Rare $K^+ \to \pi^+\ell^+\ell^-$ decays with physical mass light-quarks

Ryan Hill

P. A. Boyle, F. Erben, J. M. Flynn, V. Gülpers, R. C. Hill, R. Hodgson, A. Jüttner, F. Ó hÓgáin, A. Portelli, C. T. Sachrajda
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The RBC & UKQCD collaborations

UC Berkeley/LBNL
Aaron Meyer

University of Bern & Lund
Nils Hermansson Truedsson

BNL and BNL/RBRC
Yasumichi Aoki (KEK)
Peter Boyle (Edinburgh)
Taku Izubuchi
Chulwoo Jung
Christopher Kelly
Meifeng Lin
Nobuyuki Matsumoto
Shigemi Ohta (KEK)
Amarjit Soni
Tianle Wang

CERN
Andreas Jüttner (Southampton)
Tobias Tsang

Columbia University
Norman Christ
Yikai Huo
Yong-Chull Jang
Joseph Karpie
Bob Mawhinney
Bigeng Wang (Kentucky)
Yidi Zhao

University of Connecticut
Tom Blum
Luchang Jin (RBRC)
Douglas Stewart
Joshua Swaim
Masaaki Tomii

Edinburgh University
Matteo Di Carlo
Luigi Del Debbio
Felix Erben
Vera Gülpers
Maxwell T. Hansen
Tim Harris
Ryan Hill
Raoul Hodgson
Nelson Lachini
Zi Yan Li
Michael Marshall
Fionn Ó hÓgáin
Antonin Portelli
James Richings
Azusa Yamaguchi
Andrew Z.N. Yong

Liverpool Hope/Uni. of Liverpool
Nicolas Garron

Michigan State University
Dan Hoying

University of Milano Bicocca
Mattia Bruno

Nara Women's University
Hiroshi Ohki

Peking University
Xu Feng

University of Regensburg
Davide Giusti
Christoph Lehner (BNL)

University of Siegen
Matthew Black
Oliver Witzel

University of Southampton
Alessandro Barone
Jonathan Flynn
Nikolai Husung
Rajnandini Mukherjee
Callum Radley-Scott
Chris Sachrajda

Stony Brook University
Jun-Sik Yoo
Sergey Syritsyn (RBRC)

Motivation

- $K \to \pi\ell\bar{\ell}$ decays *via* flavour-changing neutral current
  $\to$ Highly suppressed; sensitive to new physics
- Long-distance dominated
  $\to$ Well-suited to lattice QCD techniques
- NA62 results for $K \to \pi\ell\bar{\ell}$ anticipated
  $\to$ Additional theoretical input is timely

Background

- Long-distance amplitude:

$$\mathcal{A}_\mu(q^2) = \int d^4x \langle \pi(\mathrm{p}) | T\left[J_\mu(x) H_W(0)\right] | K(\mathrm{k}) \rangle$$

- Re-expressed using EM gauge invariance[1] [2]:

$$\mathcal{A}_\mu(q^2) = -i\frac{G_F}{(4\pi)^2}\left[q^2(k+p)_\mu - \left(M_K^2 - M_\pi^2\right)q_\mu\right] \underbrace{V(z)}_{non-pert.}$$

$$V(z) = a + bz + V^{\pi\pi}(z) \qquad z = q^2/M_K^2$$

[1] JHEP 08 (1998) 004 [arXiv:hep-ph/9808289]

[2] Rev. Mod. Phys. 84, 399 (2012) [arXiv:1107.6001]

Background

- Minkowski and Euclidean spectral representations:

$$\mathcal{A}_\mu(\mathbf{k},\mathbf{p}) = +i\int_0^\infty dE\,\frac{\rho(E)}{2E}\,\frac{\langle\pi(\mathbf{p})|J_\mu|E,\mathbf{k}\rangle\langle E,\mathbf{k}|H_w|K(\mathbf{k})\rangle}{E_K(\mathbf{k})-E+i\epsilon}$$
$$-i\int_0^\infty dE\,\frac{\rho_S(E)}{2E}\,\frac{\langle\pi(\mathbf{p})|H_W|E,\mathbf{p}\rangle\langle E,\mathbf{p}|J_\mu|K(\mathbf{k})\rangle}{E-E_\pi(\mathbf{p})+i\epsilon}$$
$$I_\mu(T_a,T_b,\mathbf{k},\mathbf{p}) = -\int_0^\infty dE\,\frac{\rho(E)}{2E}\,\frac{\langle\pi(\mathbf{p})|J_\mu|E,\mathbf{k}\rangle\langle E,\mathbf{k}|H_w|K(\mathbf{k})\rangle}{E_K(\mathbf{k})-E}\left(1-e^{[E_K(\mathbf{k})-E]T_a}\right)$$
$$+\int_0^\infty dE\,\frac{\rho_S(E)}{2E}\,\frac{\langle\pi(\mathbf{p})|H_W|E,\mathbf{p}\rangle\langle E,\mathbf{p}|J_\mu|K(\mathbf{k})\rangle}{E-E_\pi(\mathbf{p})}\left(1-e^{-[E-E_\pi(\mathbf{p})]T_b}\right)$$

- T_a, T_b come from integration of normalised 4pt function:

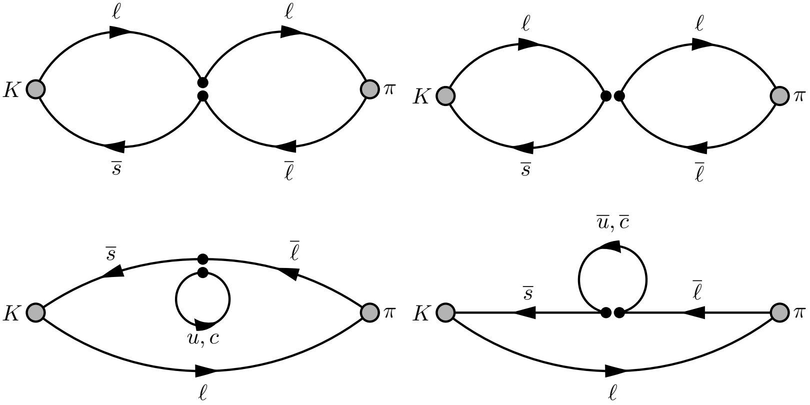
$$I_\mu(T_a,T_b,\mathbf{k},\mathbf{p}) = e^{-[E_\pi(\mathbf{p})-E_K(\mathbf{k})]t_J}\int_{t_J-T_a}^{t_J+T_b} dt_H\,\widetilde{\Gamma}_{\rm 4pt}$$

Background

$$I_\mu(T_a, T_b, \mathbf{k}, \mathbf{p}) = -\int_0^\infty dE \frac{\rho(E)}{2E} \frac{\langle \pi(\mathbf{p})|J_\mu|E, \mathbf{k}\rangle\langle E, \mathbf{k}|H_w|K(\mathbf{k})\rangle}{E_K(\mathbf{k}) - E}\left(1 - e^{[E_K(\mathbf{k})-E]T_a}\right) + \int_0^\infty dE \frac{\rho_S(E)}{2E} \frac{\langle \pi(\mathbf{p})|H_W|E, \mathbf{p}\rangle\langle E, \mathbf{p}|J_\mu|K(\mathbf{k})\rangle}{E - E_\pi(\mathbf{p})}\left(1 - e^{-[E-E_\pi(\mathbf{p})]T_b}\right)$$

- Amplitude corresponds to limit $T_a, T_b \to \infty$
- First line: π, $\pi\pi$, and $\pi\pi\pi$ on-shell intermediate states enter the spectral density (for physical masses)
  $\to E_K > E_\pi, E_{\pi\pi}, E_{\pi\pi\pi}$: Causes the T_a exponential to diverge!
- Lattice - can't take $T_a, T_b \to \infty$
  $\to$ Must remove exponentially growing terms in T_a due to intermediate states
  $\to T_a, T_b$ must be large enough for exponentials to sufficiently decay

Background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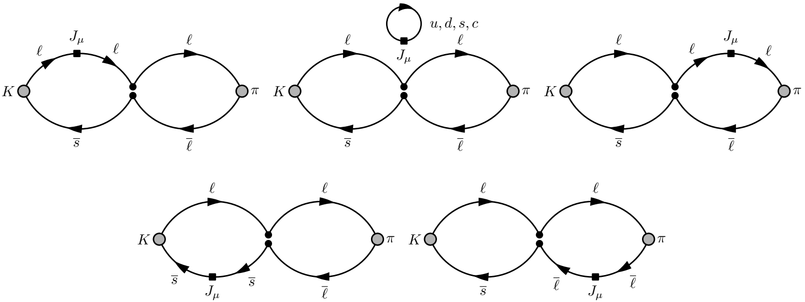
Background



Setup

Ensemble & Action properties[1]:

- $2 + 1$ flavour, $L^3 \times T = 48^3 \times 96$, $a^{-1} = 1.73$ GeV
- Physical Pion and Kaon masses
  $\rightarrow$ Expensive calculation!
  $\rightarrow$ Energy budget allows π, $\pi\pi$, $\pi\pi\pi$ intermediate states
- zMöbius Domain Wall Fermions
  $\rightarrow$ Significantly cheaper than Möbius DWF but requires a bias-correction step
  $\rightarrow$ Allows statistics to be accumulated on a cheaper estimator and then be shifted to the full Möbius action

[1] [arXiv:2202.08795]

Setup

Intermediate states:

- π IS: Significant contribution, must be removed
- $\pi\pi$ IS: Introduced by lattice artefacts, at practical values of T_a expected to be percent-level effect[1]
- $\pi\pi\pi$ IS: Compare decay widths of $K_S \to \pi\pi$ to $K_{S,+} \to \pi\pi\pi$: factor $\sim \mathcal{O}(1/500)$ further suppressed, $\pi\pi\pi$ completely negligible for forseeable future with these values of T_a[1]

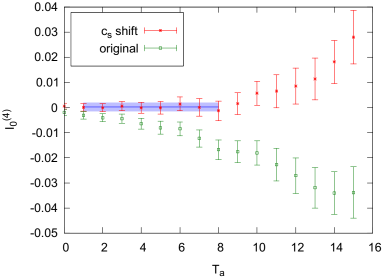
[1] Phys.Rev. D. 92 (2015) 094512 [arXiv:1507.03094]

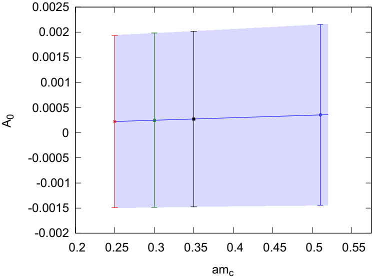
Setup

Two approaches for the removal of on-shell single-π intermediate state:

- Reconstruct the single-π contribution and explicitly subtract it
  $\rightarrow$ Stable state, can be constructed from lattice QCD correlation functions
- Shift the weak Hamiltonian with an $\bar{s}d$ scalar current,
  $H'_w(x) = H_w(x) + c_s(\mathbf{k})\bar{s}(x)d(x)$
  $\rightarrow$ $C_s(\mathbf{k})$ tuned to condition $\langle \pi(\mathbf{k}) | H'_W(0, \mathbf{k}) | K(\mathbf{k}) \rangle = 0$
  $\rightarrow$ Cancels the single-π intermediate state
  $\rightarrow$ Does not contribute to amplitude

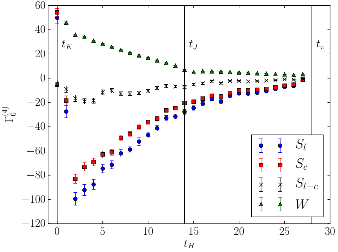
Results

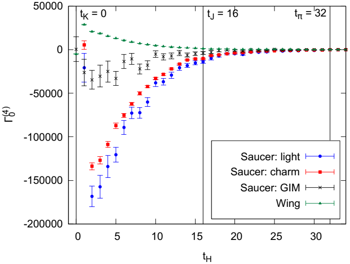




- $A_0 = 0.00035(180)$
- $V(z) = -0.87(4.44)$
- $V(z) \approx V(0) = a^+$ for our choice of kinematics
- Form factor unfortunately unresolved, but let's investigate why...

Results

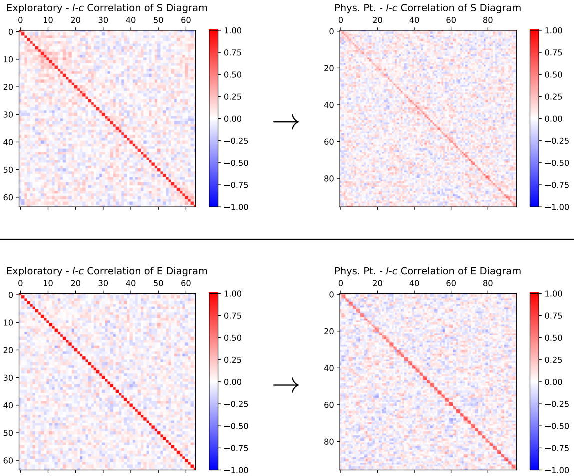




Exploratory Study

Physical-Point

- Plots show the GIM subtraction for the saucer diagram constructed from the l and c quark correlators
- GIM subtraction does not lead to a cancellation of errors with physical light masses



- Much reduced correlation between l and c loop quark diagrams at physical point due to large mass difference

Results

Statistical error cannot be overcome by square-root scaling of additional statistics alone in near future.

$\rightarrow$ Potential ways forward:

- Improvement of estimators for up- and charm-loop propagators
  $\rightarrow$ Similar to issues faced in disconnected diagrams
- Forgo explicit charm contribution to GIM loop and handle *via* different renormalisation procedure
  $\rightarrow$ Look to $K \rightarrow \pi\nu\bar{\nu}$ for lessons learned

$\rightarrow$ Combination of algorithmic improvements and next-generation computers makes a competitive lattice result appear feasible in the coming years.

Summary

- Viability of calculation demonstrated at physical point
- Next steps identified: gain control of GIM loop stochastic estimators or sidestep the explicit simulation thereof
- Challenging calculation with physical kinematics achieved. Competitive errors in the next few years?
- Related efforts: Rare $\Sigma^+ \to p\ell^+\ell^-$
  $\to$ Raoul Hodgson: Thursday 12:10 (Now!)
  $\to$ Felix Erben: Thursday 12:30