Hadronic Vacuum Polarization: A Window on the g-2 mystery

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Introduction

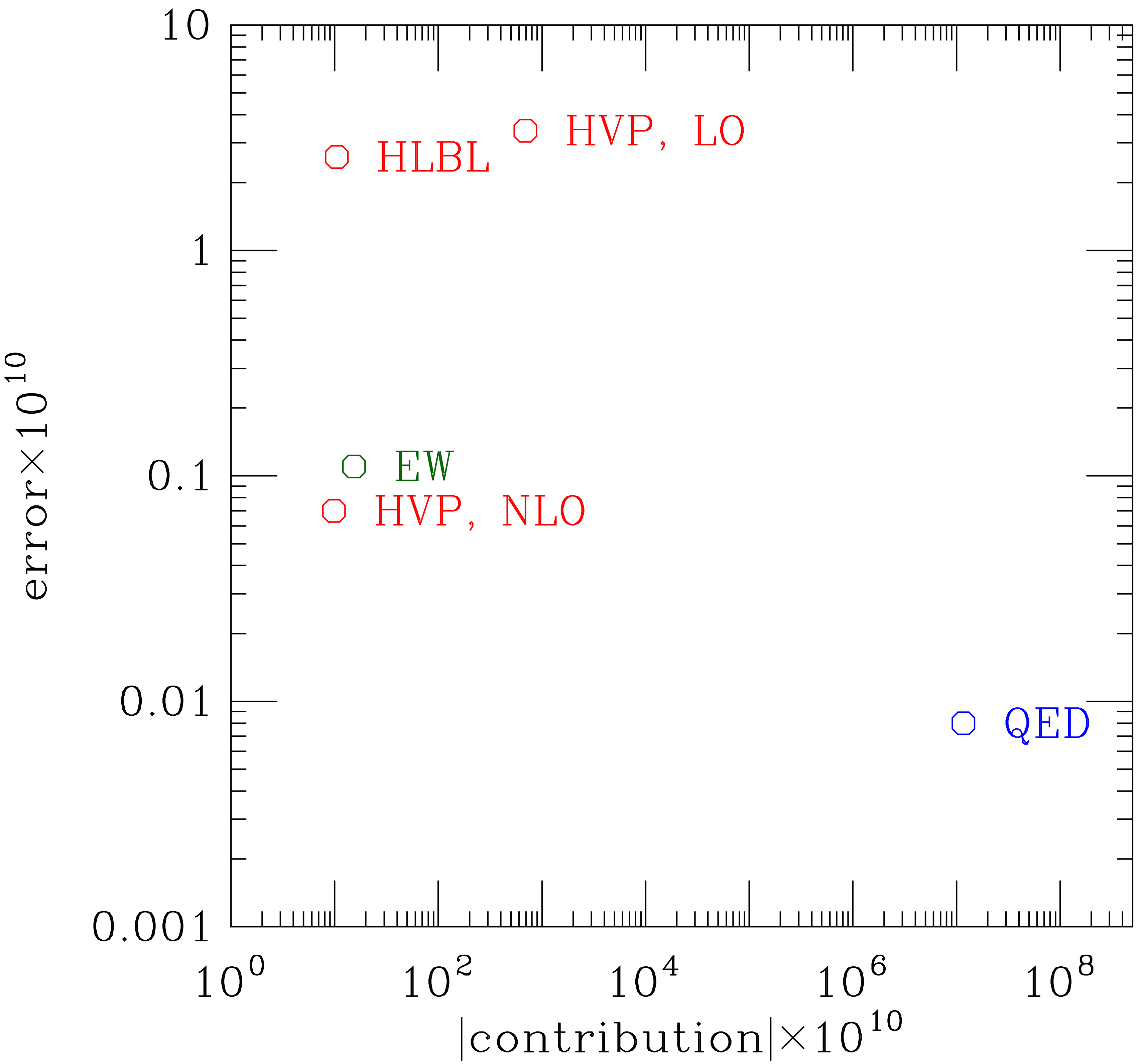
- ✦Anomalous magnetic moments of electron and muon are two of the most precisely measured quantities in physics
- ✦E821 at BNL published its final value for the muon in 2006
- ✦FNAL E989 announced its initial result in April, 2021
    - spectacular agreement with E821
    - Will continue running
    - New experiment E34 planned at J-PARC
- ✦There is ≈4.2 σ difference between data driven standard model (SM) calculation and experiment
- ✦BMW 2021 value lies between SM value and experiment
- ✦It is important to improve the precision of other lattice QCD calculations.

Theory Overview

- ✦ SM contributions come from QED (electron & muon), electroweak contributions, and hadronic contributions that involve quarks
  - all forces save gravitation contribute
- ✦ Current situation summarized by Muon g-2 Theory Initiative
  - T. Aoyama *et al*., Phys. Rept. 887 (2020, 2006.04822 [hep-ph]
- ✦ Next plot shows how the hadronic corrections dominate the error

Error vs. Contribution

- QED in blue has very small error
- Electroweak in green has larger error, but small contribution
- Hadronic contributions are all in red
  - LO Hadronic vacuum polarization largest error and 2nd largest contribution
  - HLBL 2nd largest error
- This talk on LO HVP



Lowest Order HVP

- ✦ HVP is calculated as sum of several contributions: light quark connected, strange connected, …, light disconnected, …, strong isospin breaking, electromagnetic, etc.
- ✦ $a_\mu^{ll}(\text{conn.})$ light quark connected is biggest contribution, by far
- ✦ FNAL/HPQCD/MILC: PRD **101**, 034512 (2020), 1902.04223 [hep-lat]
  - briefly recap

Lattice Ensembles

✦ In 2020, we used N_f=2+1+1 HISQ ensembles from the MILC collaboration with physical light quark masses

$\approx a$ (fm)	$am_l^{\rm sea}/am_s^{\rm sea}/am_c^{\rm sea}$	w_0/a	M_{π_5} (MeV)	$(L/a)^3 \times (T/a)$	$N_{\rm conf.}$
0.15	0.00235/0.0647/0.831	1.13670(50)	133.04(70)	$32^3 \times 48$	997
0.15	0.002426/0.0673/0.8447	1.13215(35)	134.73(71)	$32^3 \times 48$	9362
0.12	0.00184/0.0507/0.628	1.41490(60)	132.73(70)	$48^3 \times 64$	998
0.09	0.00120/0.0363/0.432	1.95180(70)	128.34(68)	$64^3 \times 96$	1557
0.06	0.0008/0.022/0.260	3.0170(23)	134.95(72)	$96^3 \times 192$	1230

✦ Have retuned 0.12 fm and added statistics for current analysis. Still adding at 0.06 fm.

$\approx a$[fm]	$N_s^3 \times N_t$	$am_l^{\rm sea}/am_s^{\rm sea}/am_c^{\rm sea}$	w_0/a	M_{π_5}(MeV)	$N_{\rm conf.}$	$N_{\rm wall}$
0.15	$32^3 \times 48$	0.002426 / 0.0673 / 0.8447	1.13215(35)	134.73(71)	9362	48
0.12	$48^3 \times 64$	0.001907 / 0.05252 / 0.6382	1.41110(59)	134.86(71)	9011	64
0.09	$64^3 \times 96$	0.00120 / 0.0363 / 0.432	1.95180(70)	128.34(68)	5384	48
0.06	$96^3 \times 128$	0.0008 / 0.022 / 0.260	3.0170(23)	134.95(72)	2621	24

Blinding

- ✦ To avoid confirmation bias in analysis, correlators are all blinded by multiplication by an unknown factor.
- ✦ Once all aspects of analysis are completed, the collaboration will decide to unblind and actual result will be available.
- ✦ **None of the plots in this talk can be used to compare values with other groups, except for one.**
- ✦ As the blinding factor is multiplicative, the percentage error in result is reasonably accurate, but preliminary.

Windows Analysis

- ✦ The statistical noise at large Euclidean time is challenging
  - RBC/UKQCD suggested using windows to achieve higher precision and allow better comparison of different calculations
    - PRL 121, 022003 (2018)
  - FNAL/HPQCD/MILC recently advocated one-sided windows with longer time extent than SD defined in PRL above.
    - 2207.04765 [hep-lat] (use such windows as part of this study)
- ✦ We have considered multiple windows and concentrate on just two here

$$\Theta\left(t, t_0, t_1, \Delta\right) = \frac{1}{2}\left[\tanh\left(\frac{t-t_1}{\Delta}\right) - \tanh\left(\frac{t-t_2}{\Delta}\right)\right]$$

Windows Considered

✦We fix $\Delta = 0.15$ fm.

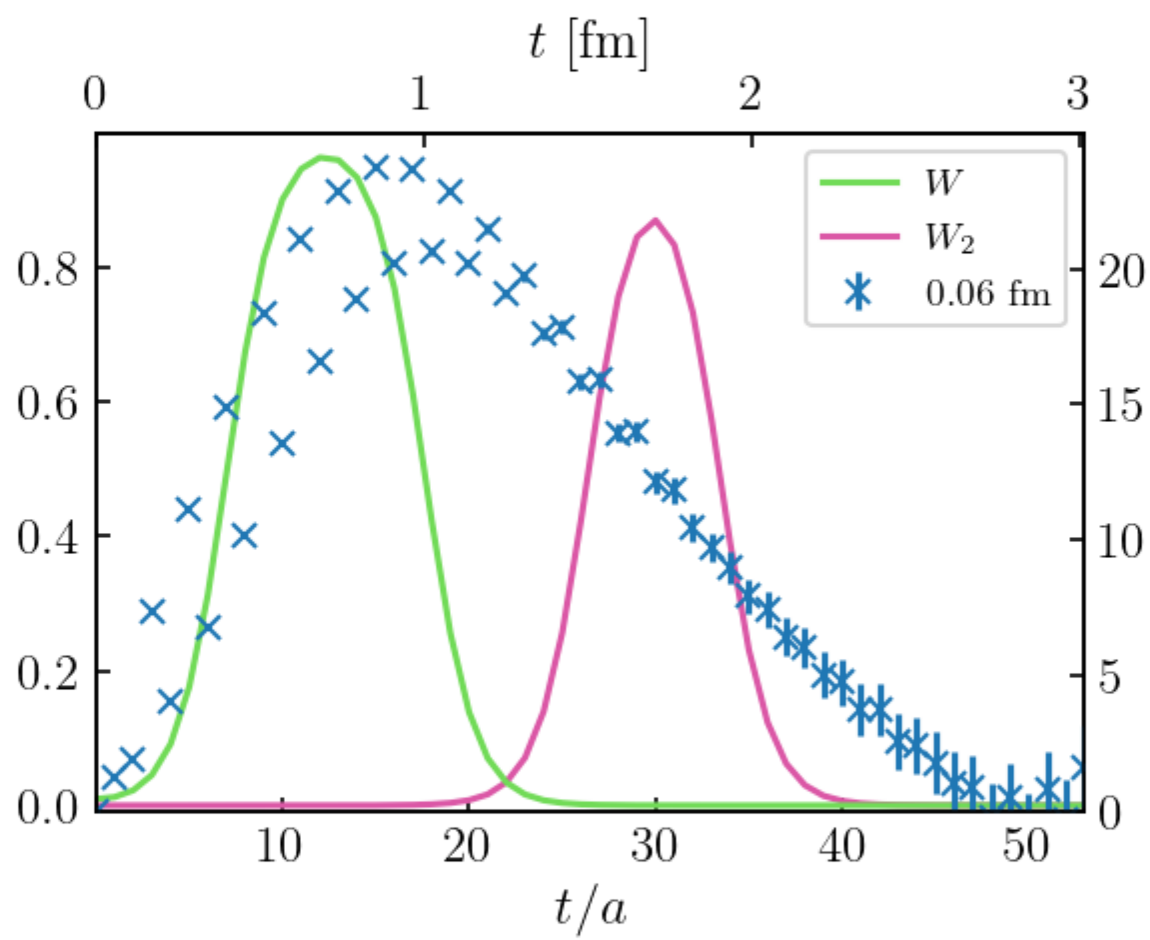
✦For the one-sided (O.S.), $t_1 = 1, 1.5, 2, 3.$

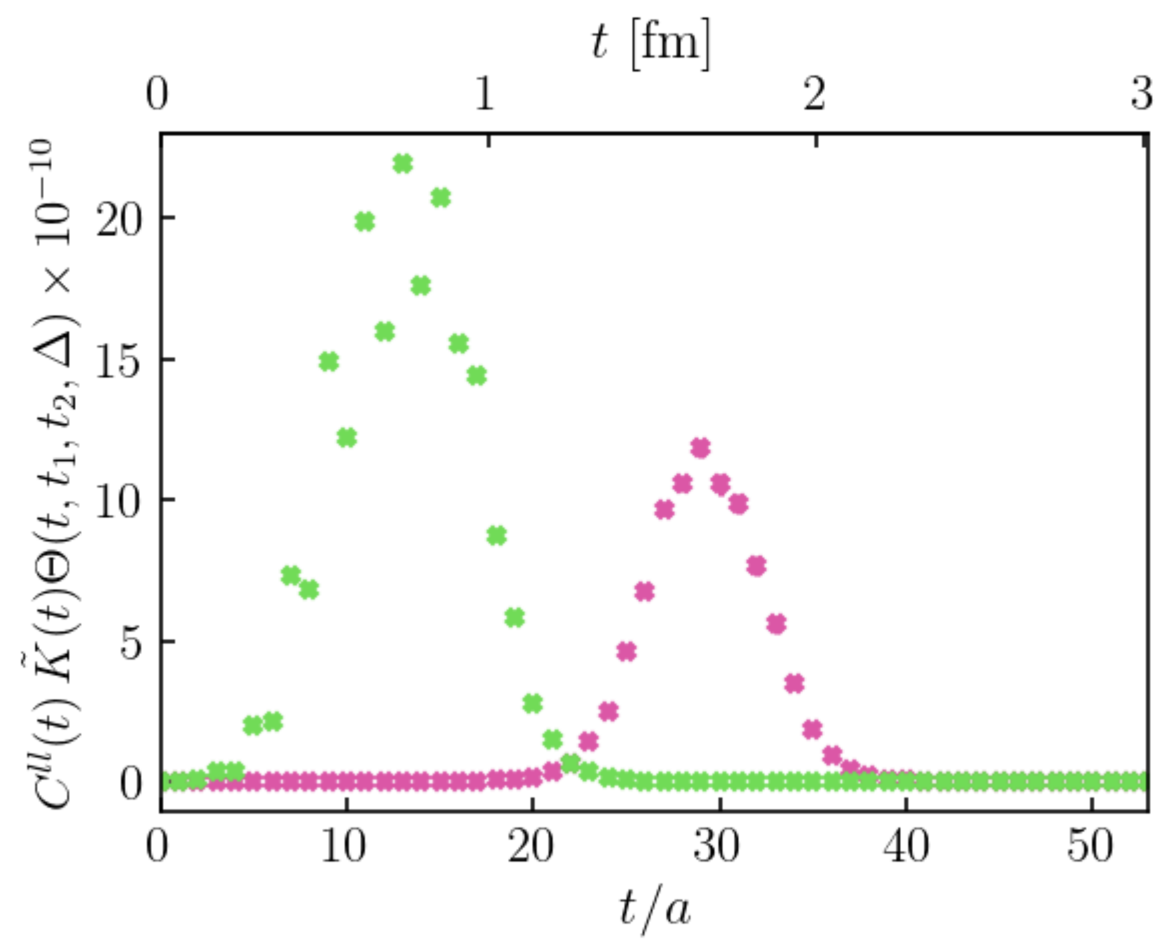
label	$[t_0, t_1]$ fm
a_μ^{SD}	$[0, 0.4]$
a_μ^{W}	$[0.4, 1]$
$a_\mu^{\mathrm{W}_2}$	$[1.5, 1.9]$
$a_\mu^{\mathrm{O.S.}}(t_1)$	$[0, t_1]$

✦Here, we only present W and W_2 (Aubin *et al*. 2204.12256 [hep-lat])

✦Each window has its own blinding factor, so can unblind independently.

Effect of Window





- ✦ Left: a_μ integrand in blue; W window factor in green; W_2 in red
- ✦ Right: integrand after multiplication by window factor
  - ✦ note effect of staggering on W

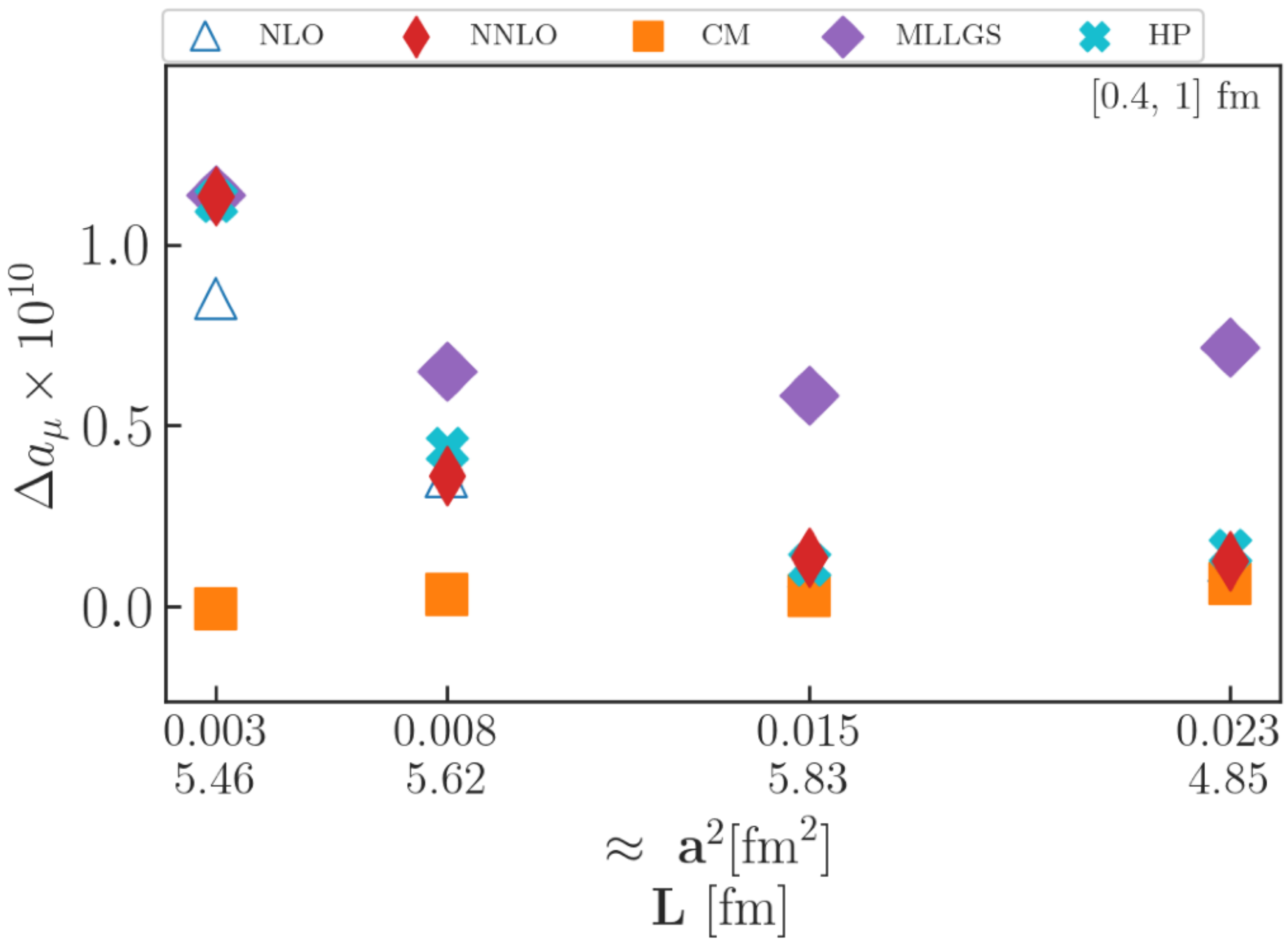
Corrections

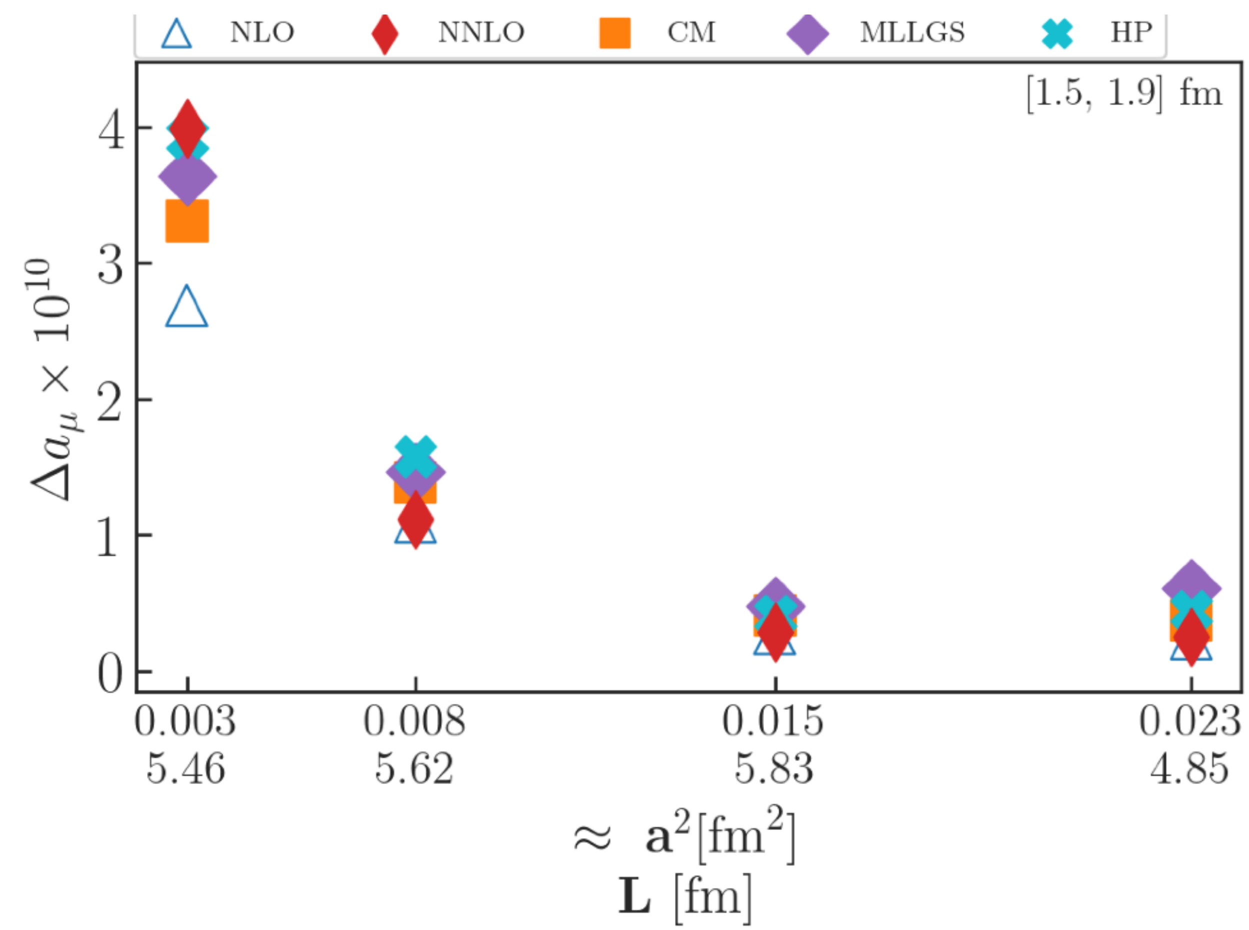
- Three corrections are applied: volume, mass mistuning, and taste breaking. (Latter is optional, see below.)
- $a_\mu(L_\infty, m_{\pi_{phys}}) = a_\mu(L_{latt}, m_{\pi_{latt,\xi_1}}, \cdots, m_{\pi_{latt,\xi_{16}}}) + \Delta_{\mathrm{FV}} + \Delta_{\mathrm{mistune}} + \Delta_{\mathrm{TB}}$
- $\Delta_{\mathrm{FV}} = a_\mu(L_\infty, m_{\pi_{latt,\xi_1}}, \cdots, m_{\pi_{latt,\xi_{16}}}) - a_\mu(L_{latt}, m_{\pi_{latt,\xi_1}}, \cdots, m_{\pi_{latt,\xi_{16}}})$
- $\Delta_{\mathrm{mistune}} = a_\mu(L_\infty, m_{\pi_{phys,\xi_1}}, \cdots, m_{\pi_{phys,\xi_{16}}}) - a_\mu(L_\infty, m_{\pi_{latt,\xi_1}}, \cdots, m_{\pi_{latt,\xi_{16}}})$
- $\Delta_{\mathrm{TB}} = a_\mu(L_\infty, m_{\pi_{phys}}, \cdots, m_{\pi_{phys}}) - a_\mu(L_\infty, m_{\pi_{phys,\xi_1}}, \cdots, m_{\pi_{phys,\xi_{16}}})$
- Correction terms calculated on each ensemble using several models

Correction Models

- ✦ We consider several models
  - Chiral Perturbation Theory (ChiPT NLO, NNLO)
  - Meyer-Lellouch-Lüscher-Gournaris-Sakurai (MLLGS)
  - Chiral Model (CM, and CM' variation)
  - Hansen and Patella (HP)
    - last is used only for finite volume correction
- ✦ We also try neglecting $\Delta_{\rm TB}$ at each lattice spacing and allowing continuum limit to eliminate taste breaking
- ✦ Don't need to use the same model for all correction terms.
  - many, many variations

Finite Volume Correction

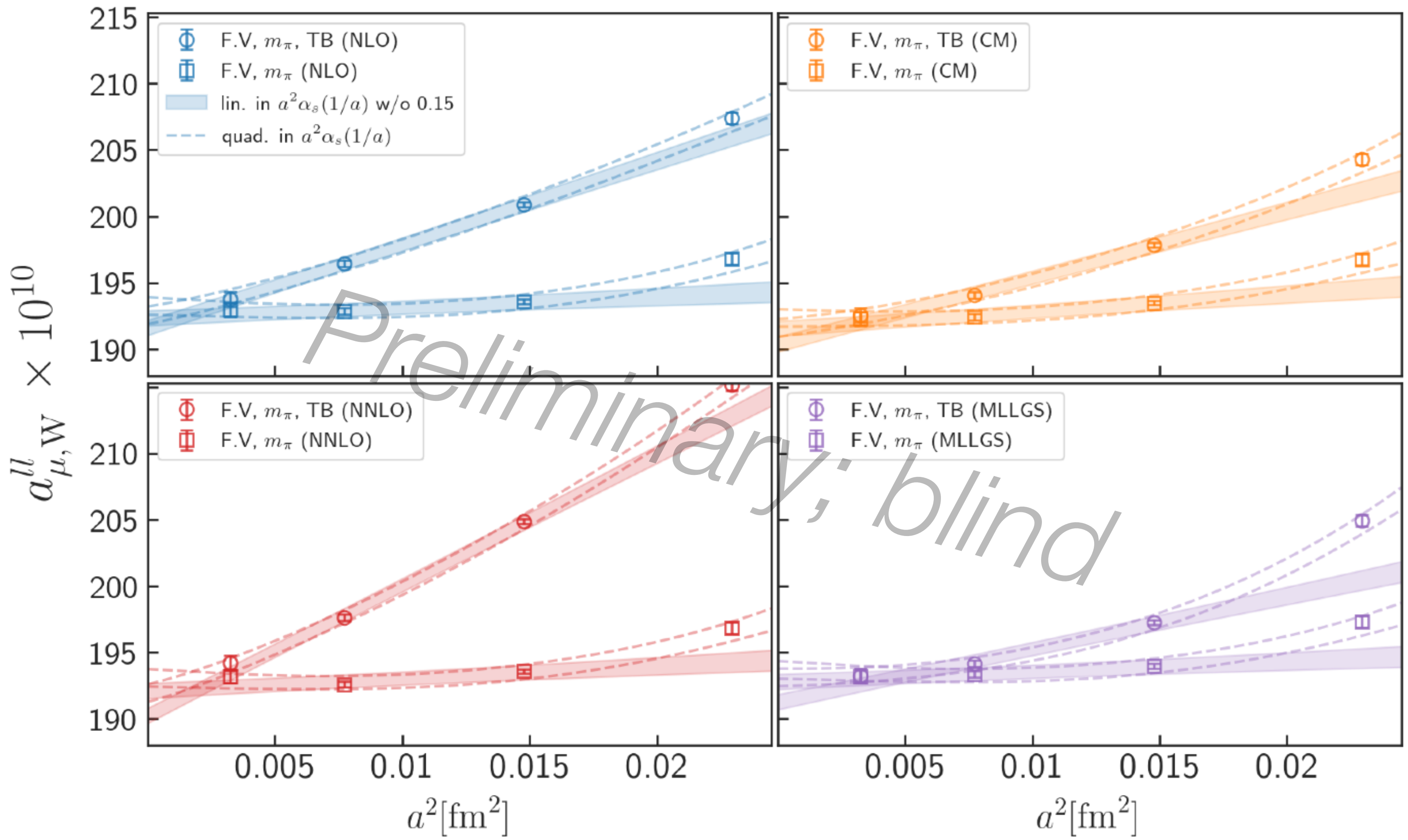




- FV correction for W (left) and W_2 (right) windows, shows much better consistency for the window at larger time advocated by Aubin et al.
- FV correction is so small at smaller volume (coarser ensembles) because taste breaking is larger there.

To Correct TB or Not?

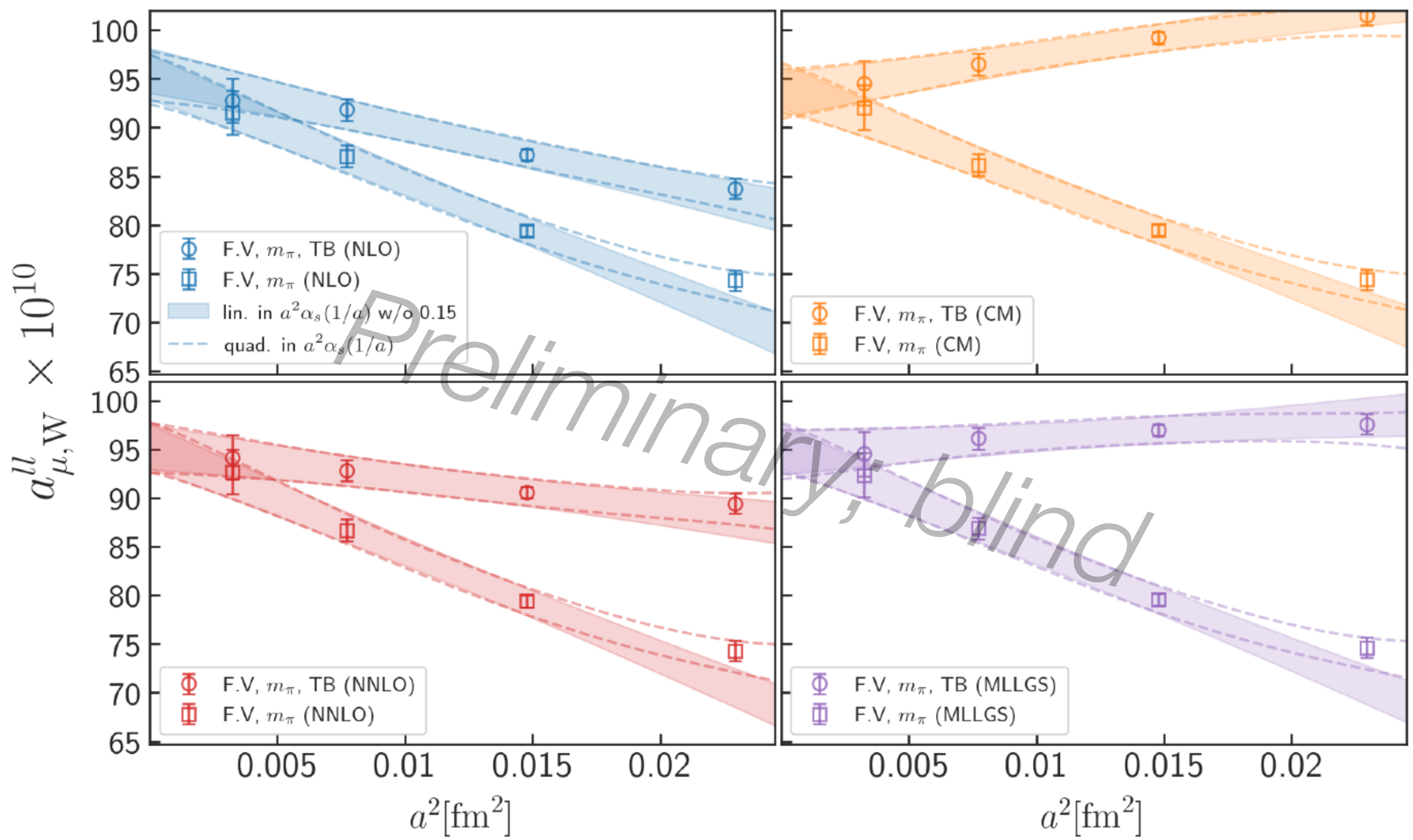
- We can allow continuum limit to remove taste breaking or remove on each ensemble.
- We see some differences as $a \to 0$ depending on model whether we include coarsest ensemble.



W window

To Correct TB or Not? II

- Lattice spacing dependence is quite different for window at larger time.
- Model corrections can differ quite a bit, but as $a \to 0$ results are more consistent, than in previous case.
- Error is also larger.



W_2 window

Bayesian Model Averaging

- ✦ Introduced by Jay and Neil, PRD **103**, 114502 (2021).
- ✦ Useful when considering multiple models (or parameter values like t_{min} in fits).

$$\text{pr}(M \mid D) \equiv \exp\left[-\frac{1}{2}\left(\chi^2_{\text{aug}}\left(\mathbf{a}^\star\right) + 2k + 2N_{\text{cut}}\right)\right]$$

gives the weight of each model in the average.

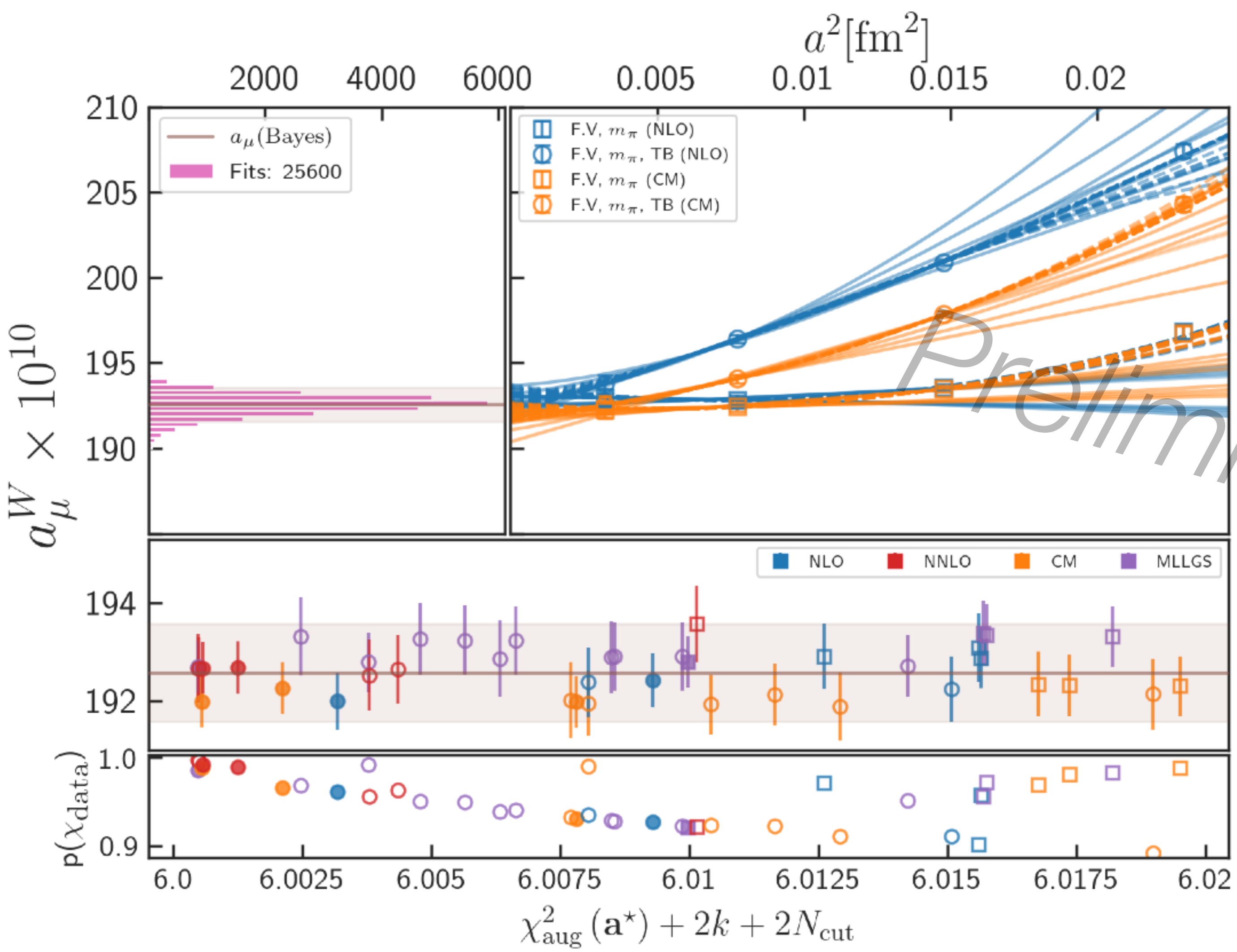
$$\left\langle a_\mu \right\rangle = \sum_i \left\langle a_\mu \right\rangle_i \text{pr}\left(M_i \mid D\right)$$

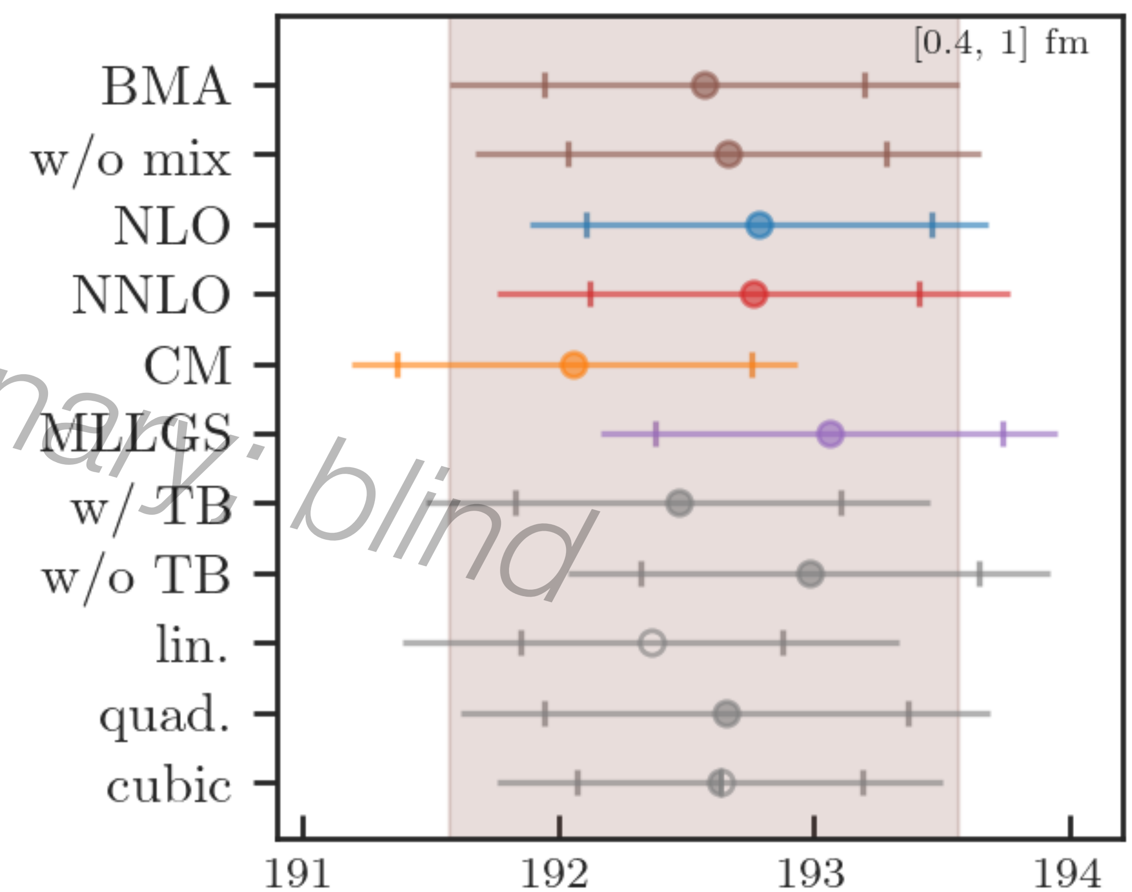
is the average over the models.

Bayesian Model Averaging II

- Many variations in how the fit is done:
  - choice of model for each correction FV, mistuning, TB
    - also no taste breaking correction
  - apply corrections to a reduced region of time
  - remove opposite parity contributions to vector-correlator that come from using staggered quarks
  - dropping some of the coarser ensembles
  - variations in the number of powers of a^2 and α_s in continuum fit
  - inclusion of sea-quark mistuning term

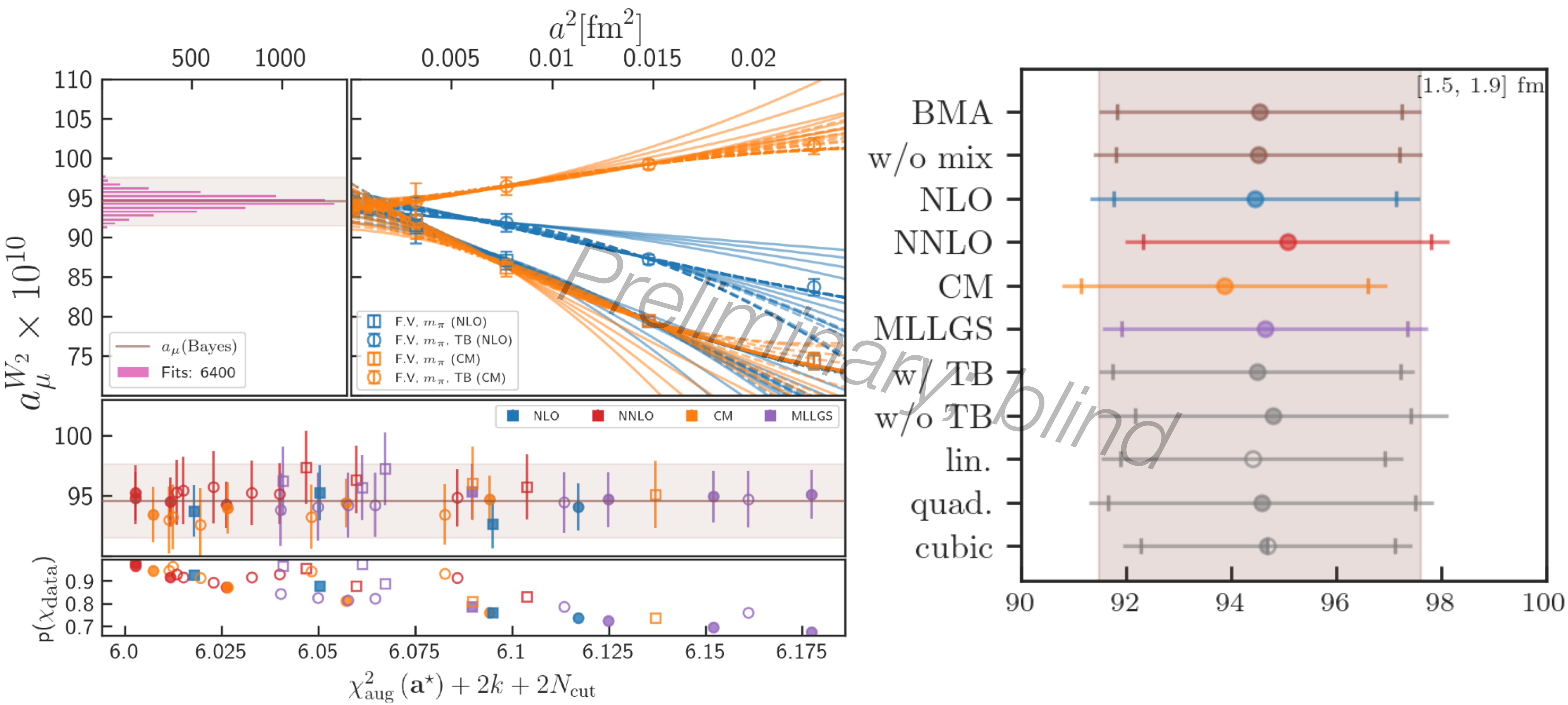
BMA for W





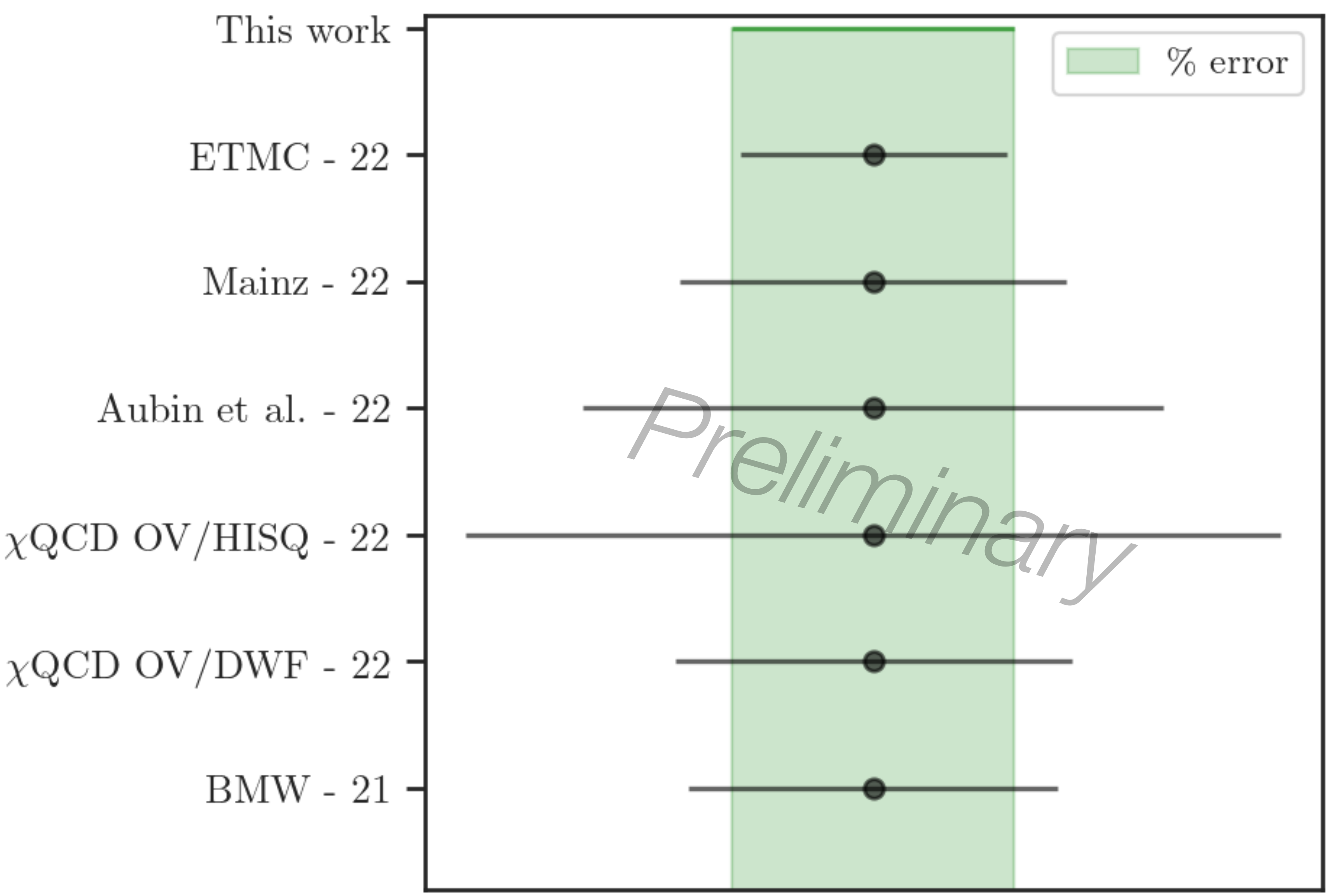
- (L) Four panels show many aspects of the various fits: histogram of 25,600 fits; examples of fits using CM and NLO chiral perturbation theory; 50 best fits; p-value for data contribution to χ^2.
- (R) Model average using only subsets of the models.

BMA for W_2



- Similar to previous slide but for the window suggested by Aubin et al.

Expected Error for $a_{\mu ll}^{W}(\text{conn.})$



- Result is blinded by a multiplicative factor so we can calculate our percentage error.
- Expect our result to be comparable in precision to recent results.

Towards a Complete Calculation

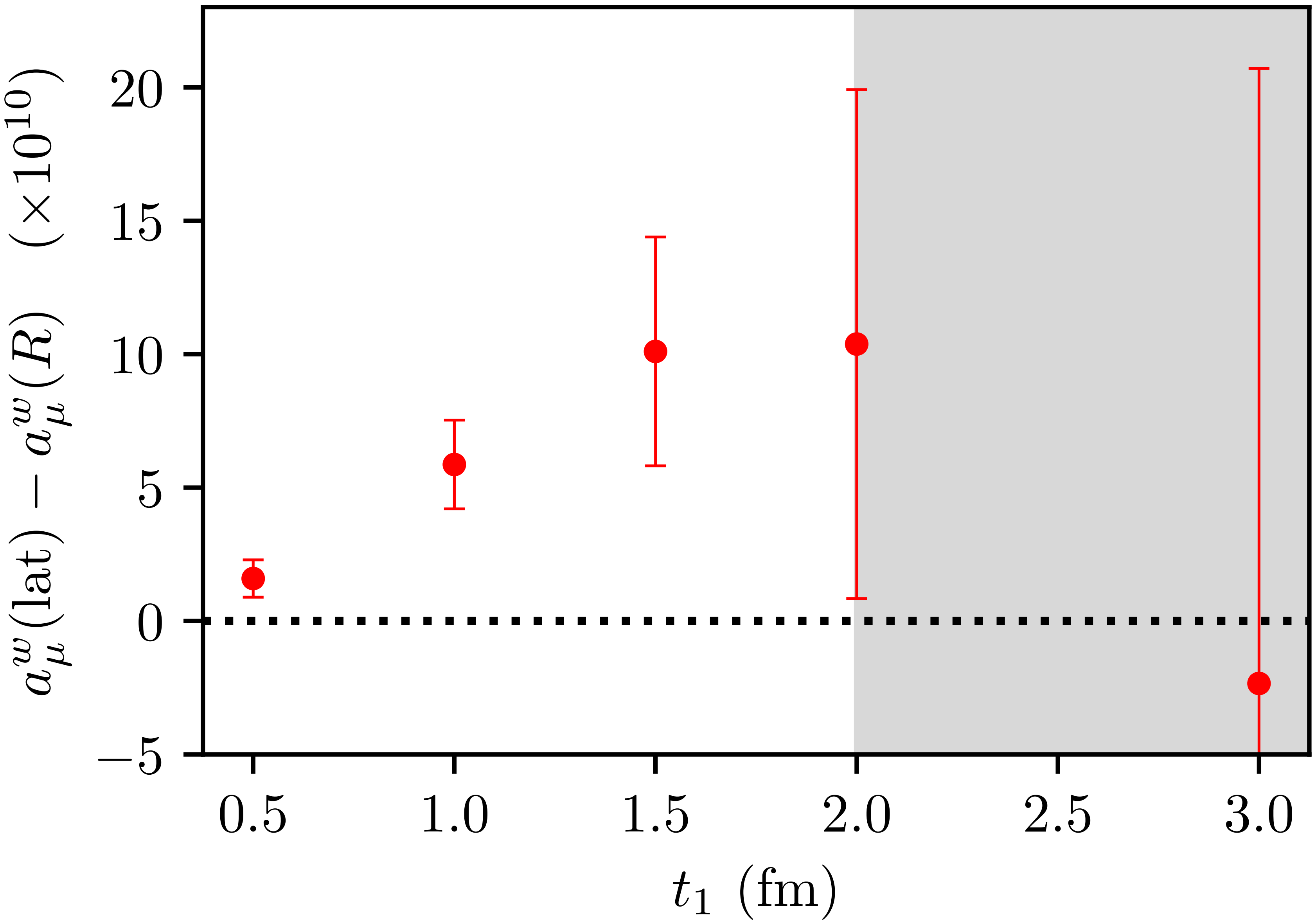
✦ Ultimate goal is a_μ, so we need:

- better scale setting
  - extending range of ensembles with gauge flow data
  - Ω baryon mass (Yin Lin)
- better statistical accuracy at large time
  - Michael Lynch's poster on low-mode improvements
  - Shaun Lahert's work on two pion states (not presented here)
    - now analyzing 0.12 fm ensemble
- strong isospin breaking
  - Curtis Peterson's analysis (not presented here)
- electromagnetic corrections
  - Gaurav Ray's talk in 20 minutes

Conclusions

- Contributions to a_μ from various windows in Euclidean time provide valuable benchmarks for lattice QCD calculations on the way to complete HVP calculation
- Stay tuned for our upcoming paper with unblinded results.
  - Expect it to be posted before Muon g-2 Theory Initiative meeting in Edinburgh.
- Do **not** quote any numbers from these blinded plots.

One sided windows



- Difference between lattice and R-ratio determination for various one-sided windows.
- From 2207.04765, using data from 2020.
- We have analyzed several windows with our updated data set