Chiral extrapolation of hadronic vacuum polarization and isospin-breaking corrections

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Colangelo, MH, Stoffer JHEP 02 (2019) 006, PLB 814 (2021) 136073 [2π disp]

Colangelo, MH, Kubis, Niehus, Ruiz de Elvira PLB 825 (2022) 136852 [2π IAM+disp]

Stamen, Hariharan, MH, Kubis, Stoffer EPJC 82 (2022) 432 [$\bar{K}K$ disp]

Colangelo, MH, Kubis, Stoffer, to appear [IB in 2π]

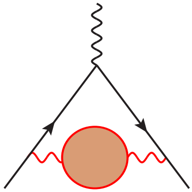
thanks to B.-L. Hoid [$\pi^0\gamma$], A. Keshavarzi [$\eta\gamma$], and L. Lellouch [IB in BMWc 2020]

Motivation

effect	$\pi^0\gamma$	$\eta\gamma$	ρ–ω mixing	FSR	M_{π^0} vs. $M_{\pi^\pm}$	total
size **in units of 10^{-10}**	4.64(4)	0.65(1)	2.71(1.36)	4.22(2.11)	$-4.47(4.47)$	7.8(5.1)

BMWc 2017, Jegerlehner

- Detailed comparison between e^+e^- data and lattice QCD
- Well-defined for total and windows, here: **what about isospin breaking?**
- Can do much better than previous estimates, but still caveats:
  - Cannot cover all channels
  - Scheme dependence
- Dominant effects:
  - **Radiative channels $\pi^0\gamma$, $\eta\gamma$**: data
  - **ρ–ω mixing**: residue in dispersive representation
  - **FSR**: scalar QED + dispersive corrections
  - **M_{π^0} vs. $M_{\pi^\pm}$ for 2π channel**: IAM + Omnès
  - **$\bar{K}K$**: resonance/threshold enhancement



Dispersive representation of 2π contribution

- Decomposition of **pion form factor**

$$F_\pi^V(s) = \underbrace{\Omega_1^1(s)}_{\text{elastic } \pi\pi \text{ scattering}} \times \underbrace{G_\omega(s)}_{\text{isospin-breaking } 3\pi \text{ cut}} \times \underbrace{G_{\text{in}}(s)}_{\text{inelastic effects: } 4\pi, \ldots}$$

- Omnès factor

$$\Omega_1^1(s) = \exp\left\{\frac{s}{\pi}\int_{4M_\pi^2}^\infty ds' \frac{\delta_1^1(s')}{s'(s'-s)}\right\}$$

$\hookrightarrow$ can get **pion-mass dependence from IAM** Guo et al. 2009

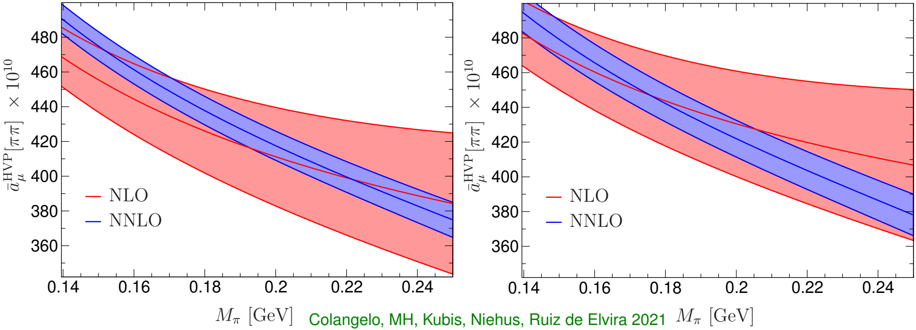
- $G_\omega(s)$ describes ρ–ω mixing in terms of residue $\epsilon_{\rho\omega}$
- $G_{\text{in}}(s)$ parameterized as normal or conformal polynomial

$\hookrightarrow$ free parameters can be matched to $\langle r_\pi^2\rangle$ (and c_π)

- Pion-mass dependence of $\langle r_\pi^2\rangle$ at two loops known Bijnens, Colangelo, Talavera 1998

$\hookrightarrow$ new LEC r_{V1}^r (from resonance saturation or lattice calculation of $\langle r_\pi^2\rangle$)

Predicting the pion-mass dependence from the IAM



Colangelo, MH, Kubis, Niehus, Ruiz de Elvira 2021

- $\bar{a}_\mu^{\rm HVP}[\pi\pi]$ only $I = 1$ correlator (with $\epsilon_{\rho\omega} = 0$)
- Free parameters:
  - LECs in $\delta_1^1(s)$: combined fit to data Colangelo, MH, Stoffer 2019 and lattice Andersen et al. 2019
  - r^r_{V1}: resonance saturation $r^r_{V1} = 2.0 \times 10^{-5}$ in concord with lattice Feng, Fu, Jin 2020
- Validated at physical point

Possible application to lattice QCD

1. **Chiral LECs as fit parameters**:
   - Describes $\pi\pi$ physics
   - Need to add $a_\mu^{\text{HVP}}[ud, I=1, \text{non-}\pi\pi] = \zeta + M_\pi^2\xi$

     $\hookrightarrow$ **infrared singularities** will be totally dominated by 2π
   - Can provide **independent constraints from other lattice calculations**: δ_1^1, F_π, $\langle r_\pi^2\rangle$
2. **Simple parameterizations**:
   - Only possible for integrated HVP or space-like integrand $\frac{\bar\Pi(-Q^2)}{Q^2} = \frac{a+bQ^2}{1+cQ^2+dQ^4}$
   - Test infrared singularities Golterman, Maltman, Peris 2017, e.g., M_π^{-2}, $\log M_\pi^2$
   - Fits to $\{a,b,c,d\}$ indicate singularity as strong as M_π^{-2} in $[0.14, 0.25]$ GeV
   - **Purely empirical finding**, no analytic approximation to full IAM nor true chiral behavior
   - Could help inform lattice fits
3. **Isospin breaking** due to pion mass difference:

$$a_\mu^{\text{HVP}}[\pi\pi]\big|_{M_{\pi^\pm}} - a_\mu^{\text{HVP}}[\pi\pi]\big|_{M_{\pi^0}} = -7.67(4)_{\text{ChPT}}(3)_{\text{polynomial}}(4)_{\langle r_\pi^2\rangle}(21)_{r_{V1}^r}[22]_{\text{total}}$$

$\hookrightarrow$ **threshold effect**, almost exclusively in LD window

ρ–ω mixing and FSR

- FSR dominated by **infrared enhanced effects**

  $\hookrightarrow$ scalar QED
- Corrections small Moussallam 2013

$$a_\mu^{\text{HVP}}[\pi\pi\gamma,\text{non-Born}] = 0.15_{\pi^+\pi^-\gamma} + 0.03_{\pi^0\pi^0\gamma} = 0.18(4)$$

- Can get FSR and $\epsilon_{\rho\omega}$ contributions from dispersive fits to 2π
- Higher-order terms $\mathcal{O}(e^2\epsilon_{\rho\omega})$ small, $\lesssim 0.1$
- Line shape matters Wolfe, Maltman 2009, we use

$$G_\omega(s) = 1 + \frac{s}{\pi}\int_{9M_\pi^2}^\infty ds' \frac{\operatorname{Im} g_\omega(s')}{s'(s'-s)}\left(\frac{1-\frac{9M_\pi^2}{s'}}{1-\frac{9M_\pi^2}{M_\omega^2}}\right)^4$$

$$g_\omega(s) = 1 + \epsilon_{\rho\omega}\frac{s}{(M_\omega - \frac{i}{2}\Gamma_\omega)^2 - s} \qquad \epsilon_{\rho\omega} \to \operatorname{Re}\epsilon_{\rho\omega} + i\operatorname{Im}\epsilon_{\rho\omega}\frac{\left(1-\frac{M_{\pi^0}^2}{s}\right)^3}{\left(1-\frac{M_{\pi^0}^2}{M_\omega^2}\right)^3}\theta(s-M_{\pi^0}^2)$$

to account for radiative channels $\rho\to\pi^0\gamma,\ldots\to\omega$

ρ–ω mixing and FSR

- Results:

$$a_\mu^{\text{HVP}}[\pi\pi, \text{FSR}, \text{Born}] = 4.24(2) \qquad a_\mu^{\text{HVP}}[\pi\pi, \rho\text{–}\omega] = 3.68(17)$$

- Separation of $\epsilon_{\rho\omega}$ into $\mathcal{O}(e^2)$ and $\mathcal{O}(\delta) = \mathcal{O}(m_u - m_d)$ Urech 1995

$$\Theta_{\rho\omega} \simeq -3\epsilon_{\rho\omega} M_V^2 \qquad \Theta_{\rho\omega}\big|_{e^2} = e^2 F_\rho F_\omega \qquad \Gamma(V \to e^+e^-) = \frac{e^4 F_V^2}{12\pi M_V}$$

- Correction actually relative to $\rho(770)$, so

$$\epsilon_{\rho\omega}\big|_{e^2} \simeq -e^2 \left(\frac{F_\omega}{M_\omega}\right)^2 \simeq -0.34(1) \times 10^{-3}$$

- With $|\epsilon_{\rho\omega}| \simeq 1.97 \times 10^{-3}$ we estimate

$$a_\mu^{\text{HVP}}[\pi\pi, \rho\text{–}\omega, e^2] = -0.64(3) \qquad a_\mu^{\text{HVP}}[\pi\pi, \rho\text{–}\omega, \delta] = 4.32(20)$$

Isospin breaking in $\bar{K}K$ channel

- Why $\bar{K}K$?
  - ϕ resonance close to $\bar{K}K$ threshold
  - **Isospin breaking from masses enhanced**
- Threshold region dominated by isoscalar form factor via ϕ Stamen et al. 2022

  $\hookrightarrow$ analyzed in terms of ϕ resonance parameters
- Significant isospin breaking in residues

$$c_\phi^{K^+K^-} = 0.977(6) \qquad c_\phi^{\bar{K}^0K^0} = 1.001(6)$$

  $\hookrightarrow$ dominant source of uncertainty
- Definition of isospin limit via self energy $(M_{K^\pm}^2)_{\text{EM}} = 2.12(18)\times 10^{-3}\,\text{GeV}^2$ from Cottingham formula

$$M_{K^\pm} = (494.58 - 3.05_\delta + 2.14_{e^2})\,\text{MeV} \qquad M_{K^0} = (494.58 + 3.03_\delta)\,\text{MeV}$$

- How close is this to popular lattice conventions?

  matches well with BMWc 2020 scheme, estimate by L. Lellouch: $M_{K^\pm} = (494.54 - 3.06_\delta + 2.19_{e^2})$ MeV, $M_{K^0} = (494.55 + 3.06_\delta)$ MeV

Isospin breaking in $\bar{K}K$ channel

- Results

$$a_\mu^{\text{HVP}}[K^+K^-, \leq 1.05\,\text{GeV}] = 18.45(20) \qquad a_\mu^{\text{HVP}}[K^0\bar{K}^0, \leq 1.05\,\text{GeV}] = 11.83(15)$$

$$a_\mu^{\text{HVP}}[K^+K^-, \text{FSR}] = 0.75(4)$$

$$a_\mu^{\text{HVP}}[K^+K^-, e^2] = -3.24(17) \qquad a_\mu^{\text{HVP}}[K^0\bar{K}^0, e^2] = -0.02(0)$$

$$a_\mu^{\text{HVP}}[K^+K^-, \delta] = 4.98(26) \qquad a_\mu^{\text{HVP}}[K^0\bar{K}^0, \delta] = -4.62(23)$$

$$a_\mu^{\text{HVP}}[K^+K^-, e^2\delta] = -0.33(1)$$

- Note:
  - K^0 self energy negligible, but indirect $\mathcal{O}(e^2)$ effect from the $K^\pm$ contribution to the ϕ spectral function
  - Remaining differences between "isospin limit" K^+K^- (16.29) and $\bar{K}^0K^0$ (16.47) due to c_ϕ and isovector form factor
- Isospin-breaking effects huge due to **threshold/resonance enhancement**

  $\hookrightarrow$ higher-order terms $\mathcal{O}(e^2\delta)$ in K^+K^- larger than in 2π!

Summing everything up

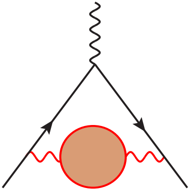
	SD window		int window		LD window		full HVP	
	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$
$\pi^0\gamma$	0.16(0)	–	1.52(2)	–	2.70(4)	–	4.38(6)	–
$\eta\gamma$	0.05(0)	–	0.34(1)	–	0.31(1)	–	0.70(2)	–
ρ–ω mixing	−0.01(0)	0.06(0)	−0.14(1)	0.97(8)	−0.48(2)	3.27(13)	−0.64(3)	4.32(20)
FSR (2π)	0.11(0)	–	1.17(1)	–	3.14(3)	–	4.42(4)	–
M_{π^0} vs. $M_{\pi^\pm}$ (2π)	0.04(1)	–	−0.09(7)	–	−7.62(14)	–	−7.67(22)	–
FSR (K^+K^-)	0.07(0)	–	0.39(2)	–	0.29(2)	–	0.75(4)	–
kaon mass (K^+K^-)	−0.29(1)	0.44(2)	−1.71(9)	2.63(14)	−1.24(6)	1.91(10)	−3.24(17)	4.98(26)
kaon mass ($\bar{K}^0K^0$)	0.00(0)	−0.41(2)	−0.01(0)	−2.44(12)	−0.01(0)	−1.78(9)	−0.02(0)	−4.62(23)
total	0.13(1)	0.09(3)	1.46(12)	1.16(20)	−2.92(16)	3.40(19)	−1.32(29)	4.68(40)
BMWc 2020	–	–	−0.09(6)	0.52(4)	–	–	−1.5(6)	1.9(1.2)

- Note: error estimates only refer to the effects included

  $\hookrightarrow$ **additional channels missing** (most relevant for SD and int window)
- Systematic error $\simeq 0.8$ from c_ϕ due to ambiguity how to define the isospin limit
- Reasonable agreement with BMWc 2020, if anything, the result would become larger with these phenomenological estimates

Conclusions

- Phenomenological estimates for **dominant isospin-breaking effects** from $\pi^0\gamma$, $\eta\gamma$, $2\pi(\gamma)$, $\bar{K}K(\gamma)$
- Chiral extrapolation for pion mass difference
- Separation into $\mathcal{O}(e^2)$ and $\mathcal{O}(m_u - m_d)$ and windows
- Caveats:
    - Other hadronic channels
    - Scheme dependence
- Cancellations especially among various $\mathcal{O}(e^2)$ effects
- Reasonable agreement with BMWc 2020, maybe some indication that $\mathcal{O}(m_u - m_d)$ is a little larger
- Some room for improvement by better matching schemes



Other channels

- **3π channel**:
  - Naive estimate for FSR by comparing to 2π: $\simeq 0.4$ Kubis, Prabhu, Schuh, work in progress
  - BaBar 2021 quotes $\rho \to 3\pi$ contribution in VMD fit ($\simeq -0.6$ in a_μ^{HVP} Boito et al. 2022), but hard to extract beyond the model context
  - ω peak scales with $1/\Gamma_\omega$ in narrow-width approximation, dependence of Γ_ω on pion mass could play a role Dax, Isken, Kubis 2018, but effect cancels out in the integral
  - Threshold effects suppressed by $(s - 9M_\pi^2)^3$, far away from ω peak
  - Main uncertainty again from residue $c_\omega^{3\pi}$, is there any isospin breaking as in $c_\phi^{\bar{K}K}$?
- ***R*-ratio from perturbative QCD**
  - QED corrections included in `rhad` Harlander, Steinhauser 2003, but $\mathcal{O}(10^{-3})$ compared to leading-order result, and $10^{-3} a_\mu^{\text{HVP}}[\geq 1.8\,\text{GeV}] \lesssim 0.05$

Hadronic vacuum polarization from lattice QCD

- **Chiral extrapolation** part of systematic error budget

  $\hookrightarrow$ extrapolation to (or interpolation around) physical quark masses

- Biggest contribution from ***I* = 1 *ud* isospin-symmetric correlator**

  $\hookrightarrow$ phenomenologically dominated by 2π channel, first correction from 4π

- ChPT not enough Golterman, Maltman, Peris 2017

$$a_\mu^{I=1} = \frac{\alpha^2}{24\pi^2}\bigg(-\log\frac{M_\pi^2}{m_\mu^2} - \frac{31}{6} + 3\pi^2\sqrt{\frac{M_\pi^2}{m_\mu^2}} + \mathcal{O}\Big(\frac{M_\pi^2}{m_\mu^2}\log^2\frac{M_\pi^2}{m_\mu^2}\Big)\bigg)$$

  $\hookrightarrow$ "convergence" in M_π/m_μ

- Need to provide information on the $\rho(770)$ resonance

  $\hookrightarrow$ **inverse-amplitude method at two-loop order**

Possible application to lattice QCD