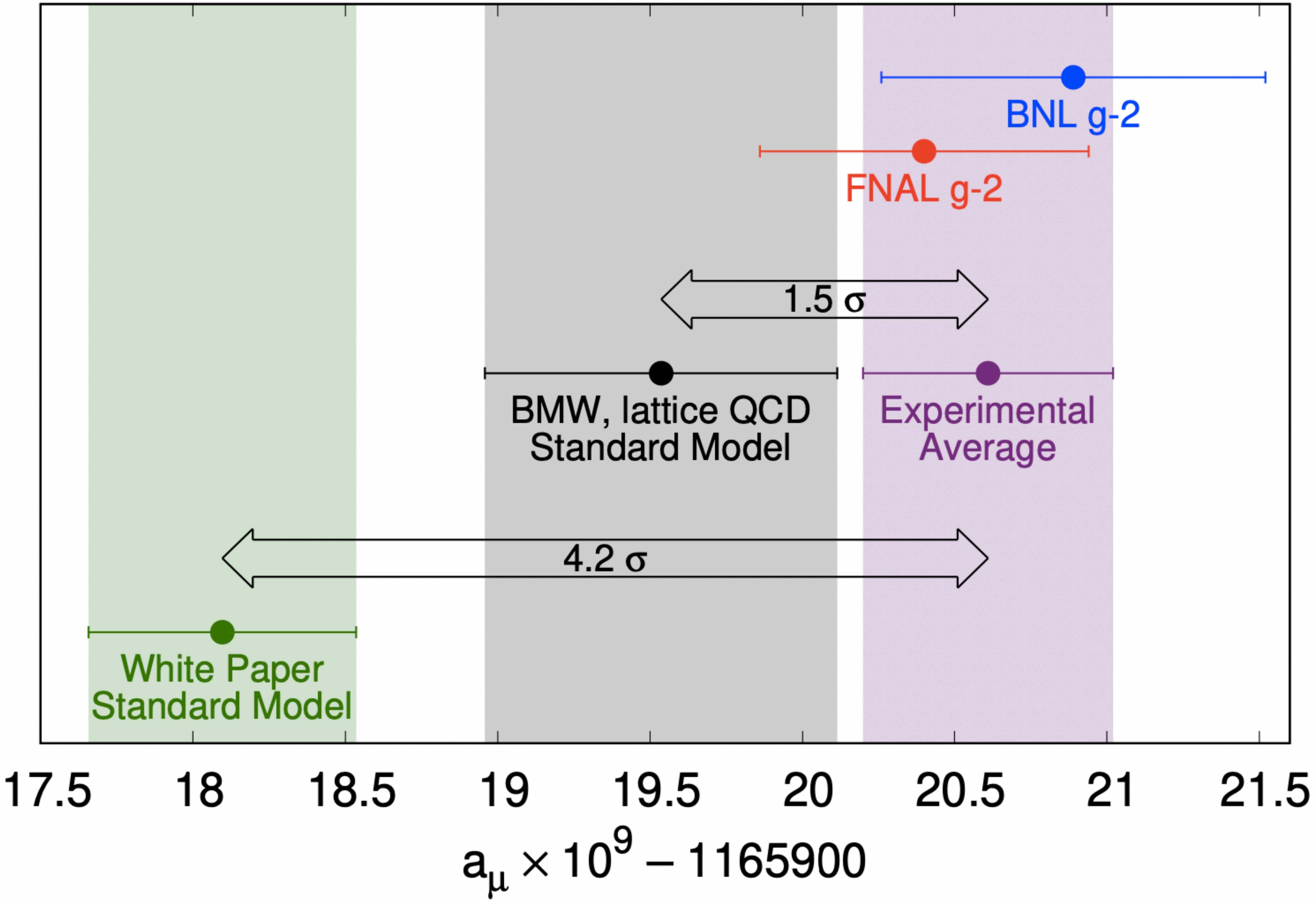
Intermediate window observable for the muon g-2 from overlap valence quarks on staggered ensembles

A. Yu. Kotov for the BMW collaboration

JÜLICH Forschungszentrum

Lattice 2022

HVP contribution to muon anomalous magnetic moment
Budapest-Marseille-Wuppertal collaboration



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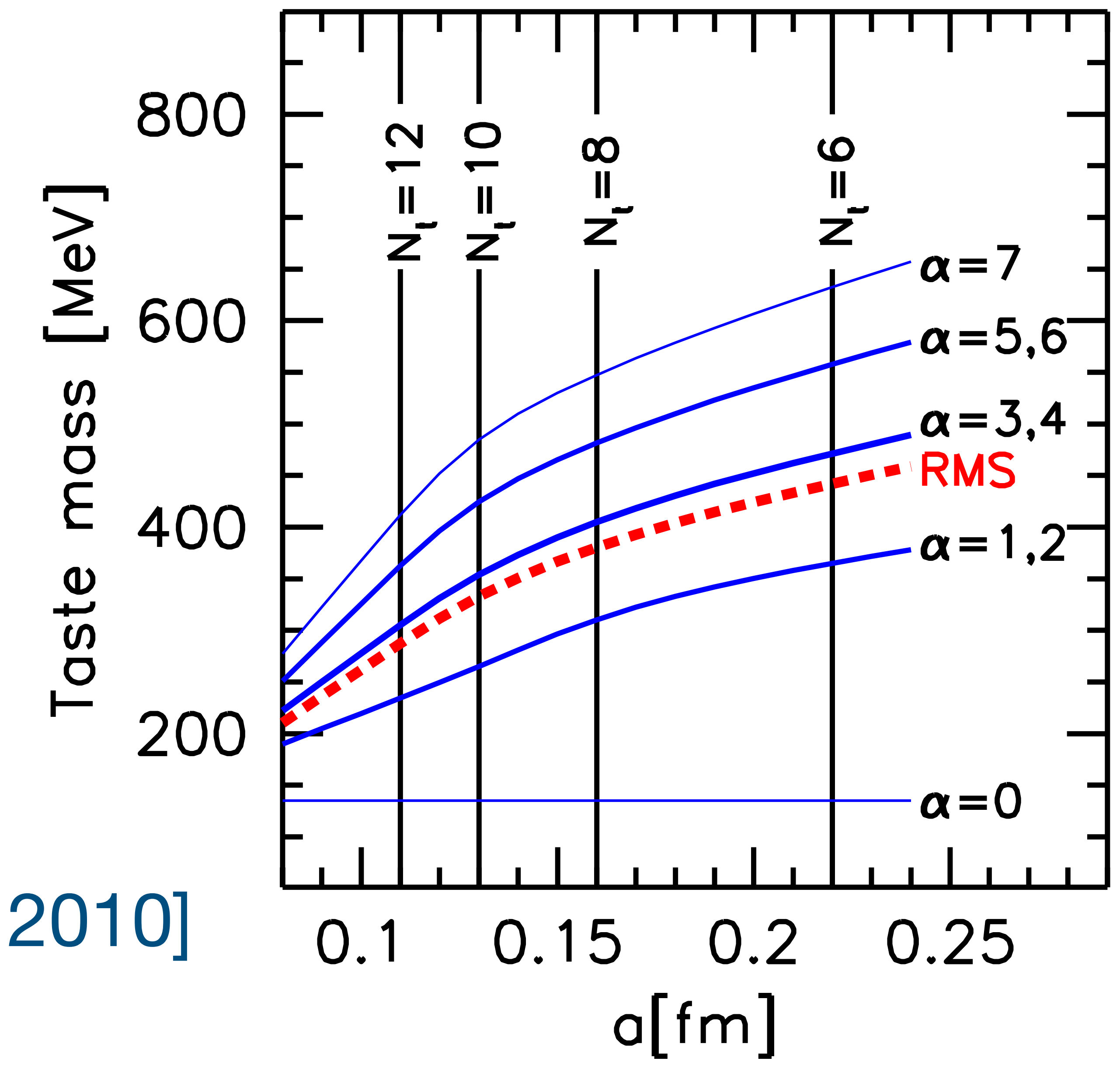

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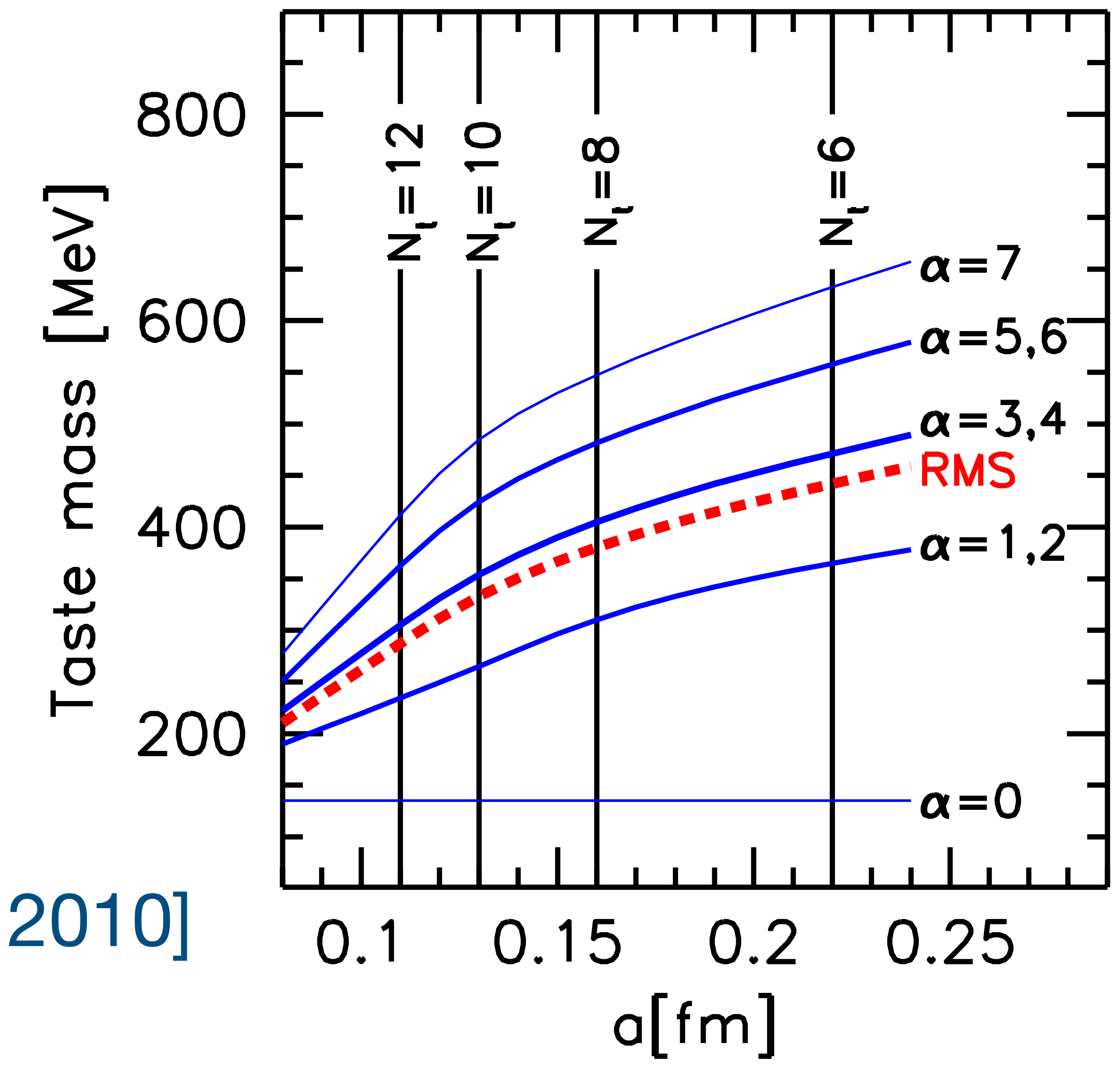

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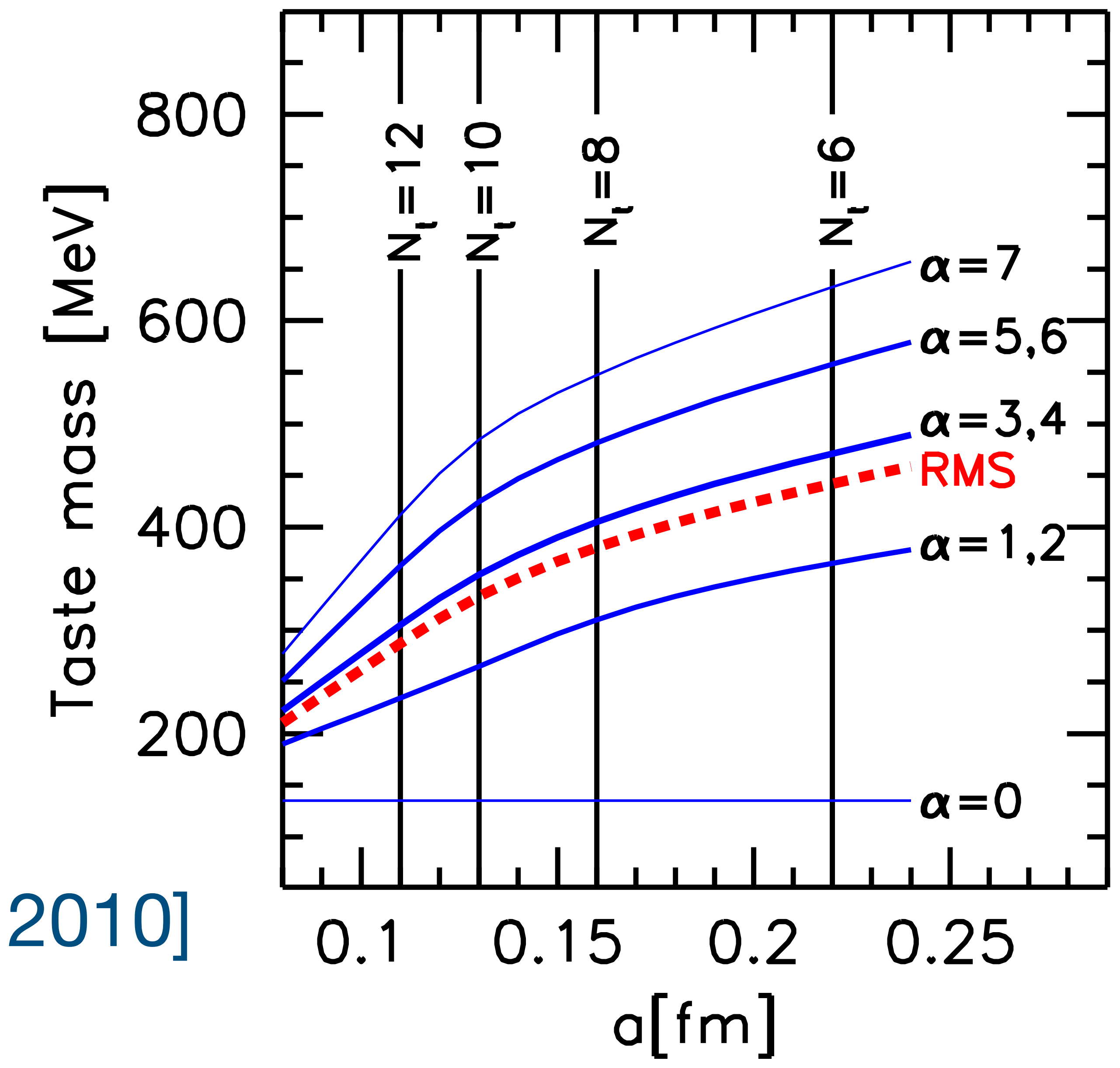

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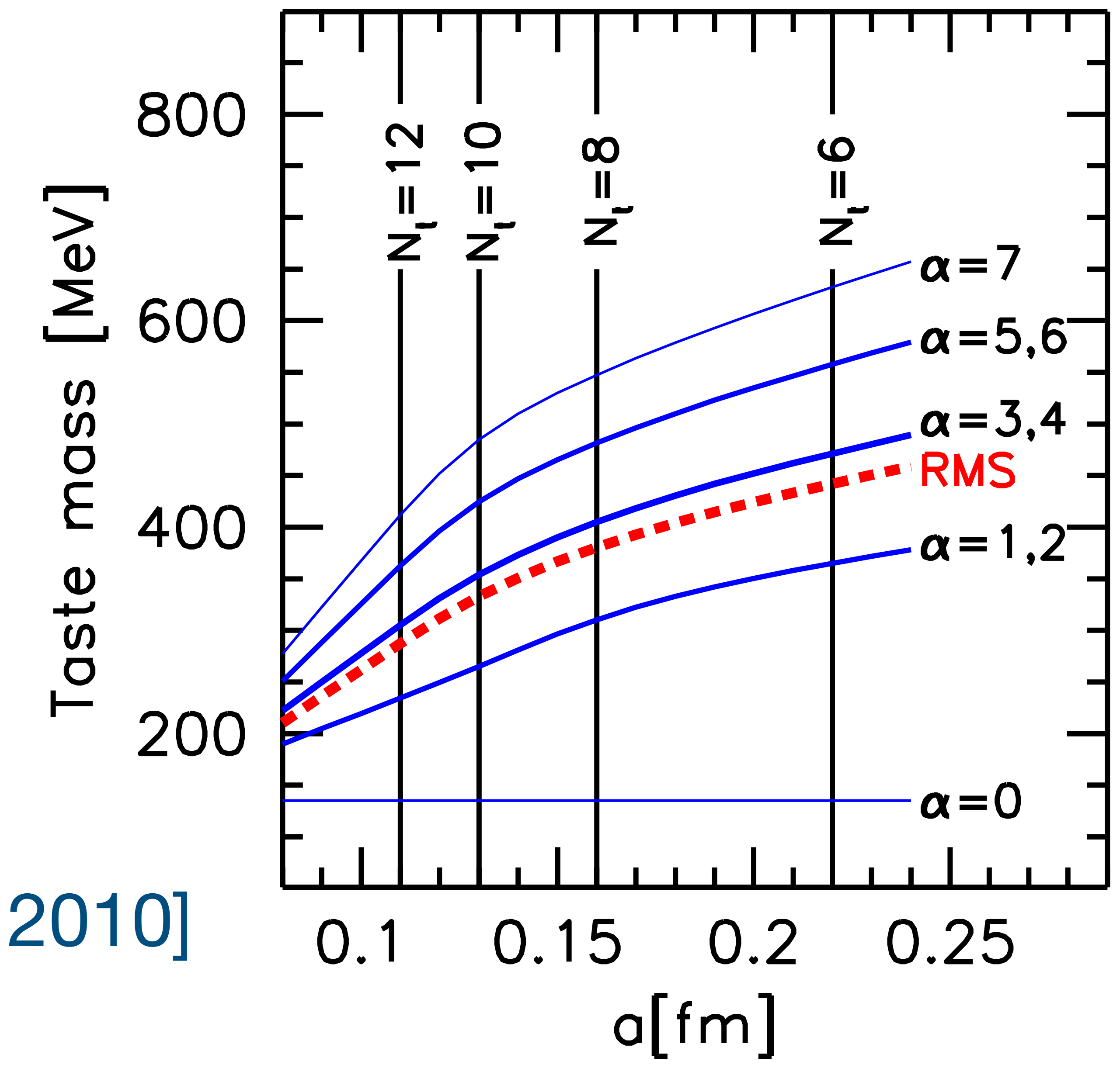

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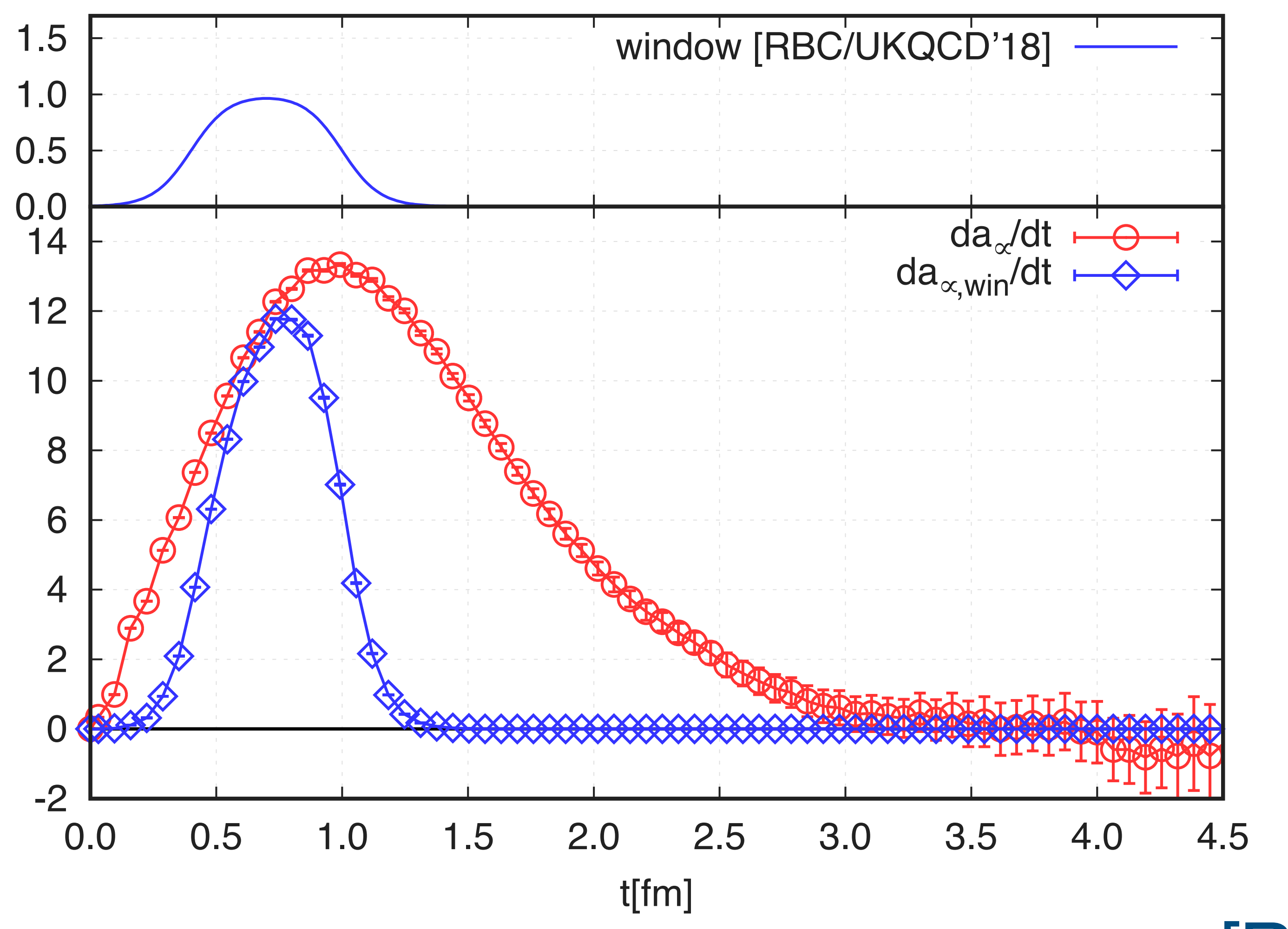

[RBC/UKQCD, 2018]

Alternative: overlap fermions

- Very expensive
- Restrict to window observable (less noisy, smaller artifacts):

$$G(t) \to G(t)W(t; t_1, t_2)$$

Setup for the crosscheck with chiral fermions

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- Mixed action: staggered sea (4-stout smearing), overlap valence:
    - 2 steps of HEX smearing [S. Capitani et al., 2006]
    - Local current $j_\mu = \bar{\psi}(x)\gamma_\mu\psi(x)$: needs renormalization
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- Lattice size $L \sim 3$ fm ($L \sim 6$ fm for standard staggered runs)

a [fm]	L x T	#conf
0.1315	24 x 48	716
0.1116	28 x 56	887
0.0952	32 x 64	1110
0.0787	40 x 80	923
0.0640	48 x 96	577

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$$\zeta(t) = \frac{\langle P(T/2)V_4(t)\bar{P}(0)\rangle}{\langle P(T/2)\bar{P}(0)\rangle}$$

$$P(t) = \sum_{\vec{x}} (\bar{\psi}_2\gamma_5\psi_1)(\vec{x}, t) \quad V_\mu(t) = \sum_{\vec{x}} (\bar{\psi}_1\gamma_\mu\psi_1)(\vec{x}, t)$$

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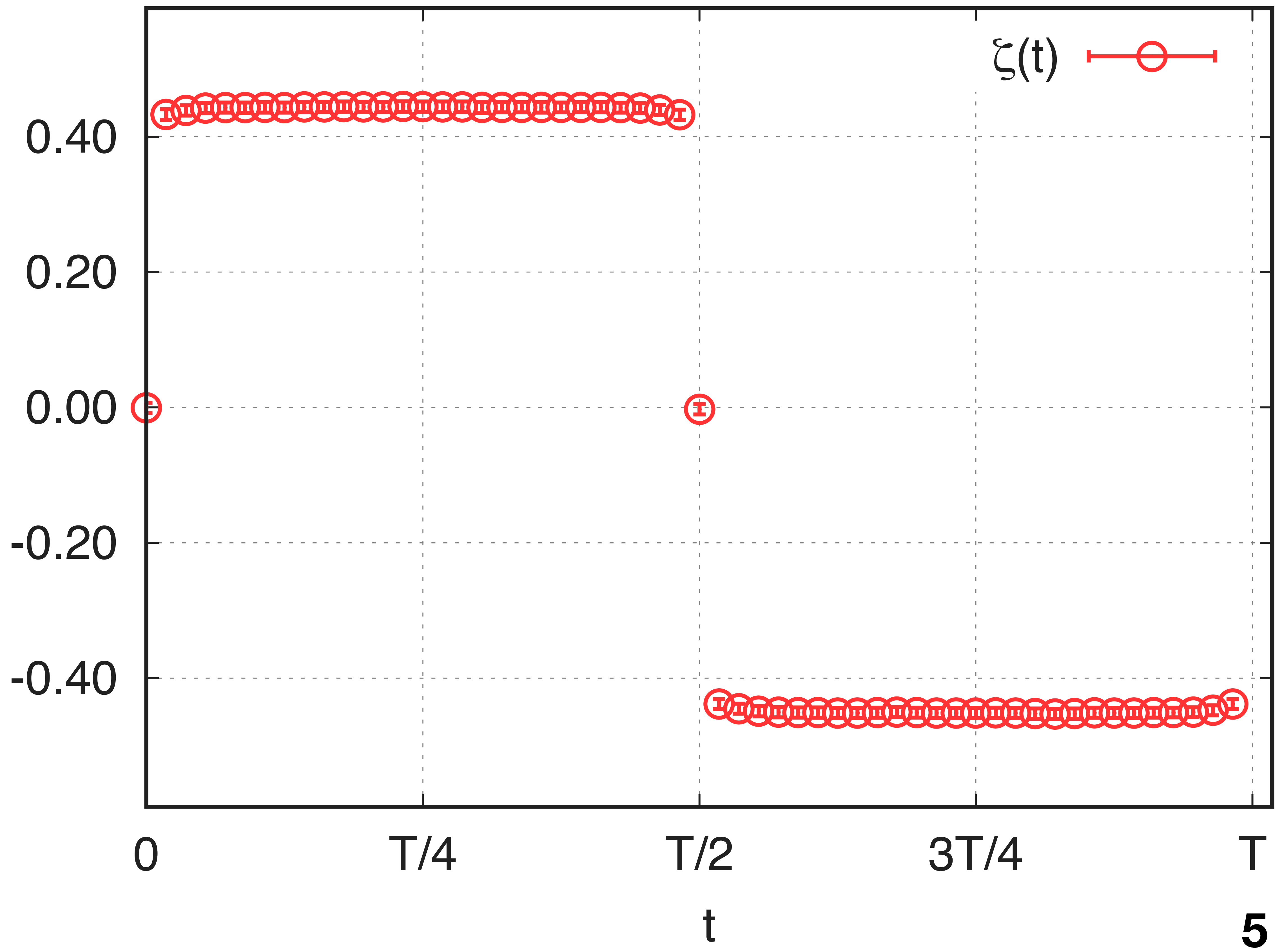
Z_V: from matching $\zeta(t)$ at some physical t

We use: $Z_V = [\zeta(T/4) - \zeta(3T/4)]^{-1}$

$a = 0.1116$ fm, 28^3x56



Low mode averaging

$$C(t,\bar{t}) \equiv C^{\mathrm{conn}}(t,\bar{t}) = -\frac{1}{3L^3}\sum_{\vec{x},\vec{\bar{x}},\mu=1,2,3} \mathrm{Re}\ \mathrm{tr}\ [J_{\mu,\vec{x},t}M^{-1}J_{\mu,\vec{\bar{x}},\bar{t}}M^{-1}]$$

$$Mv_i = \lambda_i v_i$$

$$M^{-1} = M_e^{-1} + M_r^{-1} \qquad M_e^{-1} = \sum_i \frac{1}{\lambda_i} v_i v_i^\dagger \qquad M_r^{-1} = M^{-1}\left(1 - \sum_i v_i v_i^\dagger\right)$$

$$C = C_{ee} + C_{re} + C_{rr}$$

C_{ee} is calculated exactly without stochastic sources

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- Need: low modes of $D_{\text{ov}}(m_{\text{ov}} = 0) = \frac{1}{2}\left(1 + \gamma_5 \text{sgn}\ \gamma_5 D_{\text{w}}(-m_{\text{w}})\right)$
- $D_{\text{ov}}(m_{\text{ov}} \neq 0) = \left(1 - \mu/2\right) D_{\text{ov}} + \mu, \quad \mu = m_{\text{ov}}/m_{\text{w}}$ is straightforward

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[R. Edwards, U. Heller, R. Narayanan, 2000]

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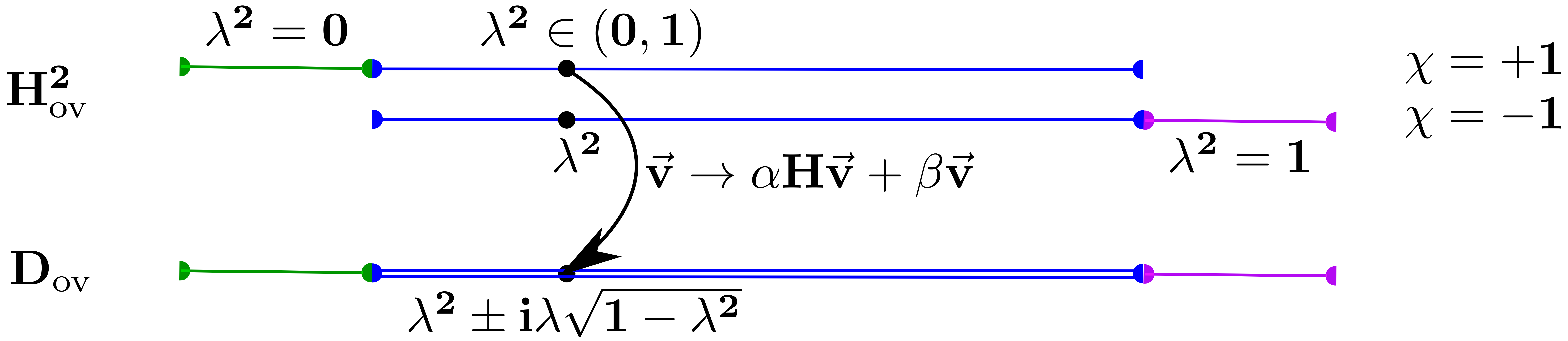
  [R. Edwards, U. Heller, R. Narayanan, 2000]
- $\lambda^2 \neq 0,\ 1$: degenerate pairs of opposite chirality
- $\lambda^2 = 0$: only for one chirality: ${\rm sign}\ Q \Longrightarrow$ Guess sign of topological charge ${\rm sign}\ Q$



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  $$V_\mu(x,\tau) = -g_0^2 \left[\partial_{x,\mu} S_G(V(\tau))\right] V_\mu(x,\tau)$$
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- Best option: Iwasaki at GF time $t = t_0/2$

[C. Alexandrou et al., 2020]

$a = 0.0787\ \mathrm{fm}$, 40^3x80, 48 configs

	$t_0/2$	t_0	$2t_0$
WIL	0.75	0.66	0.62
SYM	0.71	0.62	0.62
IWA	0.80	0.71	0.76
DBW2	0.60	0.60	0.60

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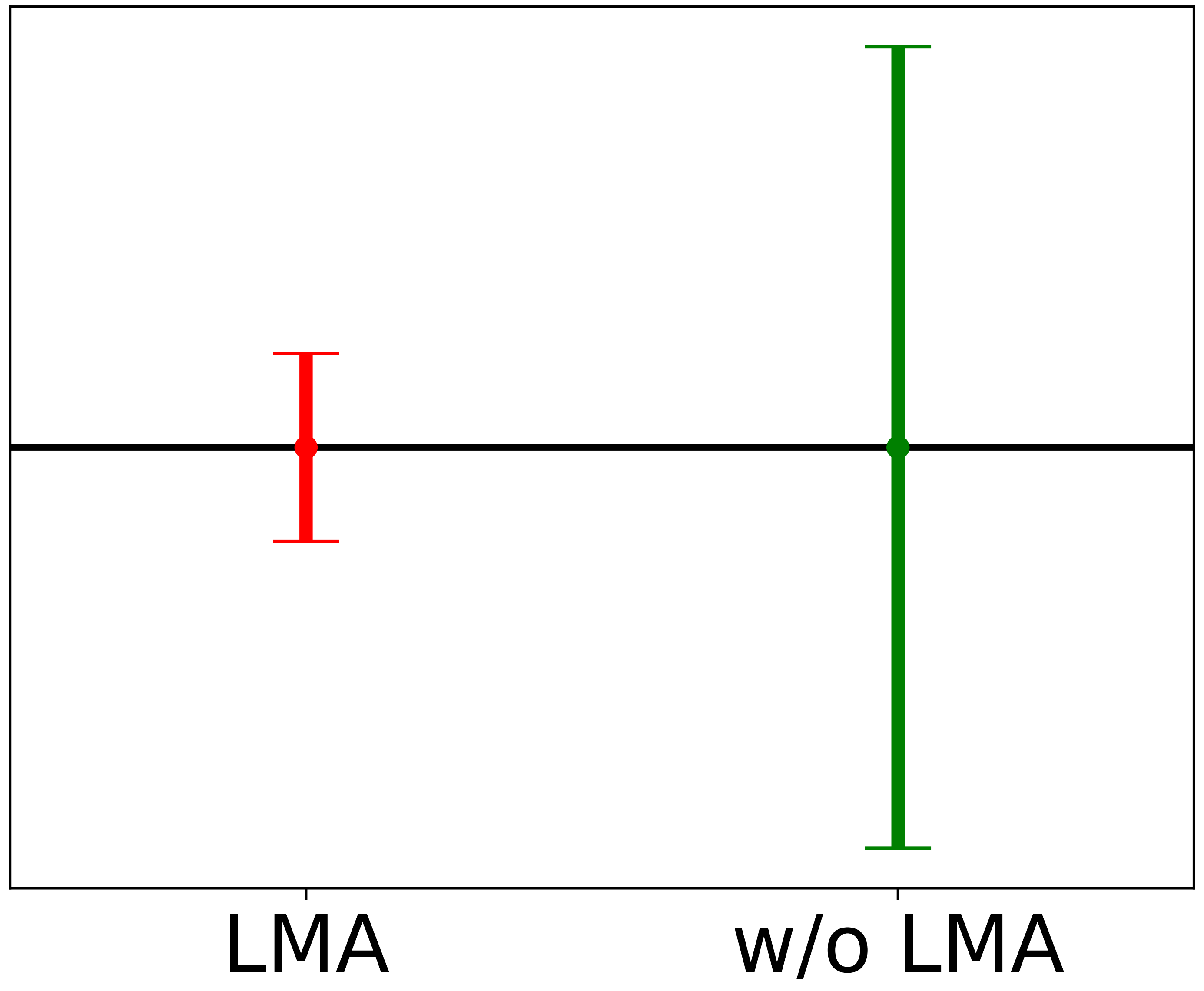
- Otherwise, determine and use eigenpairs of operator H^2_{ov} of chirality $-\mathrm{sign}\ Q_{\mathrm{GF}}$
- 512 eigenvectors of $H^2_{\mathrm{ov}} \Longrightarrow$

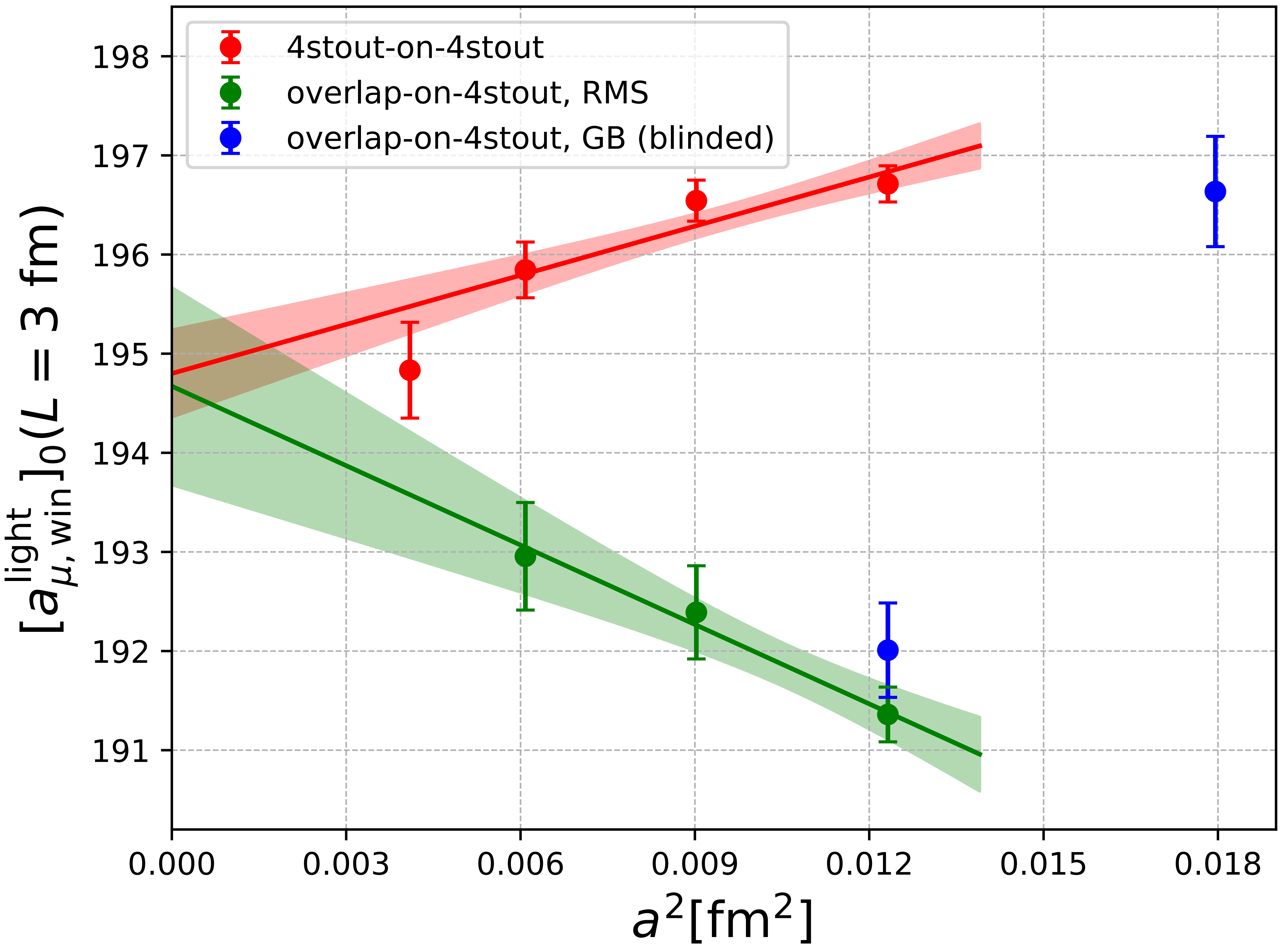
$$1024 - n_{\mathrm{zero}} \text{ eigenvectors of } D_{\mathrm{ov}}$$

- 64 sources for rest-eigen and rest-rest part



Error comparison for $a = 0.1315$ fm

Preliminary results



Staggered and Overlap (RMS):
[BMW, 2020] + new statistics

Overlap (GB): new, blinded by a factor α_{blind}

Conclusions

- Intermediate window $a_{\mu,\mathrm{win}}^{\mathrm{light}}$ from overlap valence quarks on 4 stout ensembles

- RMS pion mass matching: results agree with staggered fermions

- GB pion mass matching: LMA, finer lattices in the next runs