

# **Disconnected contribution to the LO HVP term of muon g-2 from ETMC**



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# Overview



Lattice calculation of the short and intermediate time-distance hadronic vacuum polarization contributions to the muon magnetic moment using twisted-mass fermions

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Based on results presented in [arXiv:2206.15084](https://arxiv.org/abs/2206.15084)

- Lattice setup
- Muon g-2, disconnected contributions
- Results on short and intermediate windows
- Preliminary results on long distance

Follows talk by G. Gagliardi on connected contributions, final results and conclusions.

# Twisted Mass Fermions

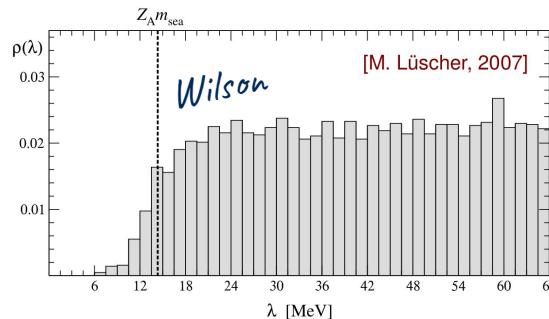
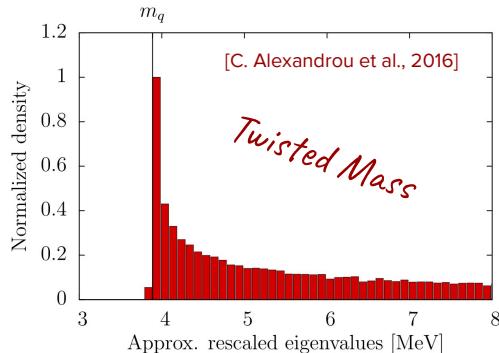


QCD invariant under chiral rotation

$$\mathcal{L}_f = \bar{\psi} (\gamma^\mu \partial_\mu + m_q) \psi \longrightarrow \mathcal{L}'_f = \bar{\psi}'_2 (\gamma^\mu \partial_\mu + m'_q + i\mu \gamma_5 \tau_3) \psi'_2$$

$$\psi'_2 = e^{i\frac{\omega}{2}\Gamma_5 \otimes \tau_3} \begin{bmatrix} \psi_u \\ \psi_d \end{bmatrix} \quad \text{with } m'_q = m_q \cos \omega, \quad \mu = m_q \sin \omega \quad \text{and} \quad m_q = \sqrt{m'^2_q + \mu^2}$$

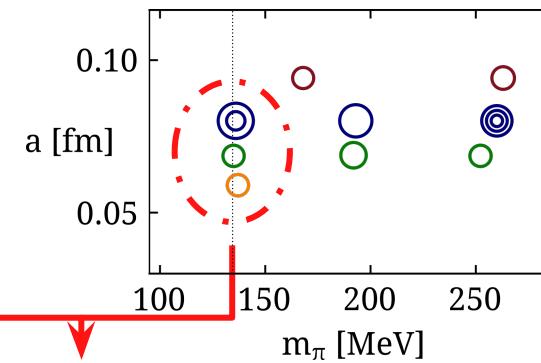
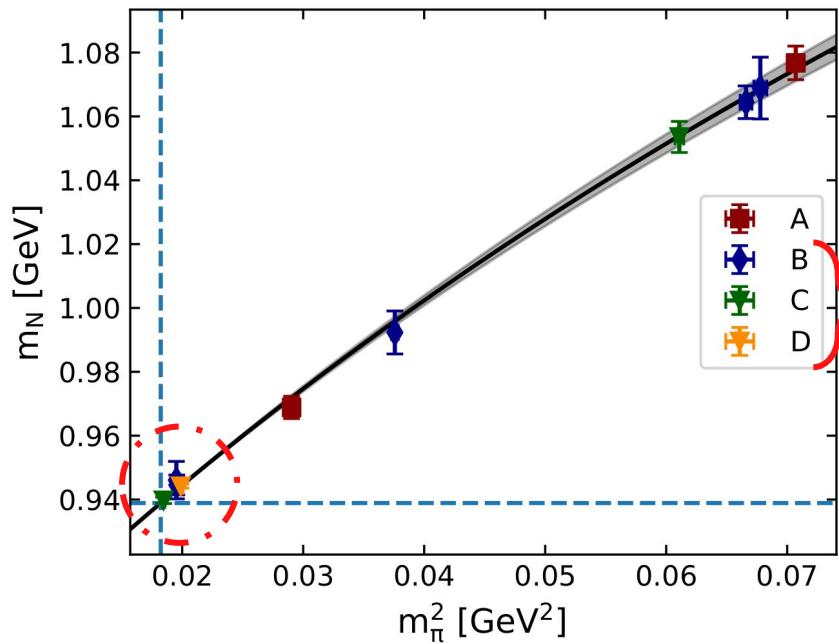
- ✓ **Automatic O(a)-improvement** at maximal twist  $\longrightarrow \omega = \arctan \frac{\mu}{m'_q} = \pi/2$  [R. Frezzotti and G. C. Rossi, 2004]
- ✓ **Protected from zero modes** with cutoff  $\mu \longrightarrow D_{\text{TM}}^\dagger D_{\text{TM}} = D^\dagger D + \mu^2$



# Simulations at the Physical Point



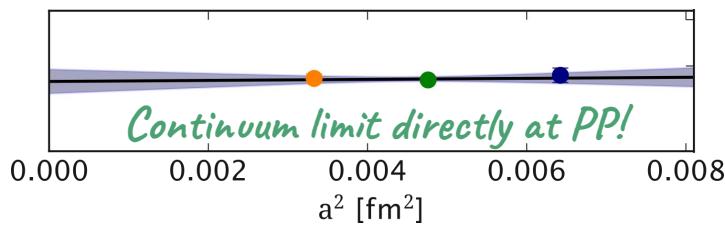
- Physical Point = “with quark masses tuned to their physical value”



$N_f = 2+1+1$   
ETMC  
ensembles  
 $M_\pi < 300$  MeV

[J. Finkenrath et al., 2022]

3 lattice spacings  
at PP ...



Continuum limit directly at PP!

# Simulations at the Physical Point



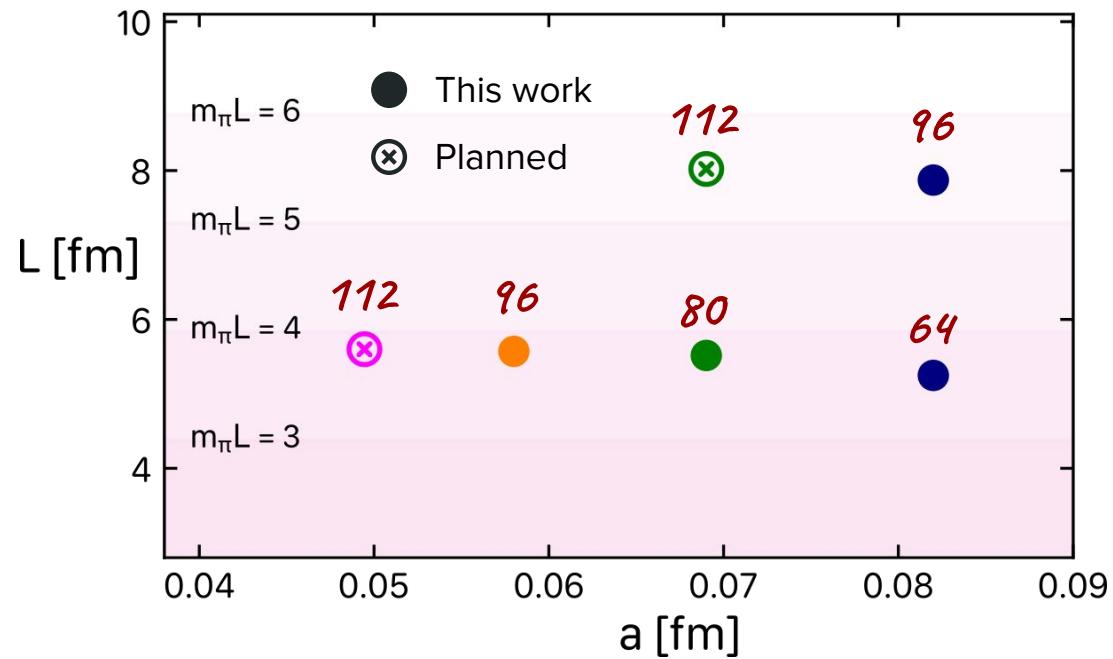
Simulating at PP is challenging and computationally demanding!

✓ No chiral extrapolation

! Finite volume effects

$$m_\pi L > 3.5$$

$$V = L^3 \times T \quad \text{and} \quad T = 2L$$



# Technical details



## Simulation parameters

ensemble	$\beta$	$V/a^4$	$a$ (fm)	$a\mu_\ell$	$M_\pi$ (MeV)	$L$ (fm)	$M_\pi L$
cB211.072.64	1.778	$64^3 \cdot 128$	0.07961 (13)	0.00072	140.2 (0.2)	5.09	3.62
cB211.072.96	1.778	$96^3 \cdot 192$	0.07961 (13)	0.00072	140.1 (0.2)	7.64	5.43
cC211.060.80	1.836	$80^3 \cdot 160$	0.06821 (12)	0.00060	136.7 (0.2)	5.46	3.78
cD211.054.96	1.900	$96^3 \cdot 192$	0.05692 (10)	0.00054	140.8 (0.2)	5.46	3.90

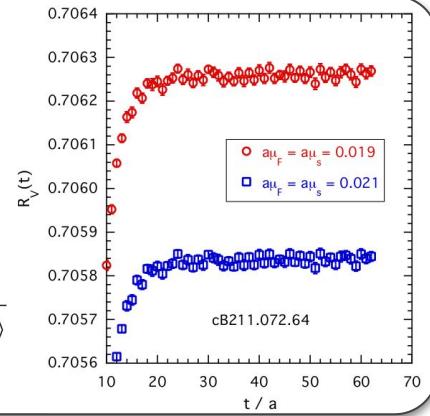
## Renormalization constants

ensemble	$Z_V$	$Z_A$
cB211.072.64	0.706378 (16)	0.74284 (23)
cB211.072.96	0.706402 (15)	0.74274 (20)
cC211.060.80	0.725405 (13)	0.75841 (16)
cD211.054.96	0.744105 (11)	0.77394 (10)

Renormalization constants determined directly on PP ensembles using hadronic observables and WI.

E.g.

$$Z_V = \lim_{\mu_F \rightarrow 0} 2\mu_F \frac{\sum_x \langle \mathcal{P}^1(x) \mathcal{P}^1(0) \rangle}{\sum_x \tilde{\partial}_\mu \langle \mathcal{A}_\mu^1(x) \mathcal{P}^1(0) \rangle}$$



# LO HVP term of Muon g-2 on the lattice



Time mom.  
represent.

$$a_\mu^{\text{HVP}} = 2\alpha_{em}^2 \int_0^\infty dt t^2 K(m_\mu t) V(t)$$

Vector  
correlator

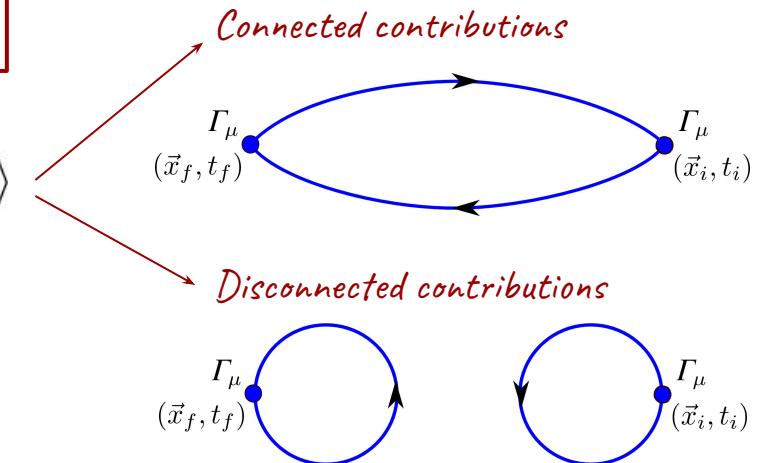
$$V(t) \equiv -\frac{1}{3} \sum_{i=1,2,3} \int d^3x \langle J_i(\vec{x}, t) J_i(0) \rangle$$

EM current  
operator

$$J_\mu(x) \equiv \sum_{f=u,d,s,c,\dots} q_f \bar{\psi}_f(x) \gamma_\mu \psi_f(x)$$

Kernel  
function

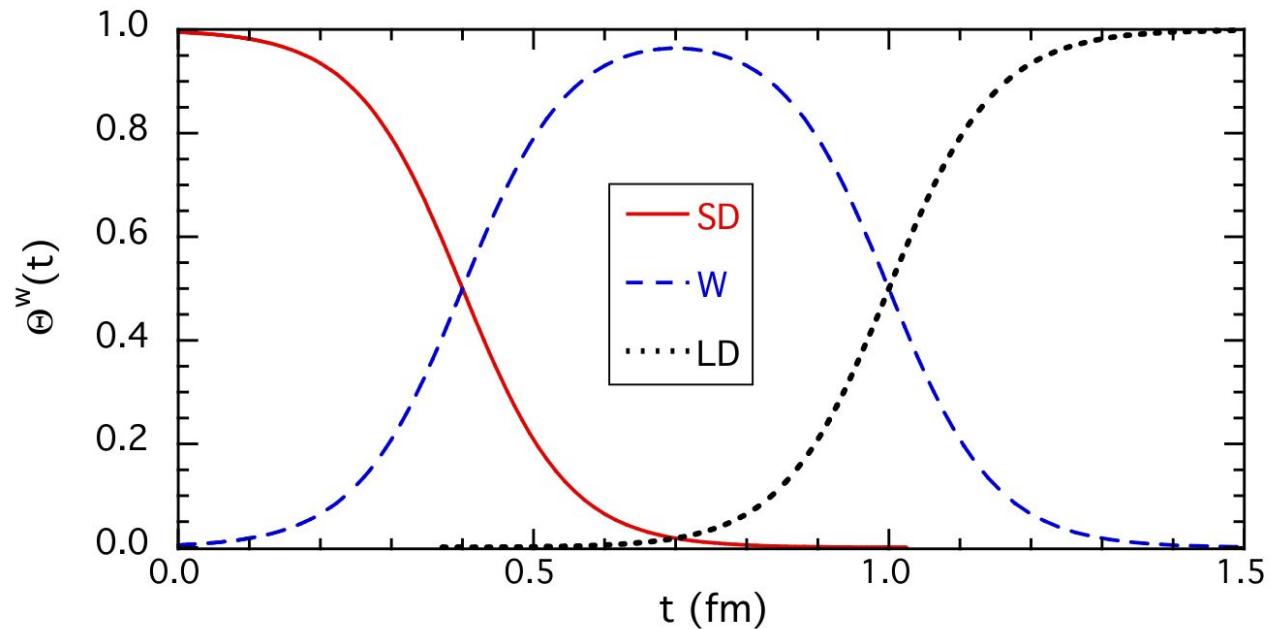
$$K(z) = 2 \int_0^1 dy (1-y) \left[ 1 - j_0^2 \left( \frac{z}{2} \frac{y}{\sqrt{1-y}} \right) \right] , \quad j_0(y) = \frac{\sin(y)}{y}$$



# The RBC/UKQCD windows



$$a_\mu^w = 2\alpha_{em}^2 \int_0^\infty dt t^2 K(m_\mu t) \Theta^w(t) V(t) \quad w = \{SD, W, LD\}$$



$$a_\mu^{\text{HVP}} = a_\mu^{\text{SD}} + a_\mu^{\text{W}} + a_\mu^{\text{LD}}$$

$$\Theta^{\text{SD}}(t) \equiv 1 - \frac{1}{1 + e^{-2(t-t_0)/\Delta}},$$

$$\Theta^{\text{W}}(t) \equiv \frac{1}{1 + e^{-2(t-t_0)/\Delta}} - \frac{1}{1 + e^{-2(t-t_1)/\Delta}}$$

$$\Theta^{\text{LD}}(t) \equiv \frac{1}{1 + e^{-2(t-t_1)/\Delta}}$$

$$t_0 = 0.4 \text{ fm} \quad t_1 = 1 \text{ fm} \quad \Delta = 0.15 \text{ fm}$$

# Disconnected contributions



Quark loops are calculated with stochastic approaches

$$L(t, \Gamma) = \sum_{\vec{x}} \text{Tr}(D^{-1}(x; x)\Gamma)$$

- Hutchinson trace estimator:  $\frac{1}{N_r} \sum_r |\xi_r\rangle \langle \xi_r| = \mathbb{1} + \mathcal{O}\left(\frac{1}{\sqrt{N_r}}\right)$  and  $\frac{1}{N_r} \sum_r |\xi_r\rangle = 0$
- Exact deflation of low-modes [A. S. Gambhir et al. (2017), 1603.05988]
- Hierarchical probing [A. Stathopoulos et al. (2013), 1302.4018]
- Spin-color dilution [W. Wilcox, (1999), hep-lat/9911013]
- One-end trick [C. McNeile and C. Michael, (2006), hep-lat/0603007.]

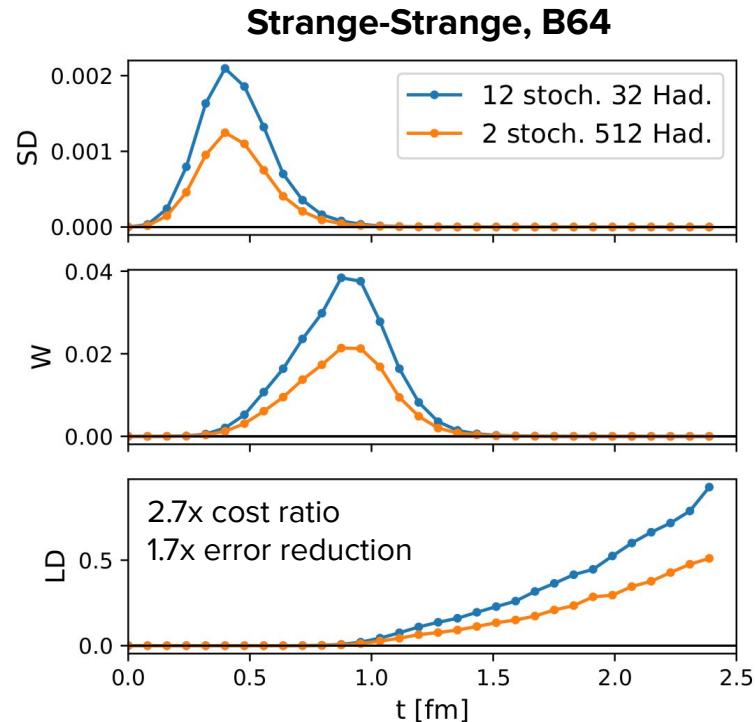
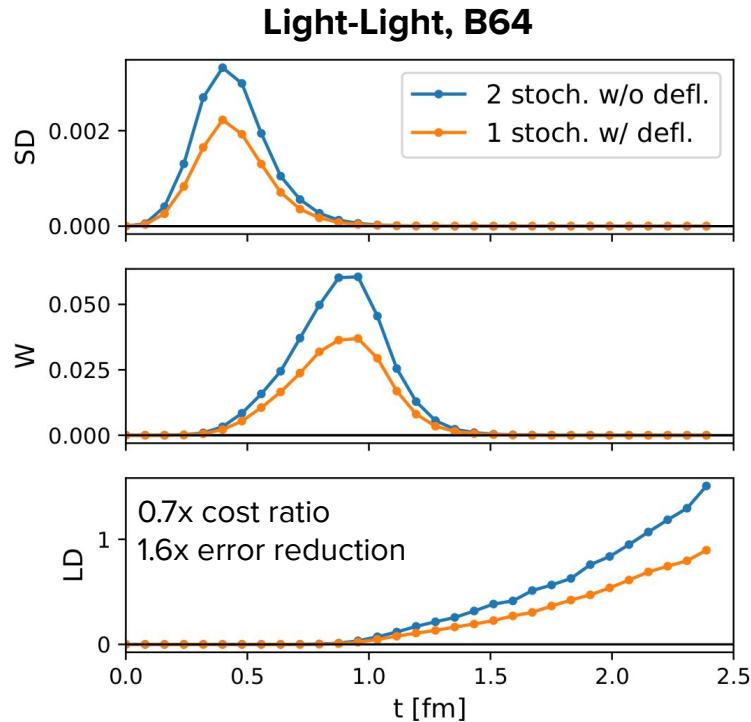
$cB211.072.64$  ( $\times 750$  confs)

Flavor	$N_{\text{def}}$	$N_r$	$N_{\text{Had}}$	$N_{\text{sc}}$	$N_{\text{inv}}$
light	200	1	512	12	6144
strange	0	1	512	12	6144
charm	0	12	32	12	4608

# Noise reduction techniques



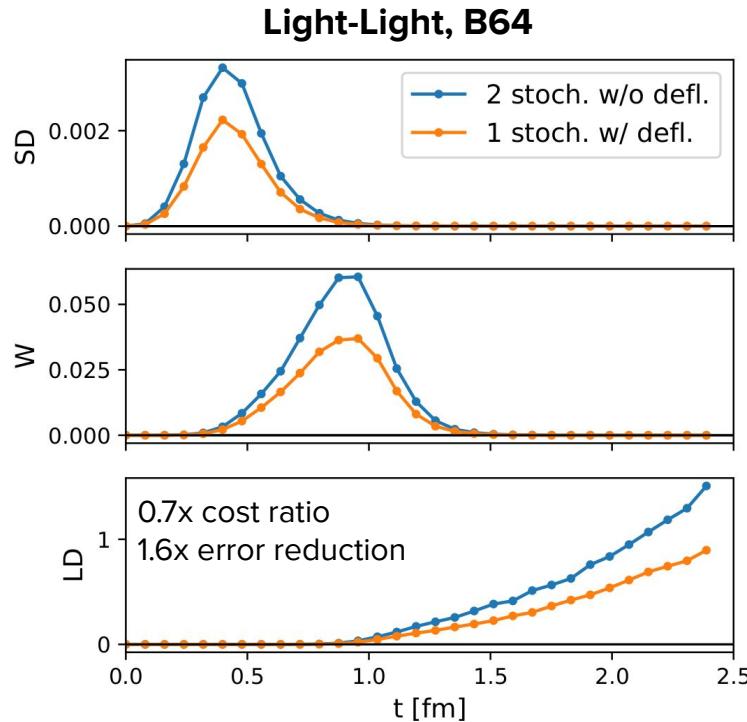
Integrand errors using different approaches



# Noise reduction techniques



Integrand errors using different approaches



- B64: 1 stoch. 200 defl. evs.
- C80: 1 stoch. 450 defl. evs.
- D96: 8 stoch. no deflation



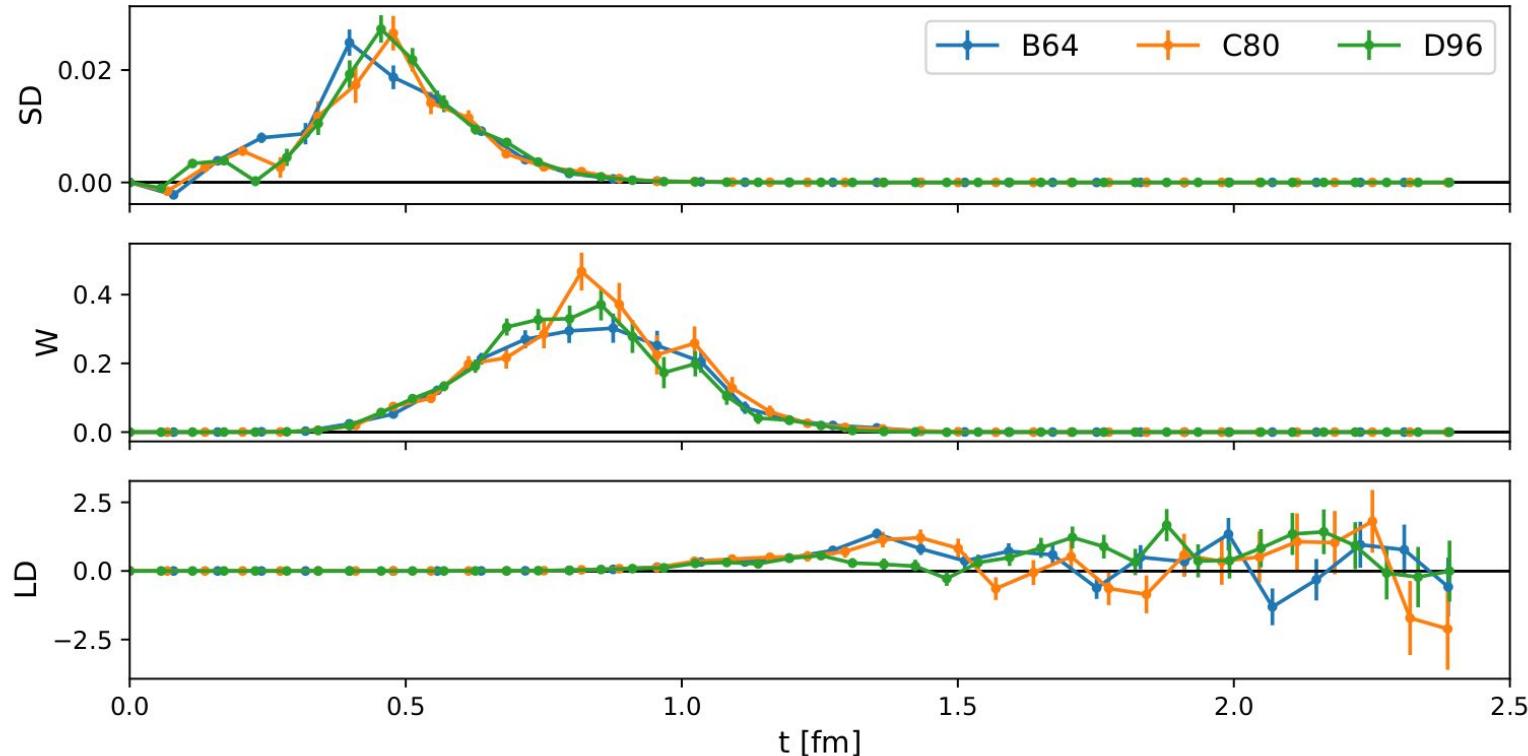
Number of evs. has to grow with the volume

- Cost of deflation grows with  $V^2$
- Memory requirements grows with  $V^2$
- **Not convenient on large lattices!**

# Bare integrand



Light - Light  
contribution



LD difficult to  
extract  
reliably

# Disconnected flavor decomposition



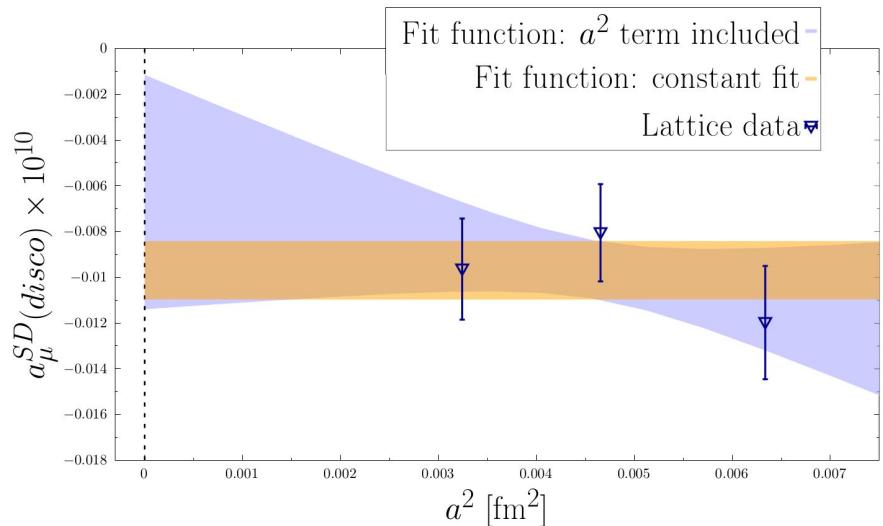
*Short distance ( $\times 10^{-12}$ )*

Ensemble	$\ell\ell$	$ss$	$cc$	$\ell s$	$\ell c$	$sc$	total
cB211.072.64	-3.36 (11)	-2.090 (58)	-1.18 (18)	+5.29 (12)	-1.52 (23)	+1.67 (17)	-1.20 (25)
cC211.060.80	-3.36 (14)	-2.090 (69)	-0.78 (13)	+5.53 (19)	-1.48 (24)	+1.37 (18)	-0.80 (21)
cD211.054.96	-3.54 (16)	-2.084 (71)	-0.71 (14)	+5.60 (19)	-1.50 (22)	+1.27 (17)	-0.96 (22)

*Intermediate window ( $\times 10^{-10}$ )*

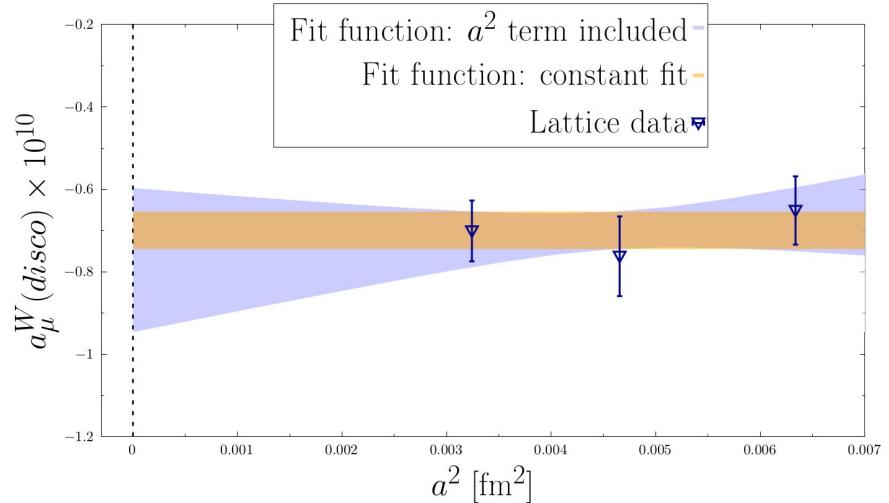
Ensemble	$\ell\ell$	$ss$	$cc$	$\ell s$	$\ell c$	$sc$	total
cB211.072.64	-1.087 (48)	-0.149 (20)	-0.030 (57)	+0.635 (48)	+0.00 (7)	-0.02 (5)	-0.651 (83)
cC211.060.80	-1.300 (63)	-0.159 (27)	-0.033 (46)	+0.726 (73)	-0.03 (9)	+0.04 (6)	-0.762 (97)
cD211.054.96	-1.201 (52)	-0.149 (28)	+0.018 (56)	+0.627 (70)	+0.02 (9)	-0.02 (7)	-0.701 (74)

# Continuum limit



$$a_\mu^{SD} (disc.) = -0.006 (5) \cdot 10^{-10}$$

0% contribution to short distance



$$a_\mu^W (disc.) = -0.77 (17) \cdot 10^{-10}$$

0.3% contribution to intermediate window

# Conclusions



Disconnected contributions are significant at the percent level

- Continuum limit directly at the physical point
- Small cut-off effects for disconnected contributions (not true for connected!)
- High systematic effects when integrating at long distance!

Ref.	$a_\mu^W (disc.)$
this work	- 0.77 (0.17)
BMW [1]	- 0.85 (0.06)
CLS/Mainz [2]	- 0.81 (0.09)
average	- 0.83 (0.05)

- [1] S. Borsanyi et al., 2021, [2002.12347](#)  
[2] M. Cè et al., 2022, [2206.06582](#)

# To be continued...

Lattice calculation of the short and intermediate time-distance hadronic vacuum polarization contributions to the muon magnetic moment using twisted-mass fermions

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Thank you for your  
attention!

Follows part 2...  
(Talk by G. Gagliardi)

# Scale setting



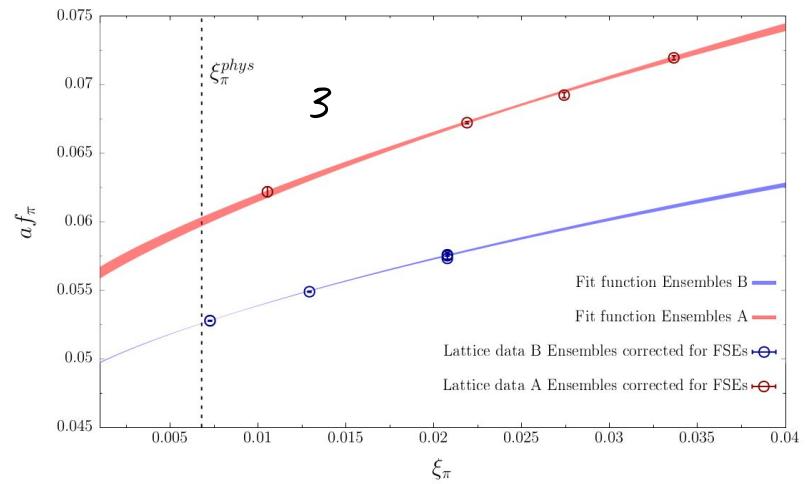
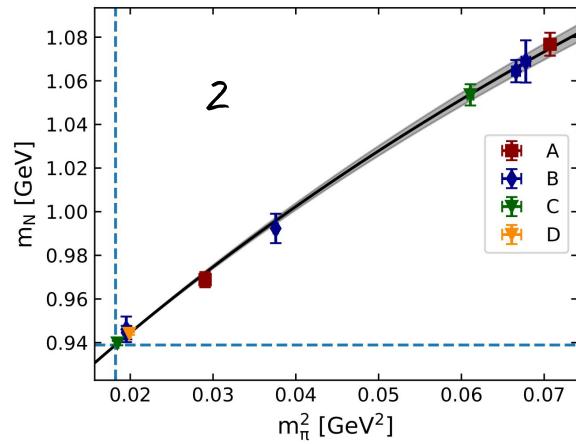
1

	A	B	C	D
Using $w_0$	0.09471 (39)	0.08161 (30)	0.06942 (26)	0.05770 (20)
Nucleon ph.	0.09300(46)	0.07984(29)	0.06862(20)	0.05686(27)
Pion decay ph.	0.09080 (53)	0.07961 (13)	0.06821 (12)	0.05692 (10)

[C. Alexandrou et al, 2021, arXiv:2104.13408]

2

3



# Scale setting



	A	B	C	D
1 Using $w_0$	0.09471 (39)	0.08161 (30)	0.06942 (26)	0.05770 (20)
2 Nucleon ph.	0.09300(46)	0.07984(29)	0.06862(20)	0.05686(27)
3 Pion decay ph.	0.09080 (53)	0.07961 (13)	0.06821 (12)	0.05692 (10)

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