Updates on the HVP computation from the BMW collaboration

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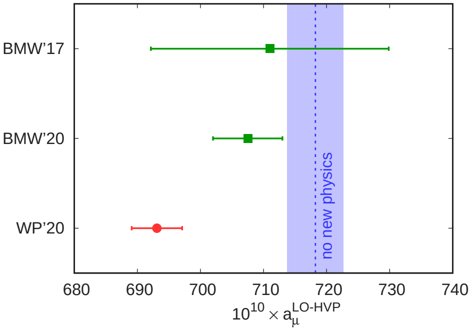
Motivation

- Tensions in $a_\mu^{\text{LO–HVP}}$
  - 4.2σ between WP'20 and experiment
  - 1.5σ between BMW'20 and experiment
  - 2.1σ between WP'20 and BMW'20

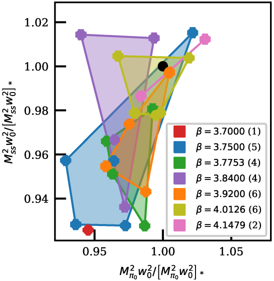


$$a = 0.048\,\mathrm{fm}$$

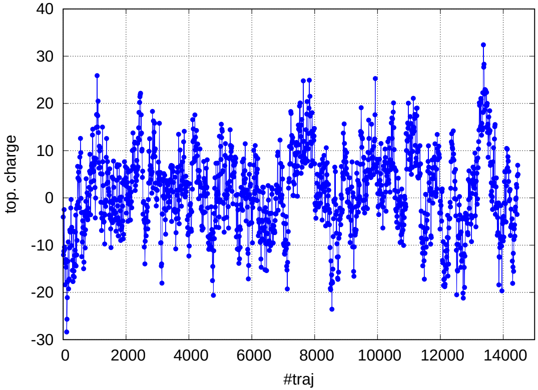
Simulations

- Tree-level Symanzik gauge action
- $N_f = 2 + 1 + 1$ staggered fermions
- stout smearing
  4 steps, $\varrho = 0.125$
- $L \sim 6$ fm, $T \sim 9$ fm
- M_π and M_K are around physical point

	β	a[fm]	$L \times T$
	3.7000	0.1315	48×64
	3.7500	0.1191	56×96
	3.7753	0.1116	56×84
	3.8400	0.0952	64×96
	3.9200	0.0787	80×128
	4.0126	0.0640	96×144
NEW $\longrightarrow$	4.1479	0.0483	128×192



Topological tunneling

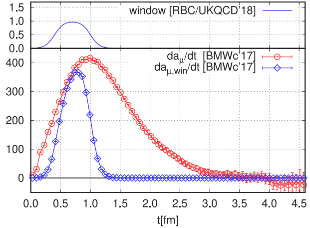


- $a = 0.0483\,\mathrm{fm}$
- Q defined at Wilson-flow time $t = w_0^2$
- $\tau_{\mathrm{int}} = 85(21)$

Window observable

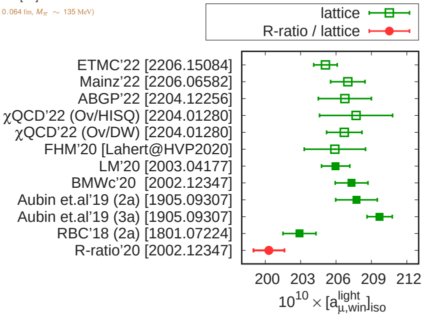
- Restrict correlator to window between $t_1 = 0.4\,\text{fm}$ and $t_2 = 1.0\,\text{fm}$

[RBC/UKQCD'18]

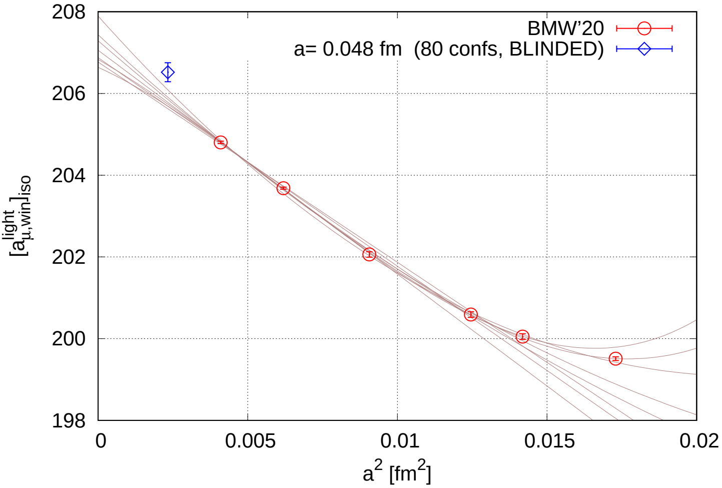


(144×96^3, $a \sim 0.064$ fm, $M_\pi \sim 135$ MeV)

- Less challenging than full a_μ
  - signal/noise
  - finite size effects
  - lattice artefacts (short & long)



Intermediate Window



Multi-operator measurements

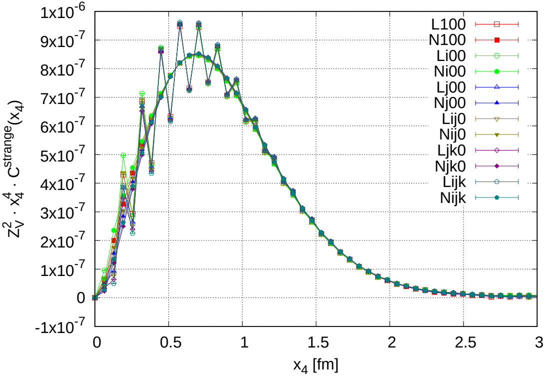
Staggered vector meson operators

Vector meson operators [Golterman '85]

OP	spin ⊗ taste		mult.	# hops	# paths	id*
1-timeslice operators						
Li00	$\gamma_i \otimes \xi_i$	local	3	0	1×1	4
L100	$\gamma_i \otimes 1$	conserved	3	1	2×1	5
Lij0	$\gamma_i \otimes \xi_i\xi_j$		6	1	2×1	10
Lijk	$\gamma_i \otimes \xi_i\xi_j\xi_k$		3	2	4×2	11
Lj00	$\gamma_i \otimes \xi_j$		6	2	4×2	15
Ljk0	$\gamma_i \otimes \xi_j\xi_k$		3	3	8×6	20
2-timeslice operators						
Ni00	$\gamma_i \otimes \xi_i\xi_4$		3	1	1×1	24
N100	$\gamma_i \otimes \xi_4$		3	2	2×2	25
Nij0	$\gamma_i \otimes \xi_i\xi_j\xi_4$		6	2	2×2	30
Nijk	$\gamma_i \otimes \xi_i\xi_j\xi_k\xi_4$		3	3	4×6	31
Nj00	$\gamma_i \otimes \xi_j\xi_4$		6	3	4×6	35
Njk0	$\gamma_i \otimes \xi_j\xi_k\xi_4$		3	4	8×24	40
Total	12		48			

* [Altmeyer *et.al.* '93], [Ishizuka *et.al.* '94]

Shape of correlator

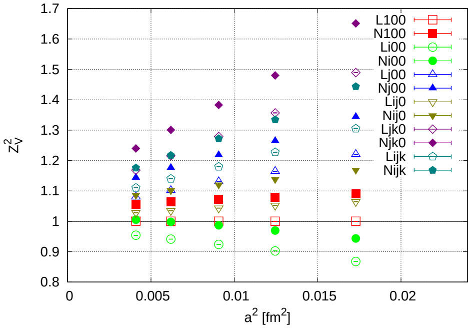


- 1-timeslice operators $\longrightarrow$ staggered oscillations
- 2-timeslice operators $\longrightarrow$ no staggered oscillations

Renormalization

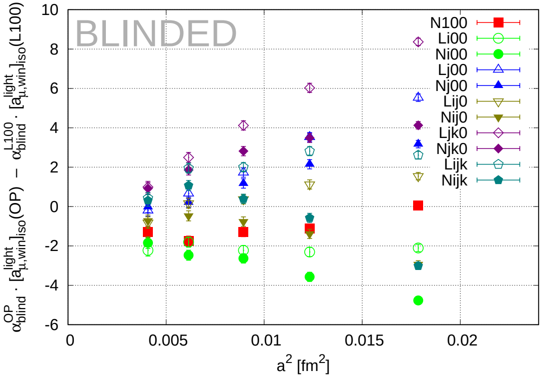
- $L100 \quad (\gamma_i \otimes 1)$ is conserved $\longrightarrow$ trivial renormalization: $Z_V(\text{L100}) = 1$
- Other tastes: $J^\mu(\text{L100}) = Z_V(\text{OP}) \cdot J^\mu(\text{OP}) + \mathcal{O}(a^2)$
- Z_V is mass-independent

$$Z_V^2(\text{OP}) \stackrel{\text{def}}{=} \frac{a_{\mu,\text{win}}^{\text{strange}}(\text{L100})}{a_{\mu,\text{win}}^{\text{strange}}(\text{OP})}$$



Collapse

- LMA implementation of all 12 operators (48 components)
- 5 lattice spacings
- 1 ensemble at each β
- 48 configurations on each ensemble



QED valence contribution

QED strategy

- Rewrite dynamical QED as quenched QED expectation values

$$\langle C\rangle_{\text{QCD+unquenched QED}} = \frac{\left\langle\left\langle C(U,A)\,\frac{\det M(U,A)}{\det M(U,0)}\right\rangle_{\text{quenched QED}}\right\rangle_{\text{QCD}}}{\left\langle\left\langle\frac{\det M(U,A)}{\det M(U,0)}\right\rangle_{\text{quenched QED}}\right\rangle_{\text{QCD}}}$$

- Expand to first order in e^2 and $\delta m = m_d - m_u$

e_v: valence charge, e_s: sea charge

  - $C(U,A) \approx C_0(U) + \frac{\delta m}{m_l}\cdot C'_m(U) + e_v\cdot C'_1(U,A) + e_v^2\cdot C''_2(U,A)$
  - $\left(\prod_{f=u,d,s,c}\frac{\det M^{(f)}[U,A]}{\det M^{(f)}[U,0]}\right)^{1/4} \approx 1 + e_s\cdot\frac{d_1(U,A)}{d_0(U)} + e_s^2\cdot\frac{d_2(U,A)}{d_0(U)}$

$\mathcal{O}(\delta m)$ sea effect vanishes

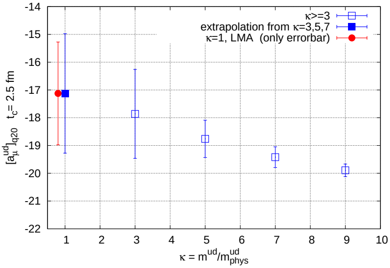
$$\langle C\rangle_{\text{QCD+QED}} = \langle C_0(U)\rangle_U + \frac{\delta m}{m_l}\cdot\langle C'_m(U)\rangle_U + \frac{e_v^2}{2}\cdot\left\langle\langle C''_2(U,A)\rangle_{A,\text{q.}}\right\rangle_U +$$
$$+ e_v e_s\cdot\left\langle\left\langle C'_1(U,A)\cdot\frac{d_1(U,A)}{d_0(U)}\right\rangle_{A,\text{q.}}\right\rangle_U +$$
$$+ \frac{e_s^2}{2}\cdot\left\langle\left(C_0(U)-\langle C_0(U)\rangle_U\right)\cdot\left\langle\frac{d_2(U,A)}{d_0(U)}\right\rangle_{A,\text{q.}}\right\rangle_U$$

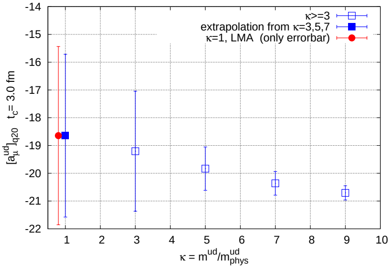
QED valence contribution

- Valence sector of quark-connected HVP

$$\left\langle \left\langle C_2''(U,A) \right\rangle_{A,\mathrm{q.}} \right\rangle_U =$$  $+$ 

- Compute as finite difference: measure $C(0)$, $C(+\frac{1}{3}e_*)$, $C(-\frac{1}{3}e_*)$
- Measure at $\kappa = \frac{m^{ud}_{\mathrm{val}}}{m^{ud}_{\mathrm{phys}}} = 3, 5, 7 \longrightarrow \kappa = 1$
- New strategy: measure directly at $\kappa = 1$ using LMA





Conclusions & Outlook

Conclusions & Outlook

- $a = 0.048\,\mathrm{fm}$
  - Will be included for $a_{\mu,\mathrm{win}}^{\mathrm{light}}$ — still blinded
  - Need to collect more statistics to include in $a_\mu^{\mathrm{LO-HVP}}$
- Multi-operators
  - $a_{\mu,\mathrm{win}}^{\mathrm{light}}$ measurements with 48 components of 12 operators
- QED at $\kappa = 1$
  - Still in development/testing phase

Staggered meson operators

- Continuum meson operators

$$\overline{\psi}\,\Gamma\,\psi \qquad \Gamma: \quad \text{combination of } \gamma \text{ matrices}$$

- Staggered meson operators [Follana *et.al.* 2007]

$$\overline{\psi}(\gamma_s \otimes \xi_t)\psi \qquad \gamma_s: \quad \text{spin}, \qquad \xi_t: \quad \text{taste}$$

- Staggered phase at $x = (x_1, x_2, x_3, x_4)$

$$\gamma_x = \gamma_1^{x_1}\,\gamma_2^{x_2}\,\gamma_3^{x_3}\,\gamma_4^{x_4} \qquad \text{only} \quad \text{mod 2} \quad \text{counts}$$

- Meson operator correspoding to spin $s = (s_1, s_2, s_3, s_4)$ and taste $t = (t_1, t_2, t_3, t_4)$:

$$\overline{\psi}(\gamma_s \otimes \xi_t)\psi \quad \longrightarrow \quad \bar{\chi}(x)\,\frac{1}{4}\,\mathrm{Tr}\left[(\gamma_t)^\dagger\,(\gamma_x)^\dagger\,\gamma_s\,\gamma_{x\oplus(s\hat{+}t)}\right]\,\chi(x\oplus(s\hat{+}t))$$

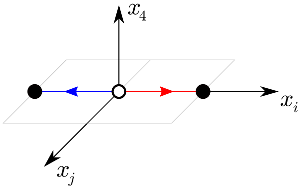
- $s\hat{+}t \equiv s + t \mod 2$ $\qquad \gamma_s = \gamma_1^{s_1}\,\gamma_2^{s_2}\,\gamma_3^{s_3}\,\gamma_4^{s_4}$
- $x \oplus \delta x$: stay within 2^4 hypercube

Staggered meson operators

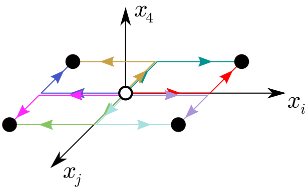
- With gauge fields: $s=(s_1,s_2,s_3,s_4)\quad t=(t_1,t_2,t_3,t_4)$

$$\bar\psi(\gamma_s\otimes\xi_t)\psi \quad\longrightarrow\quad \bar\chi(x)\,\frac{1}{4}\,\mathrm{Tr}\Big[(\gamma_t)^\dagger\,(\gamma_x)^\dagger\,\gamma_s\,\gamma_{x\tilde\pm(s\hat+t)}\Big]\;U_{x\,,\,x\tilde\pm(s\hat+t)}\,\chi(x\tilde\pm(s\hat+t))$$

- $U_{x,y}$: Average over shortest paths between x and y
- $s\hat+t\equiv s+t \mod 2$
- $x\tilde\pm\delta x$: Average over all reflections in spatial directions $\pm\delta x_1,\pm\delta x_2,\pm\delta x_3$
- Take 0 spatial momentum: $\sum_{x_1,x_2,x_3}$
- 1-timeslice: $(s\hat+t)_4=0$ $\quad\displaystyle\sum_{\mathbf{x}}\bar\chi(\mathbf{x},x_4)\;J(\mathbf{x},x_4\,;\,\mathbf{x}\tilde\pm(\mathbf{s}\hat+\mathbf{t}),x_4)\;\chi(\mathbf{x}\tilde\pm(\mathbf{s}\hat+\mathbf{t}),x_4)$



L100 $(\gamma_i\otimes 1,\quad$ conserved$)$



Lj00 $(\gamma_i\otimes\xi_j,\quad j\neq i)$

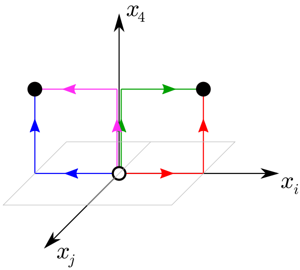
Staggered meson operators

$$\overline{\psi}(\gamma_s \otimes \xi_t)\psi \quad \longrightarrow \quad \bar{\chi}(x)\ \frac{1}{4}\,\mathrm{Tr}\Big[(\gamma_t)^\dagger\,(\gamma_x)^\dagger\,\gamma_s\,\gamma_{x\tilde{\pm}(s\hat{+}t)}\Big]\ U_{x\,,\,x\tilde{\pm}(s\hat{+}t)}\ \chi(x\tilde{\pm}(s\hat{+}t))$$

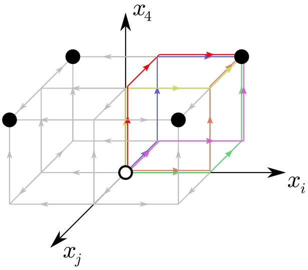
- Time-nonlocal (2-timeslice) operators: $(s\hat{+}t)_4 = 1$

$$\sum_{\mathbf{x}} \bar{\chi}(\mathbf{x}, x_4)\ J^+(\mathbf{x}, x_4\,;\,\mathbf{x}\tilde{\pm}(\mathbf{s}\hat{+}\mathbf{t}), x_4+1)\ \chi(\mathbf{x}\tilde{\pm}(\mathbf{s}\hat{+}\mathbf{t}), x_4+1)+$$

$$+\,\bar{\chi}(\mathbf{x}, x_4+1)\ J^-(\mathbf{x}, x_4+1\,;\,\mathbf{x}\tilde{\pm}(\mathbf{s}\hat{+}\mathbf{t}), x_4)\ \chi(\mathbf{x}\tilde{\pm}(\mathbf{s}\hat{+}\mathbf{t}), x_4)$$



N100 $(\gamma_i \otimes \xi_4)$



Nj00 $(\gamma_i \otimes \xi_j\xi_4,\quad j \neq i)$