as t_f goes larger. Overlap definition is taken for this work.

CP-violating angle α^1





Sea pion mass ~ 339 MeV

Sea pion mass ~ 576 MeV

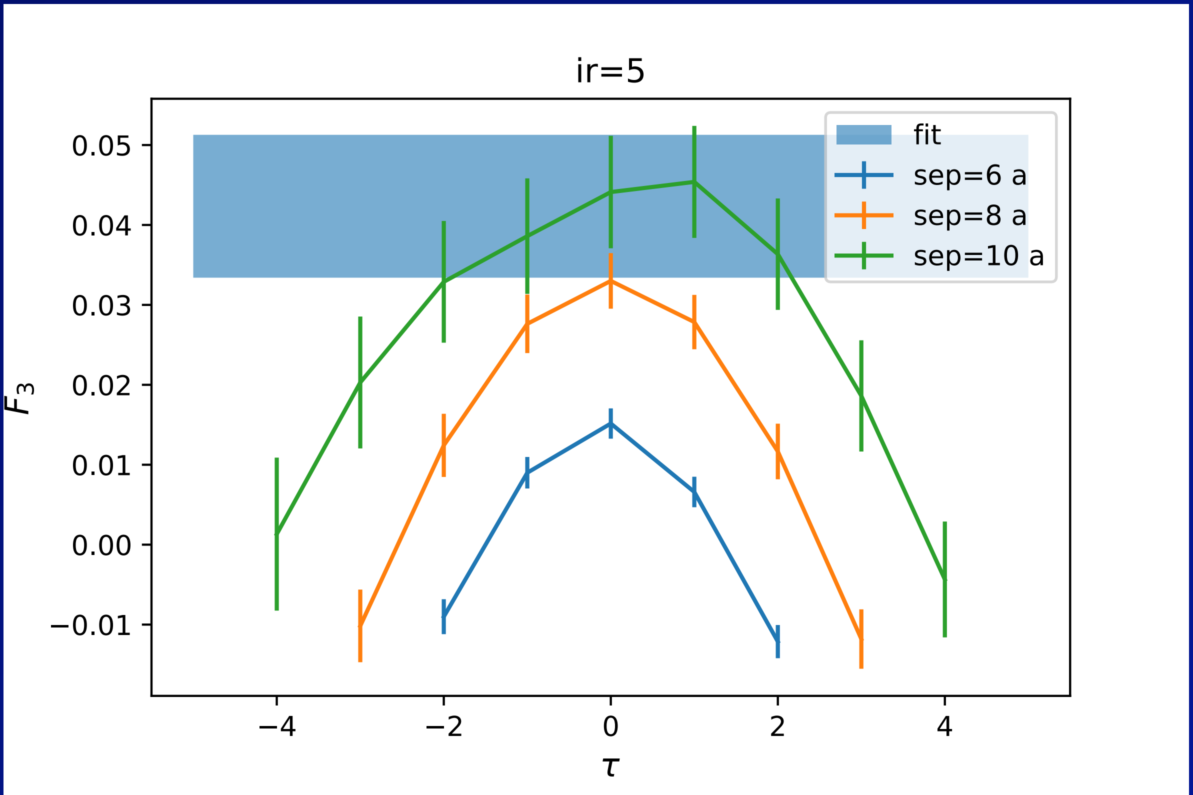
Ratio of 4-pt to 2-pt function

$$
\begin{aligned}
R_3^{Q,\mathrm{EM1}}(\Gamma_i,\gamma_4) &= \frac{\mathrm{Tr}\left[\Gamma_i G_3^Q(\gamma_4)\right]}{\mathrm{Tr}\left[\Gamma_e G_2(\vec{p}=0)\right]} \\
&= \frac{\mathrm{Tr}\left[-i\frac{1+\gamma_4}{2}\gamma_5\gamma_i\left[\alpha^1\gamma_5 W_4^{\mathrm{even}}\frac{1+\gamma_4}{2} + \frac{-i\not{p}_f+m}{2m}W_4^{\mathrm{even}}\alpha^1\gamma_5 + \frac{-i\not{p}_f+m}{2m}W_4^{\mathrm{odd}}\frac{1+\gamma_4}{2}\right]\right]}{2} \\
&= \frac{p_{f,i}}{2E_f}\left[\alpha^1 F_1 + \frac{E_f+3m}{2m}\alpha^1 F_2 + \frac{E_f+m}{2m}F_3'\right] \\
&= \frac{p_{f,i}}{2E_f}\left[\alpha^1 F_1 - \frac{E_f-m}{2m}\alpha^1 F_2 + \frac{E_f+m}{2m}\left(2\alpha^1 F_2 + F_3'\right)\right] \\
&= \frac{p_{f,i}}{2E_f}\left[\alpha^1 G_E + \frac{E_f+m}{2m}F_3\right].
\end{aligned}
$$

All negative

In the zero-momentum transfer limit, F_3 does NOT depend on α^1

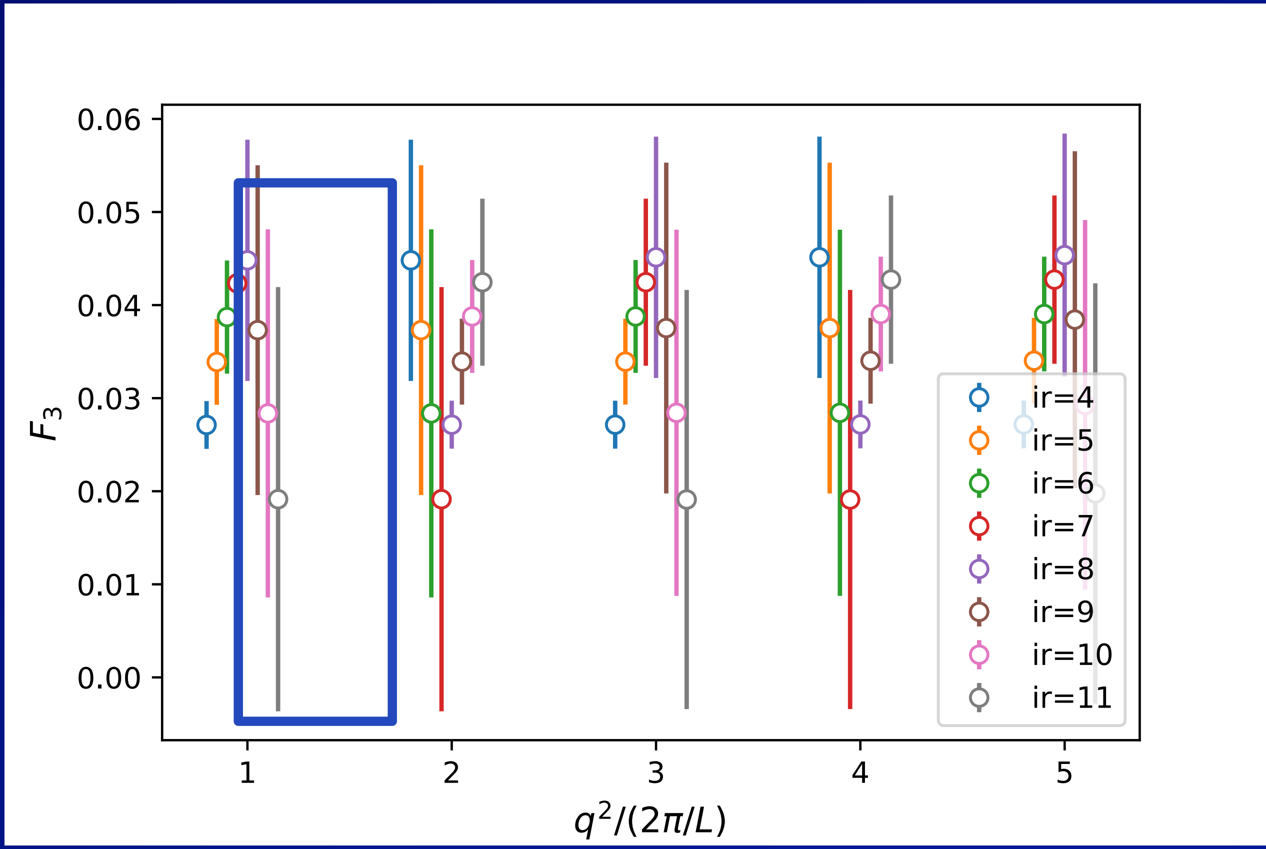
Two-state fit



1st mass, 1st momentum transfer, CDER cutoff = 5a, 24I005 lattice.

a = 0.114 fm, 24^3 x 64 DWF lattice, overlap valence, m_π = 339 MeV
8 grid sources with low mode substitution and 8 shifts in the spatial direction.

Momentum extrapolation - linear



ir=5 is OK, Linear fit in q^2

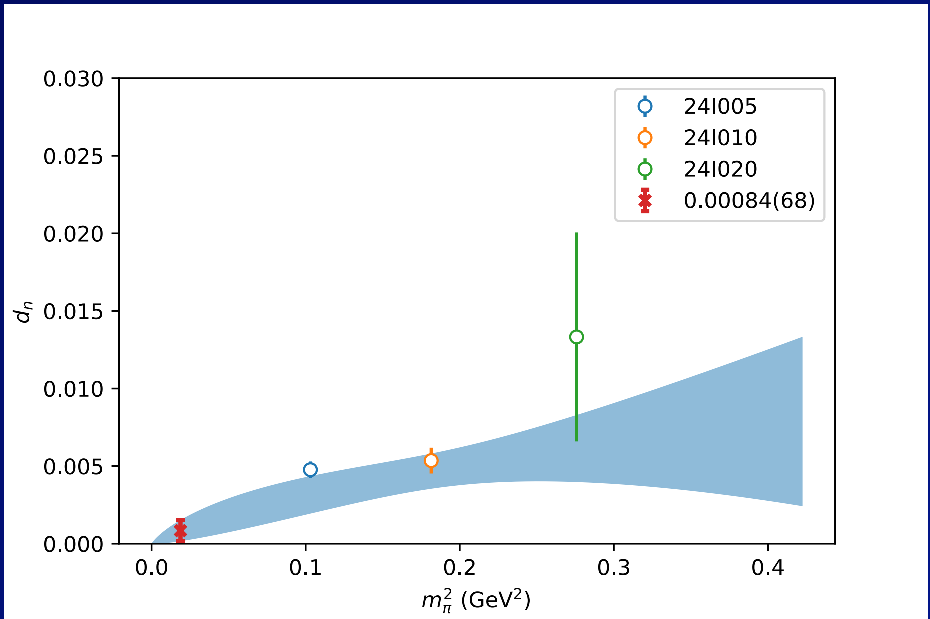
Pion masses

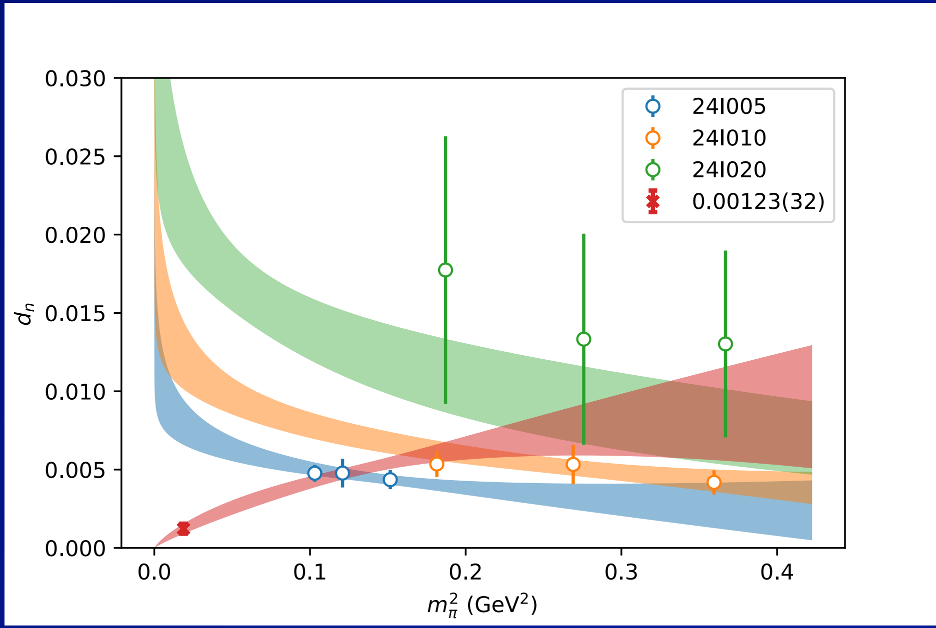
24I005:
Sea: 339 MeV
Valence (GeV): 0.321064, 0.347715, 0.389302

24I010
Sea: 432.2(1.4) MeV
Valence (GeV): 0.4260, 0.5187, 0.5996

24I020
Sea: 576.1 MeV
Valence (GeV): 0.4325, 0.5252, 0.6057

Preliminary results





$$d_n^{(PQ)} = \frac{e\,\overline{\theta}\,m_{\rm sea}}{4\pi^2 f^2}\left[F_\pi \log\left(\frac{m_\pi^2}{\mu^2}\right) + F_J \log\left(\frac{m_J^2}{\mu^2}\right)\right] + \overline{\theta}\,\frac{e}{\Lambda_\chi^2}\left[\frac{m_{\rm sea}}{2}\,c(\mu) + d\,\underline{(m_{\rm sea}-m_{\rm val})} + f q_{jl}\,\underline{(m_{\rm sea}-m_{\rm val})}\right]$$

D.O'Connell and M. J. Savage, PLB633:319 (2006)

$$c_0 m_{\pi,s}^2 \log\left(\frac{m_{\pi,v}^2}{m_N^2}\right) + c_1 m_{\pi,s}^2 + c_2\left(m_{\pi,s}^2 - m_{\pi,v}^2\right)$$

$d_n = 0.00123(32)\ \theta\ e \cdot \mathrm{fm}$

New production of the heaviest 24I020 lattice is going to finish. Currently only the results of 24I005 and 24I010 lattices are of full statistics. The extrapolated results are in the legend of the figures.

Summary and outlook

- Direct calculation at the physical point is numerically challenging.
- Chiral symmetry is of special importance in the calculation of nEDMs.
- Using overlap (chiral) fermions ensures a correct chiral limit of nEDM even at finite lattice spacings.
- Cluster Decomposition Error Reduction (CDER) will be essential for larger lattices.
- More statistics and more (partially-quenched) pion mass points will be added to have a more reliable chiral extrapolation. We have 8 times the statistics on the 24I020 lattice and 5 momentum transfers (3 currently) being analyzed. The statistical errors should be comparable to those of 24I005 and 24I010 lattices.
- Larger lattices (5.5 fm) at physical pion mass will be the next target. Systematic uncertainties (continuum and infinite volume limits) should be carefully estimated.