On the nature of light Tetraquark systems: A case study in the Scalar-Isoscalar channel

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ABSTRACT

The light scalar sector is challenging and cumbersome to study due to low-energy uncertainties in QCD calculations. The internal quark structure of these systems remains controversial, suggesting an exotic four-quark component. We systematically examined the impact of next-to-leading order perturbative terms (NLO PT) in scalar ($J^{PC} = 0^{++}$) light-quark tetraquark systems using QCD sum-rules, specifically for the so-called σ state. We used a variety of models and we found a mass prediction $0.52\,\text{GeV} \leq m_\sigma \leq 0.69\,\text{GeV}$ [1].

Introduction

▷ ***Quark model***: Proposed independently by G. Zweig [2] and M. Gell-Mann [3] in 1964 as a way to explain meson and baryon configurations, which are hadrons made up of **quarks** (q), **antiquarks** ($\bar{q}$) and **gluons** (g).

▷ ***Exotic hadrons***: Zweig-Gell-Mann's model of hadrons was not limited to the simplest compound systems, but rather constrained by the requirement of colour neutrality. Hence, the inclusion of more complex structures such as four-quark states, pentaquarks, etc.

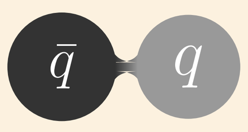

Figure 1. Meson

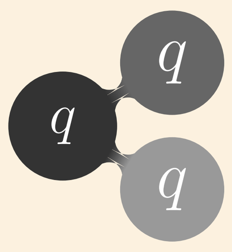

Figure 2. Baryon

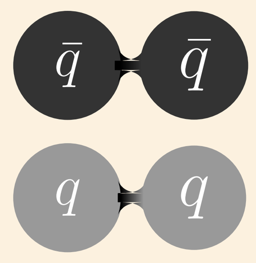

Figure 3. Tetraquarks

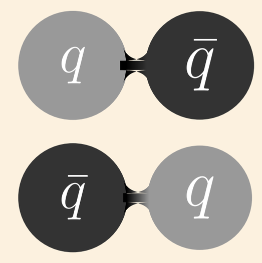

Figure 4. Meson-Meson

Laplace QCD Sum-Rules

QCD sum rules probe hadronic properties through corrrelation functions $\Pi\left(Q^2\right)$ of composite operators.
These correlation functions can be connected to the hadronic regime via a dispersion relation based on the quark-hadron duality property

$$\Pi^{\text{QCD}}(Q^2) = \frac{1}{\pi}\int_{t_0}^{\infty} dt\, \frac{\rho(t)}{t+Q^2} + \text{ subtraction terms} \qquad (1)$$

where $\rho(t)$ is the hadronic spectral function with threshold t_0.
The Borel transform operator helps suppressing excited states and unknown constants, hence a family of Laplace sum-rules are now obtained, with τ being the Borel parameter and s_0 the QCD continuum,

$$\mathcal{R}_k\left(\tau, s_0\right) = \int_{t_0}^{s_0} t^k e^{-t\tau}\rho(t)\, dt. \qquad (2)$$

Spectral Function

The correlation functions of the light scalar tetraquark systems at LO and NLO were calculated in Refs. [4] and [5], respectively. In our study (Ref. [1]) different resonance models were used for the analysis:

1. Single (SR) and Double Resonance (DR).

$$\frac{\mathcal{R}_1(\tau, s_0)}{\mathcal{R}_0(\tau, s_0)} = \begin{cases} m_\sigma^2 & \text{Single Resonance} \\ f(m_\sigma^2, m_{f_0}^2, \tau) & \text{Double Resonance} \end{cases}$$

2. SR & DR with Symmetric (SW) and Asymmetric (AW) width.

$$\frac{\mathcal{R}_1(\tau, s_0)}{\mathcal{R}_0(\tau, s_0)} = m_\sigma^2 \frac{\widetilde{W}_1\left(m_\sigma, \Gamma, \tau\right)}{\widetilde{W}_0\left(m_\sigma, \Gamma, \tau\right)} \to \widetilde{W}_n = \begin{cases} \Delta_n & \text{Symmetric} \\ W_n & \text{Asymmetric} \end{cases}$$

Analysis Methodology for Laplace Sum-Rules

For establishing an upper and lower bound on the Borel parameter τ, we used:

$$B_k = \frac{\mathcal{L}_k^{\langle\alpha GG\rangle}(\tau)}{\mathcal{L}_k^{\text{pert}}(\tau)} < \frac{1}{3}, \quad \frac{\mathcal{R}_k(\tau, s_0)/\mathcal{R}_{k-1}(\tau, s_0)}{\mathcal{R}_{k-1}(\tau, s_0)/\mathcal{R}_{k-2}(\tau, s_0)} \geq 1, \qquad k \geq 2.$$

The results from these conditions are:

$$0.2\,\text{GeV}^{-2} < \tau < 0.57\,\text{GeV}^{-2}$$

Results

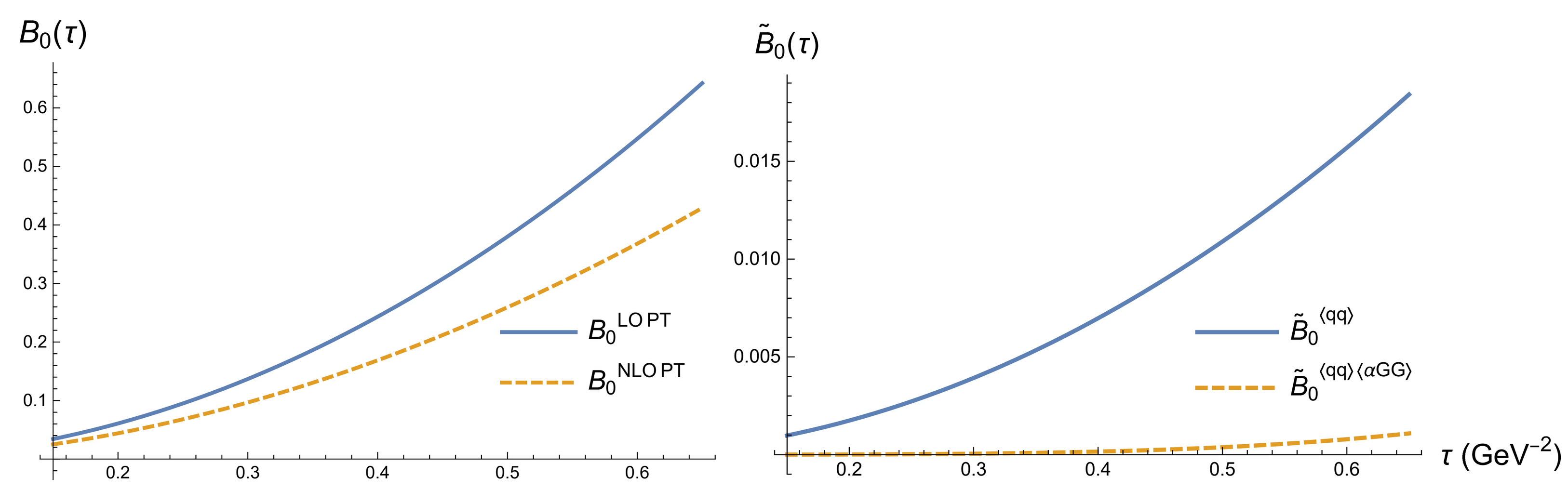

Figure 5. (Left) Ratios B_k of the gluon condensate $\langle\alpha_s GG\rangle$ contributions to LO and NLO PT terms for $k = 0, 1$. (Right) Ratio of non-PT terms contributions to LO and NLO PT terms.

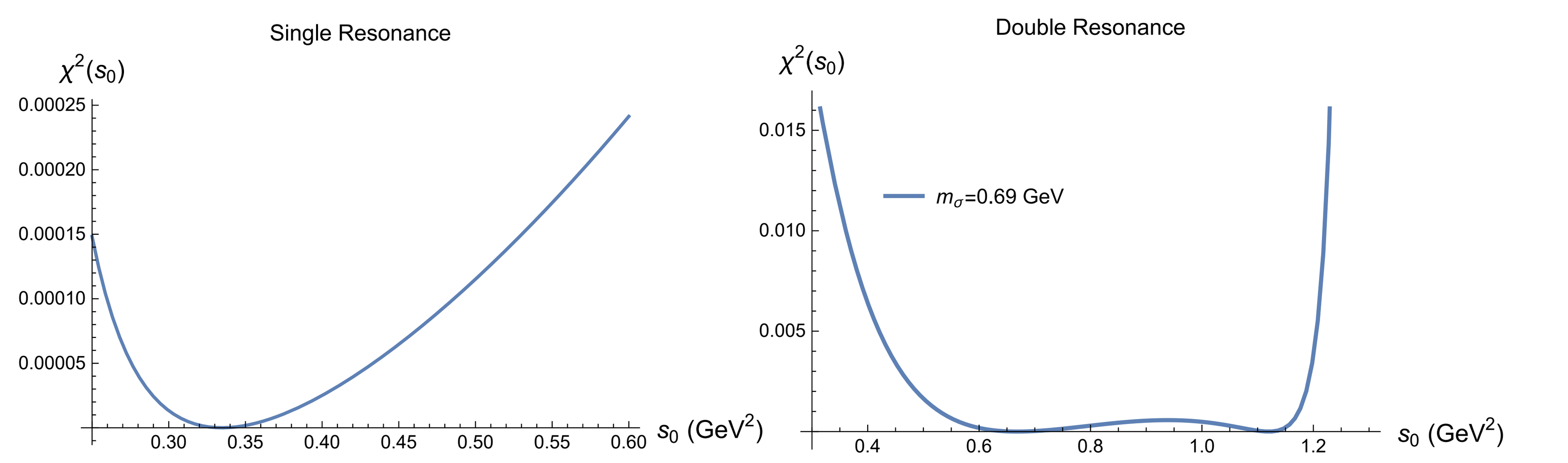

Figure 6. The residual sum of squares is shown as a function of s_0 for single and double resonance models. Optimized masses $m_\sigma^{\text{SR}} = 0.52\,\text{GeV}$ and $m_\sigma^{\text{DR}} = 0.69\,\text{GeV}$ were found.

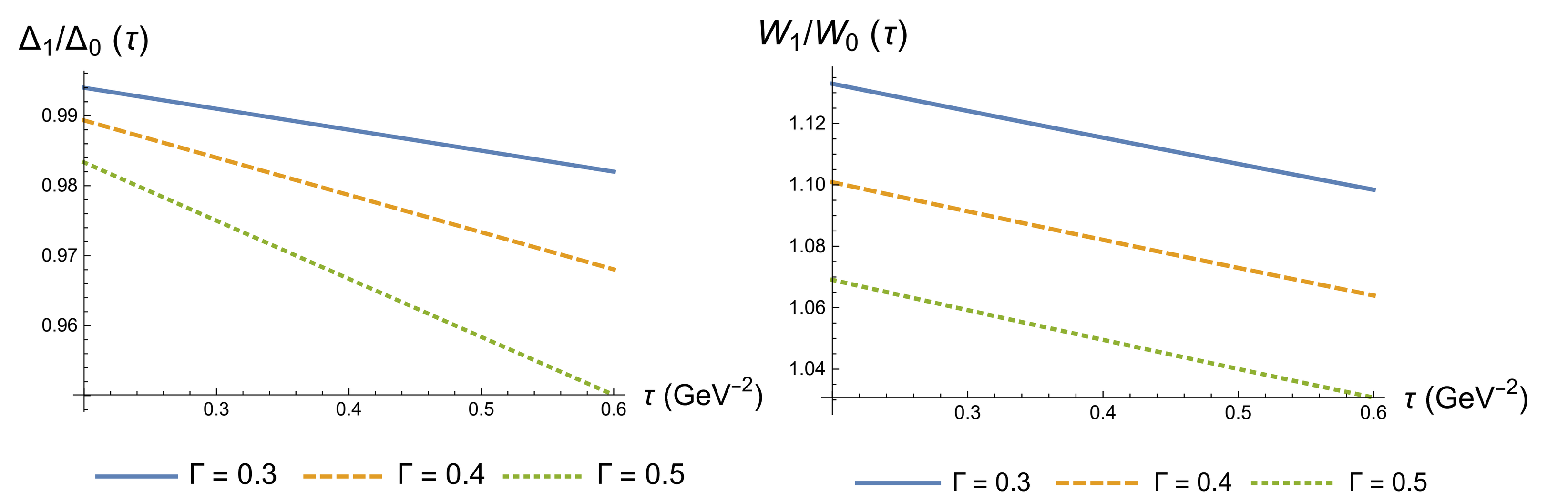

Figure 7. Width effects on the mass predictions for symmetric (left) and asymmetric (right) resonance shapes.

Conclusions

The NLO tetraquark Laplace sum-rule ratios were used to obtain predictions for the lightest scalar-isoscalar σ state in a variety of models from a tetraquark picture approach, showing remarkable stability under large NLO corrections [1]. The Borel window is expanded by these corrections, providing a stronger foundation for a sum-rule analysis. Additionally, the individual Laplace sum-rules can be used to extract the anomalous dimension for the QCD spectral function.

References

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