Calculating the QED correction to the hadronic vacuum polarisation on the lattice

Fermilab Lattice, MILC, and HPQCD collaborations

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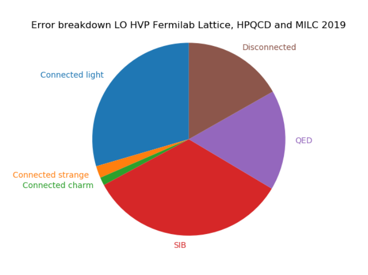
Introduction

$$10^{10} a_\mu^{\mathrm{HVP,LO}} = 699(15)_{u,d}(1)_{s,c,b}$$ [PRD.101.034512 (2020)]



The estimate used of $0(5) \times 10^{-10}$ for the residual QED correction is a significant part of the 2.2% uncertainty and almost saturates the 1% uncertainty bound on a_μ^{HVP}.

- Reducing the theoretical uncertainty of a_μ below 1% requires the inclusion of isospin breaking effects.
- This project aims to calculate the $\mathcal{O}(\alpha)$ QED isospin breaking correction to the light connected HVP, with uncertainty $< 5 \times 10^{-10}$.

$$\delta a_\mu^f \equiv a_\mu^f(m_f, Q_f) - a_\mu^f(m_f, 0)$$

Outline of the calculation

1. Measure vector current-current correlators with & without QED
2. Fit these periodic correlators to a model and replace data with model for $t > t^*$
3. Fourier transform correlators to obtain quark polarisation function
$$q^2\Pi(q^2) = a^4 \sum_t e^{iqt} \sum_x \langle j(x,t)j(0)\rangle$$
4. Integrate over kernel to obtain a_μ^{HVP} with & without QED
$$a_\mu^{\mathrm{HVP,(f)}} = \frac{\alpha}{\pi}\int_0^\infty dq^2 f(q^2)\, 4\pi\alpha Q_f^2\, \hat{\Pi}_f(q^2),$$
5. Take the difference and transform to an appropriate renormalisation scheme

We use the MILC code for measurements and Peter Lepage's `g2tools` to carry out the analysis.

Simulation Details

- Random sources + local vector operator
  - We have QED corrections at Q=2/3 to Z_V in the RI-SMOM scheme from the HPQCD collaboration. [PRD 100.114513 (2019)] - We derive Q=1/3 corrections from the Q=2/3 factors.
- multi mass inverter
- average over polarisations and charges
- We use the MILC code with QUDA - GPU accelerated. Running with A100 GPUs on CSD3 at Cambridge, UK.

Ensembles used

We've been running on 3 physical HISQ 2+ 1+1 ensembles provided by MILC.

Simulating at the physical point is expensive and noisy so we simulate at multiples of the light quark mass, $m_l = 0.5(m_u + m_d)$, and extrapolate. We measure neutral VT and PS correlators.

Ensemble	$L^3 \times T$	a[fm]	cfgs	masses
very coarse	$32^3 \times 48$	0.15	1844	m_u m_d $3/5/7m_l$ m_s
coarse	$48^3 \times 64$	0.12	967	$3/5/7m_l$ m_s
fine	$64^3 \times 96$	0.09	596	$3/5/7m_l$ m_s

- This analysis is blinded at the correlator level.
- Correlated fits across all the masses and charges on each ensemble.

Quenched QED on the lattice

We use the $\mathrm{QED_L}$ formulation in the quenched approximation, which means the sea quarks are electrically neutral. [Hayakawa & Uno (2008)]
We generate U(1) fields as follows:

1. generate a random momentum space photon field in Feynman gauge for each QCD gluon field configuration
2. zero modes are set to zero using the $\mathrm{QED_L}$ formulation.

$$A_\mu(\hat{k}_0, |\hat{k}| = 0) = 0$$

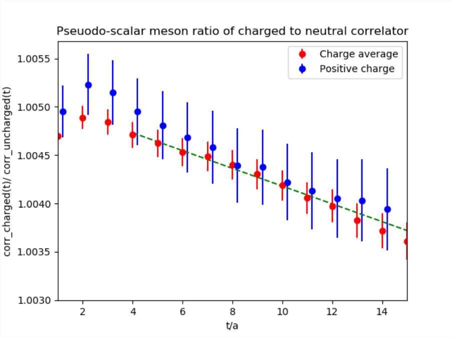
These gauge fields are exponentiated as $\exp(ieQA_\mu)$ to give a U(1) field which is then multiplied into the QCD gauge links before HISQ smearing.

Truncated Solver Method + Charge Averaging

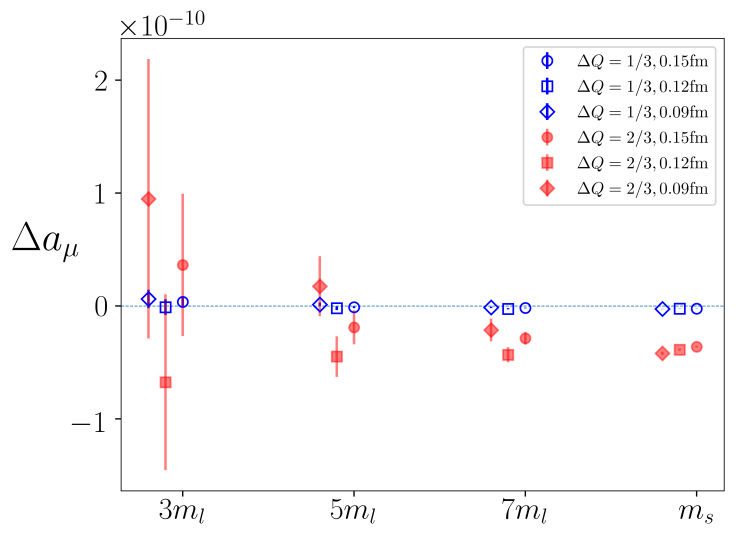
$$C(t) = \frac{1}{15}\sum_{i,i\neq j} C^{i}_{\text{sloppy}}(t) + (C^{j}_{\text{prec}}(t) - C^{j}_{\text{sloppy}}(t)) \quad ,$$

On the j timeslice do a precise solve with a smaller residual. We use $10^{-3} : 10^{-6}$ for sloppy:precise. The last two terms are to correct for potential bias incurred by using the looser residual.

[Bali et al. Comput.Phys.Commun. 181 (2010)]



Summary of results at fixed bare mass



Scheme Dependency

- To separate out the physical and unphysical effects of turning on the electric charges we have to choose a renormalisation scheme. Hadronic schemes are most commonly used.
- If we can compute quark mass shifts in this scheme then transforming our bare differences is simple,

$$\begin{aligned}\delta a_\mu &= a_\mu^{\mathrm{QCD+QED}}(m_q - \delta m_q) - a_\mu^{\mathrm{QCD}}(m_q) \\ &= \Delta a_\mu - \delta m_q \frac{\partial a_\mu}{\partial m_q}\end{aligned}$$

where Δa_μ is the fixed bare difference and δm_q is the quark mass shift (and is scheme dependent).

- To get the derivative we fit a_μ to a smooth function of m_q.
- We assume the lattice spacing does not change.

[See Tantalo's talk tomorrow and Antonin's slides]

Hadronic schemes

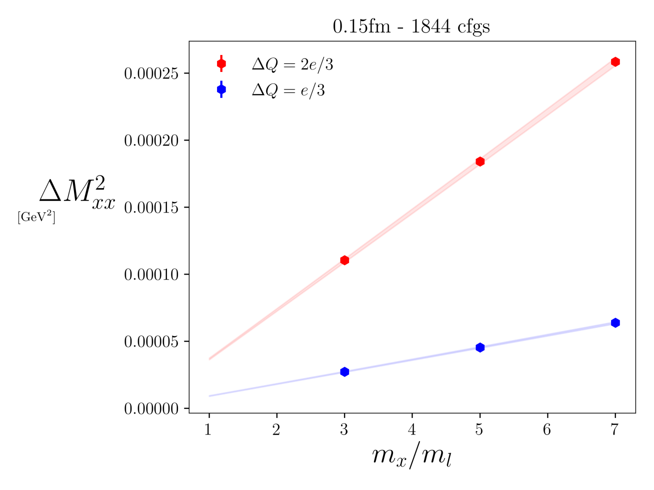
We've adopted a Dashen-like scheme following Section C of the MILC paper, [1807.05556] - 'BMW-like'.

$$M^2_{uu'} = M^2_{dd'} = M^2_{nn'} \equiv M^2_{\pi^0}$$
$$(M^2_{uu'})^\gamma = 0 = (M^2_{dd'})^\gamma$$

To go from differences at equal bare quark mass to differences at equal renormalised mass we define fractional quark mass shifts δ_u, δ_d such that

$$\Delta M^2_{uu'}(m_u) = 2Bm_l\delta_u$$
$$\Delta M^2_{dd'}(m_d) = 2Bm_l\delta_d$$

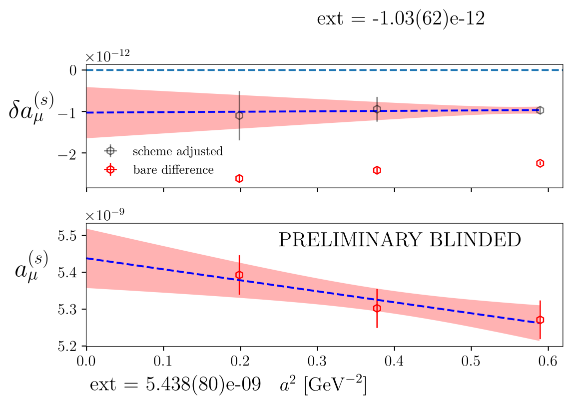
- B from LO SU(2) χPT



$$f(m_q) = c_1 m_q + c_2 m_q^2$$

Blinded Results at the strange quark mass

$$\delta a_\mu^s = a_\mu^s(m_s, -\frac{1}{3}) - a_\mu^s(m_s, 0)$$



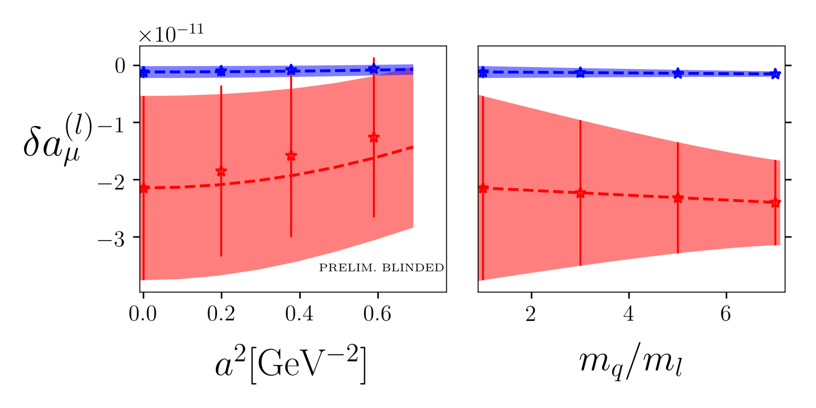
- Large noisy adjustment from δ_d - particularly on the fine ensemble - use the η_s instead?
- Extrapolation function:

$$\delta a_\mu^{(s)} = c_0 \left(1 + c_1 (a\Lambda)^2\right)$$

Blinded Result at the light quark mass

We apply the scheme adjustments before extrapolating in a, m_q.

$$\delta a_\mu^{u/d} = c_0 \left(1 + c_1 (a\Lambda)^2 + c_2 m_q\right)$$



$$\delta a_\mu^{ll}(m_q = m_l, a = 0) = \delta a_\mu^u(m_l, 0) + \delta a_\mu^d(m_l, 0) = -2.3(1.7) \times 10^{-11}$$

EM FV effects on the quark polarisation function

From Bijnens et al. [1903.10591]

$$\Delta\Pi(q^2) \sim \frac{1}{m^3 L^3} \text{ where } m = m_{\mathrm{PS}}$$

we expect FV effects to be largest on the 0.15fm ensemble,

$$m_{\eta_c} L = 2.3 \times 32 = 74 \Rightarrow \frac{1}{74^3} \ \underline{\text{tiny}}$$

$$m_{\eta_s} L = 0.53 \times 32 = 17 \Rightarrow \frac{1}{17^3} \sim \underline{0.02}\%$$

$$m_{\pi} L = 0.1 \times 32 = 3.2 \Rightarrow \frac{1}{3.2^3} \sim \underline{3}\%$$

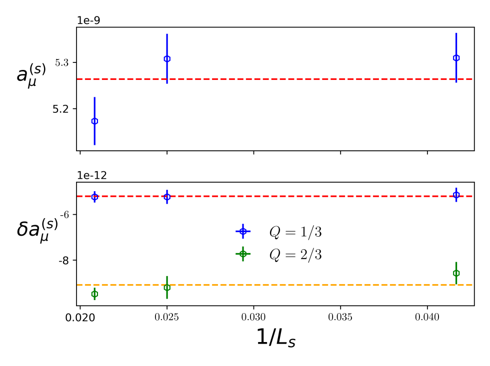
cf this to the fractional error for $\delta a_\mu^{(s)} \sim 10\%$ and $\delta a_\mu^{(l)} \sim 100\%$.
(and cf 10% systematic error because we are using quenched QED)

FV runs at m_l are a possibility although it would require lots of computer time.

Finite Volume Study at m_s

We use the following ensembles, all ~ 0.12 fm [Not Blinded]

$L^3 \times T$	L[fm]	m_l/m_s	Ncfgs
$24^3 \times 64$	2.93	1/10	400
$40^3 \times 64$	4.89	1/10	100
$48^3 \times 64$	5.82	1/27	694



Conclusions and Outlook

- We can achieve precision $\ll 5 \times 10^{-10}$ on δa_μ^{ll} with relative errors around 100%.
- Exploring different schemes - partly because we would like to compare our results with other groups
- Fits v. no Fits
- Running ongoing, Dynamical QED runs getting started.
  - Work ongoing on connected, disconnected, and SIB contributions. [eg. see C.McNeile's Schwingerfest slides]
  - Project starting on QED correction to the disconnected contribution.

Extra Information

Fitting details

Vector fits - 5 + 5 exp - wide priors

ensemble	fit range	svdcut	$\overline{\chi}^2$/Q	with noise*
very coarse	[2-23]	0.000000	0.62/1	0.67/1
coarse	[2-31]	0.000424	0.24/1	0.66/1
fine	[2-47]	0.012667	0.15/1	0.82/1

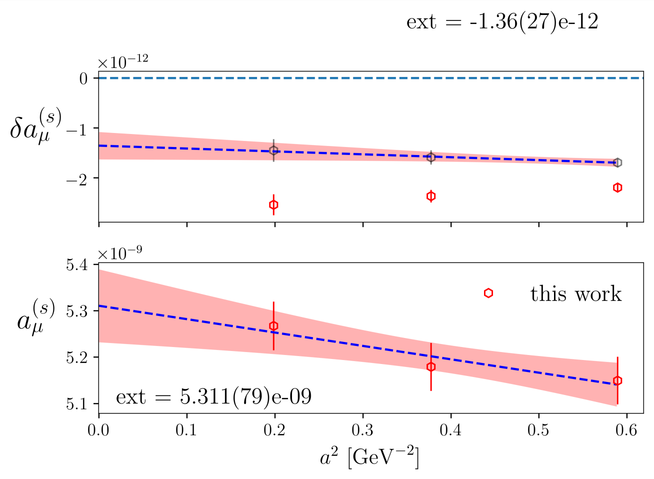
Pseudoscalar fits - 7 exp

ensemble	fit range	svdcut	$\overline{\chi}^2$/Q	with noise*
very coarse	[3-20]	5.4e-11	0.8/0.99	0.96/0.65
coarse	[4-28]	8.3e-07	0.41/1	0.99/0.52
fine	[4-40]	7.2e-05	0.2/1	0.96/0.73

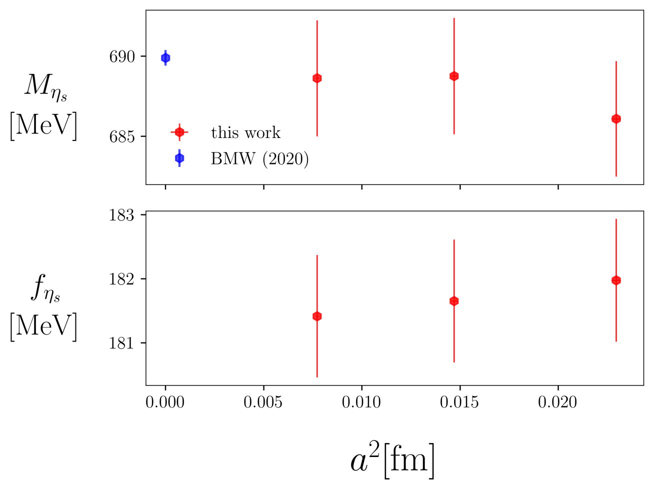
Binning increases ground state PS mass error slightly (but still much below $N^{0.5}$) and likely within 'error of the error' - looking at `UNEW` python library.

* just svd noise, no prior noise - this seems to mess up some of the fits. Needed with wide priors ?

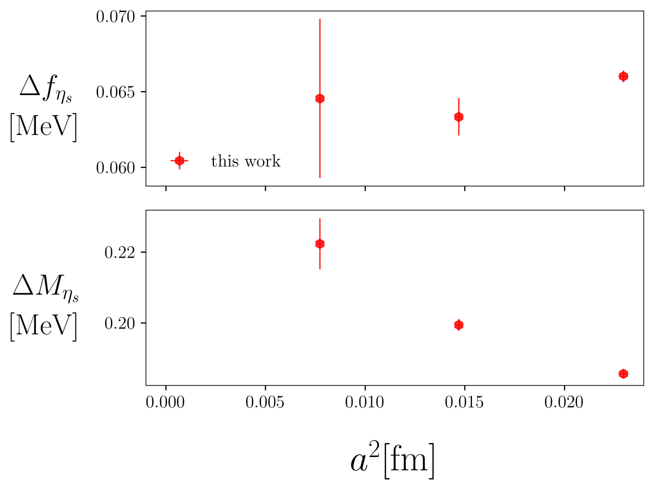
δa_μ^s in the Quark mass scheme



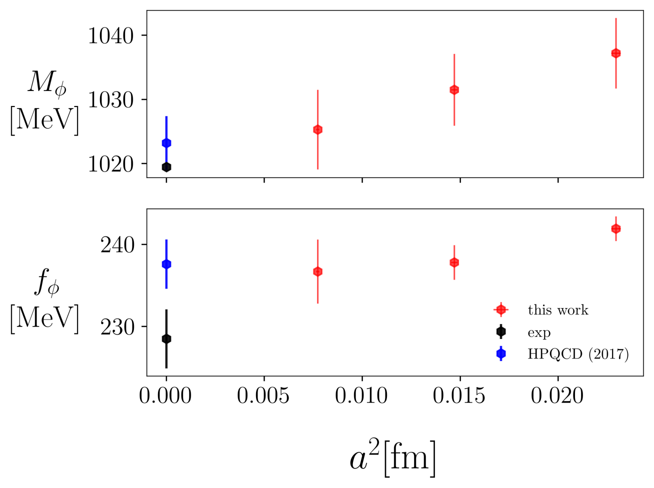
Crosschecks at m_s - η_s meson



Crosschecks at m_s - η_s meson



Crosschecks at m_s - ϕ meson



from the MILC paper [1807.05556]

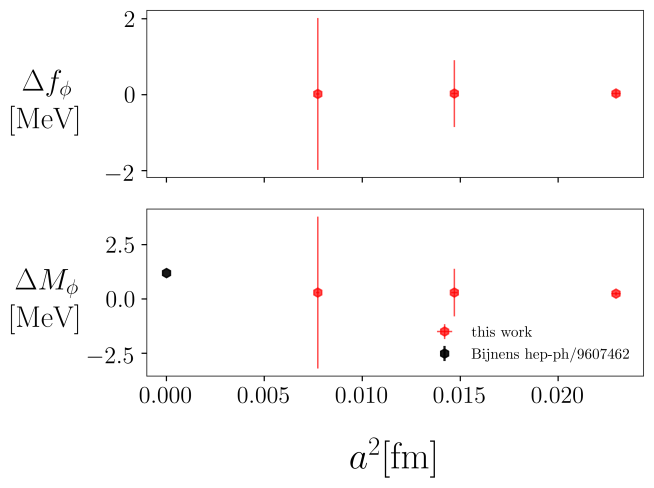
Once the strange quark mass has been renormalized, we may compute $(M^2_{K^0})^\gamma$, the EM effect on the neutral kaon, from

$$(M^2_{K^0})^\gamma = \Delta M^2_{K^0} - B_s(m_s - m_S) - B_l(m_l - m_d), \quad (43)$$

where B_s and B_l are the derivatives of $(M^2_K)^{\mathrm{QCD}}$ with respect to m_s and m_l, respectively. Unfortunately, because a large fraction of $\Delta M^2_{K^0}$ is unphysical, and removed when constructing $(M^2_{K^0})^\gamma$ in the renormalization step, the resulting systematic error in $(M^2_{K^0})^\gamma$ [or equivalently ϵ_{K^0}, Eq. (5)] is relatively large (~35%). The result is particularly sensitive to the uncertainty in the derivative B_s.

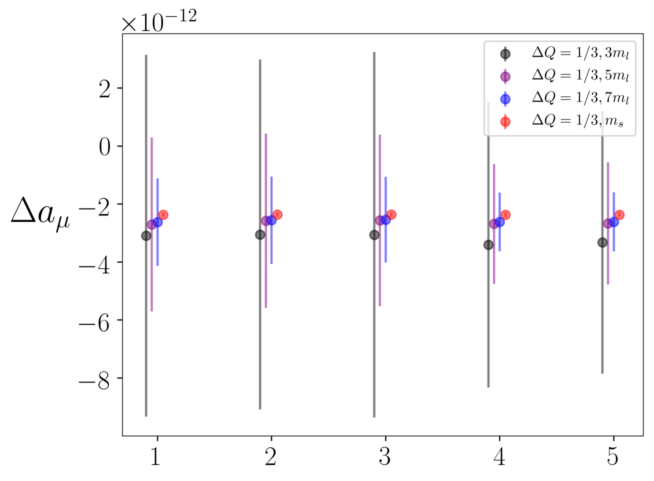
This is similar to what we see at m_s. δ_d has large errors when extrapolated to m_l. We're thinking about ways to reduce the error on δ_d and what other schemes (like ETM) might have better systematics.

Crosschecks at m_s - ϕ meson



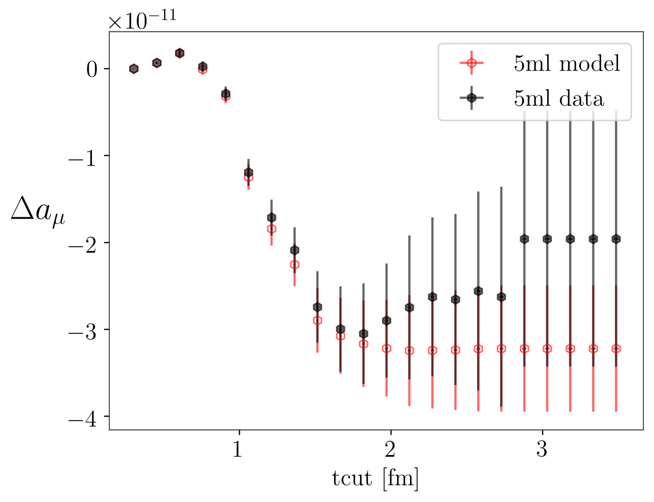
Varying the fitting range

Coarse ensemble, binsize=2, fitrange=[tmin, T-tmin+1]

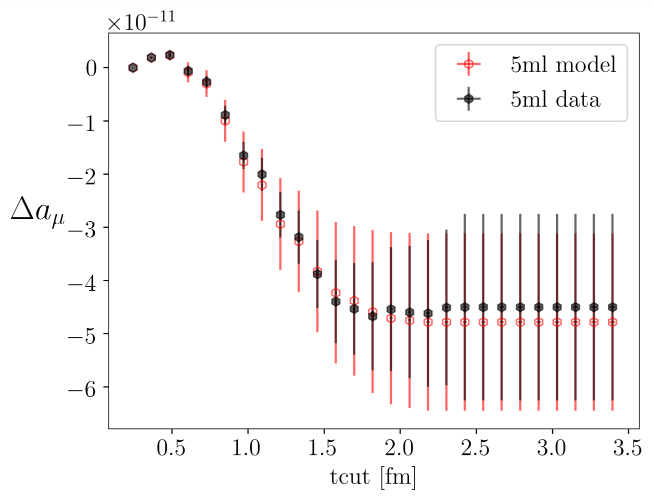


tcut dependence

5ml Very Coarse ensemble, tmin=2, binsize=2, $\Delta Q = 2/3$



5ml Coarse ensemble, tmin=2, binsize=2, $\Delta Q = 2/3$



tcut dependence

5ml Fine ensemble, tmin=2, binsize=1, $\Delta Q = 2/3$