Approaching partons on a Euclidean Lattice

Ken Wilson Award Acceptance

YONG ZHAO

LATTICE CONFERENCE 2022

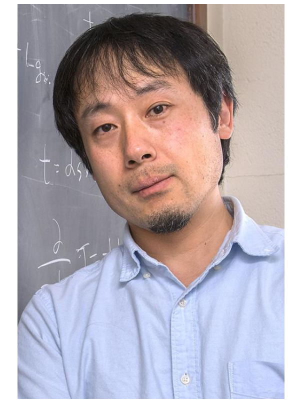
AUG 12, 2022

Argonne NATIONAL LABORATORY

Collaborators

Shohini Batthacharya
(BNL)

Markus Ebert
(MPI)



Yoshitaka Hatta
(BNL)

Xiangdong Ji
(UMD)

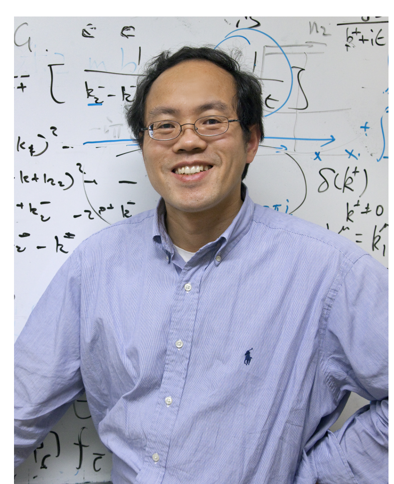
Kyle Lee
(LBNL)

Yizhuang Liu
(Jagellonian U)

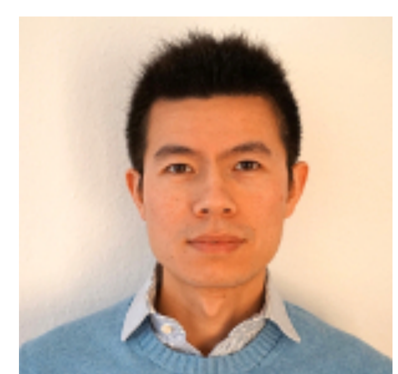
Iain Stewart
(MIT)

Stella Schindler
(MIT)

Wei Wang
(Shanghai Jiao Tong U)



Feng Yuan
(LBNL)



Jianhui Zhang
(Beijing Normal U)

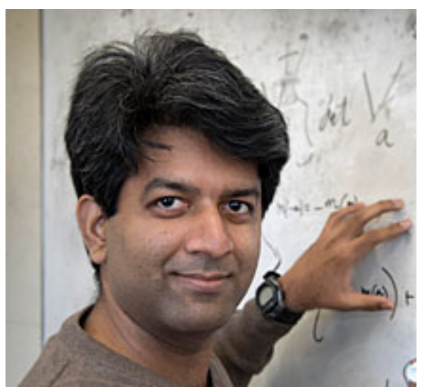
Collaborators


Xiang Gao (ANL)


Andrew Hanlon (BNL)


Jack Holligan (UMD)


Taku Izubuchi (RIKEN, BNL)


Nikhil Karthik (WM & JLab)


Swagato Mukherjee (BNL)


Peter Petrezcky (BNL)

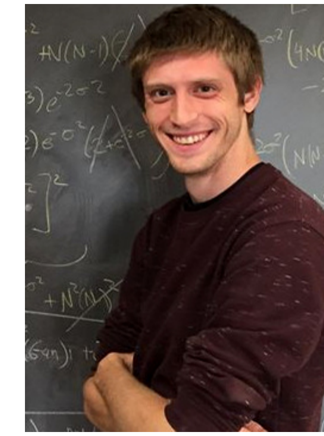

Philipp Scior (BNL)


Charles Shugert (SBU)


Sergey Syritsyn (SBU)


Artur Avkhadiev (MIT)


Phiala Shanahan (MIT)


Michael Wagman (FNAL)


William Detmold (MIT)


Anthony Grebe (MIT)


David C.-J. Lin (National Yang Ming Chiao-Tung U)

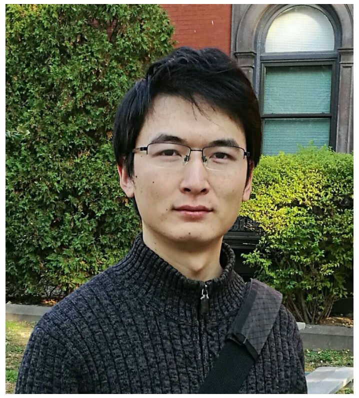

Robert Perry (National Yang Ming Chiao-Tung U)


Issaku Kanamori (RIKEN, R-CCS)


Santanu Mondal (LANL)

Collaborators

Jiunn-Wei Chen
(National Taiwan U)



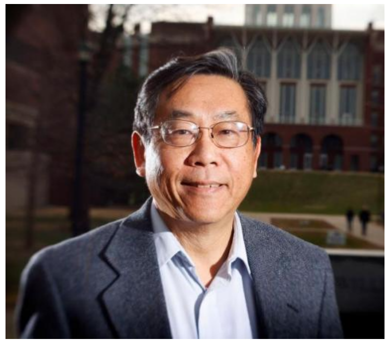
Luchang Jin
(U Connecticut)

Huey-Wen Lin
(MSU)

Yusheng Liu
(T-D Lee Institute)

Andreas Schäfer
(U Regensburg)

Yi-Bo Yang
(ITP, Beijing)



Keh-Fei Liu (U Kentucky)

Raza Sufian (JLab)

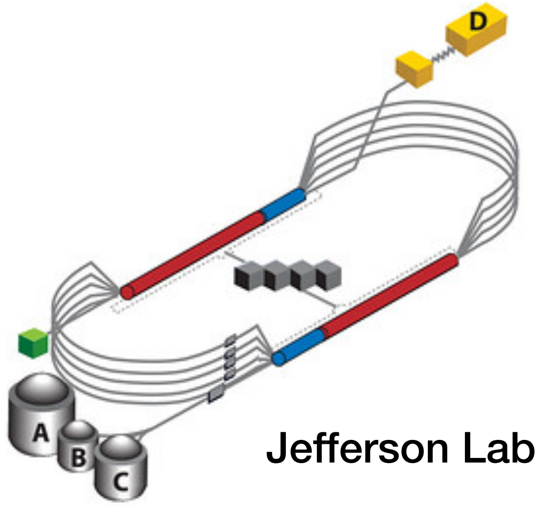
Special thanks to

The KWLA Selection Committee
The IAC of the Lattice2022 Conference

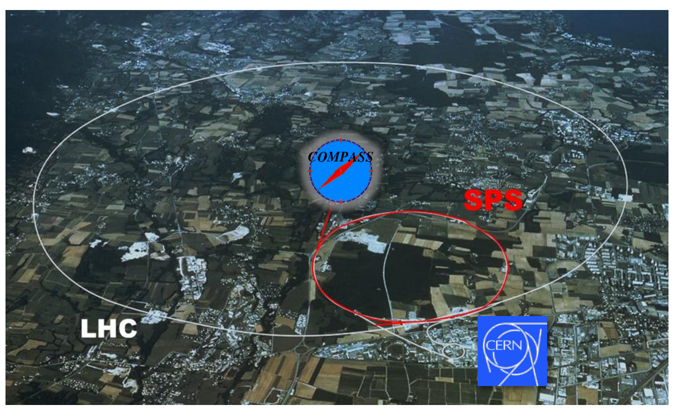
My mentors and colleagues at UMD, LBNL, MIT, BNL and ANL

My parents, sister and Lilian

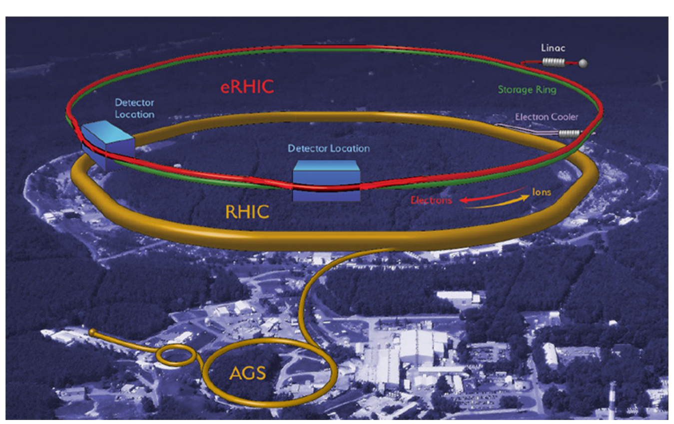
3D Imaging of the Nucleon



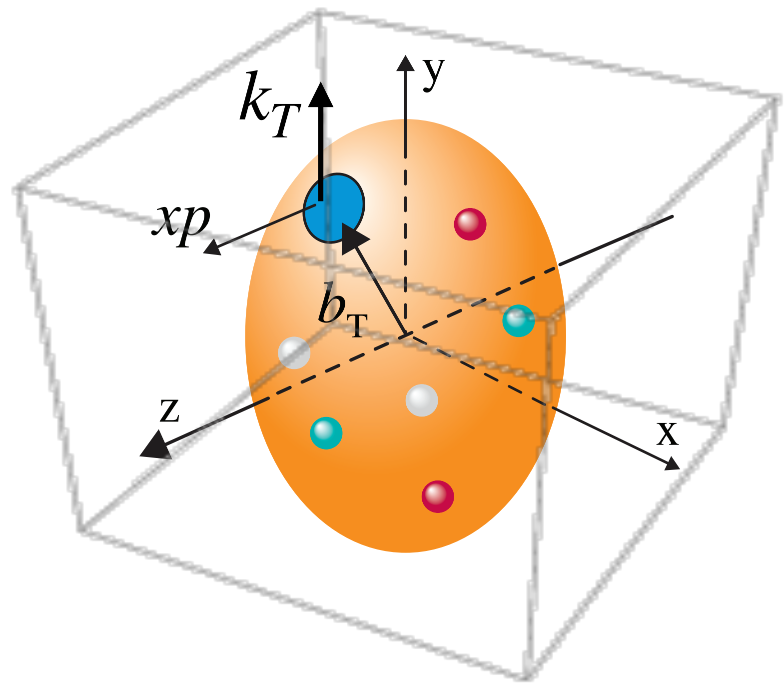
Jefferson Lab 12 GeV

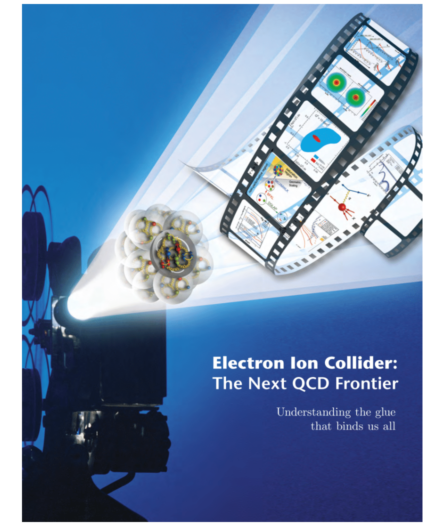


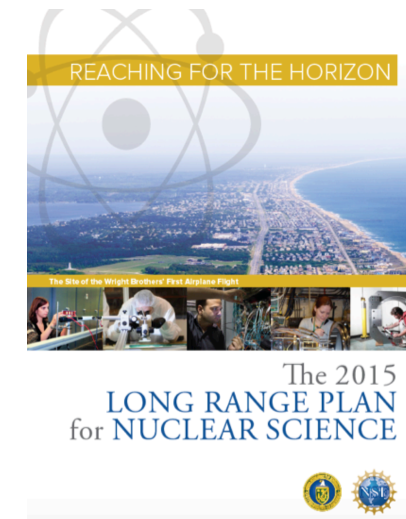
COMPASS, CERN

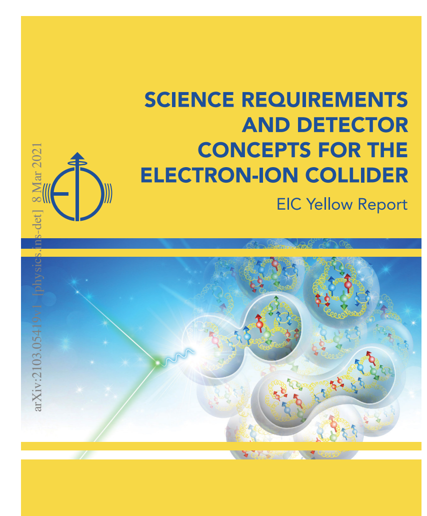


The Electron-Ion Collider, BNL

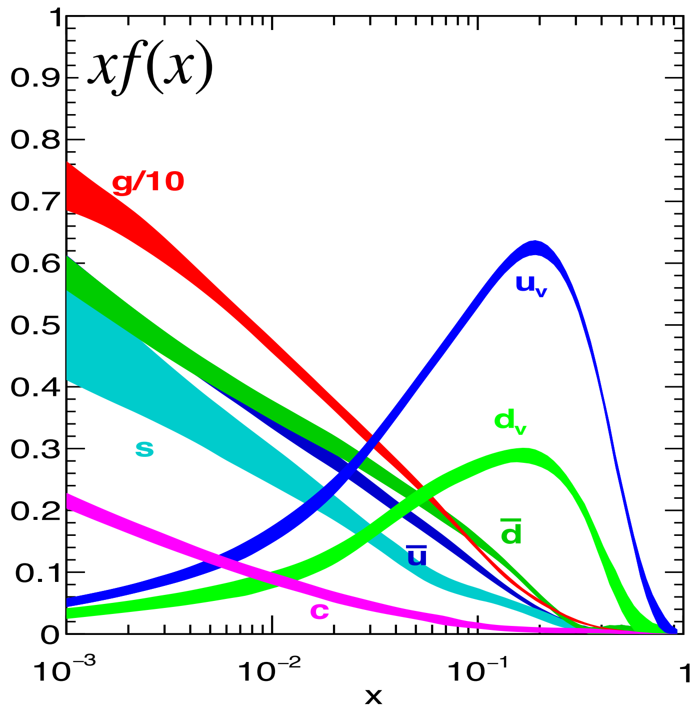








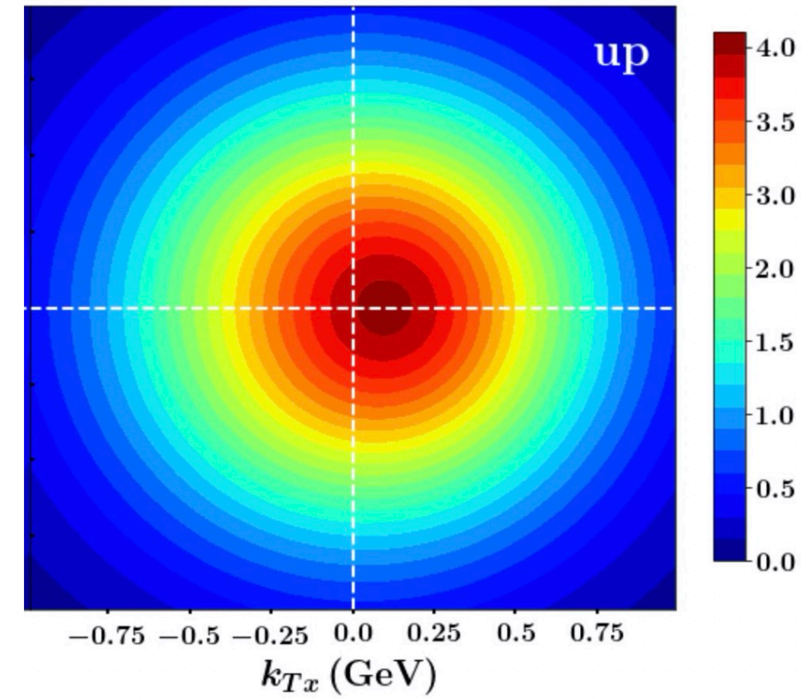
Parton Distribution Functions (PDFs)



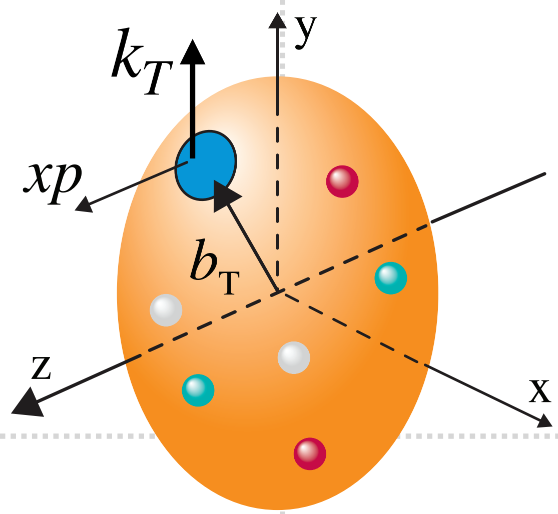
NNPDF, EPJ C77 (2017)

$\int d^2 \overrightarrow{k}_T$

Transvers momentum distributions (TMDs)



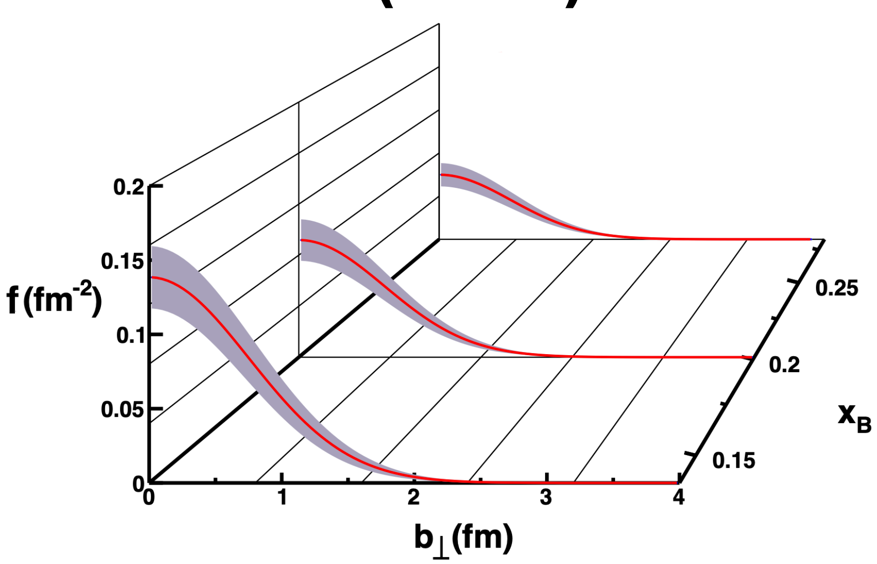
Cammarota, et al. (JAM), PRD **102** (2020).



$\int d^2 \overrightarrow{b}_T$

$\int d^2 \overrightarrow{b}_T$

Generalized parton distributions (GPDs)



W. Armstrong et al., arXiv: 1708.00888.

$\int d^2 \overrightarrow{k}_T$

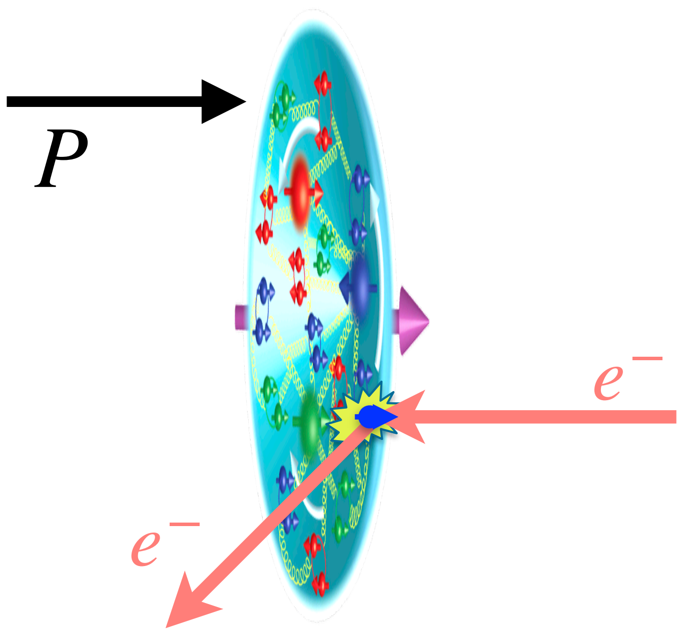
Wigner distributions/ Generalized TMDs

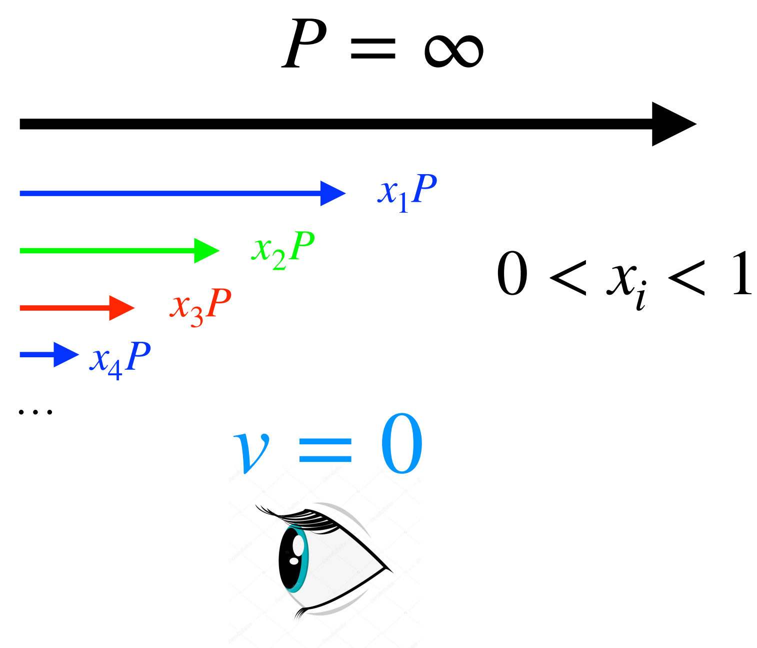
$$W(x, \overrightarrow{k}_T, \overrightarrow{b}_T)$$

Can we calculate all of them in lattice QCD?

The parton model

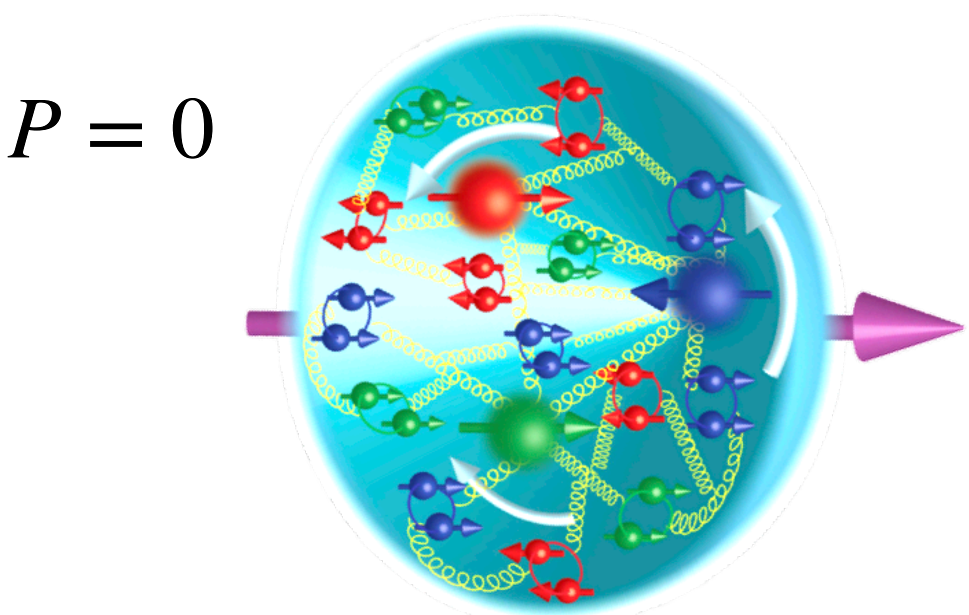
Infinite momentum frame picture:

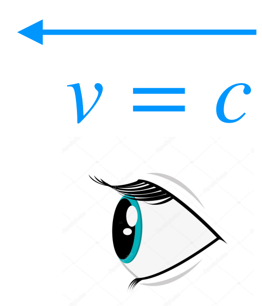




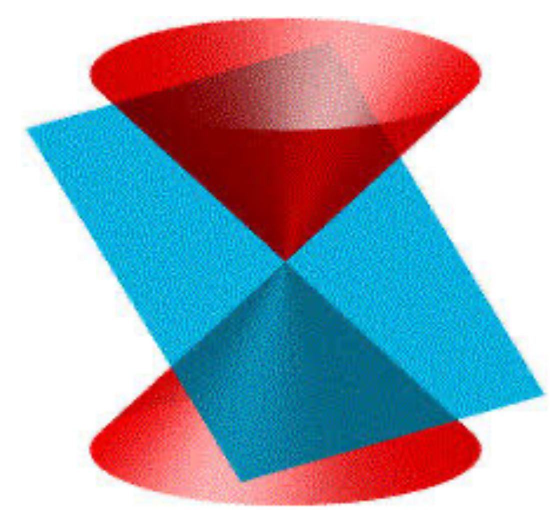
Richard P. Feynman

Light-cone picture:

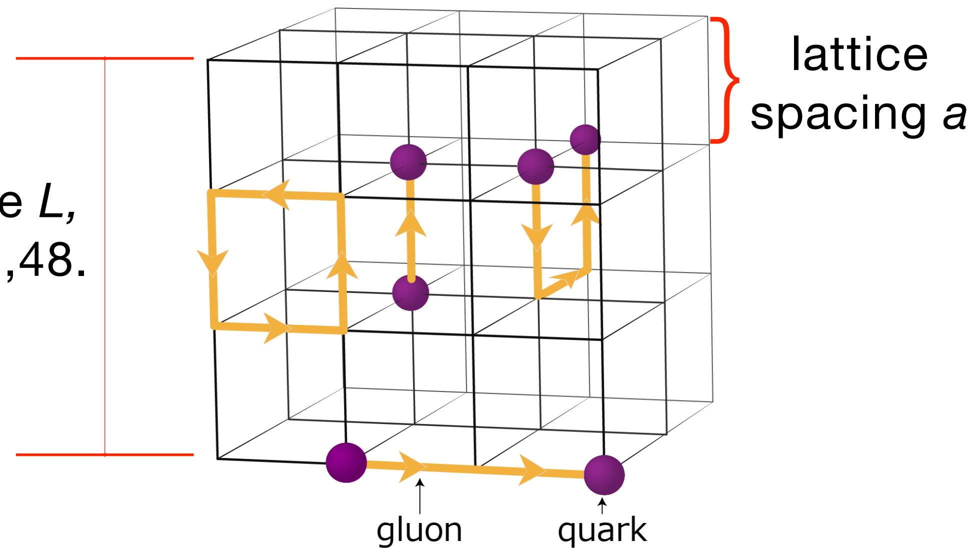




Light-cone quantization



Simulating partons on the lattice



Imaginary time: $t \to i\tau$

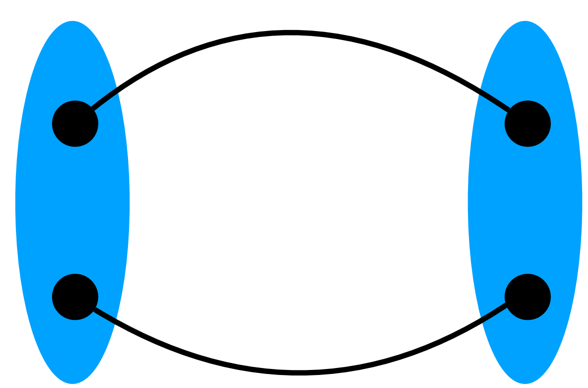
- $P = \infty$ **hadron state?** ✗ $P \ll \frac{2\pi}{a}$!
  $t = 0$
- **Light-cone correlations?** ✗
  $z + ct = 0$
  $z - ct \neq 0$
  Real-time sign problem 🙁

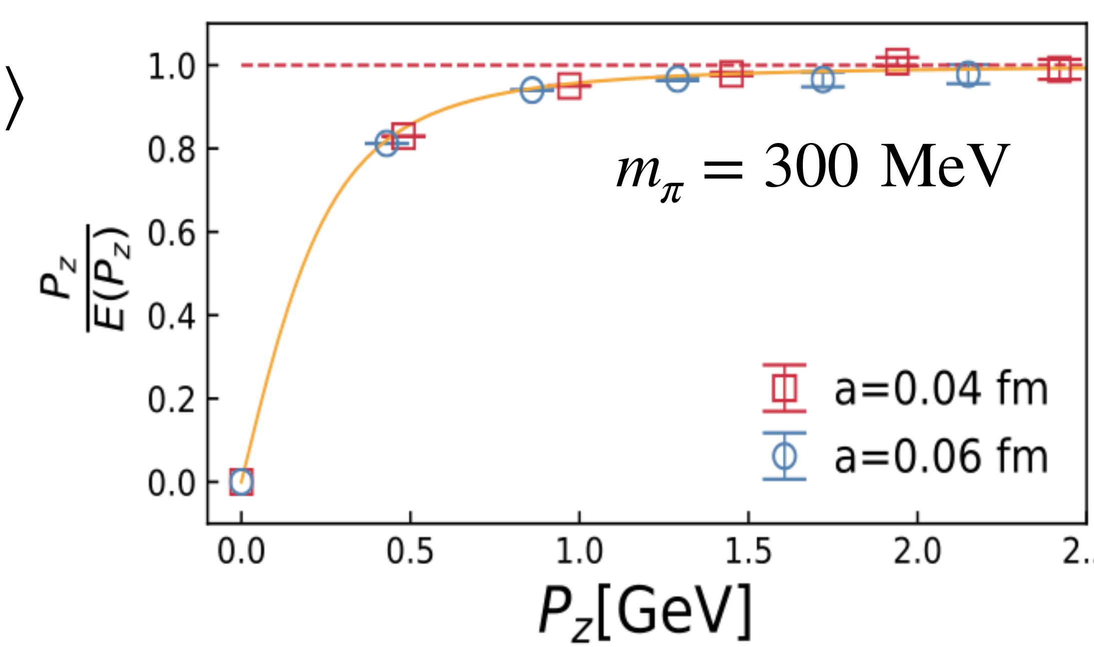
Nevertheless, it is possible to approach the Feynman parton picture by simulating a boosted hadron on the lattice 😇

e.g. $C_{\text{2pt}}^{ss'}(t_s; \mathbf{P}) = \langle \pi_s(\mathbf{x}_0, t_s) \pi_{s'}^{\dagger}(\mathbf{P}, 0) \rangle$

Gaussian momentum smearing

G. S. Bali et al., Phys.Rev.D **93** (2016).





X. Gao, N. Karthik, YZ et al., Phys.Rev.D **102** (2020).

Large momentum expansion and matching

Euclidean observable	Partonic observable
$\tilde{Q}(P^z, \Lambda_{\text{UV}}) \equiv \langle P \vert \tilde{O}(\Lambda_{\text{UV}}) \vert P \rangle$	$Q(\mu) \equiv \langle P \vert O(\mu) \vert P \rangle$
Λ_{UV}: ultraviolet (UV) cutoff, $\sim \dfrac{2\pi}{a}$	μ: $\overline{\text{MS}}$ scale. No P^z dependence.
$(P^z \ll \Lambda_{\text{UV}})$ $\tilde{O}$ $\xrightarrow{\infty \text{ Lorentz boost}}$	O $(P^z \gg \Lambda_{\text{UV}})$

$$\tilde{Q}(P^z, \Lambda_{\text{UV}}) = \boxed{Q(\mu)\ ?} + c_1 \frac{\Lambda_{\text{QCD}}}{P^z} + c_2 \frac{\Lambda^2_{\text{QCD}}}{P_z^2} + \ldots$$

✘ $\because$ $P^z \ll \Lambda_{\text{UV}}$ and $P^z \gg \Lambda_{\text{UV}}$ usually do not commute.

Large momentum expansion and matching

Euclidean observable	Partonic observable
$\tilde{Q}(P^z, \Lambda_{\rm UV}) \equiv \langle P \mid \tilde{O}(\Lambda_{\rm UV}) \mid P \rangle$	$Q(\mu) \equiv \langle P \mid O(\mu) \mid P \rangle$
$\Lambda_{\rm UV}$: ultraviolet (UV) cutoff, $\sim \dfrac{2\pi}{a}$	μ: $\overline{\rm MS}$ scale. No P^z dependence.
$(P^z \ll \Lambda_{\rm UV})$ $\tilde{O}$ $\xrightarrow{\infty \text{ Lorentz boost}}$ O $(P^z \gg \Lambda_{\rm UV})$	

$$\tilde{Q}(P^z, \Lambda_{\rm UV}) = C\left(\frac{\mu}{P^z}, \frac{\Lambda_{\rm UV}}{\mu}\right) \otimes Q(\mu) + c_1 \frac{\Lambda_{\rm QCD}}{P^z} + c_2 \frac{\Lambda_{\rm QCD}^2}{P_z^2} + \ldots$$

Perturbative matching

Power corrections

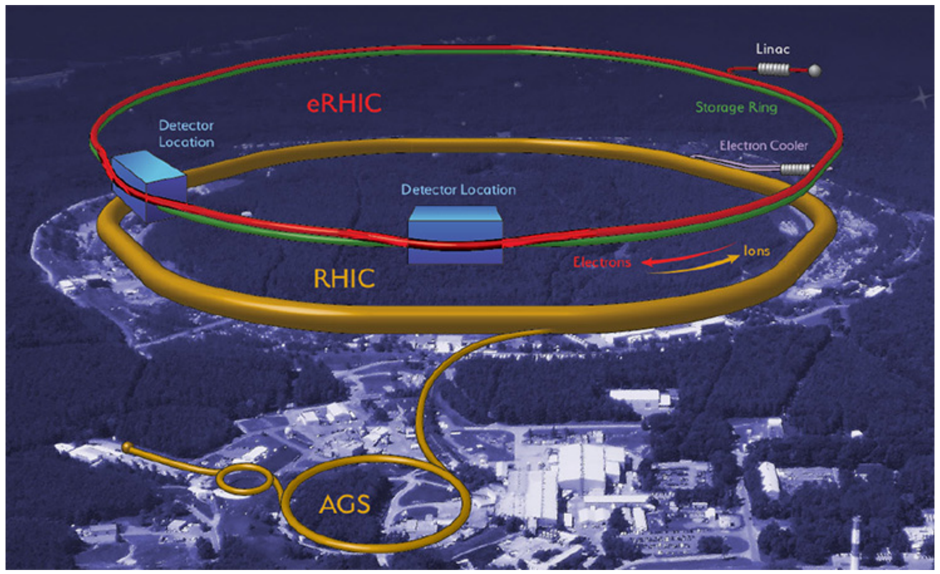
"Large-momentum effective theory (LaMET)":

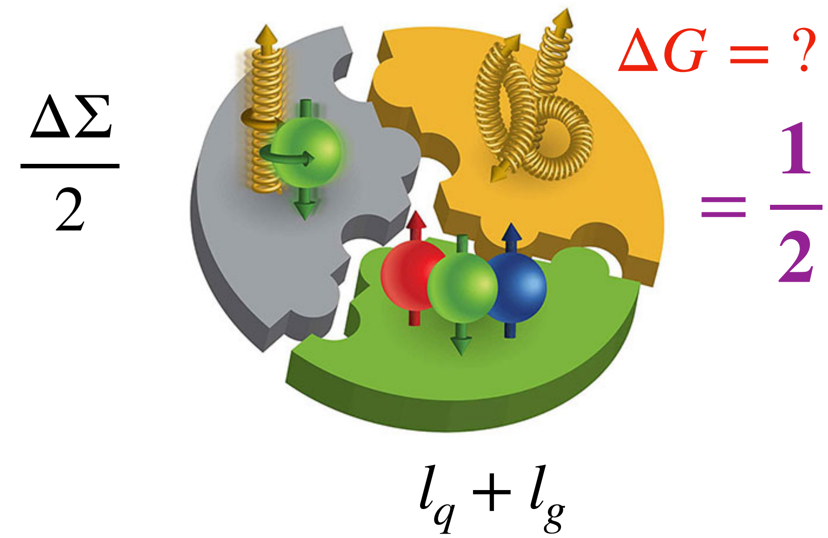
a recipe for systematically controlled calculation of parton physics

- X. Ji, Phys. Rev. Lett. 110 (2013); SCPMA57 (2014).
- X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, Rev.Mod.Phys. 93 (2021).

The gluon helicity ΔG

RHIC spin program and EIC





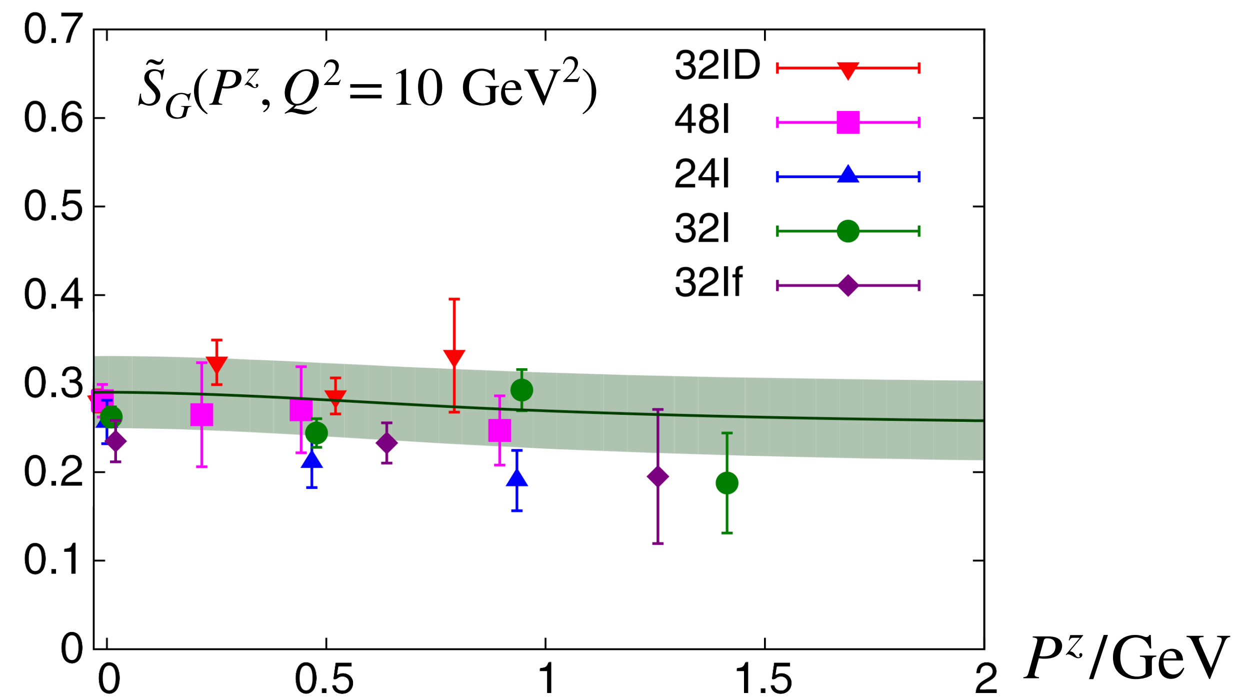
$$\Delta G(\mu) = \left.\frac{\langle PS|(\mathbf{E}\times\mathbf{A})^3|PS\rangle}{2S^+}\right|_{A^+=0}$$

$$\tilde{S}_G(P^z,\mu) = \left.\frac{\langle PS|(\mathbf{E}\times\mathbf{A})^3|PS\rangle}{2S^z}\right|_{\nabla\cdot\mathbf{A}=0}$$

$$\tilde{S}_G(P^z,\mu) = C\otimes(\Delta\Sigma,\Delta G) \; + \mathcal{O}(\Lambda^2_{\rm QCD}/P_z^2)$$

- X. Ji, J.-H. Zhang, and **YZ**, Phys. Rev. Lett. **111** (2013);
- Y. Hatta, X. Ji and **YZ**, Phys.Rev.D **89** (2014);
- X. Ji, J.-H. Zhang, and **YZ**, Phys.Lett.B **743** (2015).

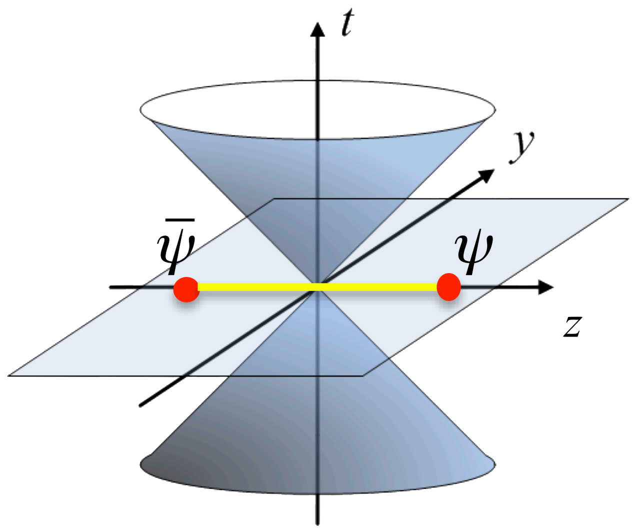
The first lattice result



Y.-B. Yang, R. Sufian, **YZ**, et al. Phys. Rev. Lett. **118** (2017)

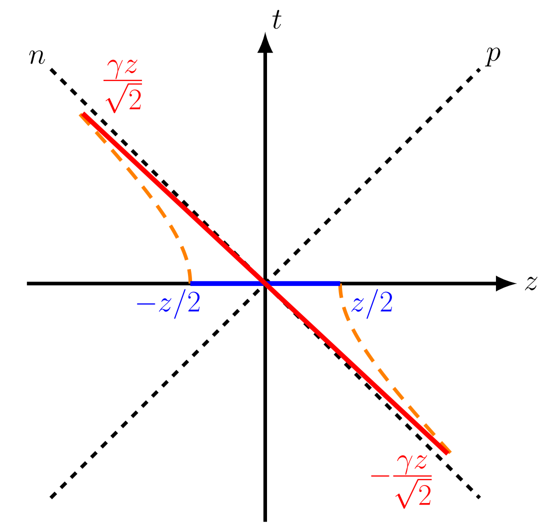
Benchmark: lattice calculation of the PDFs

A quasi-PDF $\tilde{f}(x, P^z)$ to expand from



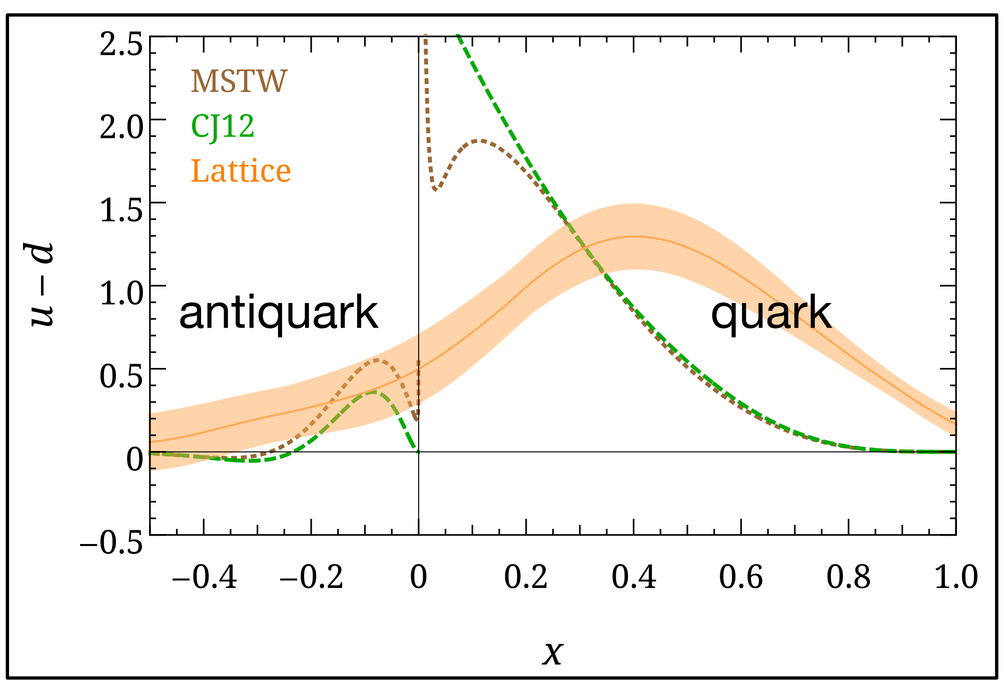
X. Ji, Phys. Rev. Lett. **110** (2013)

Lorentz boost



LaMET expansion: $\tilde{f}(y, P^z, \Lambda_{\rm UV}) = \int_{-1}^{1} \frac{dx}{|x|} C\left(\frac{y}{x}, \frac{\mu}{P^z}, \frac{\Lambda_{\rm UV}}{\mu}\right) f(x, \mu) \; + \; \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_z^2}\right)$

X. Xiong, X. Ji, J.-H. Zhang and YZ, Phys.Rev.D 90 (2014).

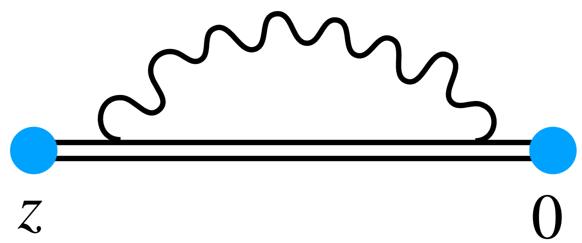


First exploratory lattice calculation

H.W. Lin et al., (LP3), Phys.Rev.D 91 (2015).

Lattice renormalization

The linear divergences became a roadblock



$$= \delta m(a)|z| \propto \frac{|z|}{a} \qquad \tilde{f}(x, P^z, \Lambda_{\rm UV}) \propto \alpha_s \Lambda_{\rm UV} + \ldots$$

Multiplicative renormalizability of the quasi-PDF operator was proven:

$$O_B^\Gamma(z, a) = \bar{\psi}_0(z)\Gamma W_0[z,0]\psi_0(0) = e^{-\delta m(a)|z|}\, Z_O(a) O_R^\Gamma(z)$$

- Ji, Zhang and YZ, Phys.Rev.Lett. **120** (2018);
- Ishikawa, Ma, Qiu and Yoshida, Phys.Rev.D **96** (2017);
- Green, Jansen and Steffens, Phys.Rev.Lett. **121** (2018).

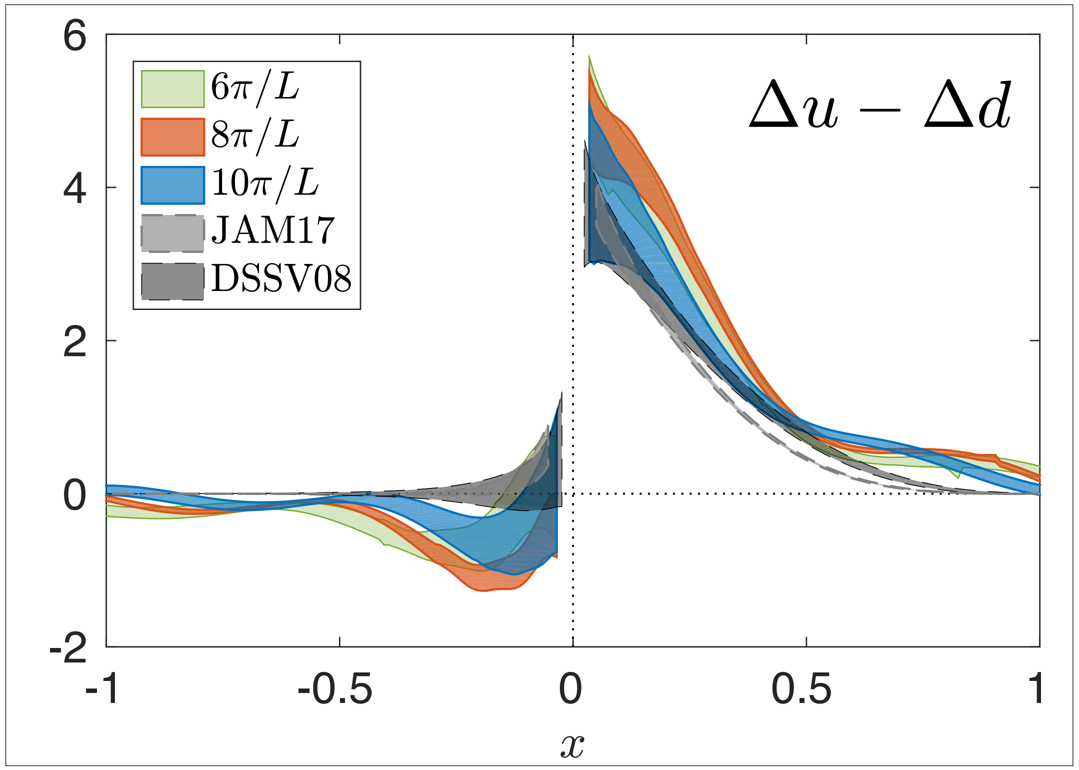
Non-perturbative lattice renormalization became possible:

$$\tilde{f}_X(x, P^z, \tilde{\mu}) = \int_{-\infty}^{\infty} \frac{dz}{2\pi}\, e^{iz(xP^z)} \lim_{a\to 0} \frac{\tilde{h}(z, P^z, a)}{Z_X(z, \tilde{\mu}, a)}$$

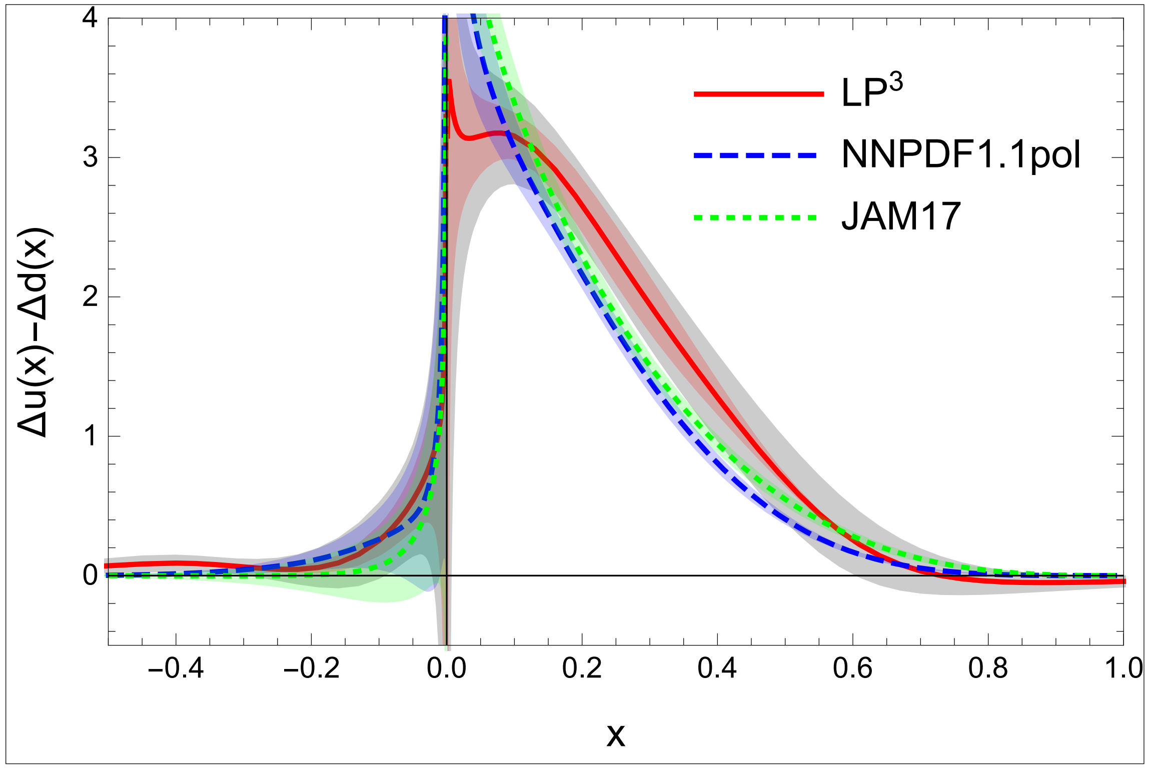
RIMOM (fixed gauge): $Z_X = \langle q \| O^\Gamma(z) \| q \rangle$	**Ratio schemes:** $Z_X = \langle P_0^z \| O^\Gamma(z) \| P_0^z \rangle$
• Constantinou and Panagopoulos, Phys.Rev.D **96** (2017); • C. Alexandrou et al., Nucl.Phys.B **923** (2017); • I. Stewart and YZ, Phys.Rev.D **97** (2018); • J.-W. Chen, YZ et al., (LP3), Phys.Rev.D **97** (2018).	• K. Orginos et al., Phys.Rev.D **96** (2017); • V. Braun, A. Vladimirov and Zhang, Phys.Rev.D **99** (2019); • Z. Fan, X. Gao et al., Phys.Rev.D **102** (2020).

Lattice renormalization

Encouraging results under the RIMOM scheme:



Alexandrou et al. (ETMC), Phys.Rev.Lett. **121** (2018)



H.W. Lin, YZ, et al. (LP3), Phys.Rev.Lett. **121** (2018)

However, the RIMOM scheme introduces non-perturbative effects at large *z*:

$$\tilde{f}_{\rm RI}(x, P^z, \tilde{\mu}) = \int_{-\infty}^{\infty} \frac{dz}{2\pi}\, e^{iz(xP^z)} \lim_{a\to 0} \frac{\tilde{h}(z, P^z, a)}{Z_{\rm RI}(z, \tilde{\mu}, a)}$$

$$Z_{\rm RI} = \langle q \,|\, O^{\Gamma}(z) \,|\, q \rangle$$

X. Gao, N. Karthik, YZ, et al., Phys.Rev.D **102** (2020).

Hybrid renormalization scheme

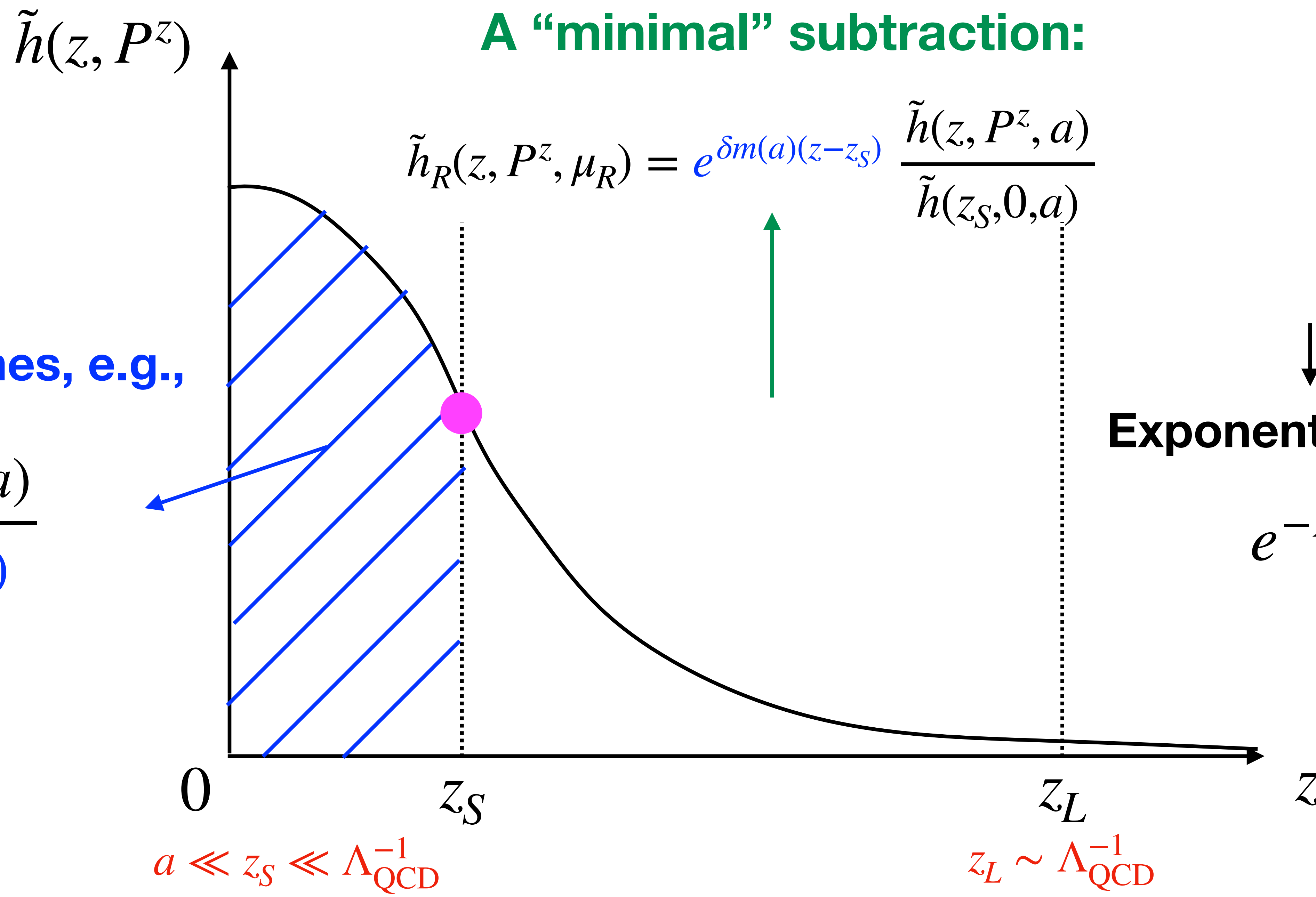
X. Ji, **YZ**, et al., Nucl.Phys.B 964 (2021).

A “minimal” subtraction:

$$\tilde{h}_R(z, P^z, \mu_R) = e^{\delta m(a)(z-z_S)} \frac{\tilde{h}(z, P^z, a)}{\tilde{h}(z_S, 0, a)}$$

Ratio schemes, e.g.,

$$\frac{\tilde{h}(z, P^z, a)}{\tilde{h}(z, 0, a)}$$



Exponential decay

$$e^{-\bar{\Lambda}|z|}$$

Hybrid renormalization scheme

A "minimal" subtraction: $z > z_S$

$$\tilde{h}_R(z, P^z, \mu_R) = e^{\delta m(a)(z-z_S)} \frac{\tilde{h}(z, P^z, a)}{\tilde{h}(z_S, 0, a)} \quad \Rightarrow \quad e^{-\bar{m}_0(z-z_S)} \frac{\tilde{h}_0^{\overline{\rm MS}}(z, P^z, \mu)}{h_0^{\overline{\rm MS}}(z_S, 0, \mu)}$$

$\bar{m}_0$:

- UV renormalon ambiguity ($\sim \Lambda_{\rm QCD}$) in the definition of $\tilde{h}_0^{\overline{\rm MS}}$
- Leading to a linear power correction $\sim \bar{m}_0/P^z$

X. Ji, YZ, et al., Nucl.Phys.B **964** (2021).

Gauge-invariant determination of δm and $\bar{m}_0$:

- Self renormalization

  Y. Huo, et al. (LPC), Nucl.Phys.B **969** (2021).

- Static potential and ratios of $\tilde{h}_0^{\overline{\rm MS}}$ at different z

  X. Gao, YZ, et al., Phys.Rev.Lett. **128** (2022).

Utilizing OPE at short distance

$$h_0^{\overline{\rm MS}}(z, 0, \mu) = C_0(\alpha_s(\mu), z^2\mu^2) + \mathcal{O}(z^2\Lambda_{\rm QCD}^2)$$

Wilson coefficient:
Known to NNLO with 3-loop anomalous dimension

- Chen, Zhu and Wang, Phys.Rev.Lett. **126** (2021);
- Li, Ma and Qiu, Phys.Rev.Lett. **126** (2021);
- V. Braun and K. G. Chetyrkin, JHEP 07 (2020).

Perturbative matching

$$f(x,\mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} \bar{C}\left(\frac{x}{y}, \frac{\mu}{\mathbf{y}P^z}, \frac{\tilde{\mu}}{\mu}\right) \tilde{f}(y, P^z, \tilde{\mu}) \; + \; \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(xP^z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-x)P^z)^2}\right)$$

- Rigorous derivation of the exact form of matching formula.

T. Izubuchi, X. Ji, L. Jin, I. Stewart, and YZ, Phys.Rev.D **98** (2018)

- Nonsinglet NNLO matching for $\overline{\rm MS}$ and hybrid schemes.

  - Chen, Zhu and Wang, Phys.Rev.Lett. **126** (2021);
  - Li, Ma and Qiu, Phys.Rev.Lett. **126** (2021);
  - X. Gao, YZ, et al., Phys.Rev.Lett. **128** (2022).

- Direct power expansion in parton momenta in *x*-space.

Reliable prediction within [$x_{\rm min}$, $x_{\rm max}$] at a given finite P^z !

Short-distance factorization in coordinate space

OPE: $$\tilde{h}(\lambda = zP^z, z^2\mu^2) = \sum_{n=0}^{\infty} C_n(z^2\mu^2)\frac{(-i\lambda)^n}{n!}a_n(\mu) + \mathcal{O}(z^2\Lambda^2_{\rm QCD}),$$

T. Izubuchi, X. Ji, L. Jin, I. Stewart, and **YZ**, Phys.Rev.D **98** (2018)

Model-independent calculation of the lowest moments with finite $\lambda_{max}=z_{max}\,p^z_{max}$.

$$z_{\rm max} \ll \Lambda^{-1}_{\rm QCD}$$

Ioffe-time pseudo distribution:

- A. Radyushkin, Phys.Rev.D **96** (2017);
- K. Orginos et al., Phys.Rev.D **96** (2017).

$$\tilde{h}(\lambda, z^2\mu^2) = \int_0^1 d\alpha\ \mathscr{C}(\alpha, z^2\mu^2)\ h(\alpha\lambda, \mu) + \mathcal{O}(z^2\Lambda^2_{\rm QCD}),$$

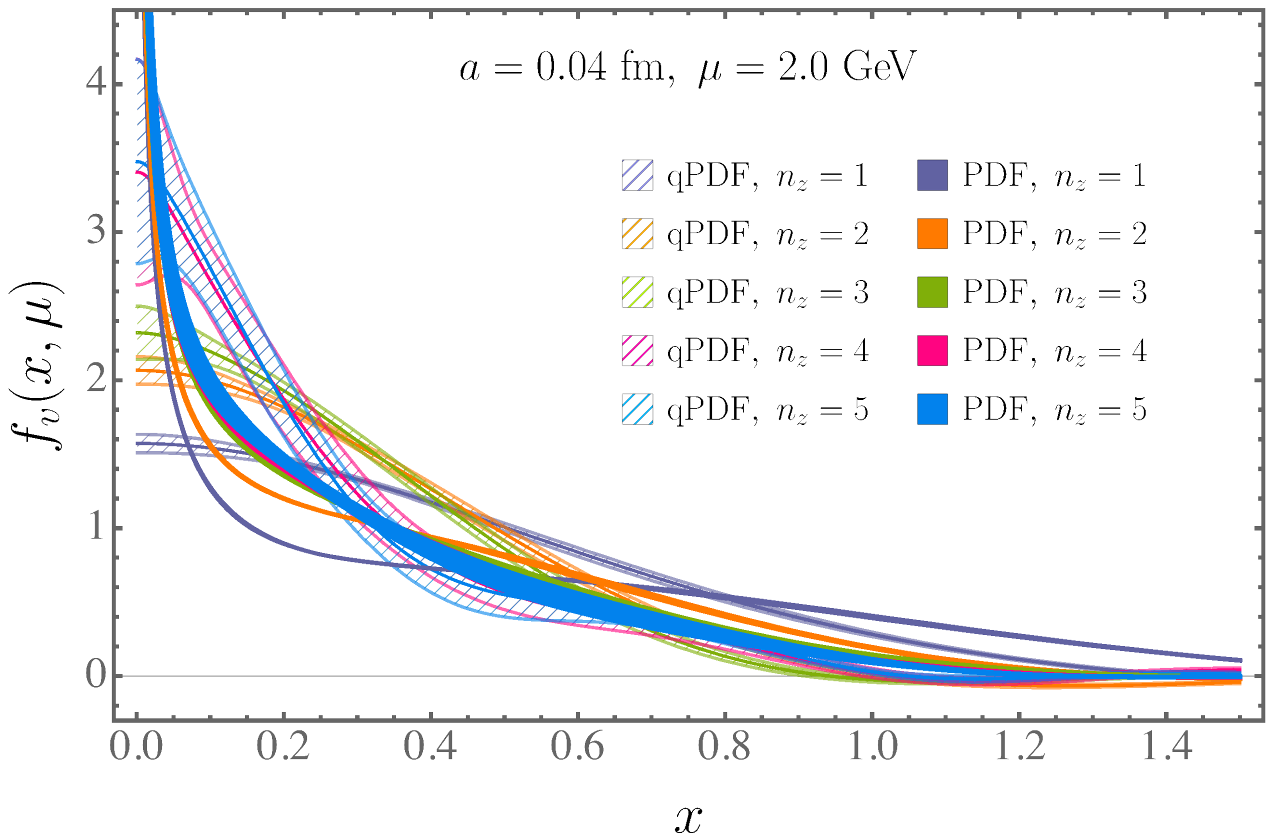
$$f(x,\mu) = \int_{-\infty}^{\infty}\frac{d\lambda}{2\pi}\ e^{-ix\lambda}\ h(\lambda,\mu)$$

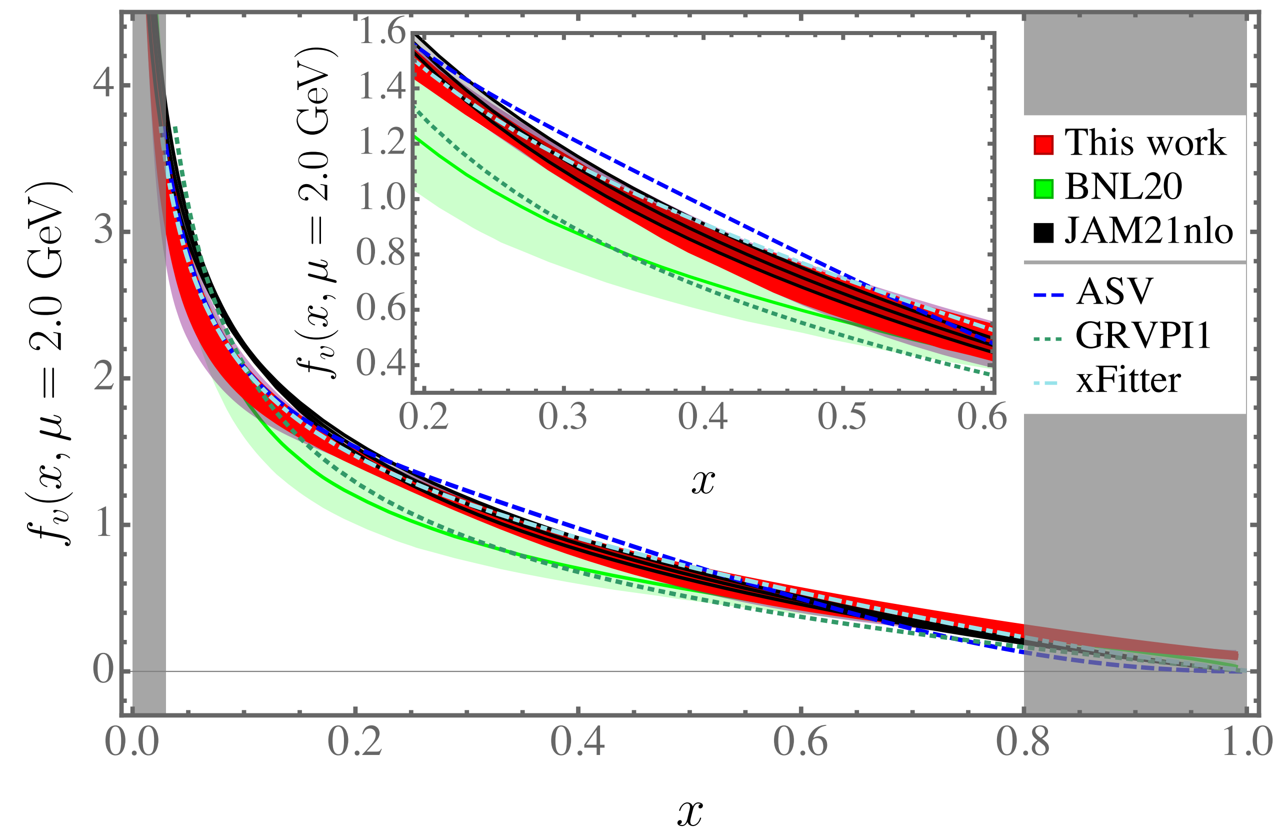
- **Model-independent** calculation of light-cone correlation $h(\lambda,\mu)$ up to λ_{max};
- $h(\lambda,\mu)$ decays slowly (power law), needs very large λ for a controlled Fourier transform;
- With not very large λ_{max}, needs **assumptions** to obtain x-dependence, e.g., $f(x) \propto x^a(1-x)^b(1+c\sqrt{x}+\ldots)$, orthonormal polynomials, and neural networks, etc..

State-of-the-art calculation of pion valence PDF

Gao, Hanlon, Mukherjee, Petreczky, Scior, Syritsyn and YZ, PRL **128**, 142003 (2022).

Super fine lattice spacing (*a*=0.04 fm and 0.06 fm), high momentum (P^z=2.42 GeV v.s. m_π=300 MeV), high statistics, first NNLO matching





See Dr. Xiang Gao's parallel talk on Wed.

Global fits at NLO

- **JAM21nlo**, Phys.Rev.Lett. **127** (2021);
- **xFitter (2020)**, Phys.Rev.D **102** (2020);
- **ASV**, Phys.Rev.Lett. **105** (2010);
- **GRVPI1,** Z. Phys. C **53** (1992).

Short-distance factorization at NLO, with same data:

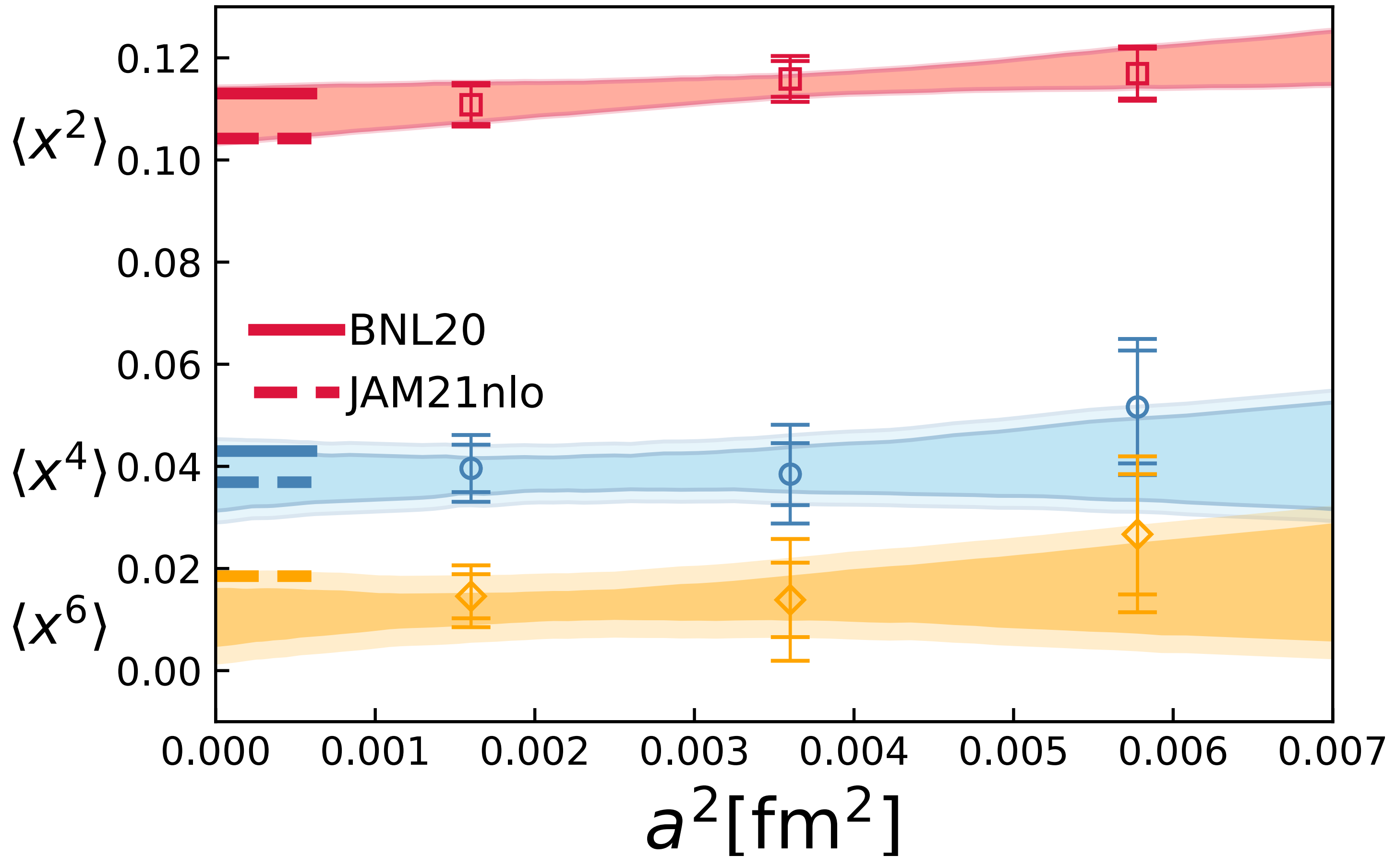
BNL20, X. Gao, N. Karthik, YZ, et al., Phys.Rev.D **102** (2020).

State-of-the-art calculation of pion valence PDF

Gao, Hanlon, Karthik, Mukherjee, Petreczky, Scior, Shi, Syritsyn, YZ and Zhou, arXiv: 2208.02297.

Continuum extrapolation with a=0.04 fm and 0.06 fm, m_π=300 MeV and a=0.076 fm, m_π=140 MeV lattice ensembles, at NNLO

Mellin moments from OPE



Towards better systematic control

- **Lattice simulation:** larger P^z (excited states), spacing $a \to 0$ (renormalization), physical m_π, lattice size $L \to \infty$, etc.

- **Perturbative theory:** all current results are obtained with fixed-order matching. End-point region uncertainty underestimated due to large logs.

  - x-space: $$f(x,\mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} \bar{C}\left(\frac{x}{y}, \frac{\mu}{yP^z}, \frac{\tilde{\mu}}{\mu}\right) \tilde{f}(y, P^z, \tilde{\mu}) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{(xP^z)^2}, \frac{\Lambda^2_{\rm QCD}}{((1-x)P^z)^2}\right)$$

    Resummation of $\alpha_s \ln[\mu^2/(2xP^z)^2], \quad \alpha_s \ln(1-x)$

  - Coordinate space: $$\tilde{h}(\lambda = zP^z, z^2\mu^2) = \sum_{n=0}^{\infty} C_n(z^2\mu^2) \frac{(-i\lambda)^n}{n!} a_n(\mu) + \mathcal{O}(z^2\Lambda^2_{\rm QCD}),$$

X. Gao, K. Lee, and YZ et al., Phys.Rev.D **103** (2021).

Resummation of $\alpha_s \ln[\mu^2 z^2], \quad \alpha_s^m \ln^n N$

- **Renormalons and power corrections:**

  Renormalon resummation improves $\bar{m}_0$ determination and perturbative convergence.

  J. Holligan, X. Ji, et al., submitted to journal.

Towards better systematic control

- **Lattice simulation:** larger P^z (excited states), spacing $a \to 0$ (renormalization), physical m_π, lattice size $L \to \infty$, etc.

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$$f(x,\mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} \bar{C}\left(\frac{x}{y}, \frac{\mu}{yP^z}, \frac{\tilde\mu}{\mu}\right) \tilde f(y, P^z, \tilde\mu) \; + \; \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(xP^z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-x)P^z)^2}\right)$$

Resummation of $\alpha_s \ln[\mu^2/(2xP^z)^2], \quad \alpha_s \ln(1-x)$

  - Coordinate space:

$$\tilde h(\lambda = zP^z, z \; \ldots \; \infty \; \ldots \; (-i\lambda)^n \; \ldots \; z^2\Lambda_{\rm QCD}^2) \, ,$$

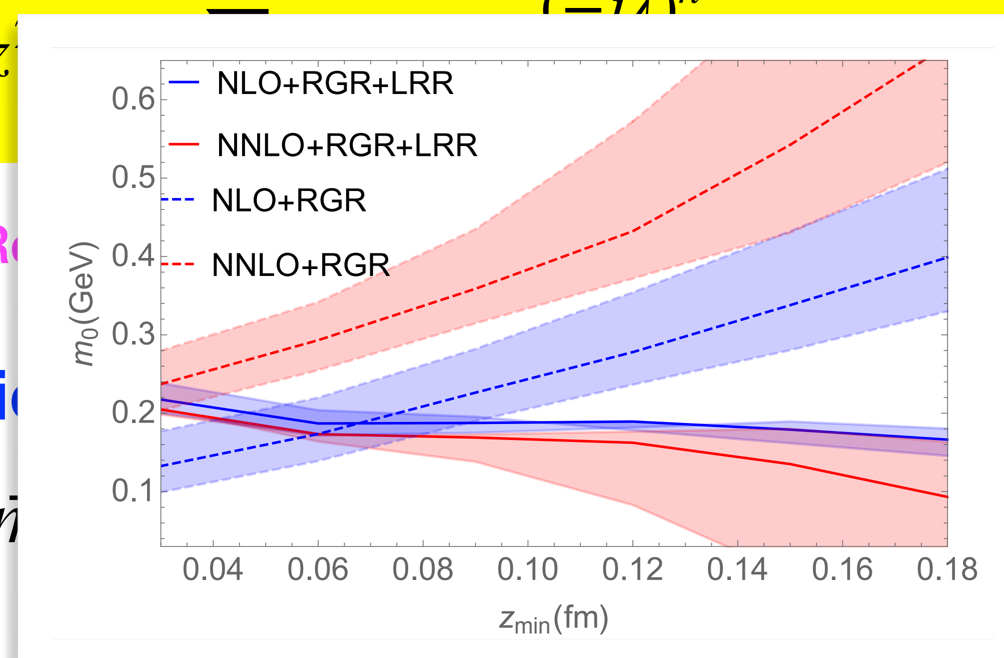
X. Gao, K. Lee, and YZ et al., Phys.Rev.D **103** (2021).

Re

- **Renormalons and power correcti**

  Renormalon resummation improves $\tilde{}$
  convergence.

  J. Holligan, X. Ji, et al., submitted to journal.



See Dr. Jack Holligan's parallel talk on Thu.

See the talks in Hadron Structure parallel session.

Other proposals:

Pseudo distribution

- A. Radyushkin, Phys.Rev.D **96** (2017);
- K. Orginos et al., Phys.Rev.D **96** (2017).

OPE of Compton form factor

- A Chambers et al. (QCDSF), Phys.Rev.Lett. **118** (2017);
- A Hannaford-Gunn et al. (CSSM/QCDSF/UKQCD), Phys.Rev.D 105 (2022).

Heavy quark OPE (HOPE)

- Detmold and Lin, Phys.Rev.D **73** (2006);
- Detmold, Lin, YZ et al. (HOPE), Phys.Rev.D **104** (2021).

Short-distance OPE of current-current correlator

- Braun and Müller, Eur.Phys.J.C **55** (2008);
- Ma and Qiu, Phys.Rev.Lett. **120** (2018).

Hadronic tensor

K.-F. Liu, Phys.Rev.Lett. **72** (1994).

Notable new results:

Light-cone distribution amplitudes

- Detmold, Grebe, YZ et al. (HOPE), Phys.Rev.D **105** (2022)
- J. Hua et al. (LPC), arXiv: 2201.09173;
- X. Gao, N. Karthik, YZ, et al., arXiv: 2206.04084

Unpolarized and helicity gluon PDFs

- Fan and Lin, Phys.Lett.B **823** (2021);
- T. Khan et al. (HadStruc), Phys.Rev.D **104** (2021);
- C. Erger, R. Sufian et al. (HadStruc), arXiv: 2207.08733.

Light flavor separation of proton PDFs

- C. Alexandrou et al. (ETMC), Phys.Rev.D **104** (2021);
- C. Alexandrou et al. (ETMC), Phys.Rev.Lett. **126** (2021).

Strange and charm quark PDFs

- R. Zhang, H.-W. Lin and B. Yoon, Phys.Rev.D **104** (2021).

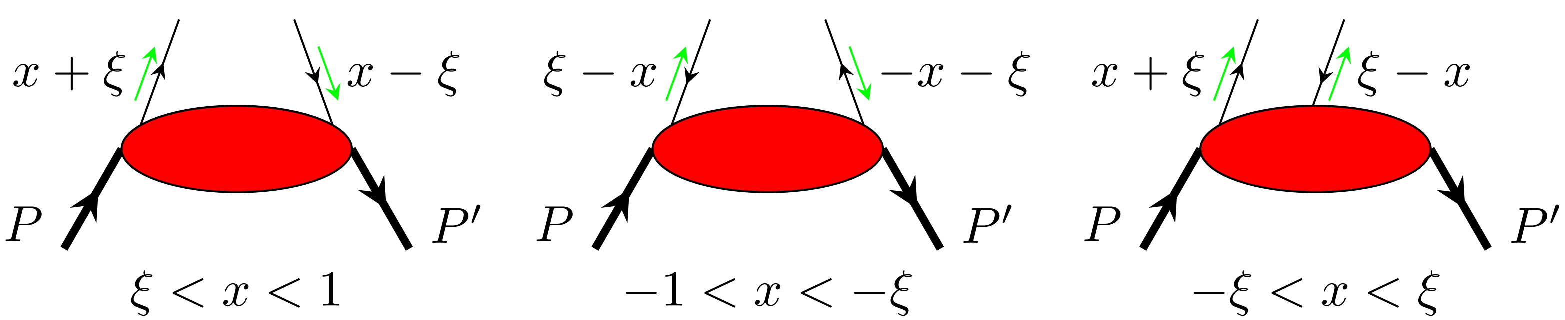
…

GPDs

Similar to the calculation of PDFs:

$$F(x,\xi,t,\mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\xi}{y}, \frac{\mu}{y\bar{P}^z}, \frac{\tilde{\mu}}{\mu}\right) \tilde{F}(y,\xi,t,\bar{P}^z,\tilde{\mu})$$
$$+\mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{(x\bar{P}^z)^2}, \frac{t}{(x\bar{P}^z)^2}, \frac{\Lambda^2_{\rm QCD}}{((1-x)\bar{P}^z)^2}, \frac{t}{((1-x)\bar{P}^z)^2}, \frac{\Lambda^2_{\rm QCD}}{((x\pm\xi)\bar{P}^z)^2}\right)$$

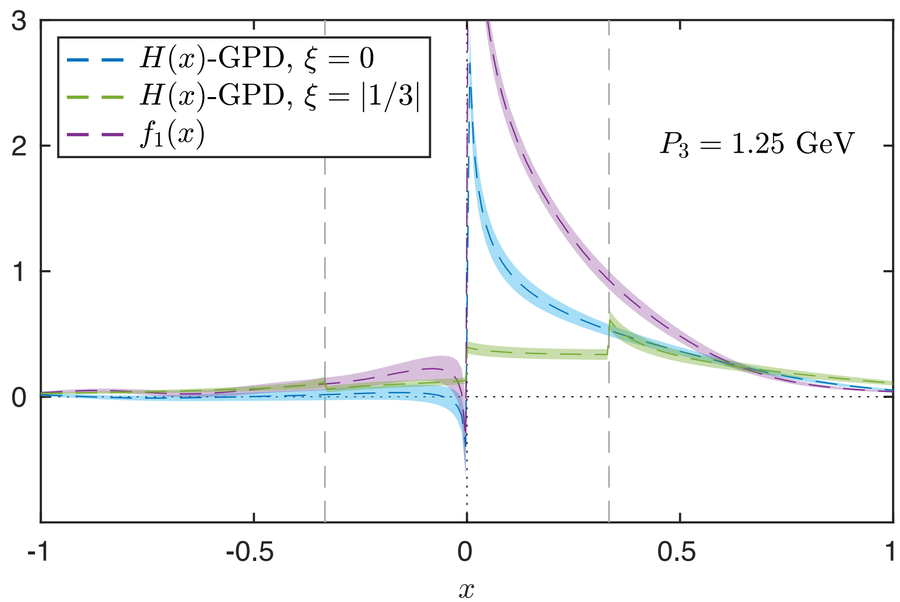
- Y.-S. Liu, YZ et al., Phys.Rev.D **100** (2019);
- Ma, Zhang and Zhang, 2202.07116.



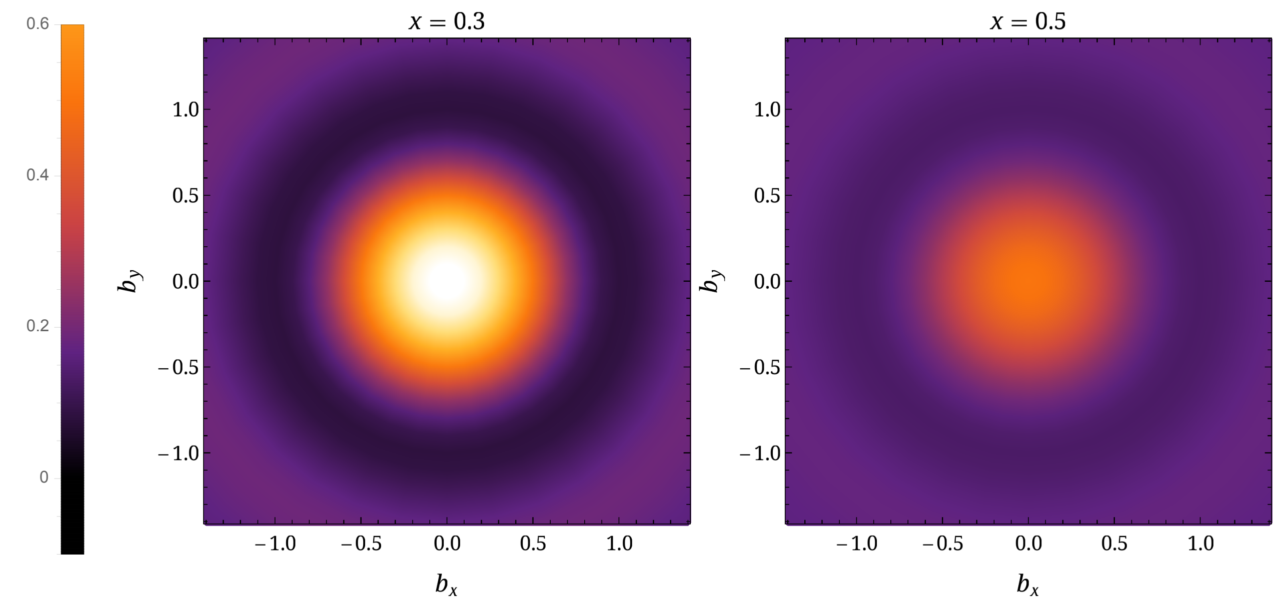
Reliable prediction of (x, ξ, t) dependence within [$x_{\rm min}$, $x_{\rm max}$] and $|x\pm\xi| > \delta$ at given finite P^z.

GPDs

First lattice calculations using the RIMOM scheme and NLO matching:



C. Alexandrou et al. (ETMC), Phys.Rev.Lett. **125** (2020).



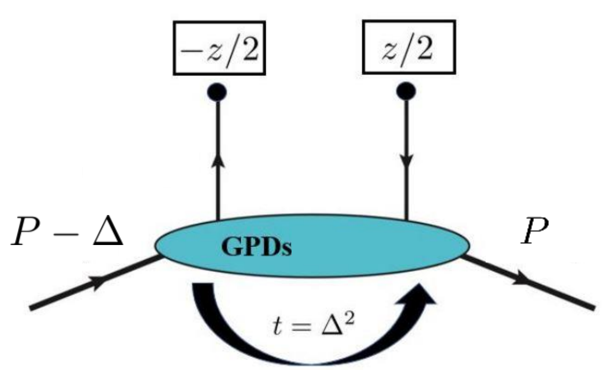
H.-W. Lin, Phys.Rev.Lett. **127** (2021).

First attempt for twist-3 GPDs also made:

S. Bhattacharya et al. (ETMC), Phys.Rev.D **102** (2020).

Recent advancement in extracting GPDs from less computationally expensive lattice matrix elements in the asymmetric frame.

Bhattacharya, Cichy, Constantinou, Dodson, Gao, Metz, Mukherjee, Scapellato, Steffens, and YZ, work in progress.



See Dr. Shohini Bhattacharya and Martha Constantinou's parallel talks on Thu.

TMDs

- **Lorentz invariant method**
  - Primary efforts focused on ratios of TMD *x*-moments

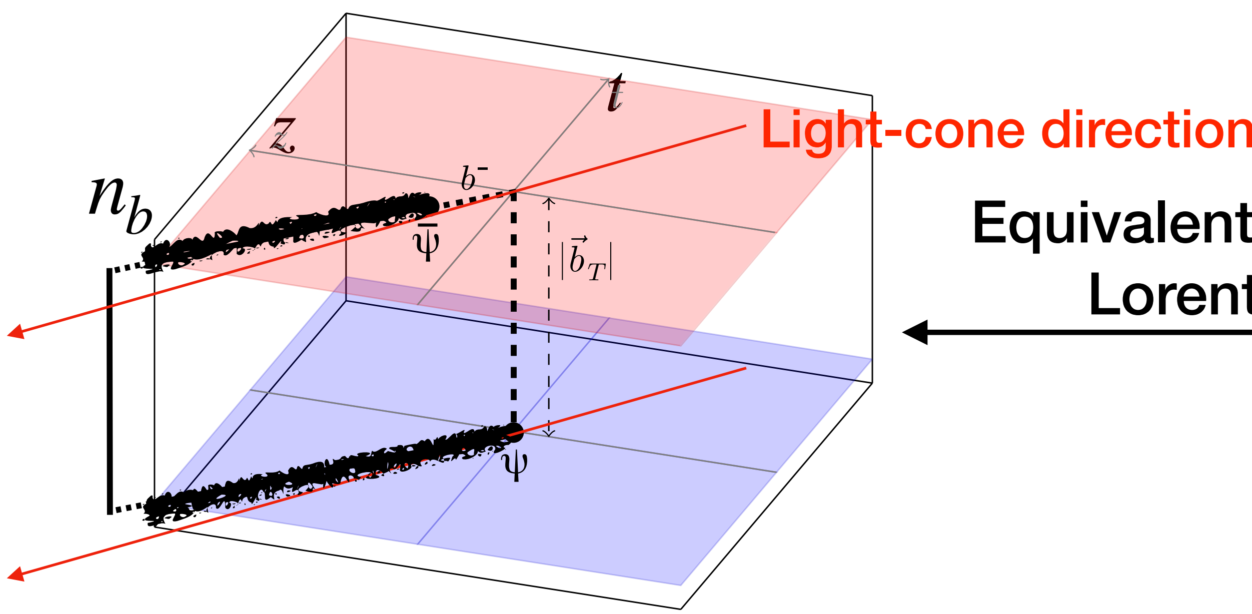
    Musch, Hägler, Engelhardt, Negele, Schäfer, et al., Eur.Phys.Lett. **88** (2009), Phys.Rev.D **83** (2011), Phys.Rev.D **85** (2012), Phys.Rev.D **93** (2016), arXiv:1601.05717, Phys.Rev.D **96** (2017)

- **Quasi TMDs**
  - One-loop studies of quasi beam and soft functions
    - Ji, Sun, Xiong and Yuan, Phys.Rev.D **91** (2015);
    - Ji, Jin, Yuan, Zhang and YZ, Phys.Rev.D **99** (2019);
    - Ebert, Stewart, YZ, JHEP **09** (2019);
    - Vladimirov and Schäfer, PRD 101 (2020).
  - Method to calculate the Collins-Soper kernel
    - Ji, Sun, Xiong and Yuan, Phys.Rev.D **91** (2015);
    - Ebert, Stewart, YZ, Phys.Rev.D **99** (2019).
  - Method to calculate the soft function, and thus the x and b_T dependence of TMDs
    - Ji, Liu and Liu, Nucl.Phys.B **955** (2020), Phys.Lett.B **811** (2020);
  - Derivation of factorization formula

    Ebert, Schindler, Stewart and YZ, JHEP **04** (2022).

Quasi TMD

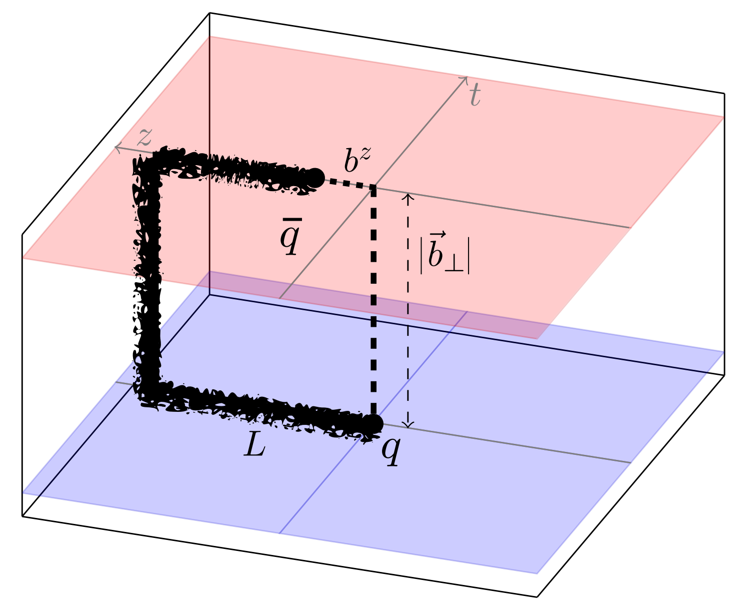
- **Beam function B:**



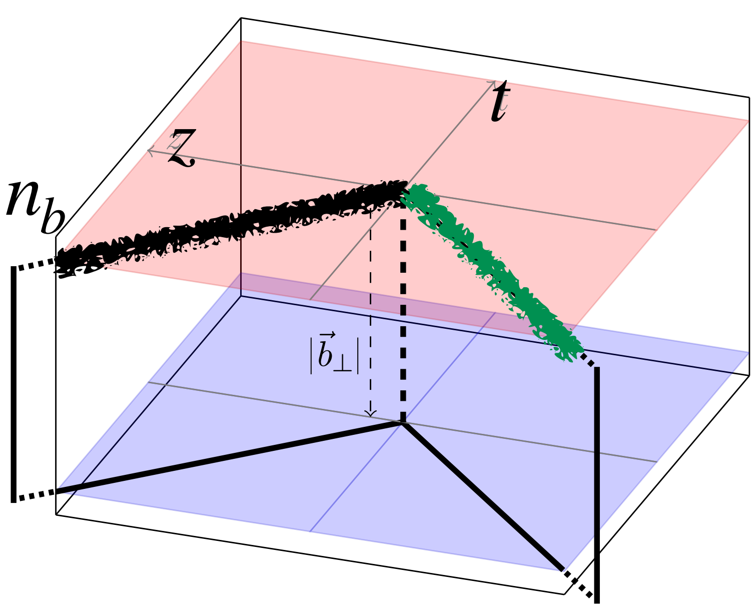
$$n_b^\mu(y_B) \equiv (-e^{2y_B}, 1, 0_\perp)\,, \qquad -y_B \gg 1$$

Equivalent under large Lorentz boost

- **Quasi beam function :**



- **Soft function S:**



Quasi TMD: $$\tilde{f}_i = \lim_{-y_B \gg 1} \lim_{\epsilon \to 0} \tilde{Z}_{\rm UV} \frac{B_i}{\sqrt{S^q}}$$

Different orders of UV limits

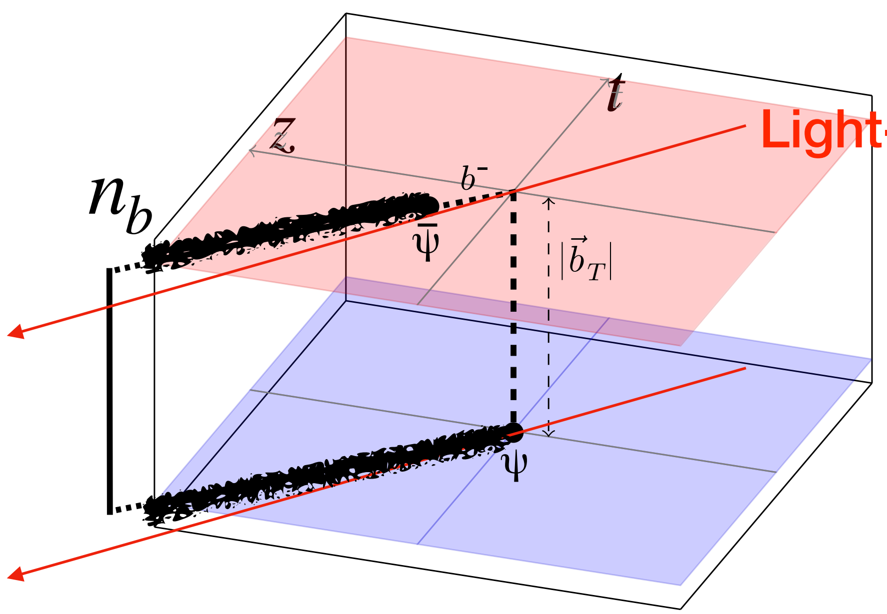
Perturbative matching in LaMET!

TMD: $$f_i = \lim_{\epsilon \to 0} Z_{\rm UV} \lim_{y_B \to -\infty} \frac{B_i}{\sqrt{S^q}}$$

Ebert, Schindler, Stewart and YZ, JHEP **04** (2022).

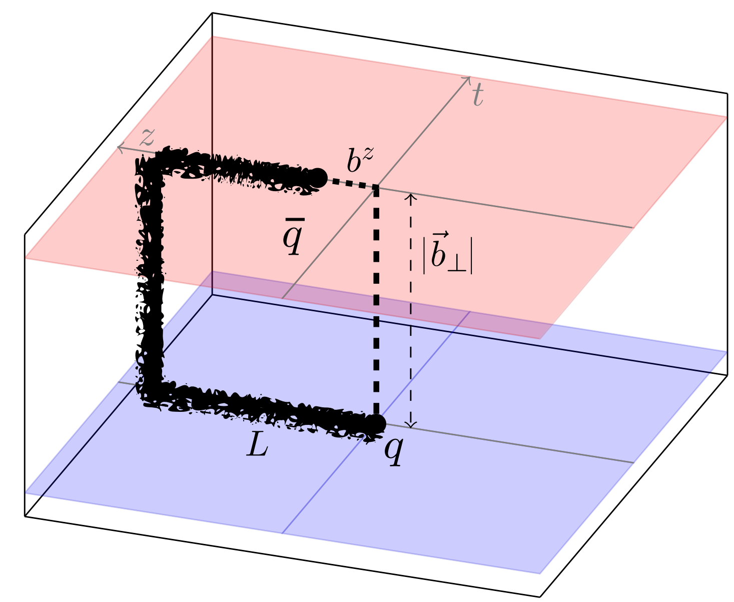
Quasi TMD

- **Beam function B:**



Light-cone direction

- **Quasi beam function :**



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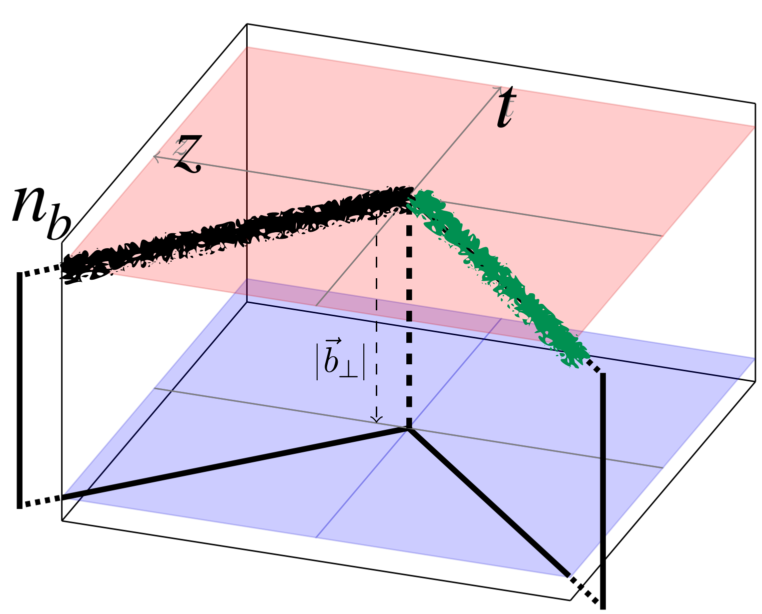

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Ebert, Schindler, Stewart and YZ, JHEP **04** (2022).

Factorization relation

$$\frac{\tilde{f}_{i/p}^{\text{naive}[s]}(x, \mathbf{b}_T, \mu, \tilde{P}^z)}{\sqrt{S_r^q(b_T, \mu)}} = C\left(\mu, x\tilde{P}^z\right) \exp\left[\frac{1}{2}K(\mu, b_T)\ln\frac{(2x\tilde{P}^z)^2}{\zeta}\right] \times f_{i/p}^{[s]}(x, \mathbf{b}_T, \mu, \zeta)\left\{1 + \mathcal{O}\left[\frac{1}{(x\tilde{P}^z b_T)^2}, \frac{\Lambda_{\text{QCD}}^2}{(x\tilde{P}^z)^2}\right]\right\}$$

Reduced soft function ✓

X. Ji, Y.-S. Liu and Y. Liu, Nucl.Phys.B 955 (2020), Phys.Lett.B 811 (2020).

$K(\mu, b_T)$: Collins-Soper evolution kernel

Matching coefficient:

- Independent of spin;
  - Vladimirov and Schäfer, Phys.Rev.D **101** (2020);
  - Ebert, Schindler, Stewart and YZ, JHEP **09** (2020);
  - Ji, Liu, Schäfer and Yuan, Phys.Rev.D **103** (2021).
- No quark-gluon or flavor mixing, which makes gluon calculation much easier.

One-loop matching for gluon TMDs:

Ebert, Schindler, Stewart and YZ, JHEP **08** (2022).

Lattice calculations of TMD physics

$$\frac{\tilde{f}_{i/p}^{\text{naive}[s]}(x, \mathbf{b}_T, \mu, \tilde{P}^z)}{\sqrt{S_r^q(b_T, \mu)}} = C\left(\mu, x\tilde{P}^z\right) \exp\left[\frac{1}{2}K(\mu, b_T)\ln\frac{(2x\tilde{P}^z)^2}{\zeta}\right] \times f_{i/p}^{[s]}(x, \mathbf{b}_T, \mu, \zeta)\left\{1 + \mathcal{O}\left[\frac{1}{(x\tilde{P}^z b_T)^2}, \frac{\Lambda_{\text{QCD}}^2}{(x\tilde{P}^z)^2}\right]\right\}$$

- ✳ **Collins-Soper kernel;**

$$K(\mu, b_T) = \frac{d}{d\ln\tilde{P}^z}\ln\frac{\tilde{f}_{i/p}^{\text{naive}[s]}(x, \mathbf{b}_T, \mu, \tilde{P}^z)}{C\left(\mu, x\tilde{P}^z\right)}$$

- ✳ **Flavor separation;**
- ✳ **Spin-dependence, e.g., Sivers function;**

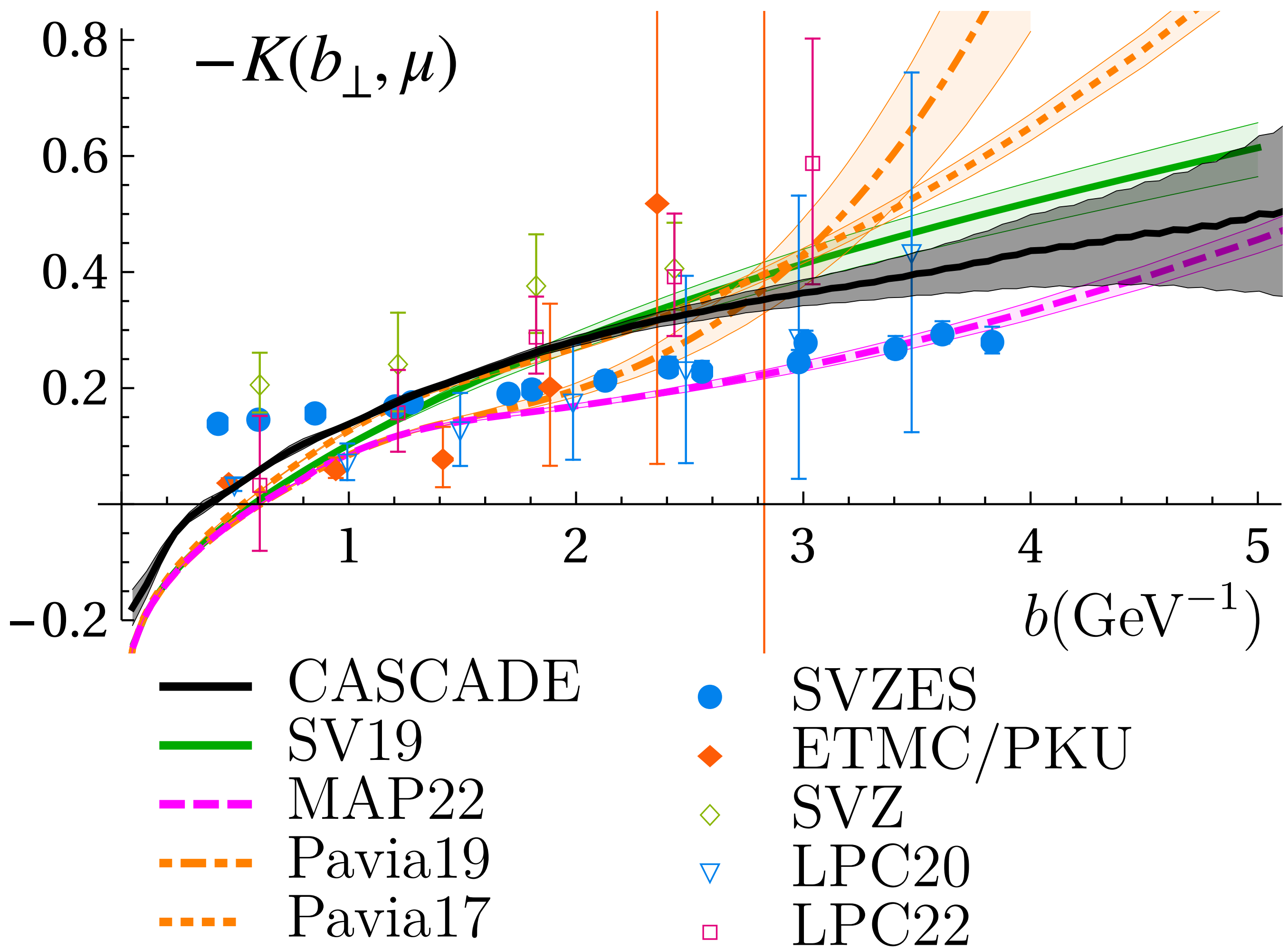
$$\frac{f_{i/p}^{[s]}(x, \mathbf{b}_T)}{f_{j/p}^{[s']}(x, \mathbf{b}_T)} = \frac{\tilde{f}_{i/p}^{\text{naive}[s]}(x, \mathbf{b}_T)}{\tilde{f}_{j/p}^{\text{naive}[s']}(x, \mathbf{b}_T)}$$

- ✳ **Full TMD kinematic dependence in $(x, \mathbf{b}_T)$.**
- ✳ **Twist-3 PDFs from small b_T expansion of TMDs.**

Ji, Liu, Schäfer and Yuan, PRD 103 (2021).

Collins-Soper kernel for TMD evolution

Comparison between lattice results and global fits



Approach	Collaboration
Quasi beam functions	P. Shanahan, M. Wagman and YZ (SWZ21), Phys. Rev.D **104** (2021)
Quasi TMD wavefunctions	Q.-A. Zhang, et al. (LPC20), Phys.Rev.Lett. **125** (2020).
	Y. Li et al. (ETMC/PKU 21), Phys.Rev.Lett. **128** (2022).
	M.-H. Chu et al. (LPC22), arXiv: 2204.00200
Moments of quasi TMDs	Schäfer, Vladmirov et al. (SVZES21), JHEP **08** (2021)

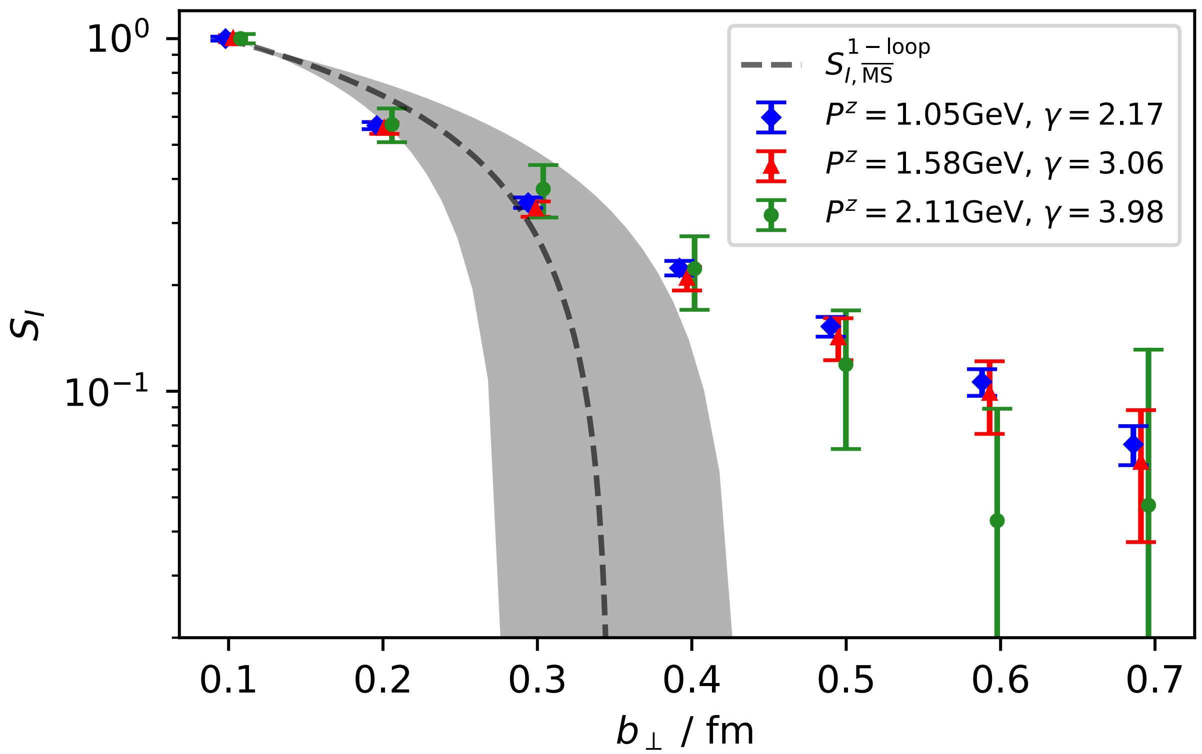
MAP22: Bacchetta, Bertone, Bissolotti, et al., 2206.07598
SV19: I. Scimemi and A. Vladimirov, JHEP 06 (2020) 137
Pavia19: A. Bacchetta et al., JHEP 07 (2020) 117
Pavia 17: A. Bacchetta et al., JHEP 06 (2017) 081
CASCADE: Martinez and Vladimirov, 2206.01105

Reduced soft factor for full TMD calculation

$$\langle \pi(-P) | j_1(b_T) j_2(0) | \pi(P) \rangle \overset{P^z \gg m_\pi}{=} S_q^r(b_T, \mu) \int dx dx' \; H(x, x', \mu) \times \Phi^\dagger(x, b_T, P^z) \Phi(x', b_T, P^z)$$

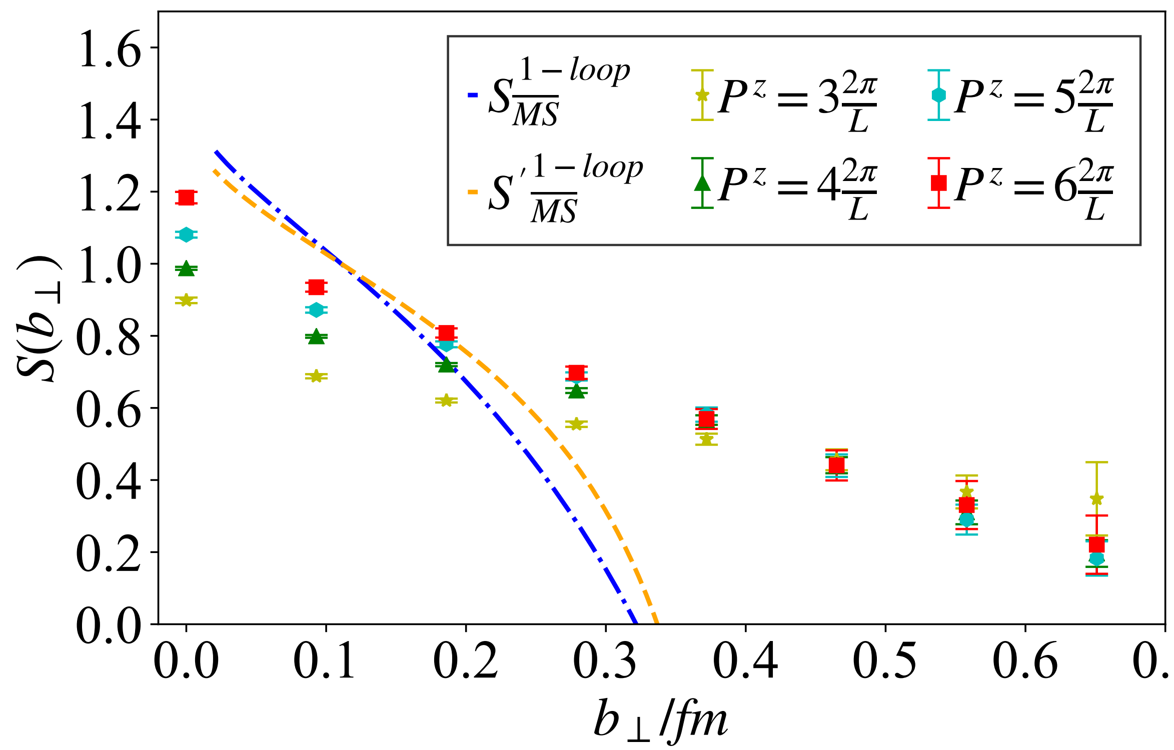
Φ: Quasi-TMD wave function

$a = 0.10$ fm, $m_\pi = 547$ MeV, $P^z_{\max} = 2.11$ GeV



Q.-A. Zhang, et al. (LPC), Phys.Rev.Lett. **125** (2020).

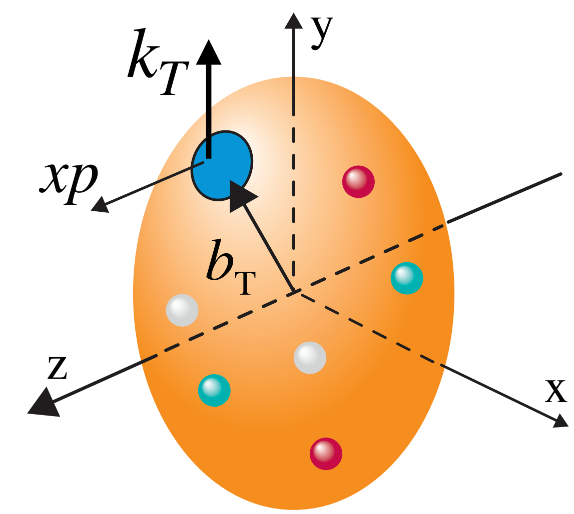
$a = 0.09$ fm, $m_\pi = 827$ MeV, $P^z_{\max} = 3.3$ GeV



Y. Li et al. (ETMC/PKU), Phys.Rev.Lett. **128** (2022).

Both calculations were done at LO accuracy.

Conclusion



Holy grail

Lattice QCD demonstration
of systematic control

PDFs

GPDs

TMDs

Wigner distributions

Theory development:

- Renormalization;
- Perturbative matching, higher order correction and resummation;
- Power corrections (and renormalons).