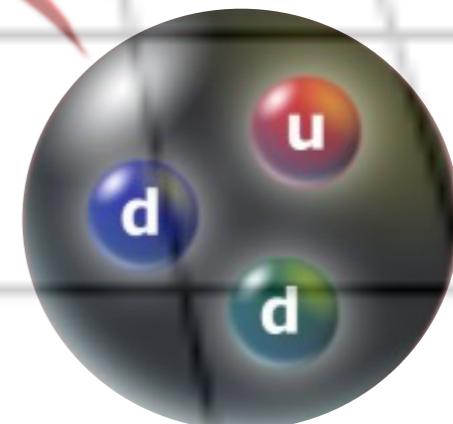




Resolving the NN controversy: a direct comparison of methods



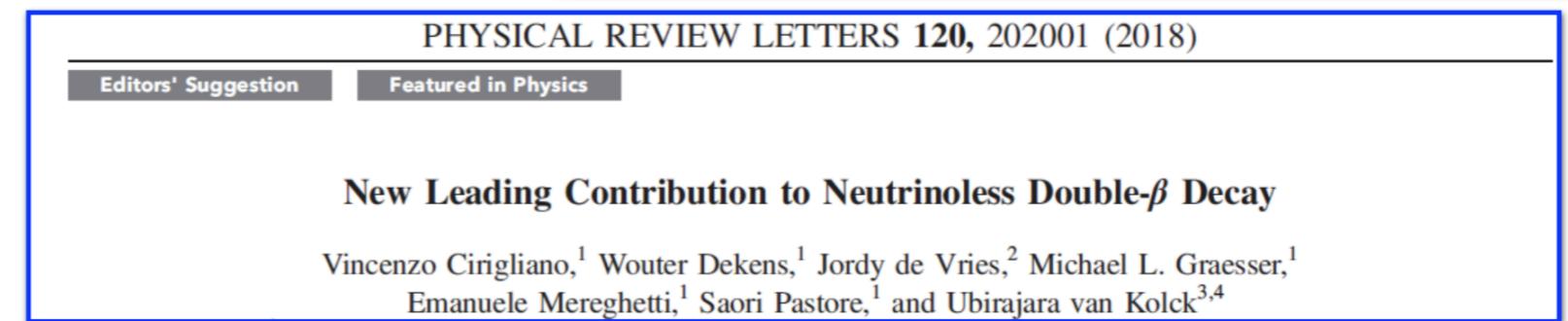
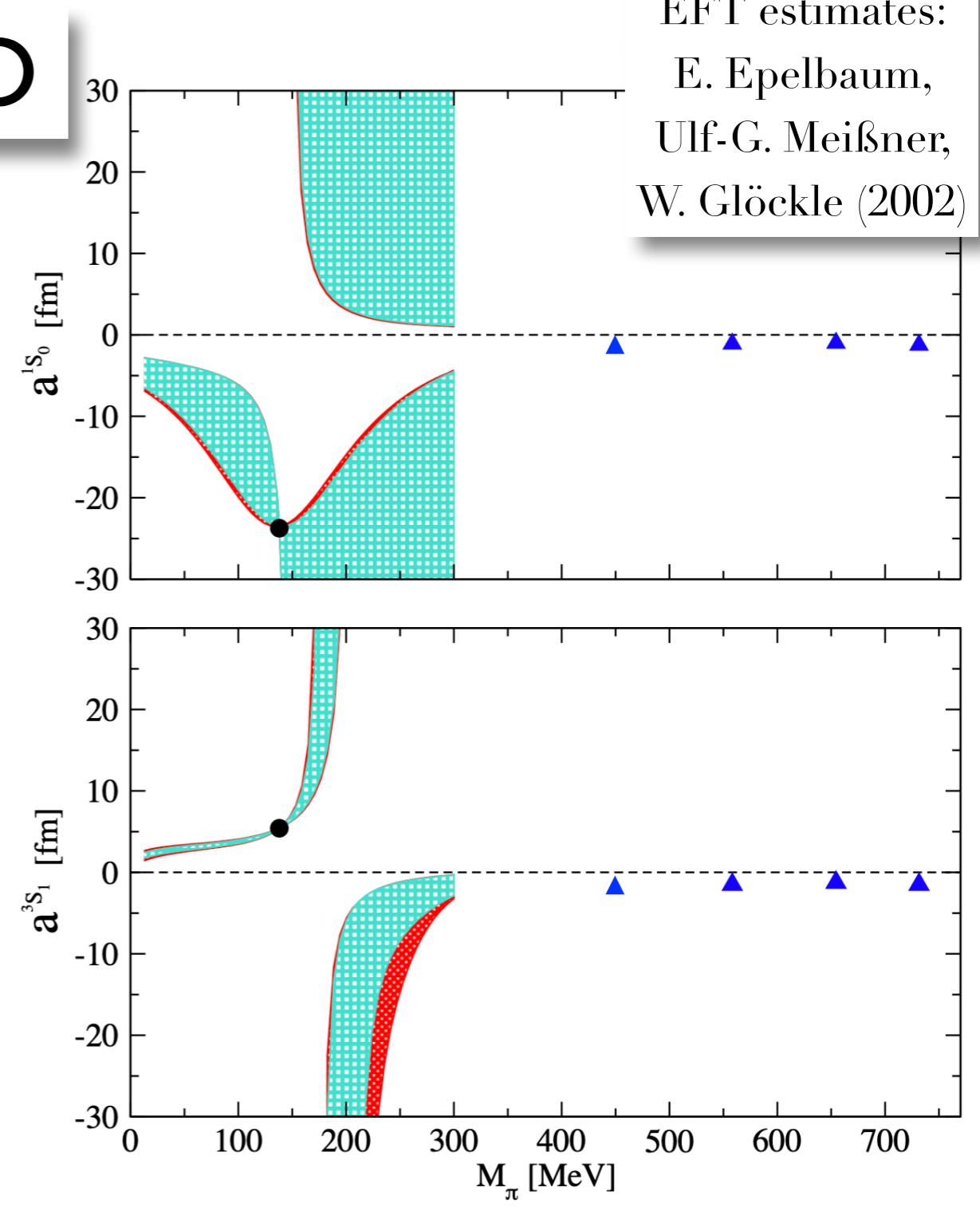
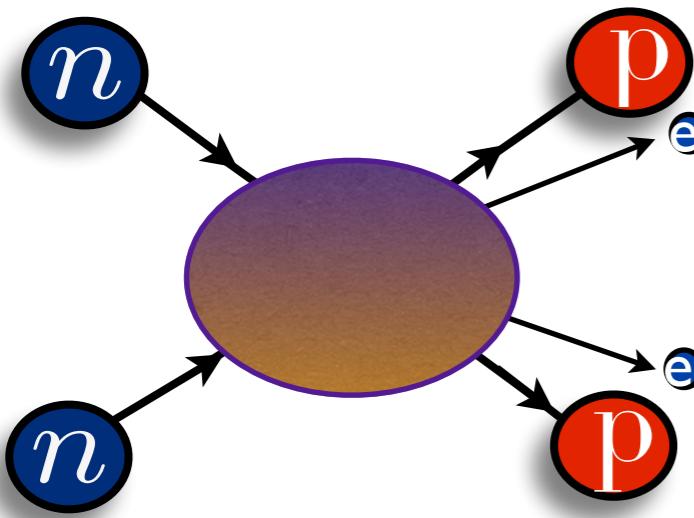
Amy Nicholson
UNC, Chapel Hill

Lattice 2022, Bonn, Germany
Aug 12, 2022

sLapHnn
Collaboration

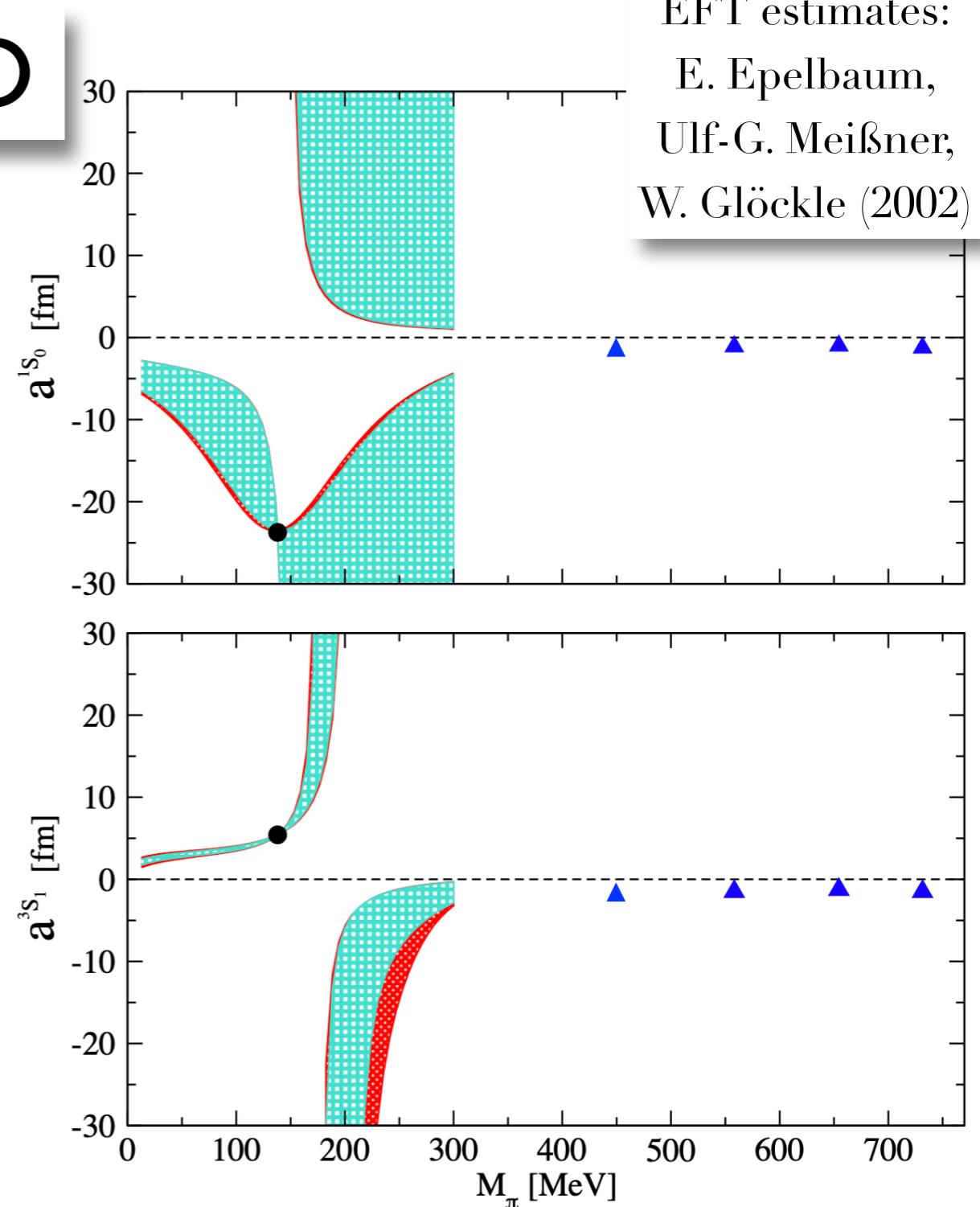
NN scattering from LQCD

- Build quantitative connection between QCD & nuclear physics
 - requires interplay between LQCD & many-body approaches
 - NN scattering should be a benchmark
 - Phase shifts required for infinite volume matching of NN MEs



NN scattering from LQCD

- Build quantitative connection between QCD & nuclear physics
 - requires interplay between LQCD & many-body approaches
 - NN scattering should be a benchmark
 - Phase shifts required for infinite volume matching of NN MEs
- Must have full control over 2-body systems
 - How do we project onto desired states?
 - How do we disentangle signals from closely spaced energy levels?



Lüscher Method

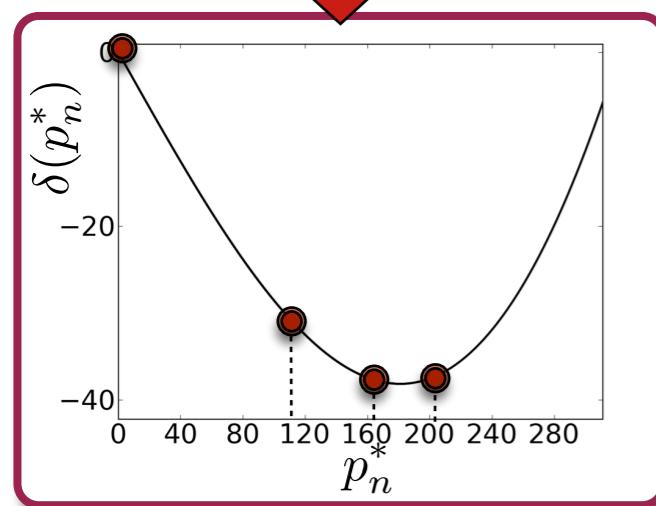
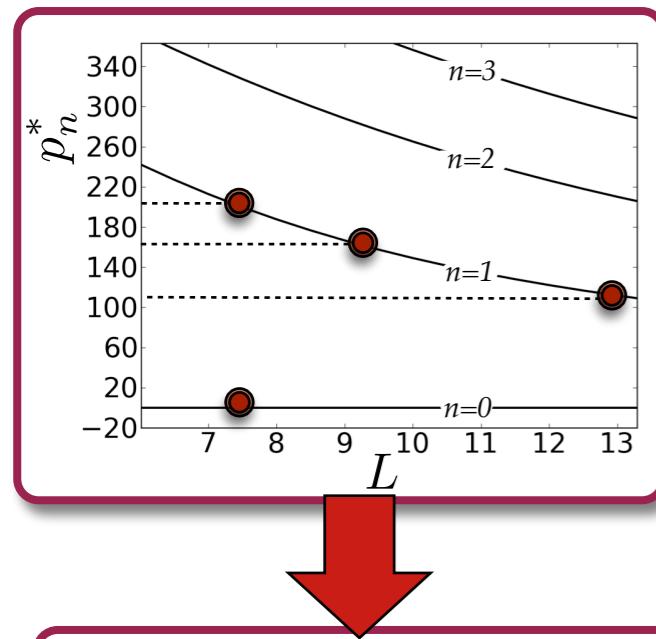
Potential Method

Two methods for
computing phase shifts

Two methods for computing phase shifts

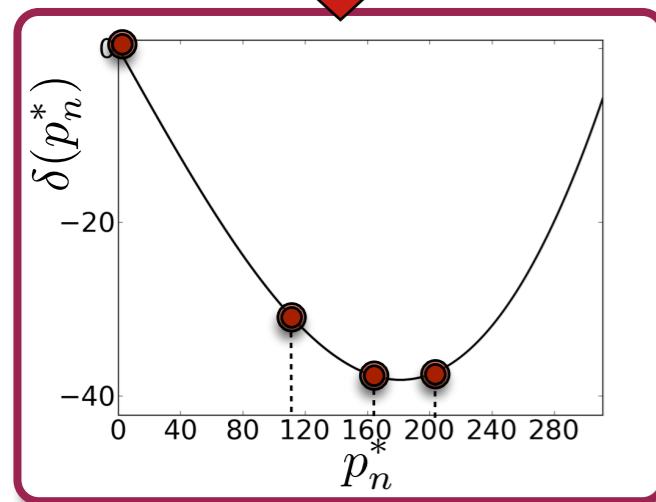
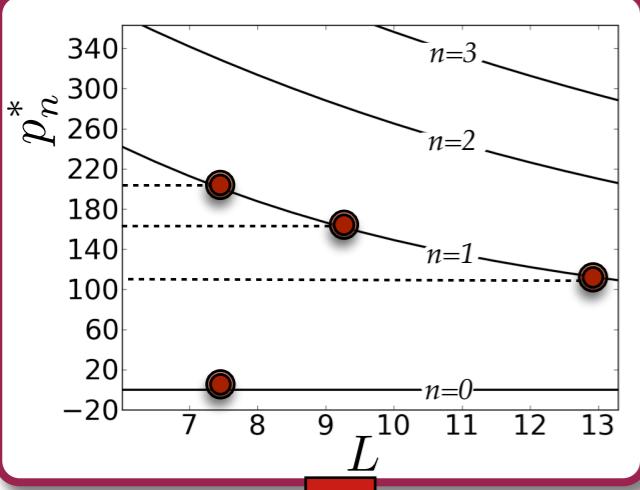
Lüscher Method

Potential Method



Two methods for computing phase shifts

Lüscher Method



Potential Method

Schrodinger Eq:
$$\left\{ -H_0 - \frac{\partial}{\partial t} \right\} R(\mathbf{r}, t) = \int d^3 r' U(\mathbf{r}, \mathbf{r}') R(\mathbf{r}', t)$$

$$R(\mathbf{r}, t) = \frac{C_{NN}(\mathbf{r}, t)}{(C_N(\mathbf{r}, t))^2}$$

(same input as Lüscher)

In practice: $U(\mathbf{r}, \mathbf{r}') = V_C(\mathbf{r})\delta(\mathbf{r} - \mathbf{r}') + \mathcal{O}(\nabla_{\mathbf{r}}^2/\Lambda^2)$



Lüscher Method

Potential Method

History: are there bound states at $m_\pi \sim 800$ MeV?

Lüscher Method



Potential Method



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History: are there bound states at $m_\pi \sim 800 \text{ MeV}$?

$$U(\mathbf{r}, \mathbf{r}') = V_C(\mathbf{r})\delta(\mathbf{r} - \mathbf{r}') + \mathcal{O}(\nabla_{\mathbf{r}}^2/\Lambda^2)$$

Uncontrolled systematics

History: are there bound states at $m_\pi \sim 800$ MeV?

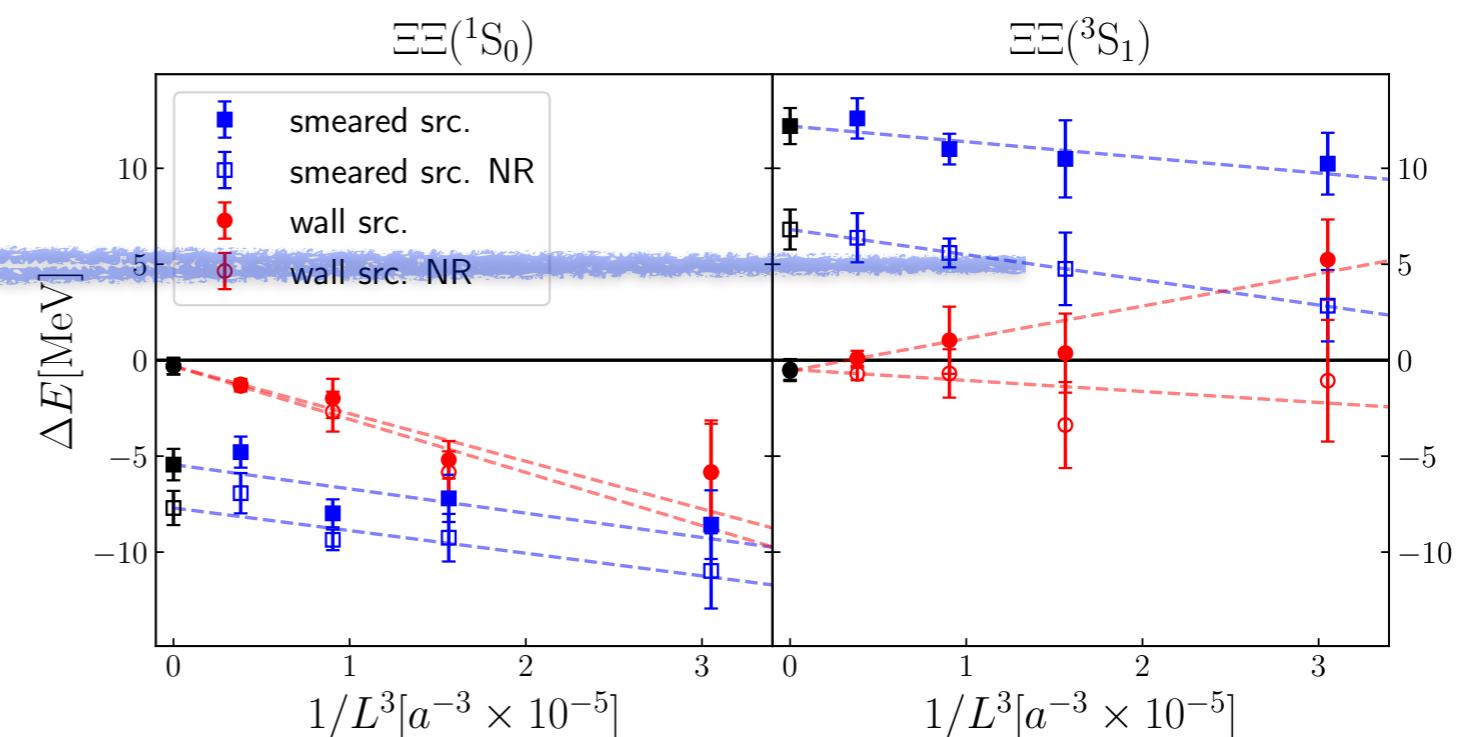
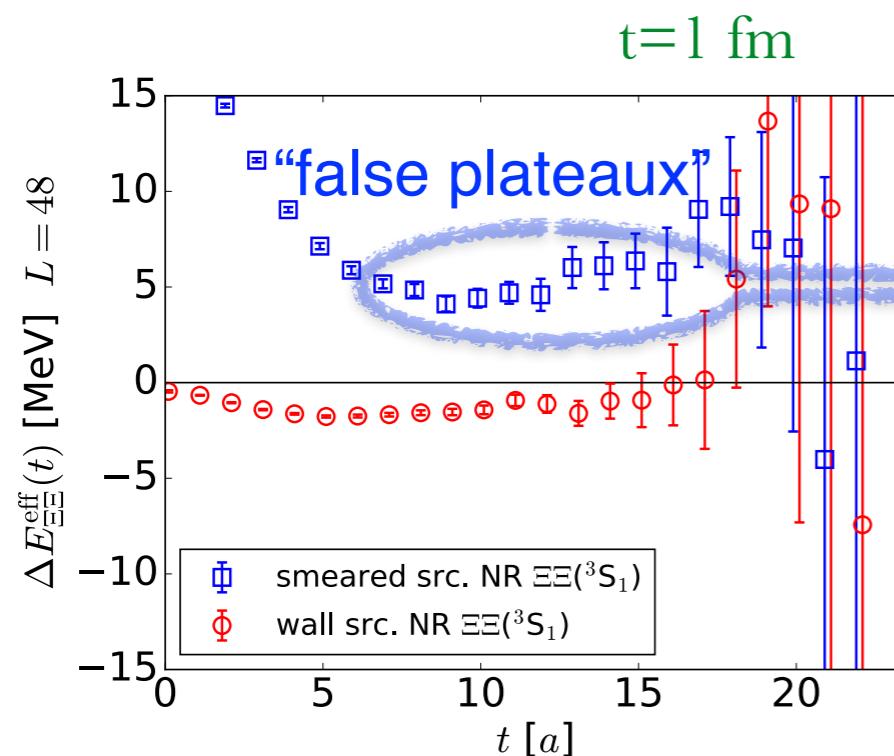
Lüscher Method

Potential Method



HAL QCD Consistency Checks [1703.07210]

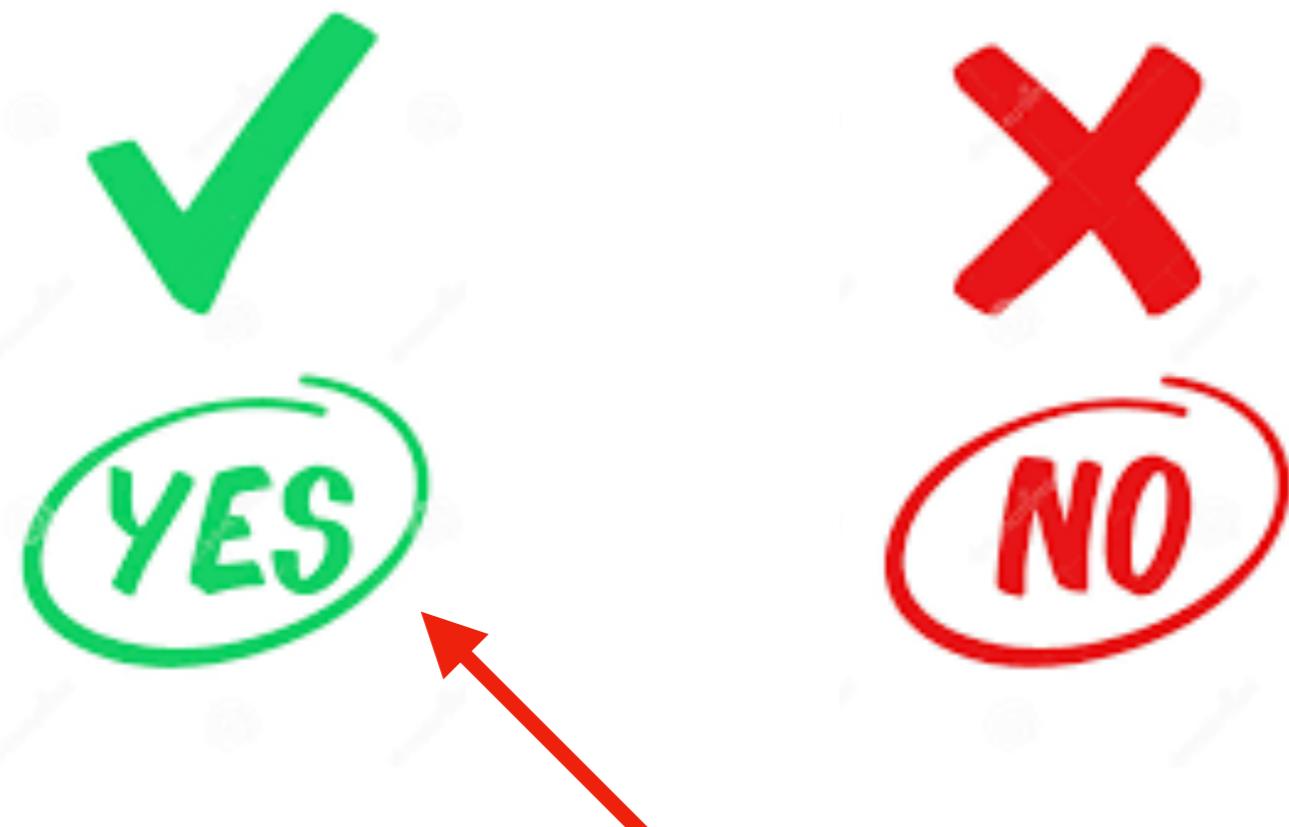
HAL QCD operator dependence of spectrum [1607.06371]



Lüscher Method

Potential Method

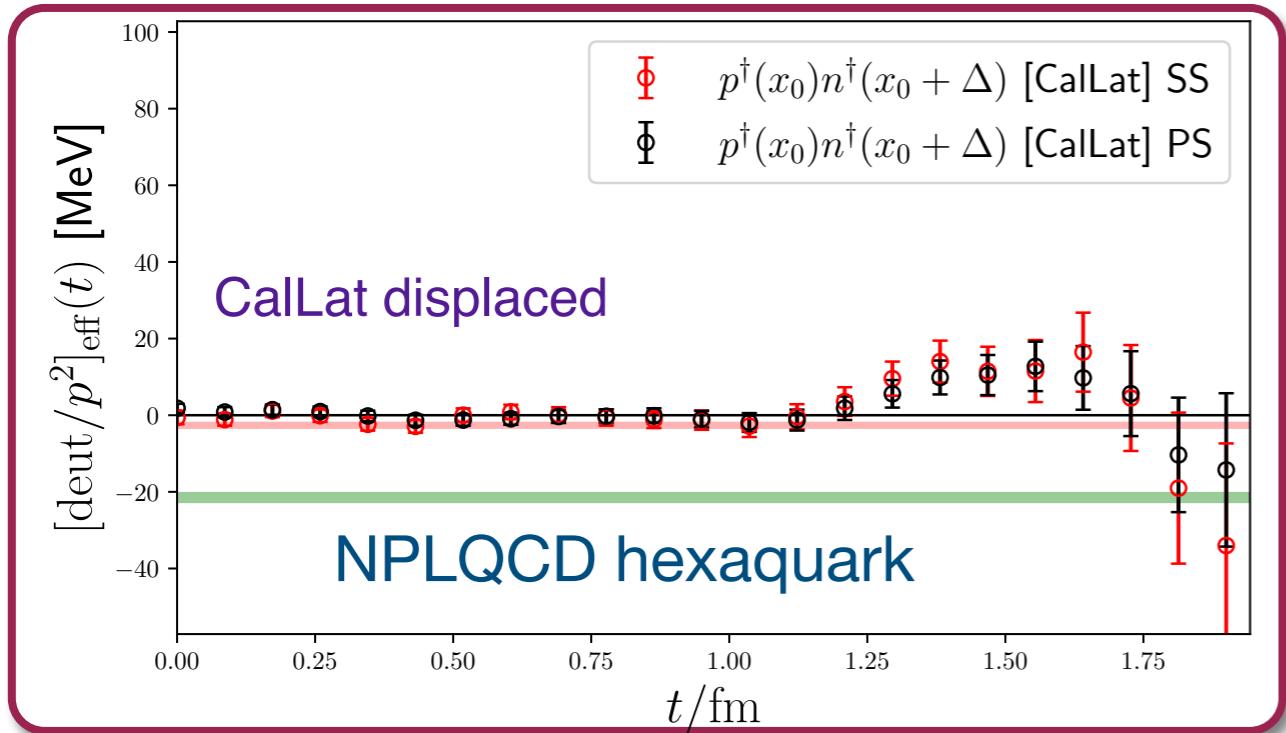
History: are there bound states at $m_\pi \sim 800$ MeV?



Older calculations used off-diagonal correlators having local, hexaquark ops \rightarrow momentum space ops

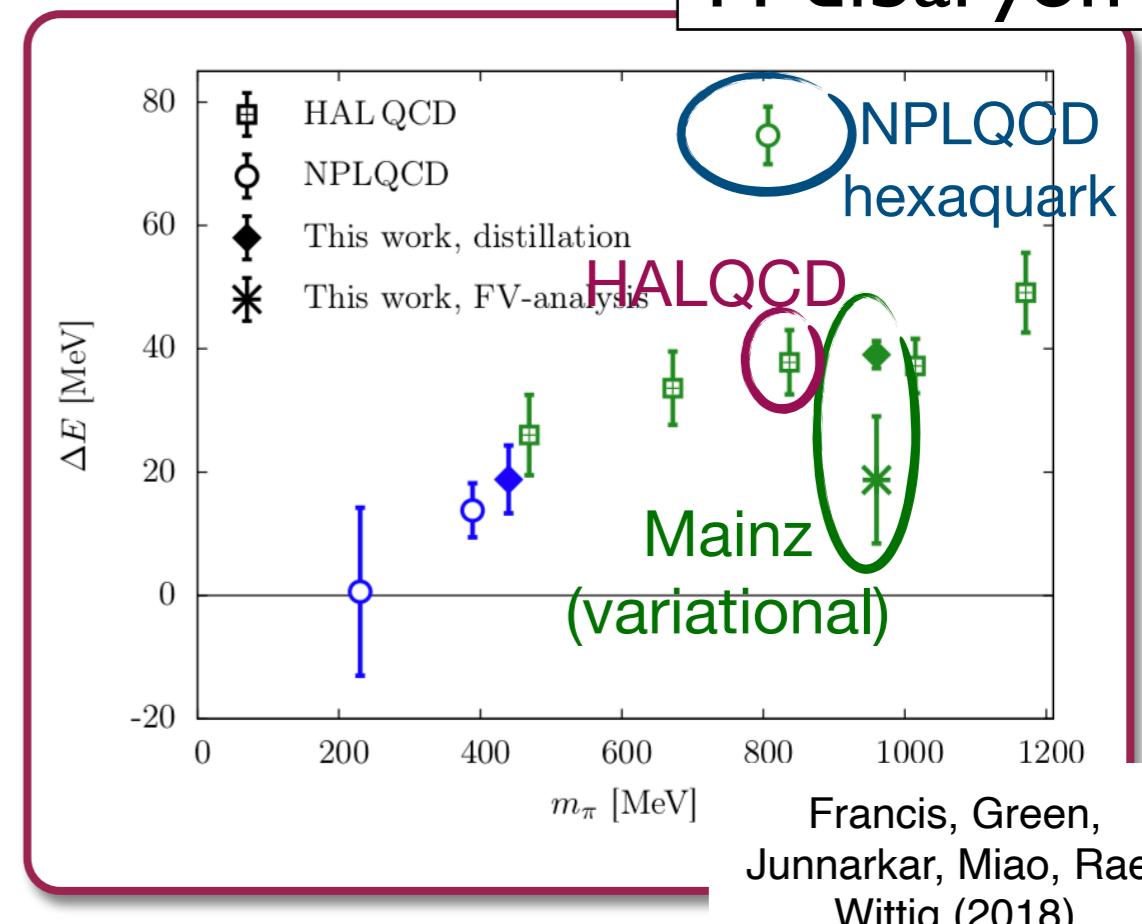
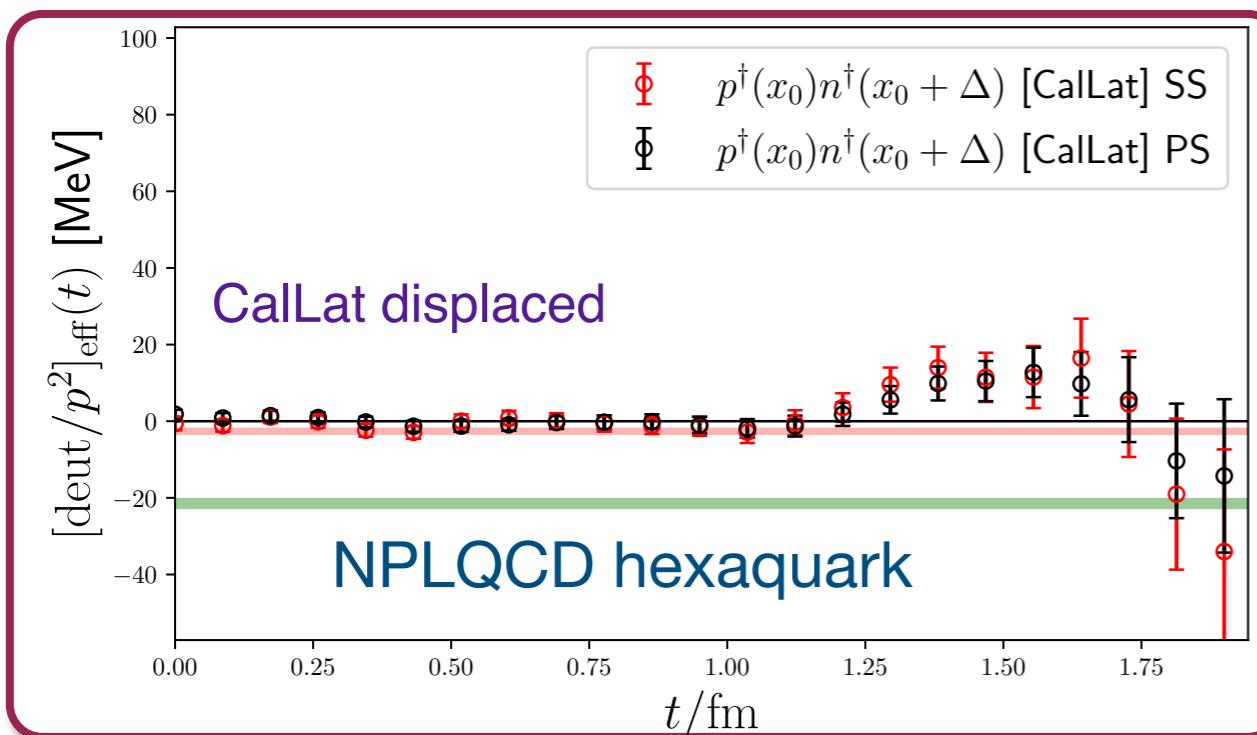
More recent Lüscher calculations

More recent Lüscher calculations



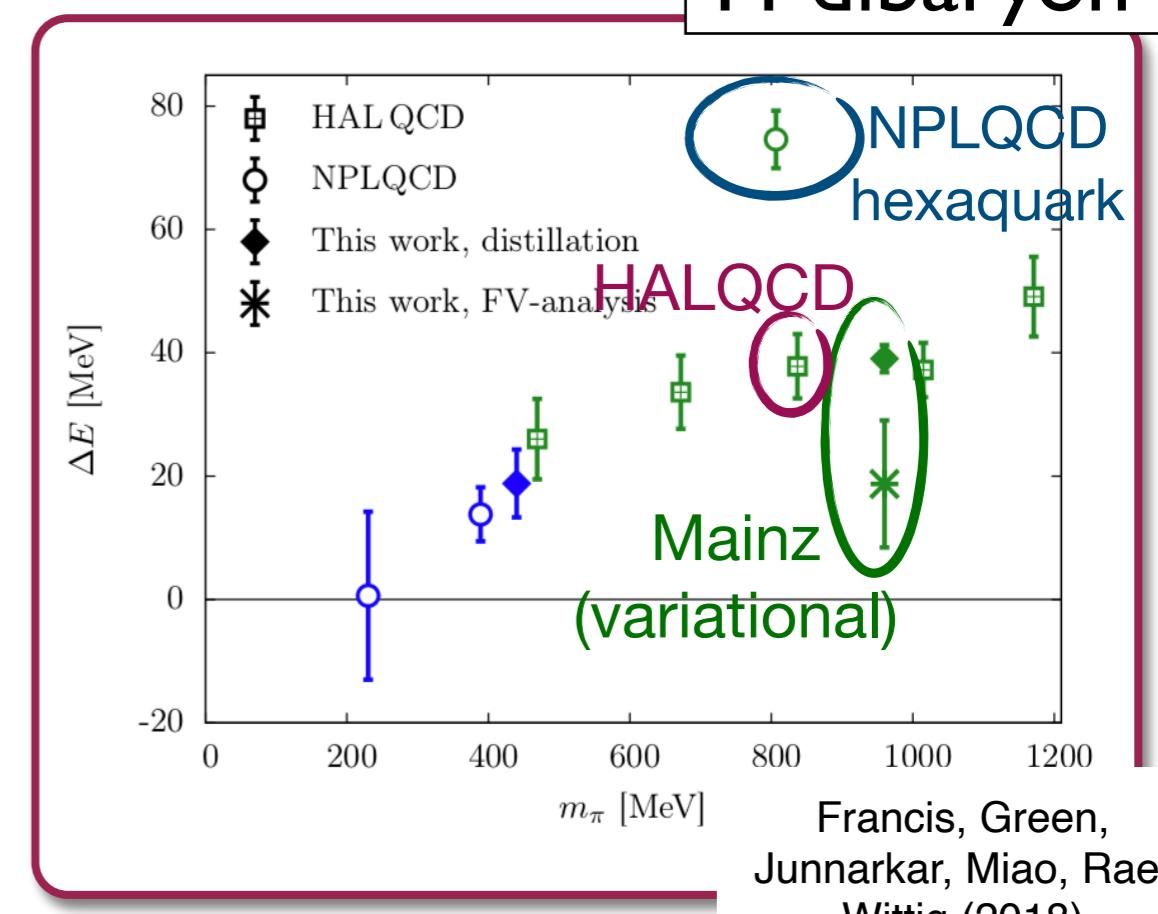
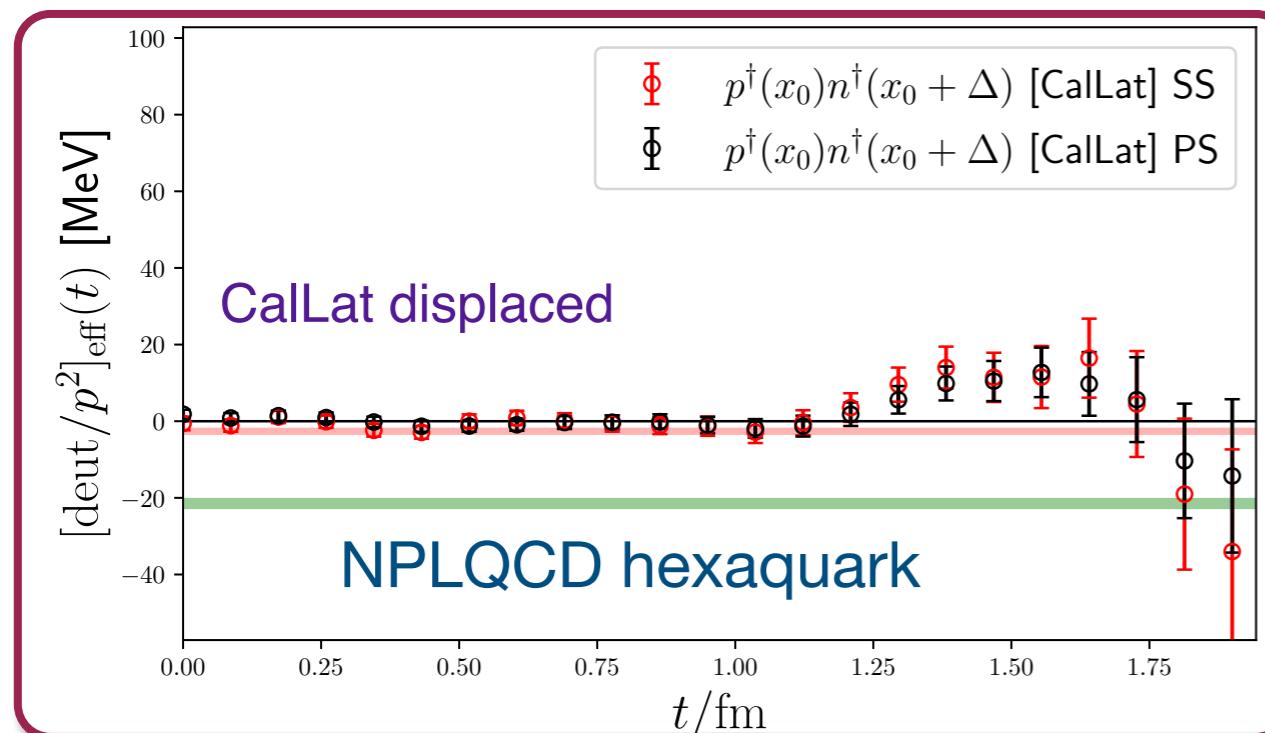
H-dibaryon

More recent Lüscher calculations

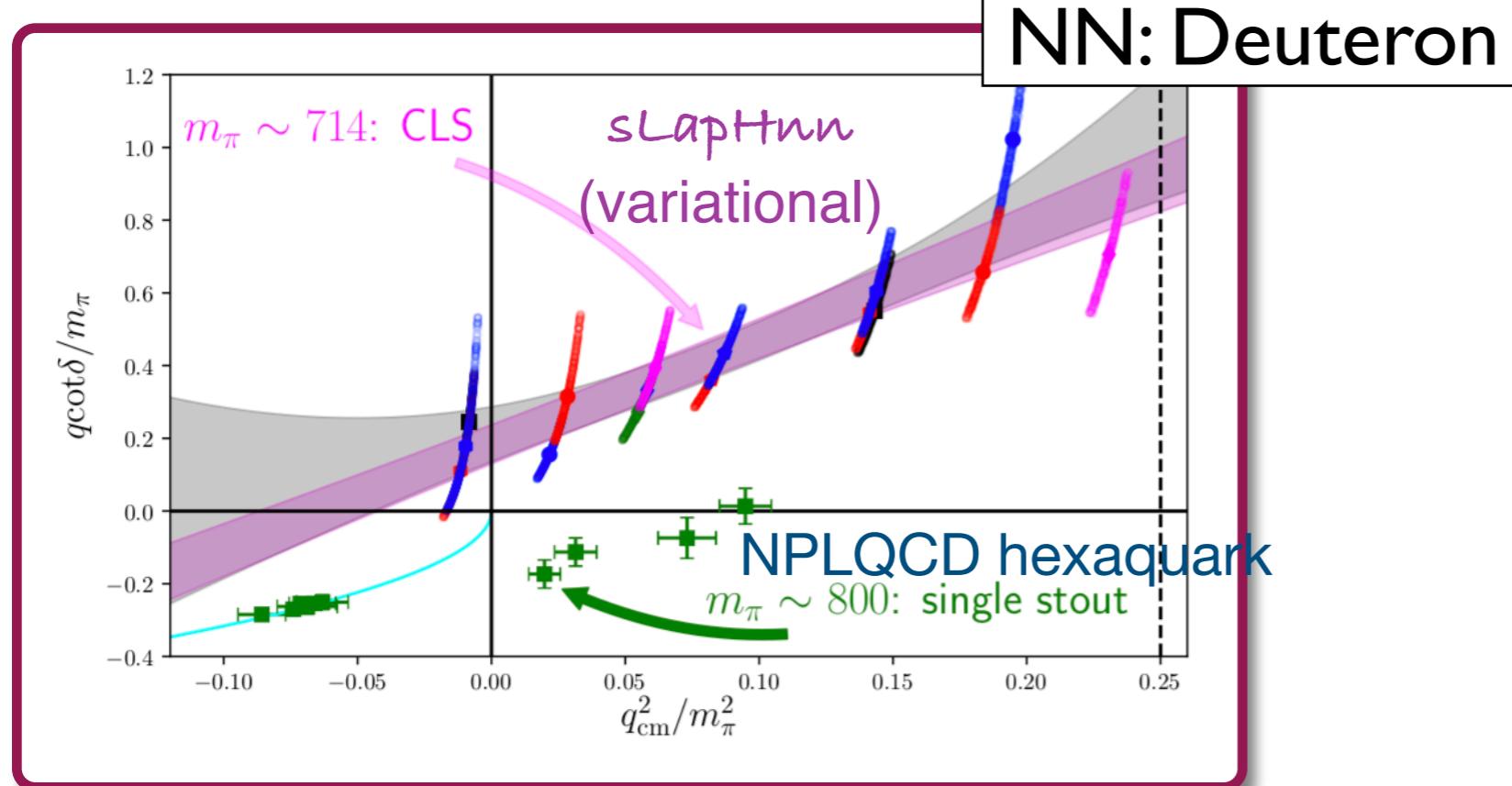


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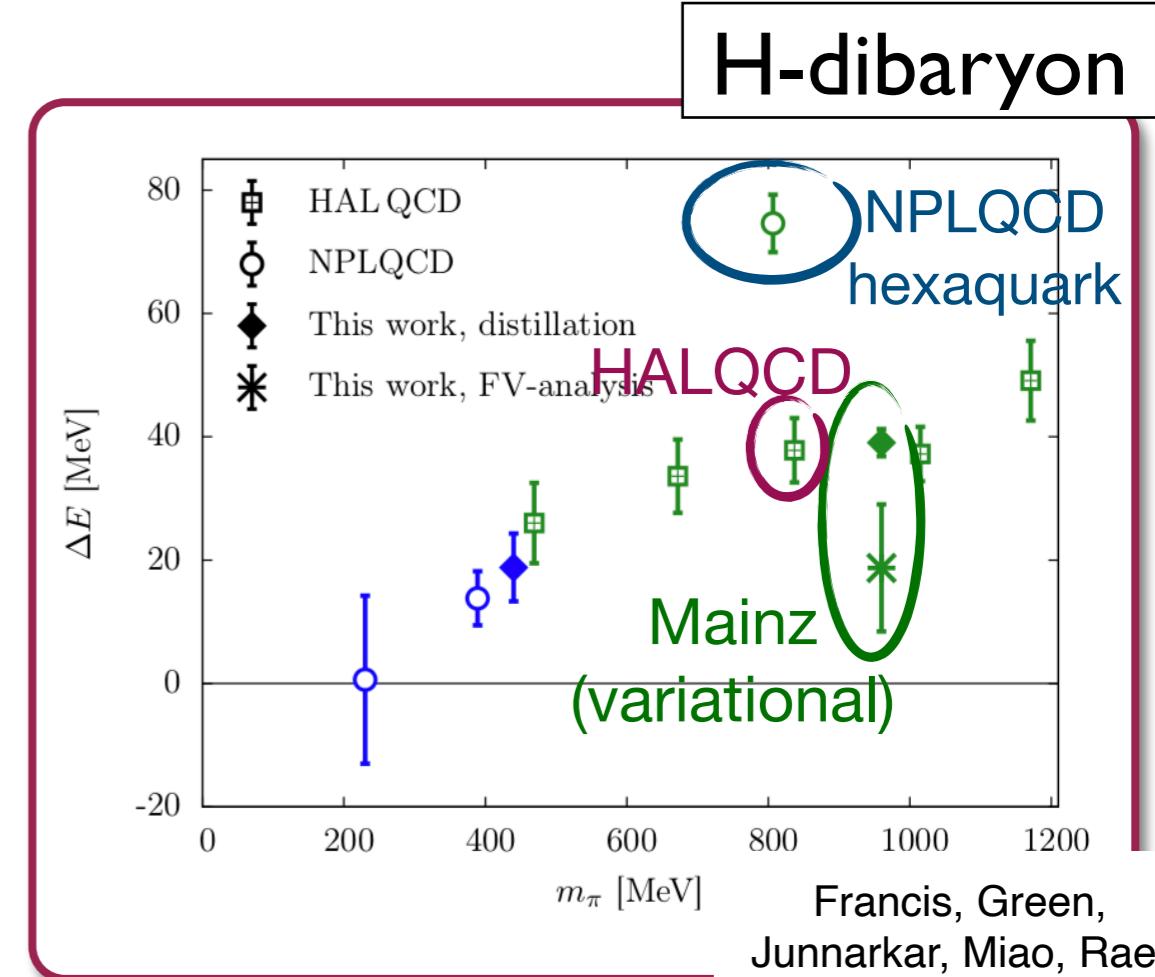
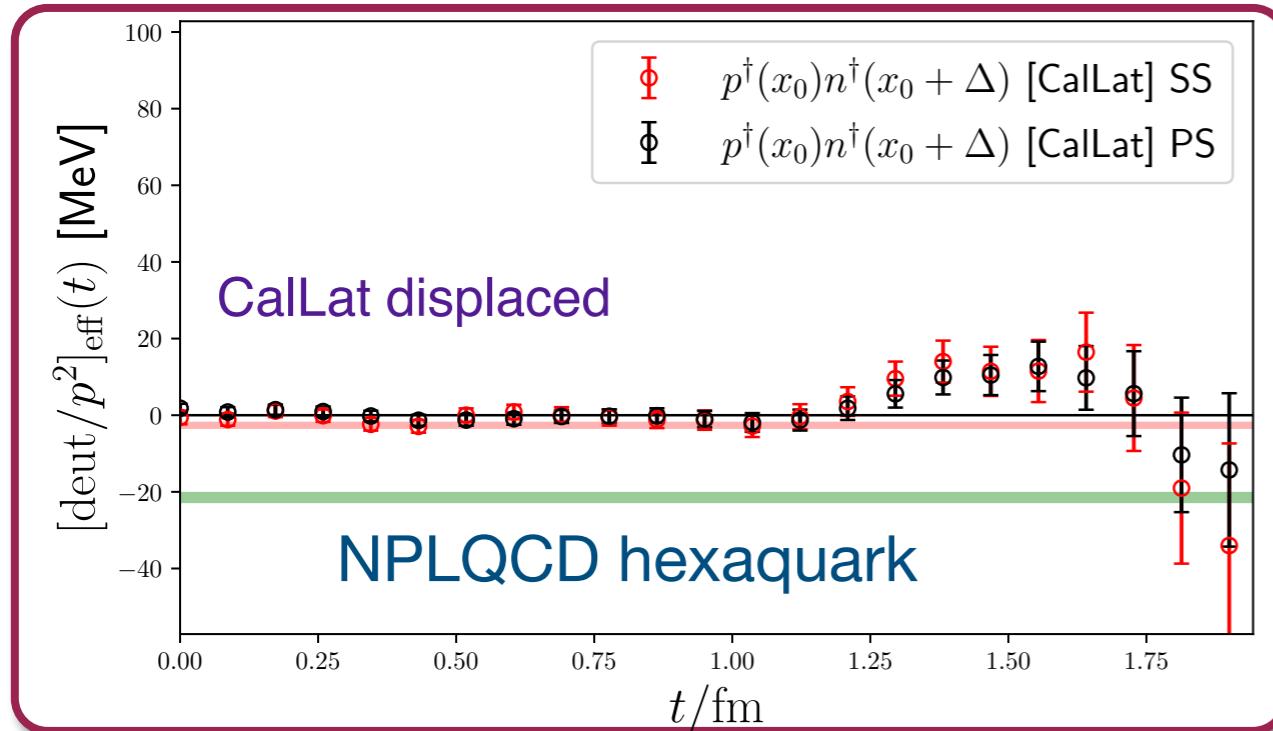


Francis, Green,
Junnarkar, Miao, Rae,
Wittig (2018)

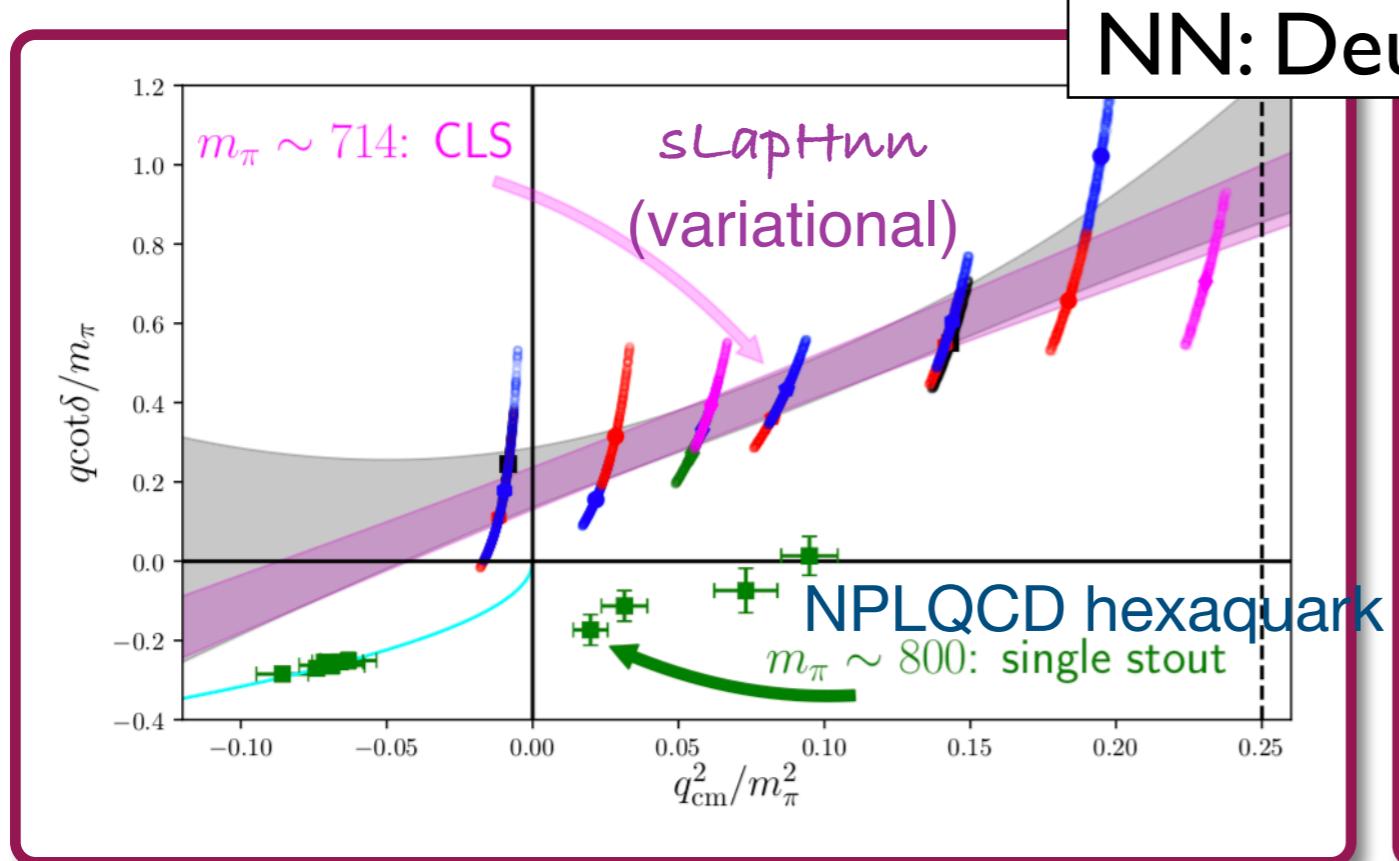


sLapHnn (2021)

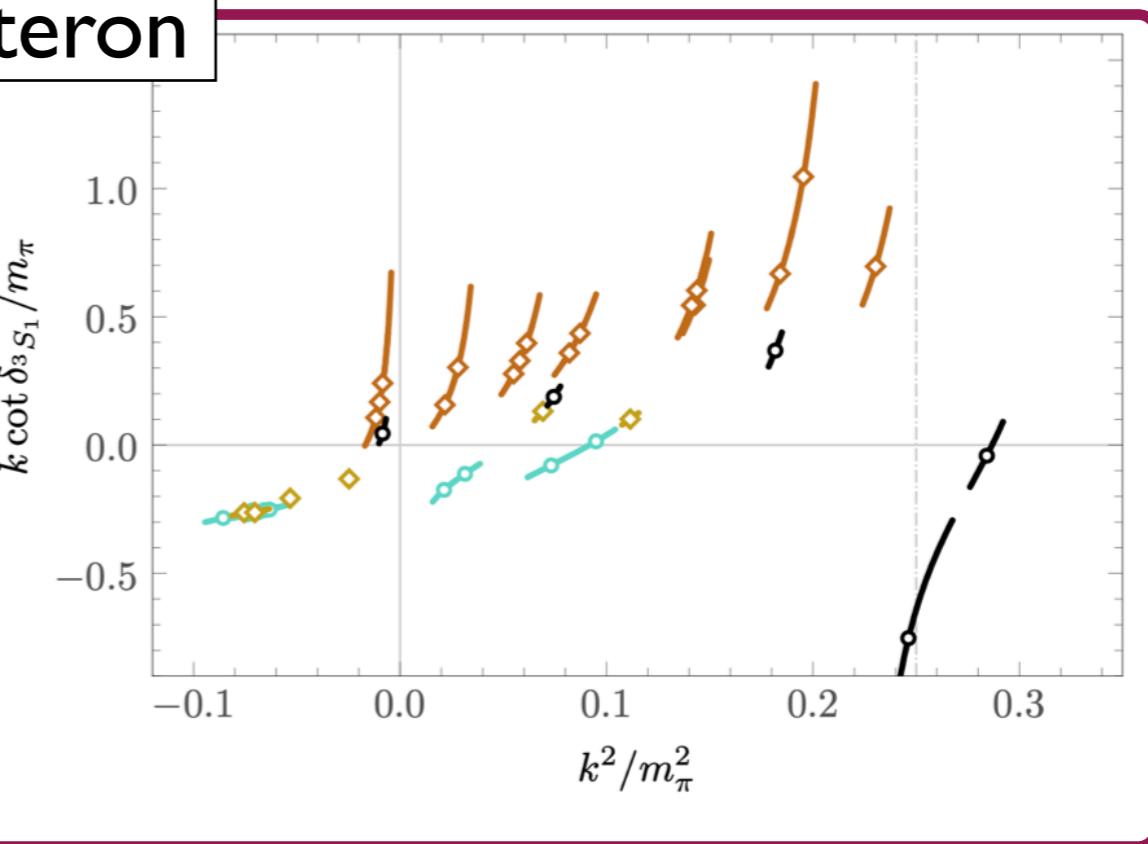
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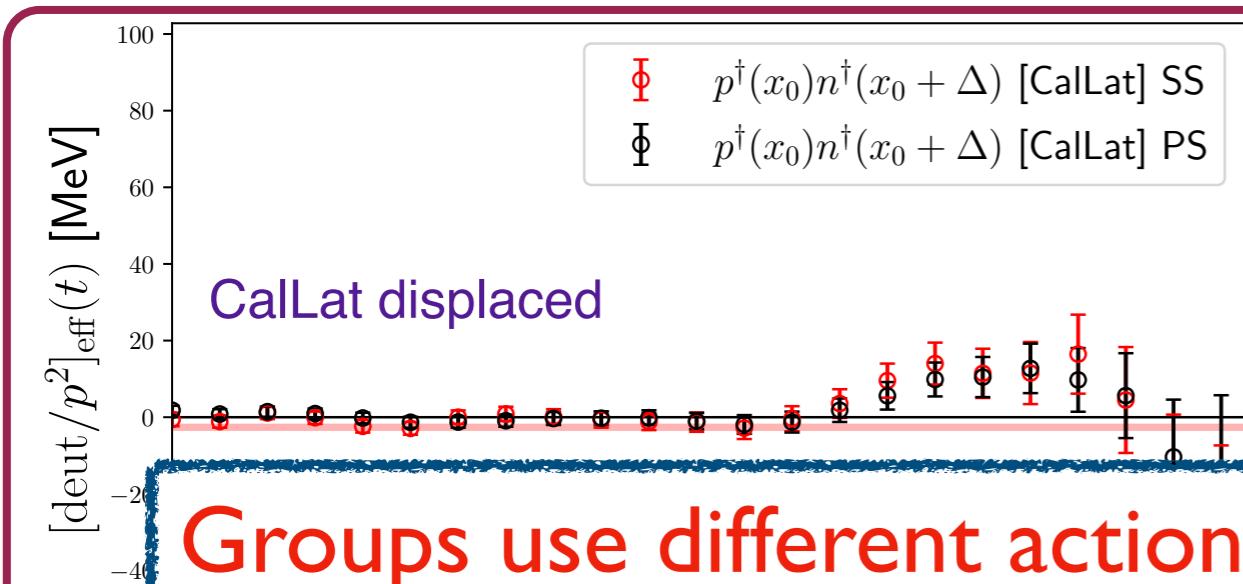


NPLQCD variational (2022)

H-dibaryon

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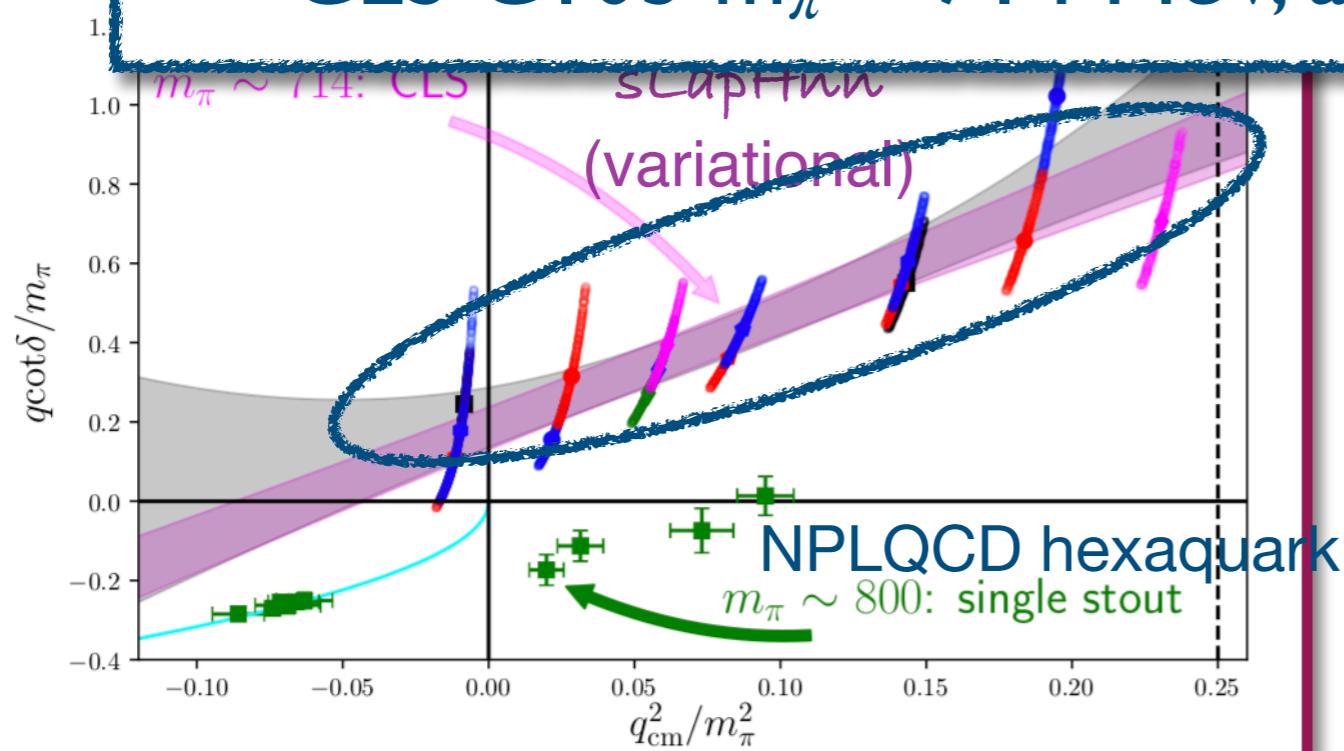
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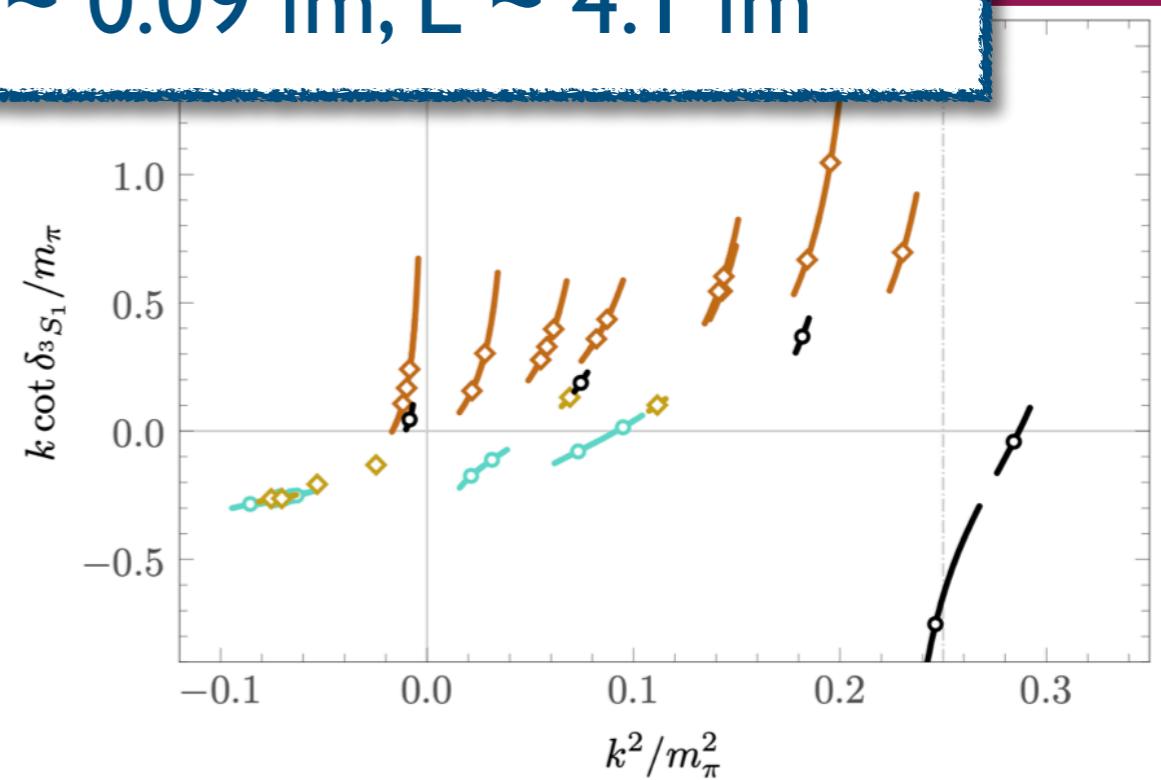
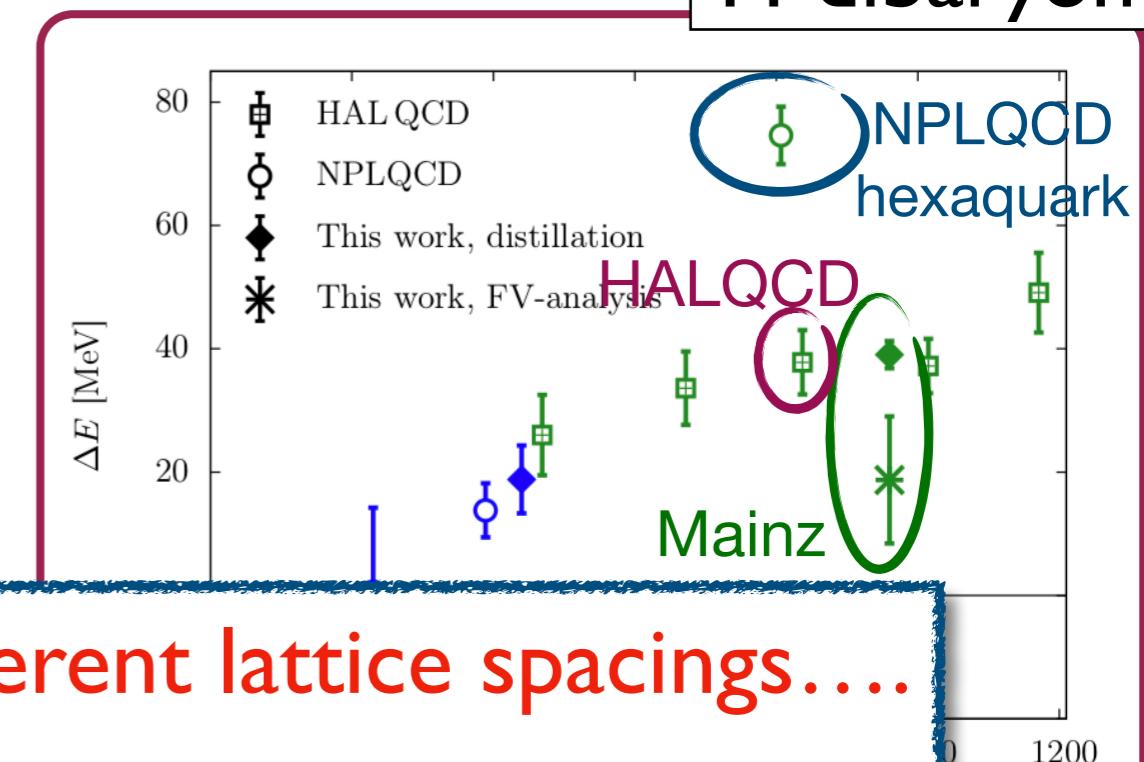
Groups use different actions, different lattice spacings....

We should test all methods on a single ensemble

CLS CI03 $m_\pi \sim 714$ MeV, $a \sim 0.09$ fm, $L \sim 4.1$ fm



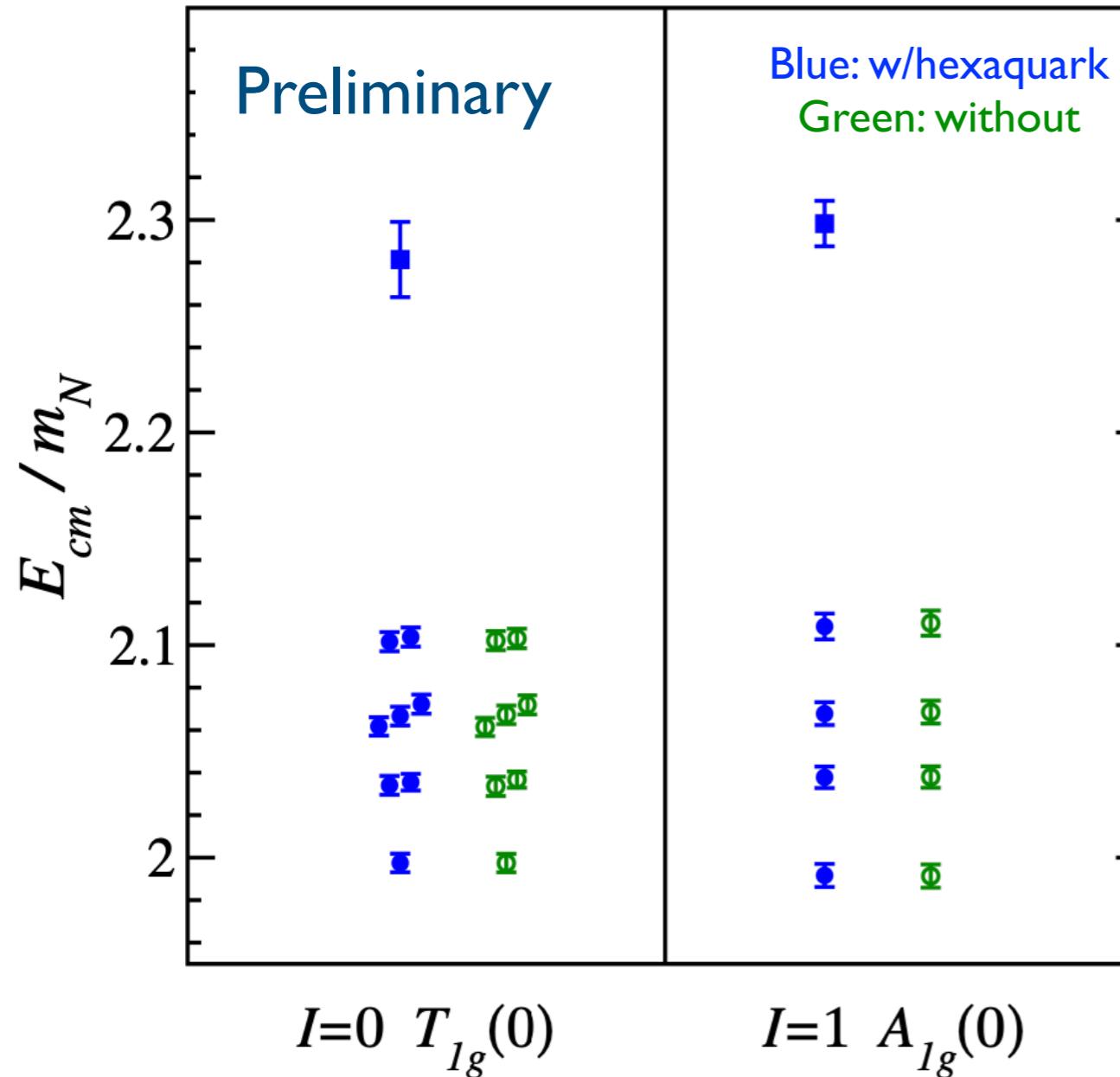
scapTHnn (2021)



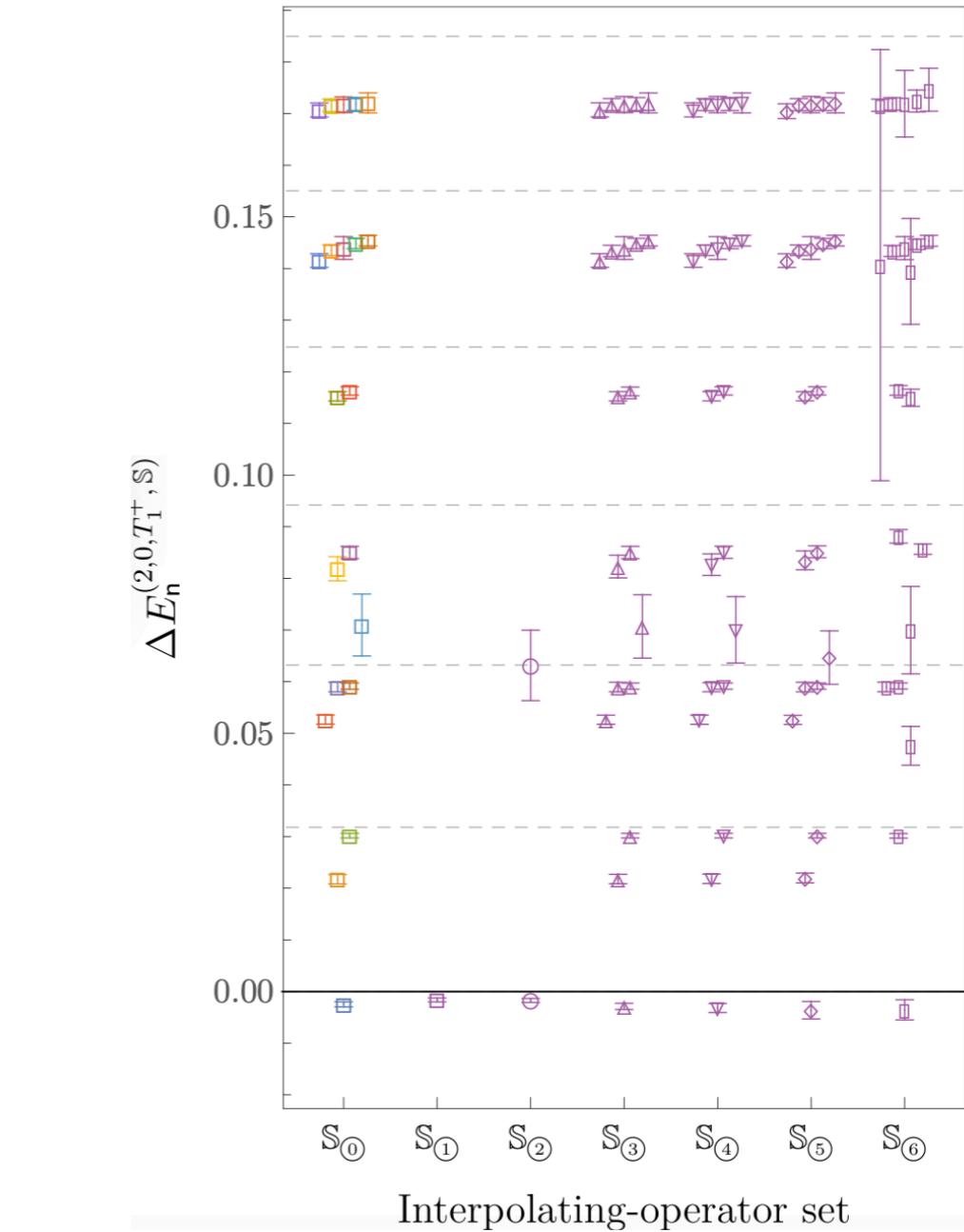
NPLQCD variational (2022)

Do we need the hexaquark operator in our variational basis?

Ground state does not change when hexaquark is removed, or more/less momentum states used



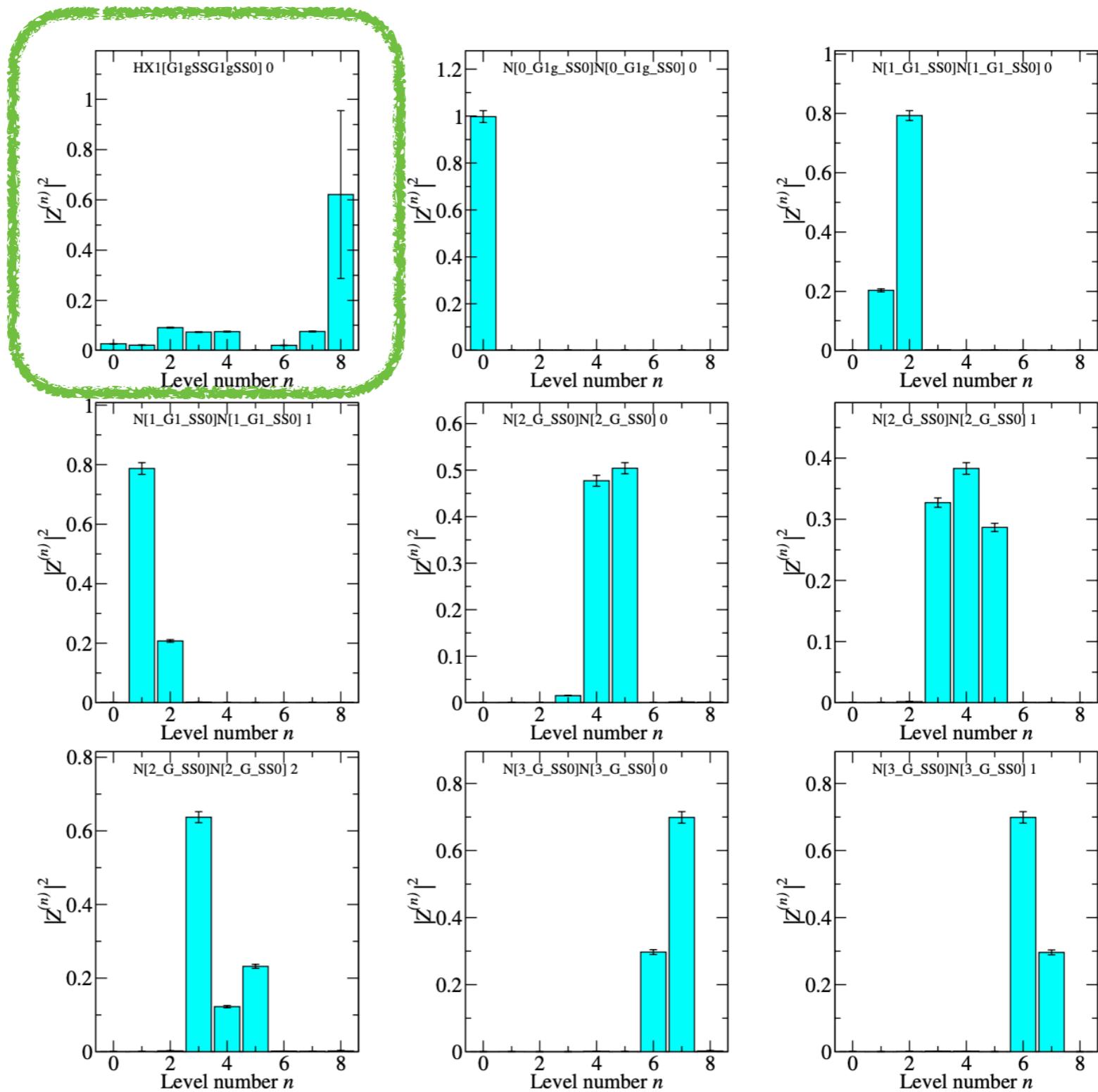
slapHnn



NPLQCD variational: arXiv:2108.10835

Do we need the hexaquark operator in our variational basis?

hexaquark has very poor overlap with lowest-lying states



Off-diagonal correlators: NPLQCD Toy Model

Three closely spaced energy levels. Two operators used to create off-diagonal correlator with overlaps:

$$Z_A = (\epsilon, \sqrt{1 - \epsilon^2}, 0) \quad Z_B = (\epsilon, 0, \sqrt{1 - \epsilon^2})$$

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$$Z_A = (\epsilon, \sqrt{1 - \epsilon^2}, 0) \quad Z_B = (\epsilon, 0, \sqrt{1 - \epsilon^2})$$

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Zero overlap: (essentially) impossible

Off-diagonal correlators: NPLQCD Toy Model

Three closely spaced energy levels. Two operators used to create off-diagonal correlator with overlaps:

More realistic:

$$Z_A = (\epsilon_1, \sqrt{1 - \epsilon_1^2 - \epsilon_2^2}, \epsilon_2) \quad Z_B = (\epsilon_1, \epsilon_2, \sqrt{1 - \epsilon_1^2 - \epsilon_2^2})$$

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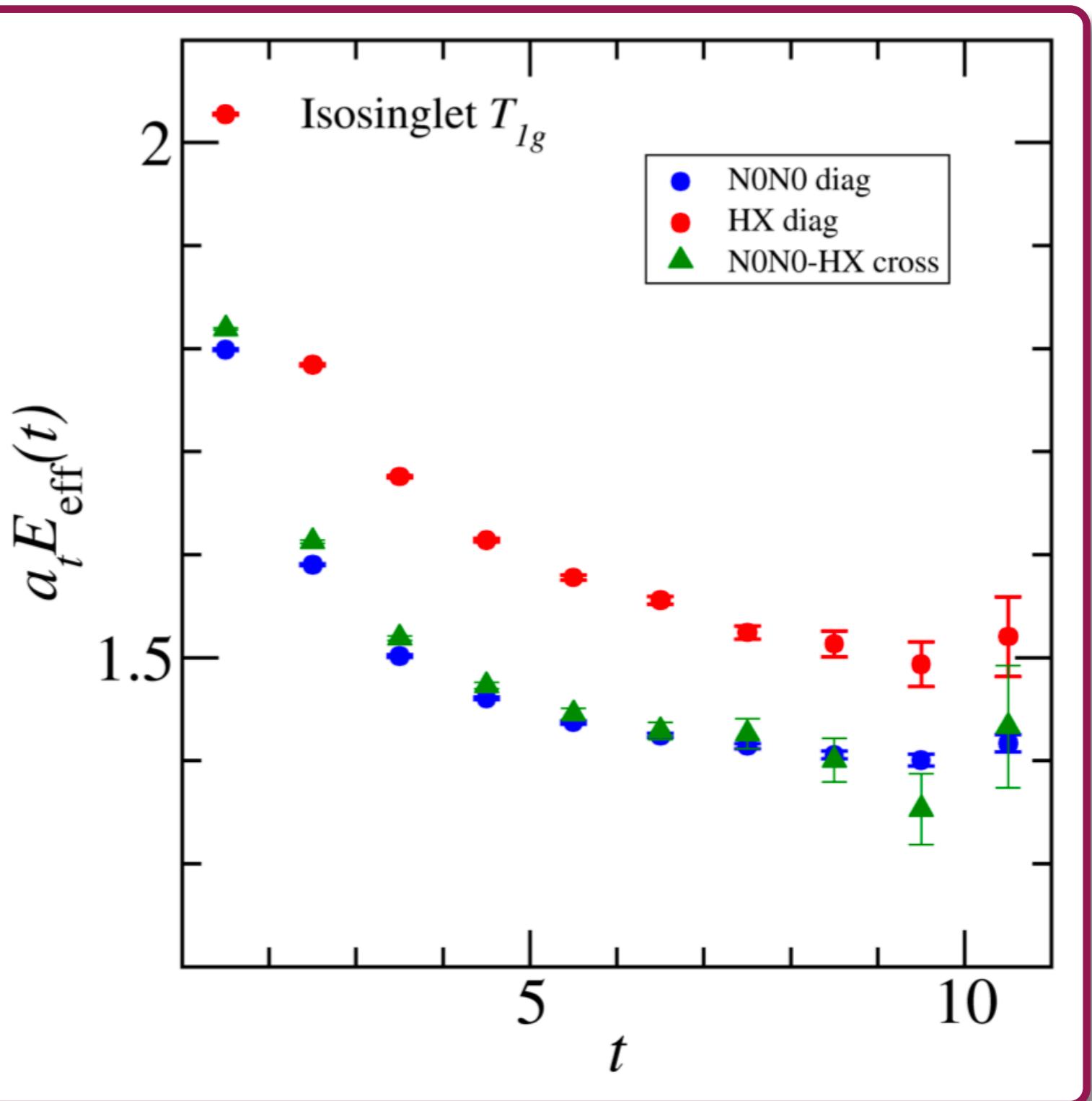
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Off-diagonal correlators dominated by higher states at intermediate times

Do we need the hexaquark operator in our variational basis?

We see no difference in g.s. energy using off-diagonal correlator



Pionless EFT: elastic excited states

$$\mathcal{L}_{\text{eff}} = \psi^\dagger \left(i\partial_\tau + \frac{\nabla^2}{2M} \right) \psi + g_0 (\psi^\dagger \psi)^2$$

$$\langle pq | \mathcal{T} | p' q' \rangle = \frac{\delta_{pp'} \delta_{qq'} + \frac{g_0}{V} \delta_{p+q,p'+q'}}{\sqrt{\xi(p)\xi(q)\xi(q')\xi(p')}}$$

$$\xi(p) \equiv 1 + \frac{\Delta(q)}{M}$$

Endres, Kaplan, Lee,
Nicholson (2011)

Pionless EFT: elastic excited states

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- Valid for energies $\ll m_\pi$ (same requirement as Luscher)

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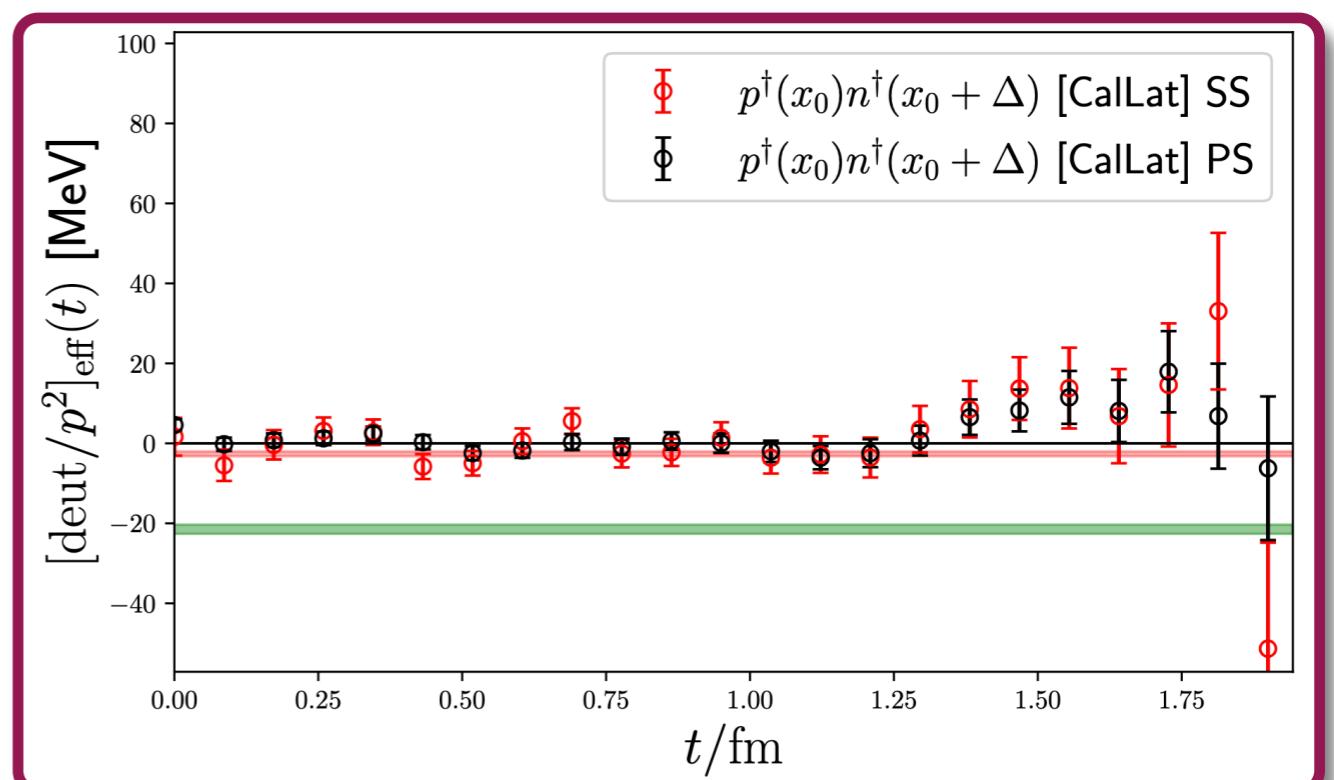
- Two point-like nucleons interact via contact interactions
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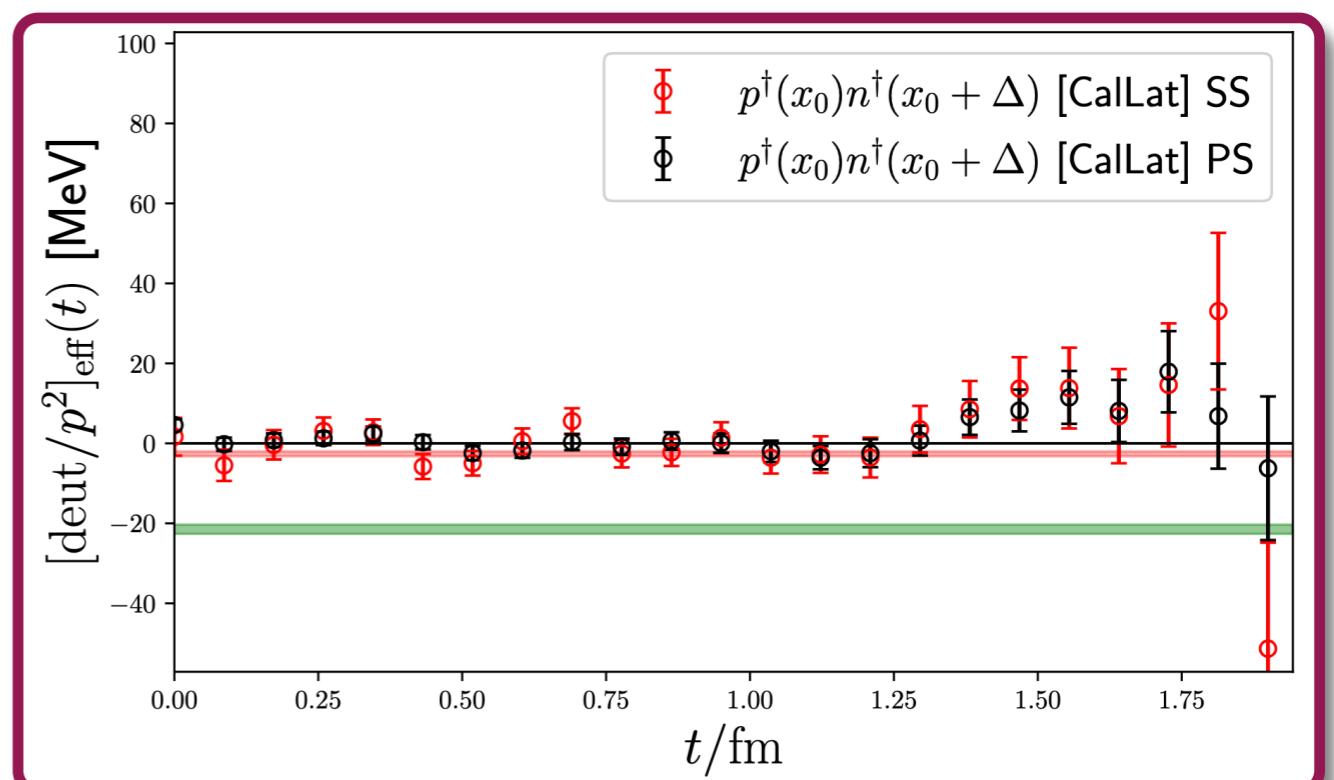
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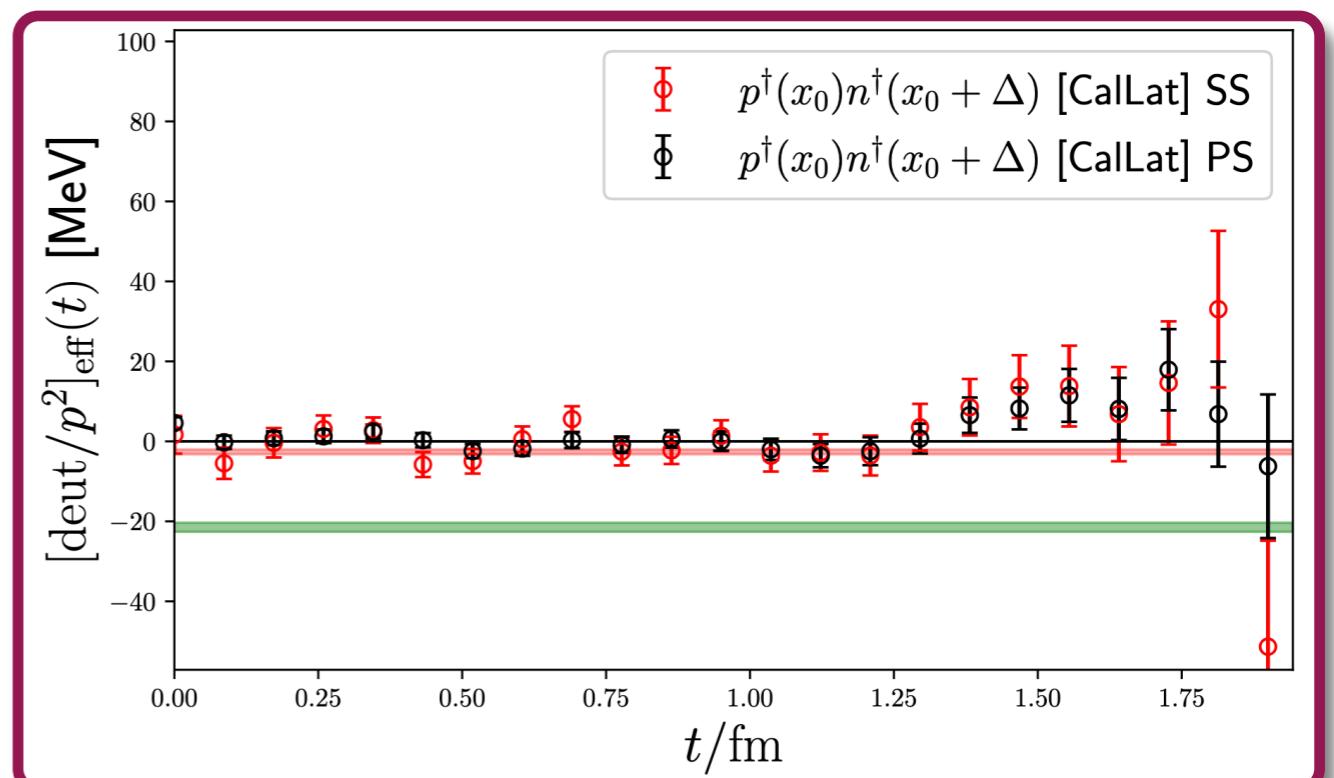
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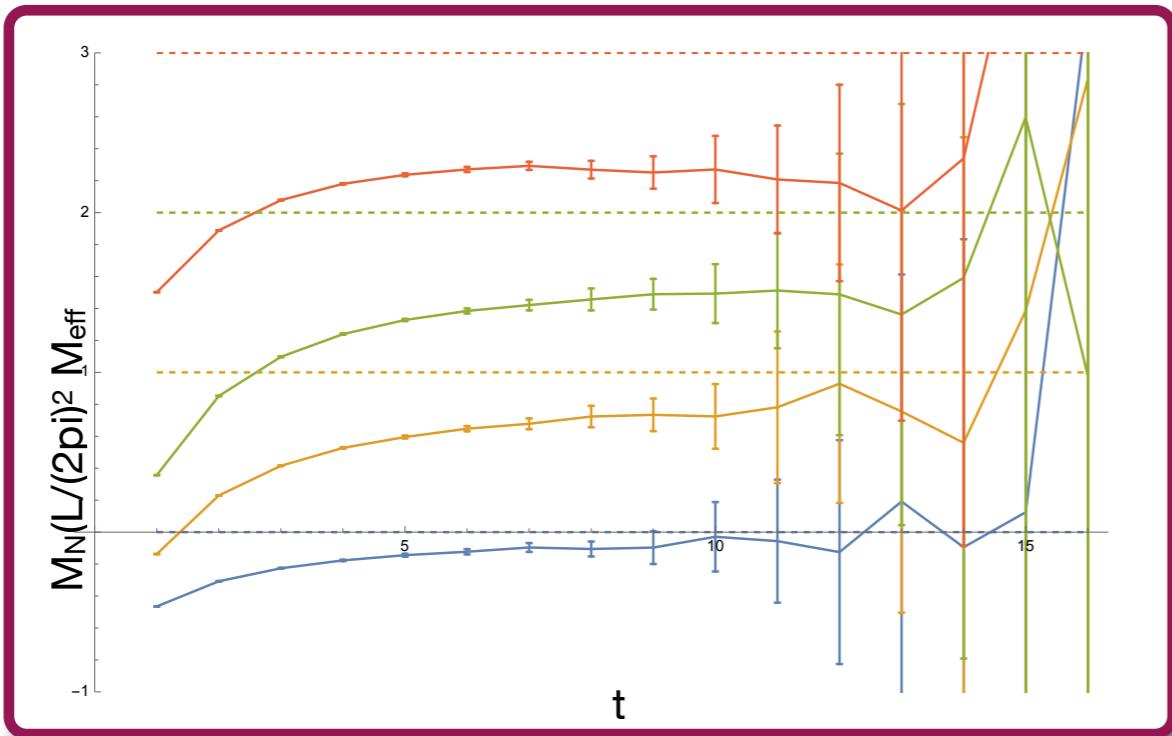
- Can tune interaction to test what we should expect to see for a system with and without a deeply bound state
- Consider smoothly varying phase shifts which obey an ERE within the relevant energy regime



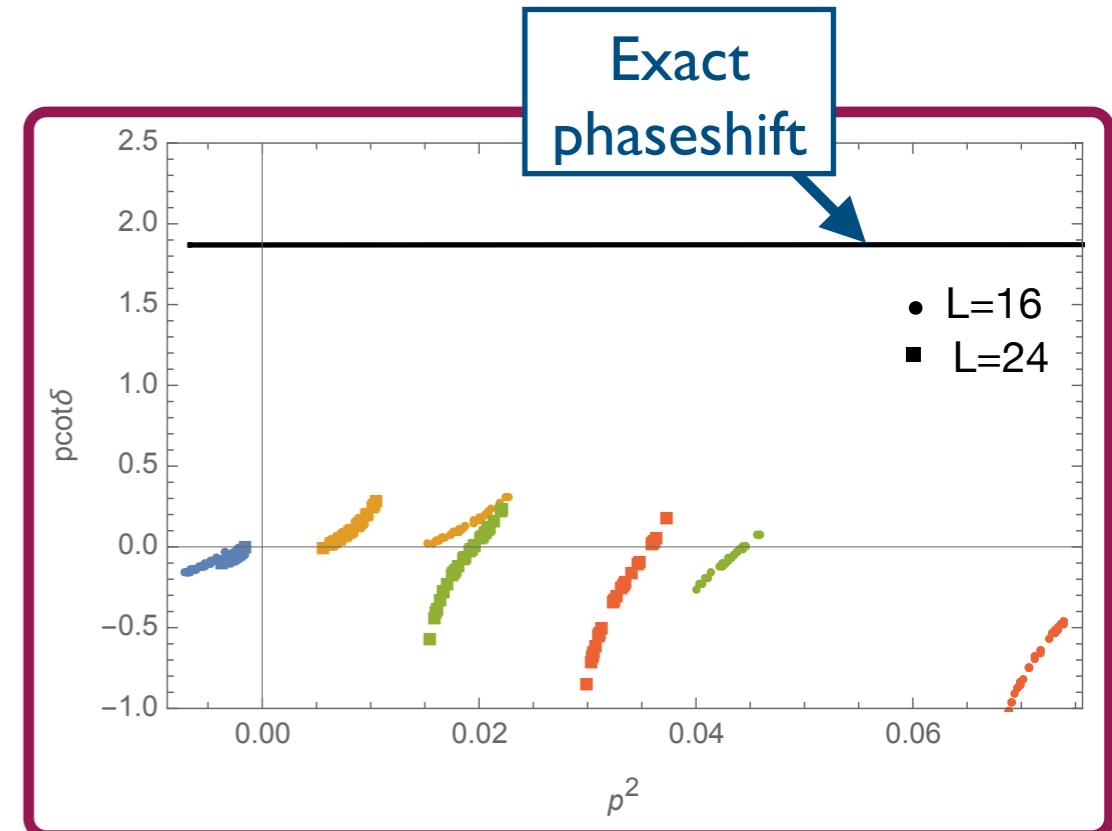
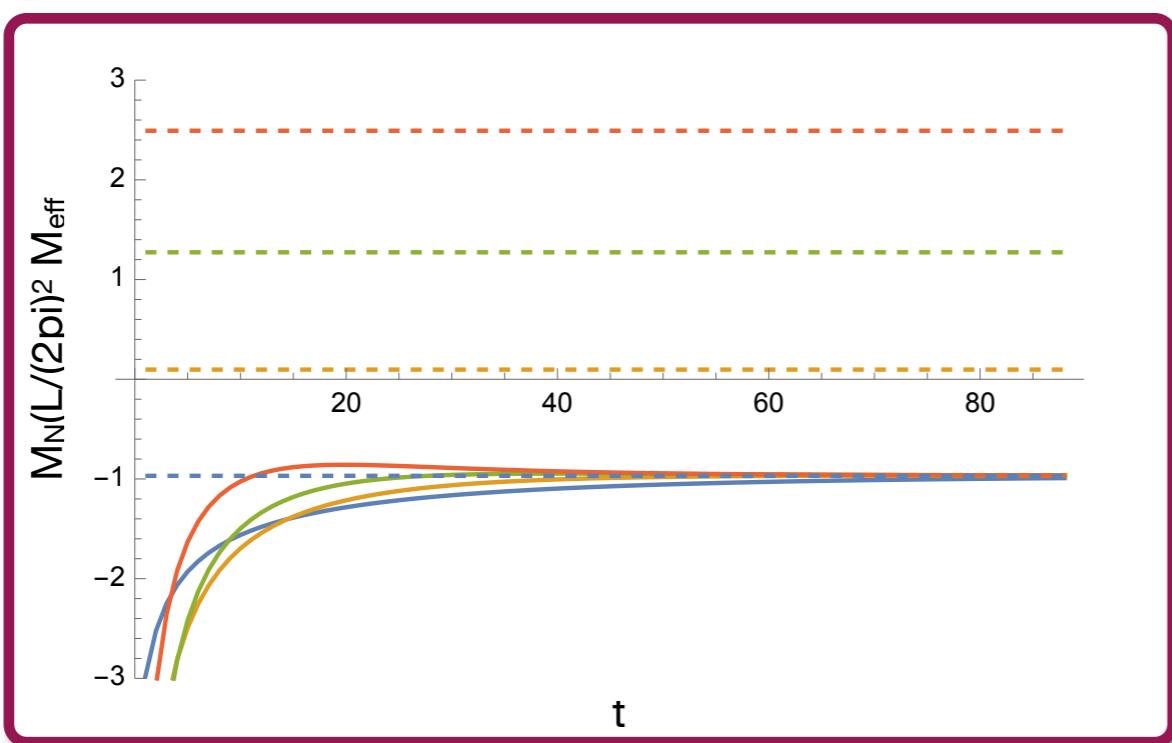
Local (hexaquark) source
→ momentum space sink

Dashed lines give exact spectrum

No Bound State



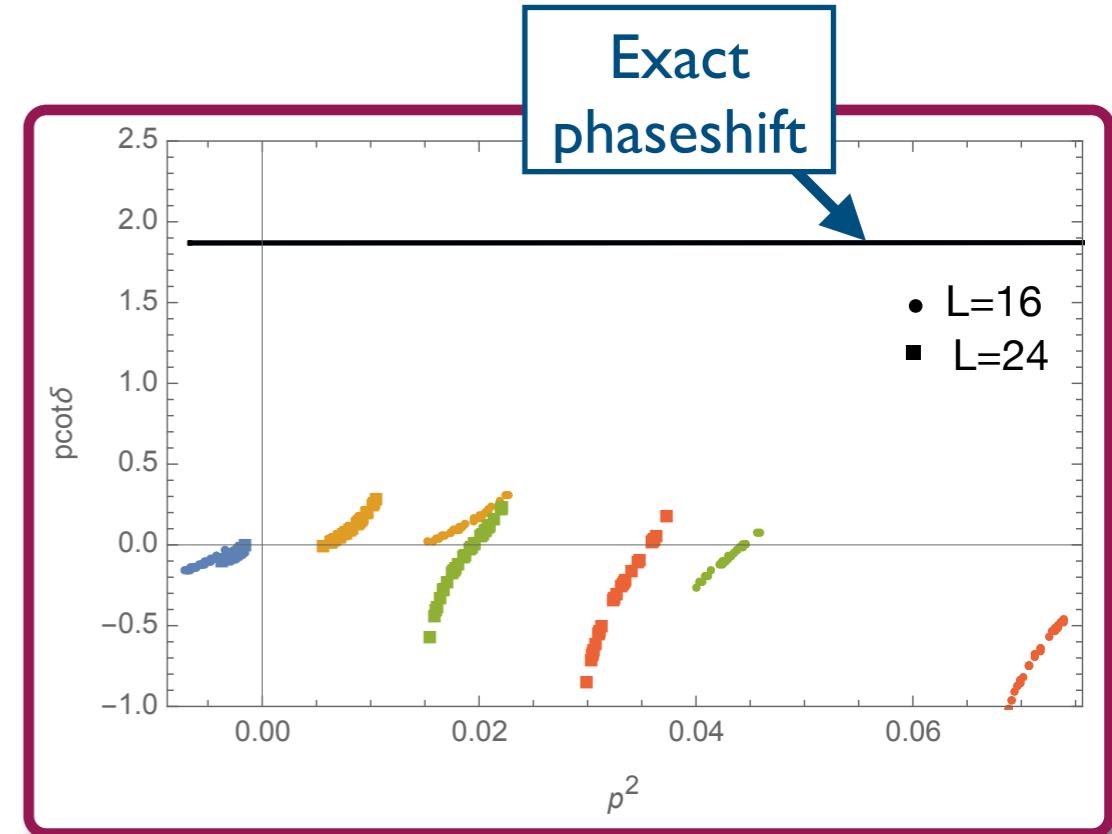
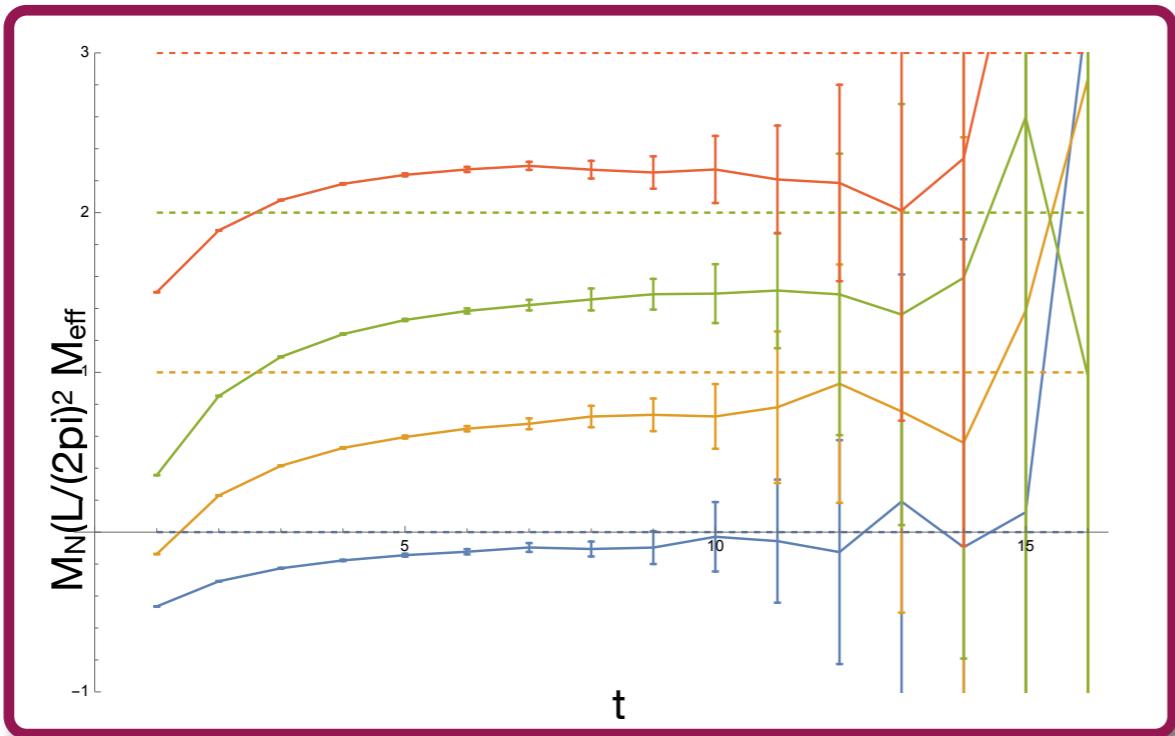
Bound State



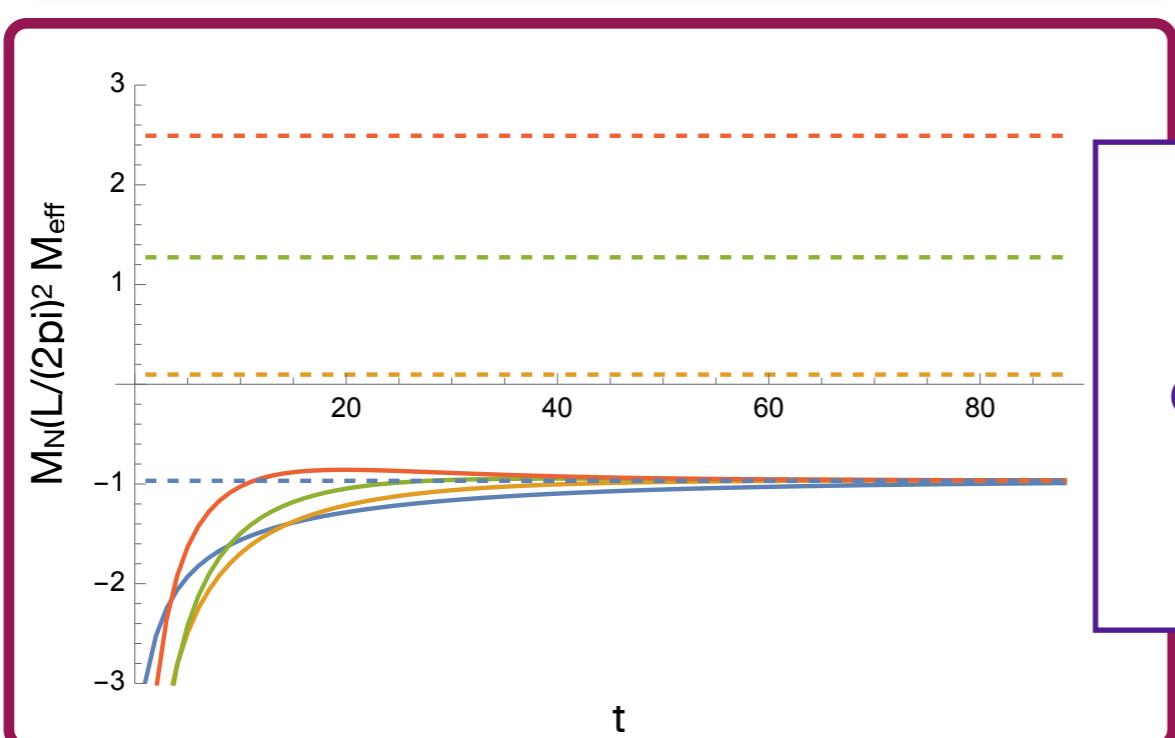
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Bound State

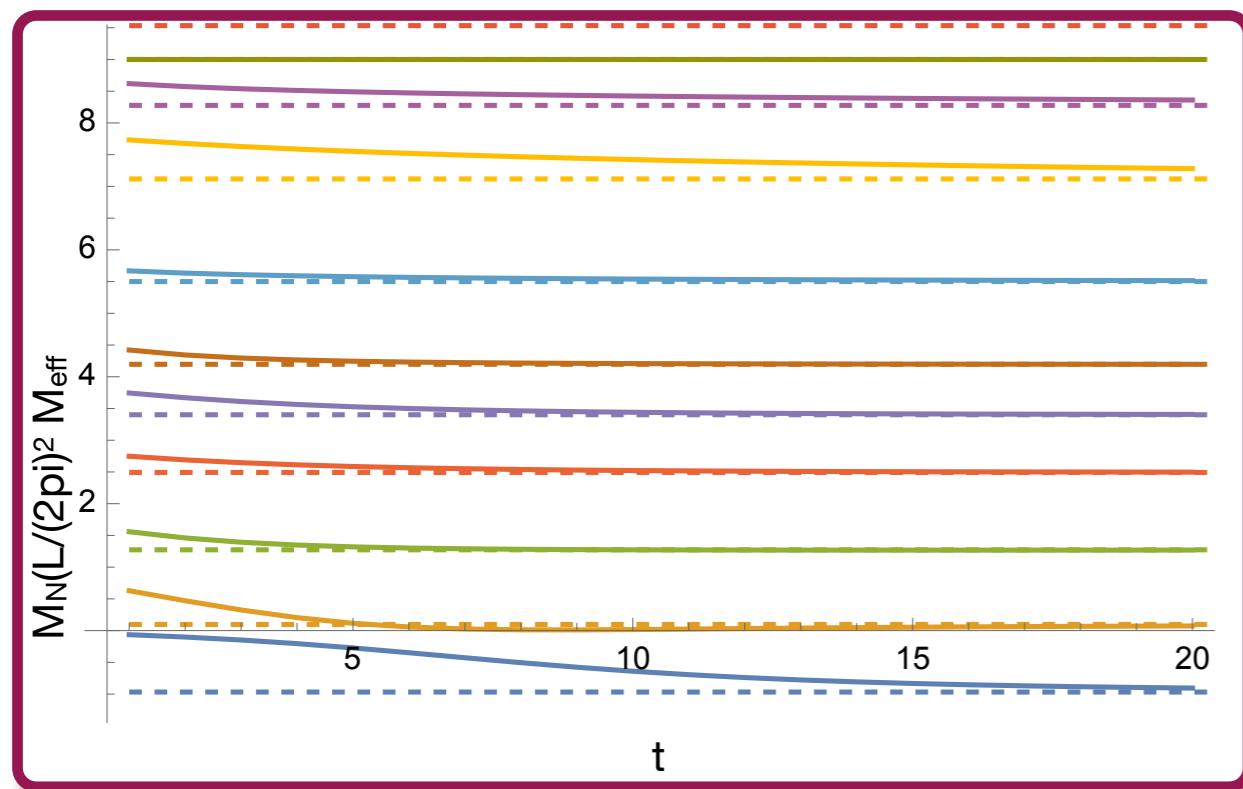


Even if the system has a deeply bound state the hexaquark correlator approaches the ground state very slowly and can be deceptive when noise is added

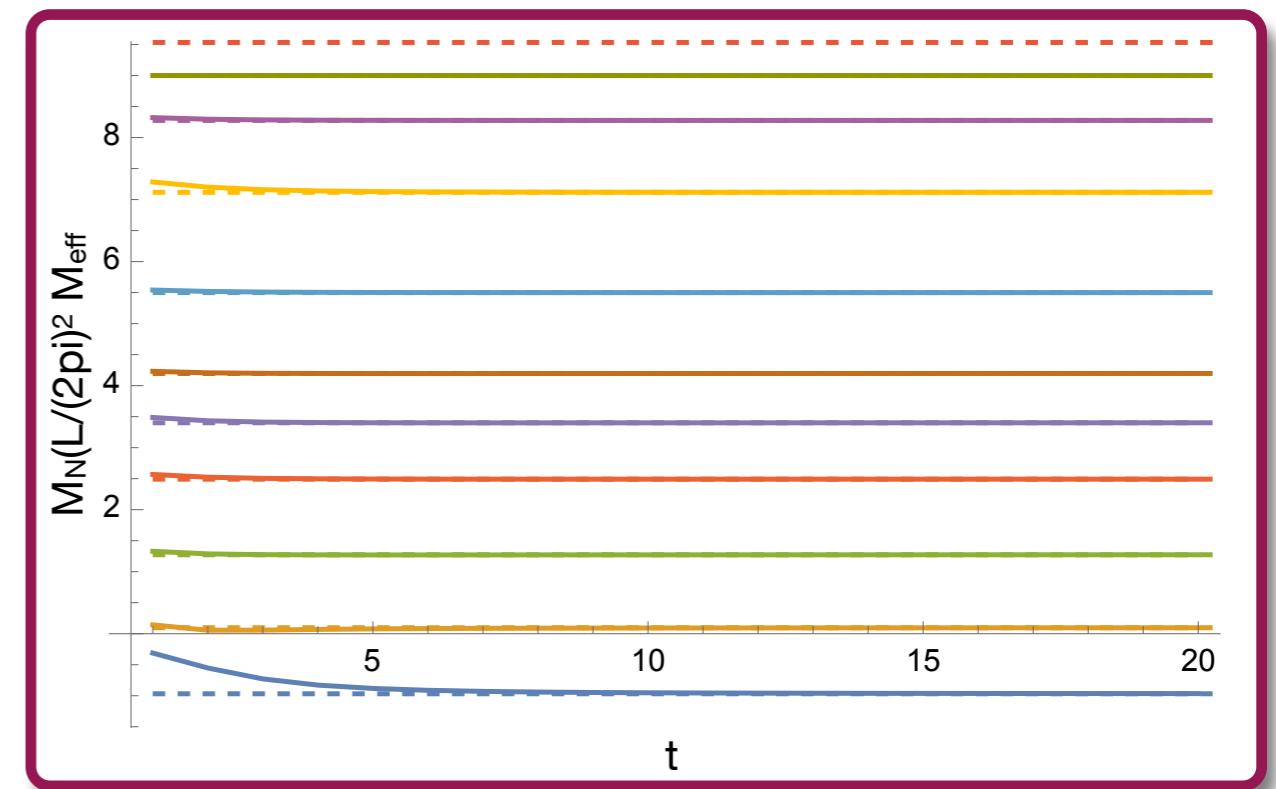
Momentum space source ➡
momentum space sink

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Bound State



Variational: 10 ops, no hexaquark

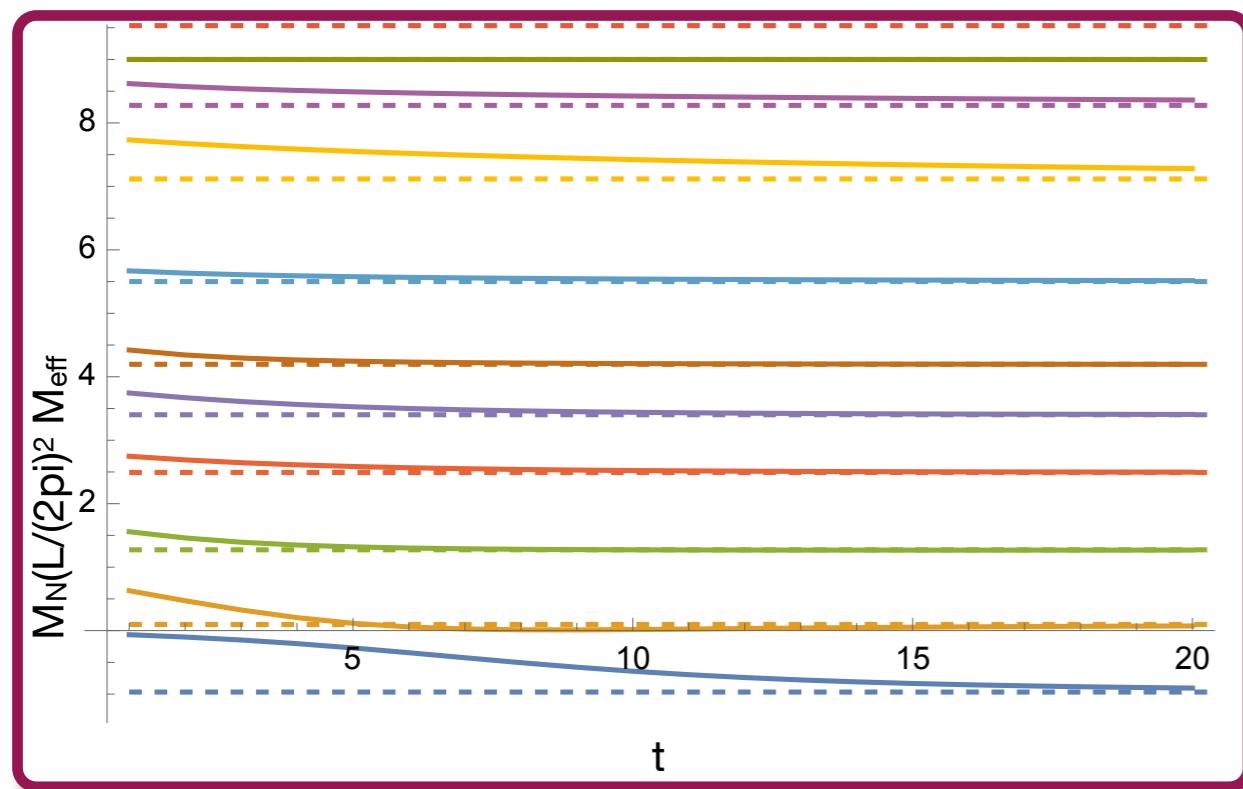


Variational: 30 ops, no hexaquark

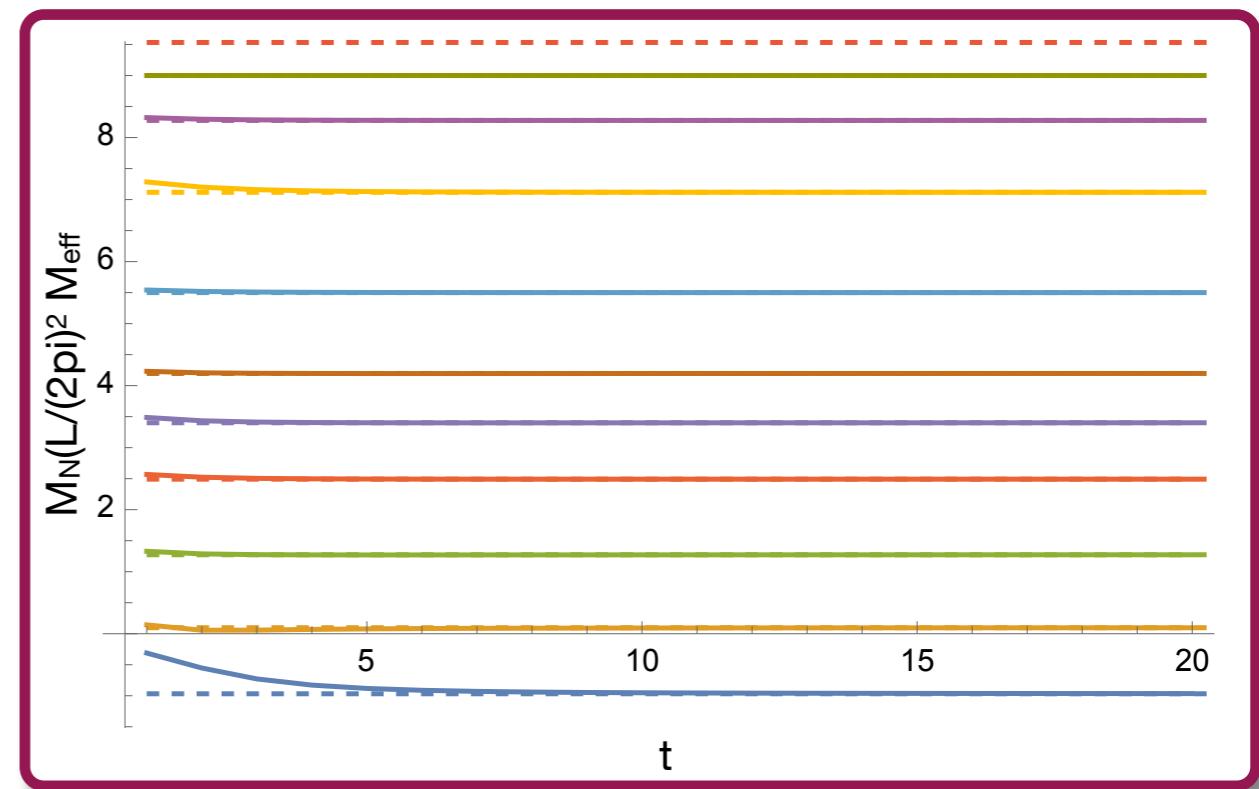
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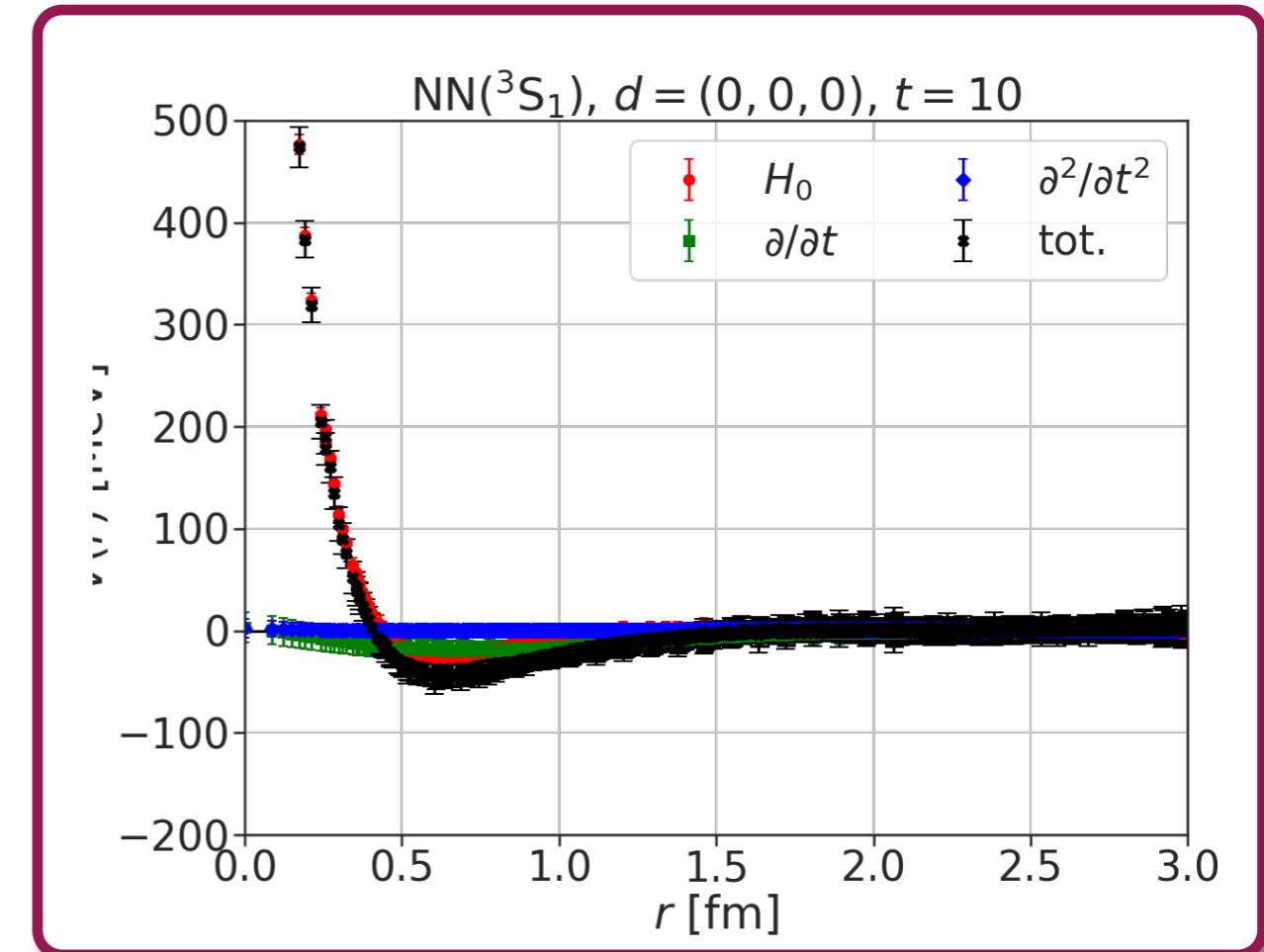
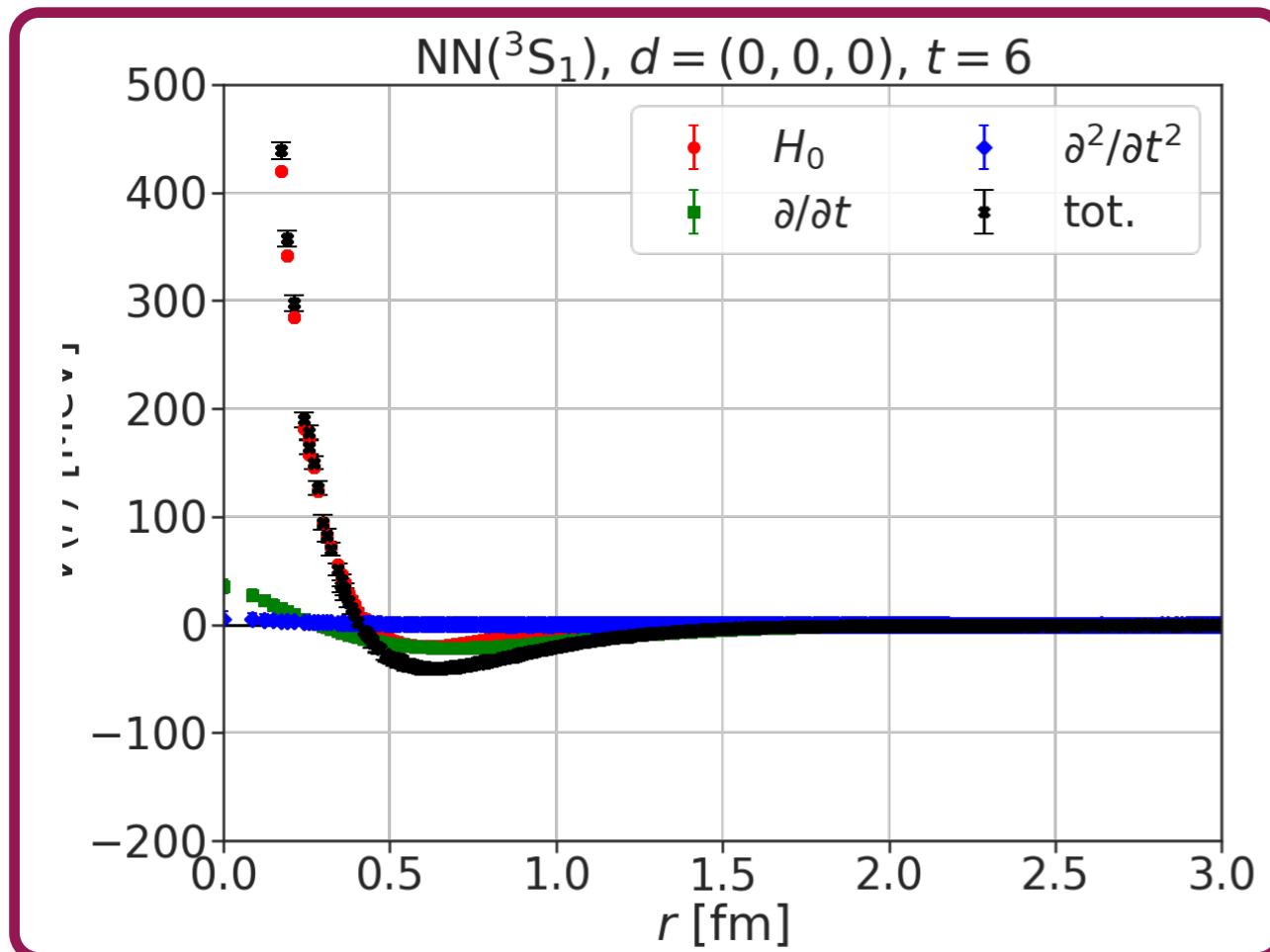
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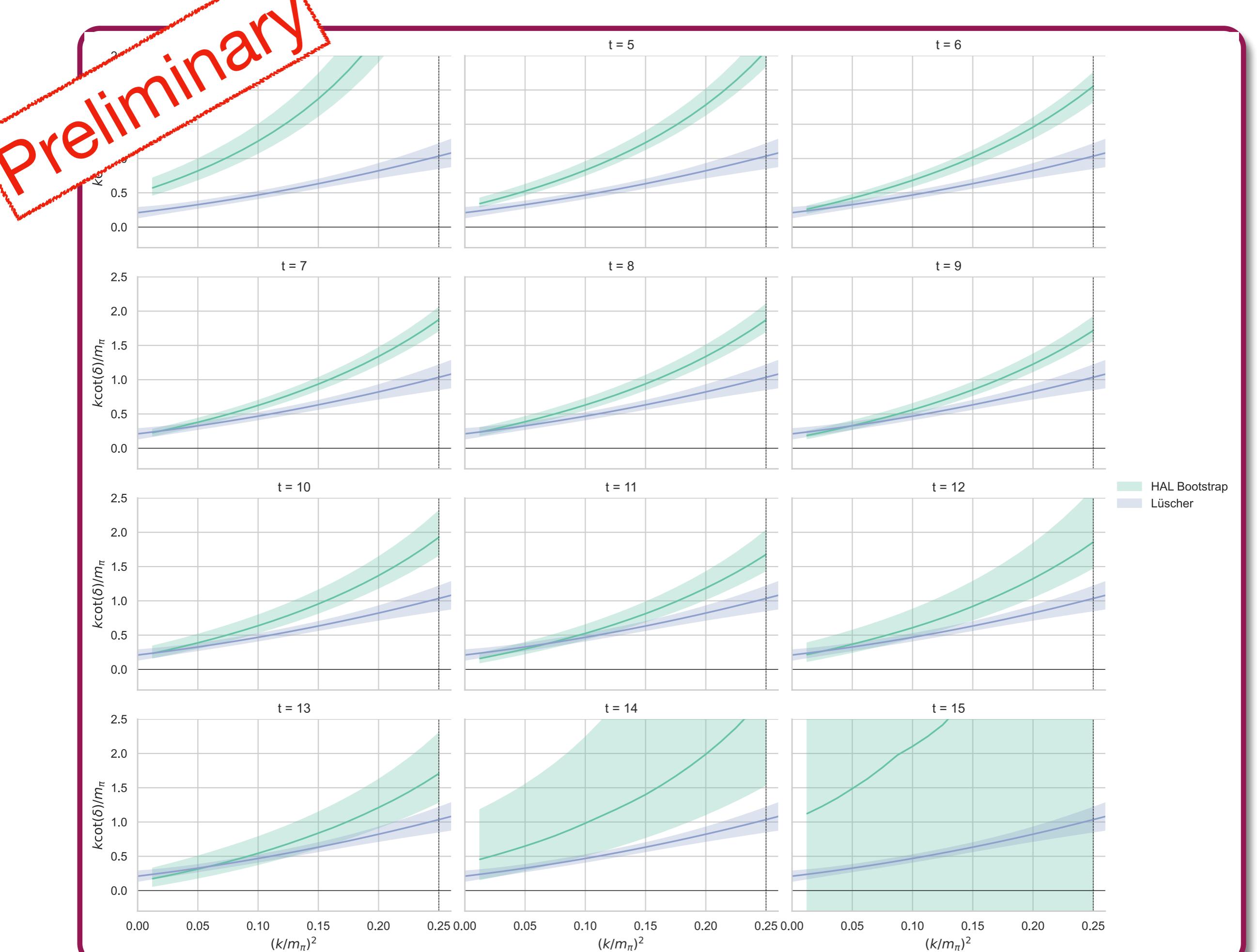
Even if the system has a deeply bound state the variational method with momentum ops only works well

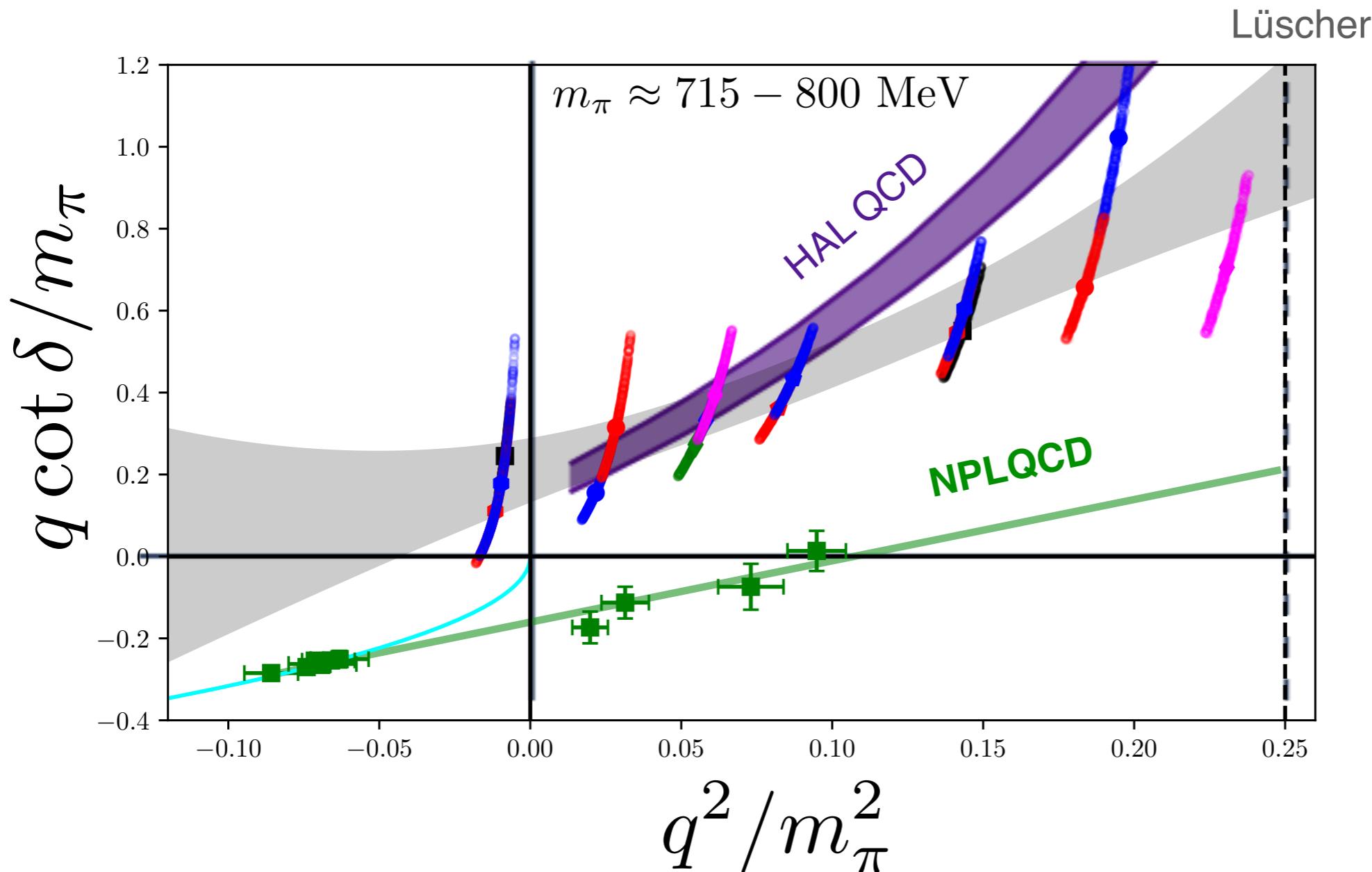
Preliminary

HALQCD Potential Method

CLS ensemble: $m_\pi \sim 714$ MeV, $a \approx 0.086$ fm, $L = 48$





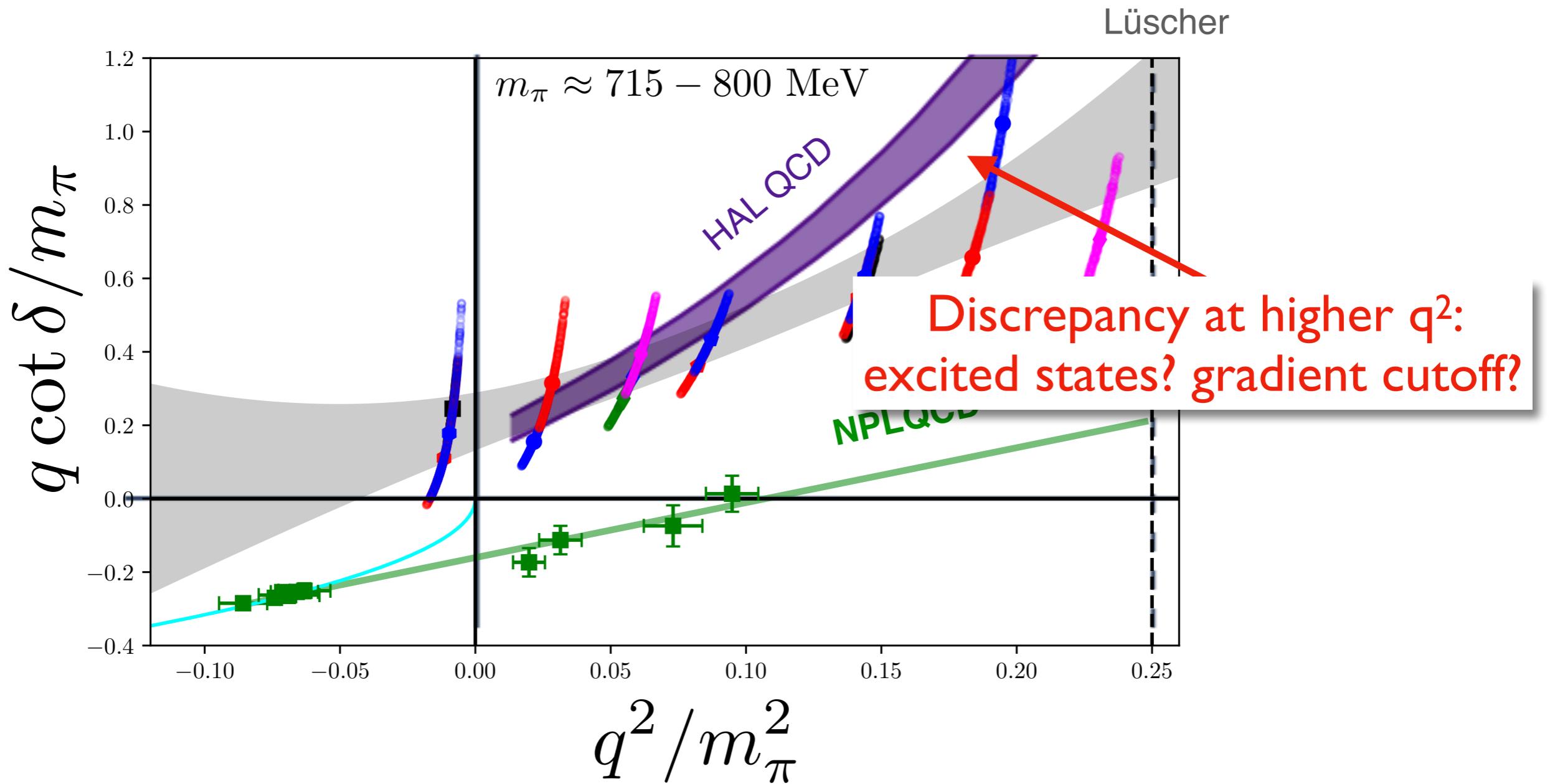


Scattering Theory Refresher

$$T \propto \frac{1}{q \cot \delta - iq}$$

bound state : $\lim_{q \rightarrow 0} q \cot \delta < 0$

no bound state : $\lim_{q \rightarrow 0} q \cot \delta > 0$



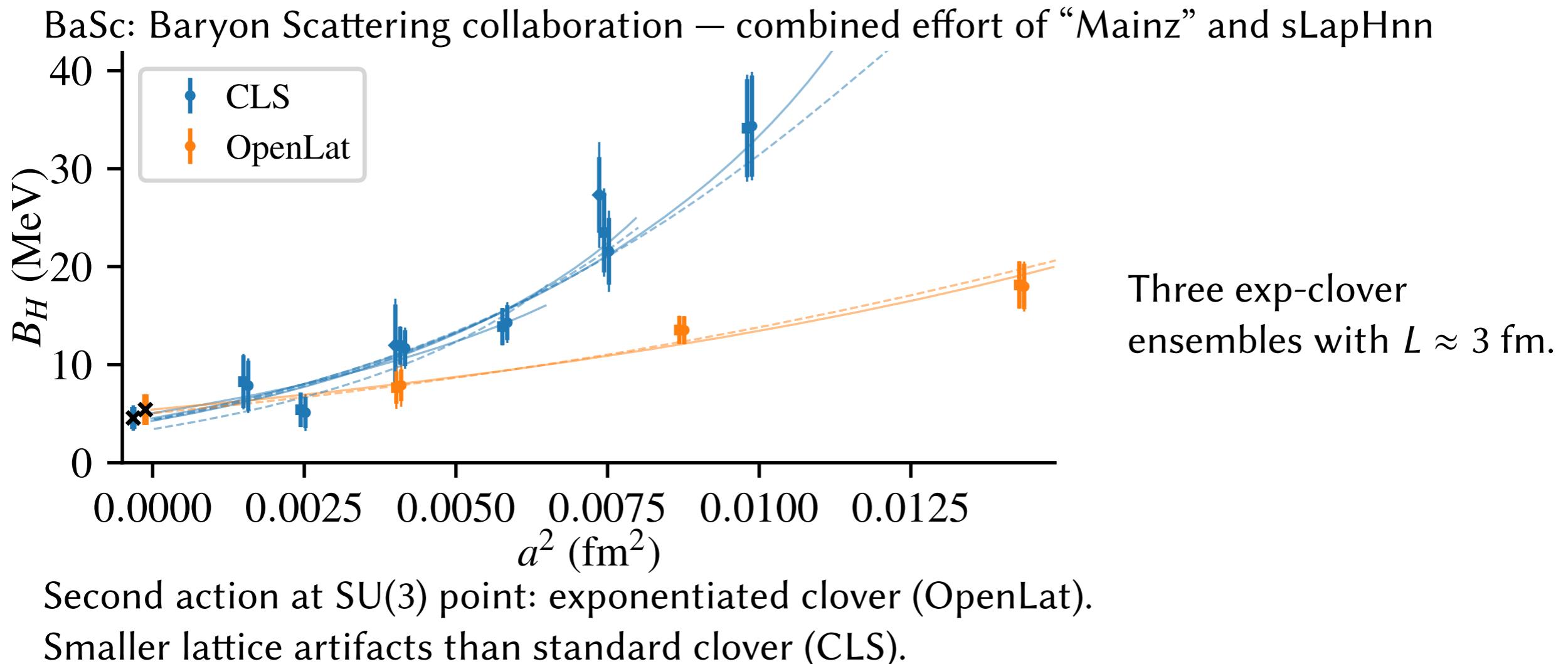
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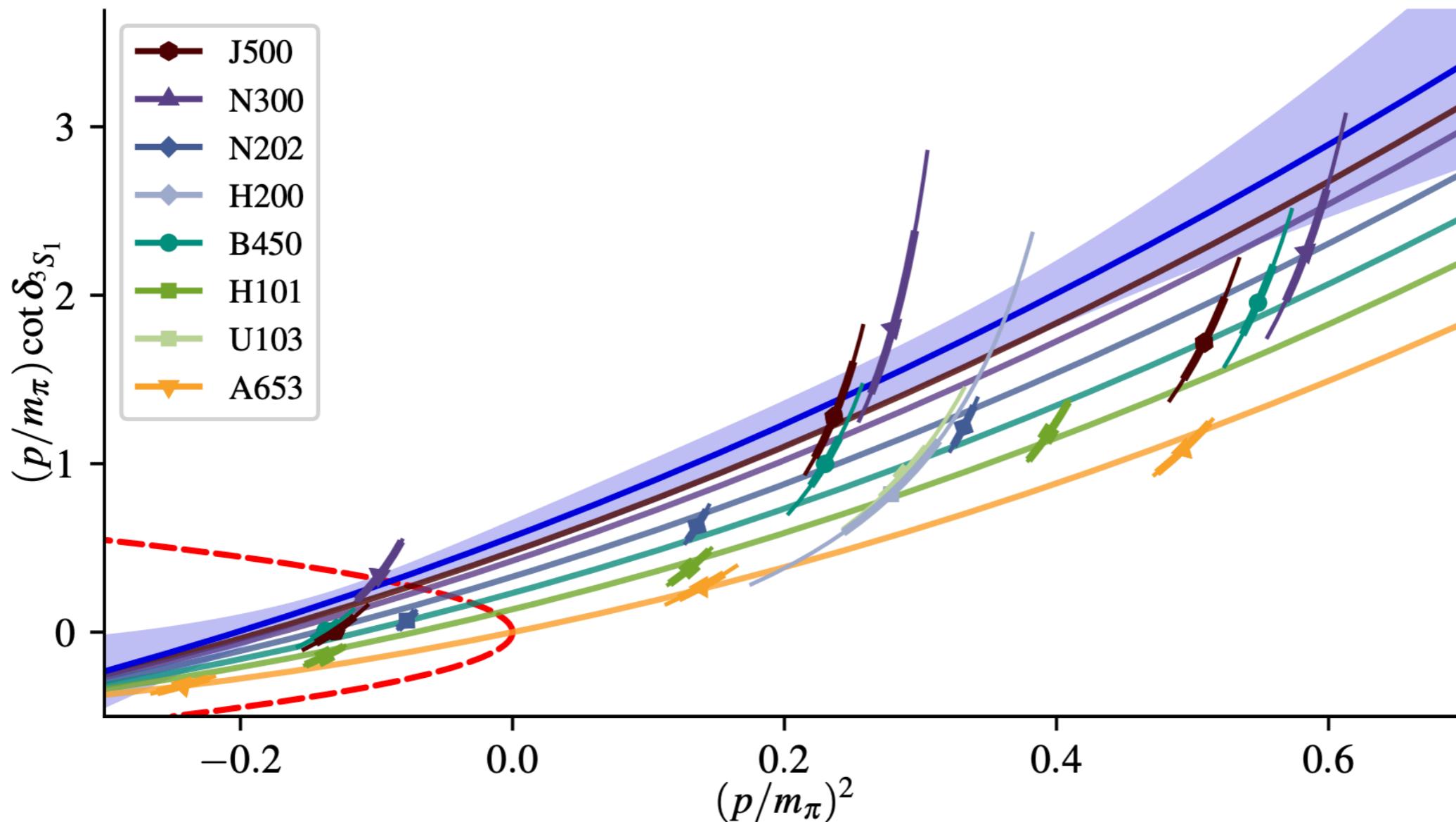
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H dibaryon: $a \rightarrow 0$ universality (PRELIMINARY)



$$m_\pi = m_K = m_\eta \approx 420 \text{ MeV}.$$



Jeremy R. Green,
 Andrew D. Hanlon, Parikshit M. Junnarkar, Hartmut Wittig

NN scattering with sLapH

- **Controversy between methods: working to benchmark at $m_\pi \sim 800$ MeV**
- **Preponderance of evidence now shows that there is no bound state at heavy pion mass**
- **Hexaquark and off-diagonal correlators are not necessary, and can be misleading**
- **Preliminary results show that the HALQCD potential method agrees well with Lüscher at low momenta**
- **Possible systematics at higher q^2**

Ben Hörz (Intel)
Dean Howarth (LLNL)
Enrico Rinaldi (RIKEN)
Andrew Hanlon (BNL)
Chia Cheng Chang (RIKEN/LBNL)
Christopher Körber (Bochum/LBNL)
Evan Berkowitz (Jülich)
John Bulava (DESY)
M.A. Clark (NVIDIA)
Wayne Tai Lee (Columbia)
Kenneth McElvain (LBNL)
Colin Morningstar (CMU)
Amy Nicholson (UNC)
Pavlos Vranas (LLNL)
André Walker-Loud (LBNL)

