Improving Lattice QCD Calculation of the Collins-Soper Kernel

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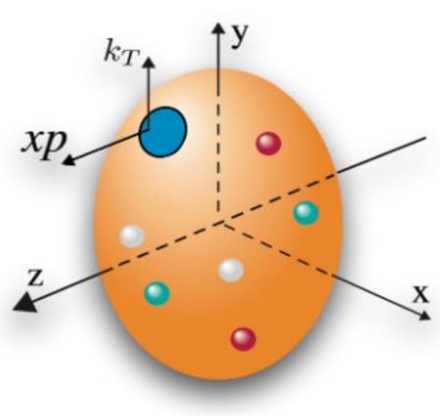
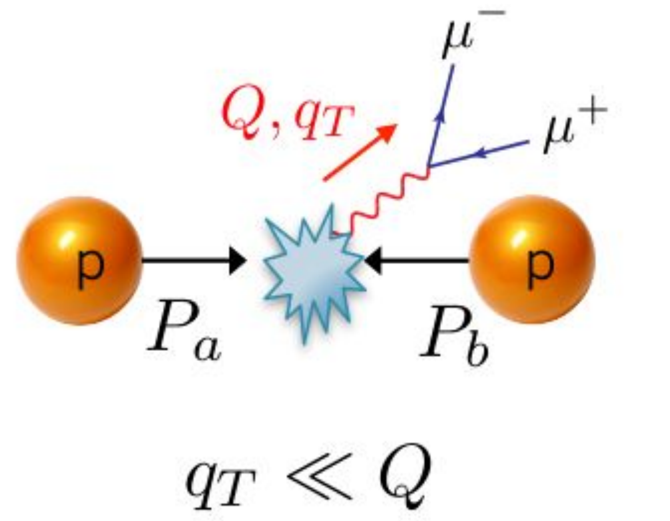
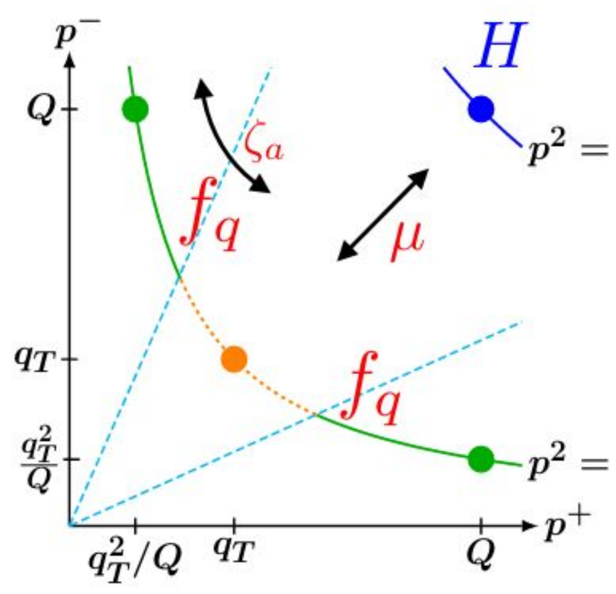
The Collins-Soper (CS) Kernel

Transverse motion of partons in hadrons gives rise to Transverse Momentum-dependent Distributions (TMDs).



TMDs appear in factorized cross-sections of processes sensitive to transverse momenta of partons – e.g. in Drell-Yan:





Figures from Iain Stewart's plenary talk at Lattice 2021 [modified for clarity]

Renormalization of rapidity divergences in TMDs leads to the Collins-Soper scale ζ and the evolution equation with a **Collins-Soper (CS) Kernel:**

Fourier conjugate to parton's transverse momentum.

$$\zeta \frac{\mathrm{d}}{\mathrm{d}\zeta} \ln f_{i/h}(x, \mathbf{b}_\mathrm{T}; \mu, \zeta) = \frac{1}{2}\gamma_\zeta^i(\mu, b_\mathrm{T}).$$

Fourier-transformed TMD for parton *i* in hadron *h*.

Collins-Soper (CS) Kernel (independent of external state *h*).

Even if μ is perturbative, $\gamma_\zeta^i(\mu, b_\mathrm{T})$ is non-perturbative at large b_T.

Non-perturbative modeling in global fits is significant for $b_\mathrm{T} \gtrsim 0.2$ fm.

Non-perturbative CS kernel is a possible target for lattice QCD.

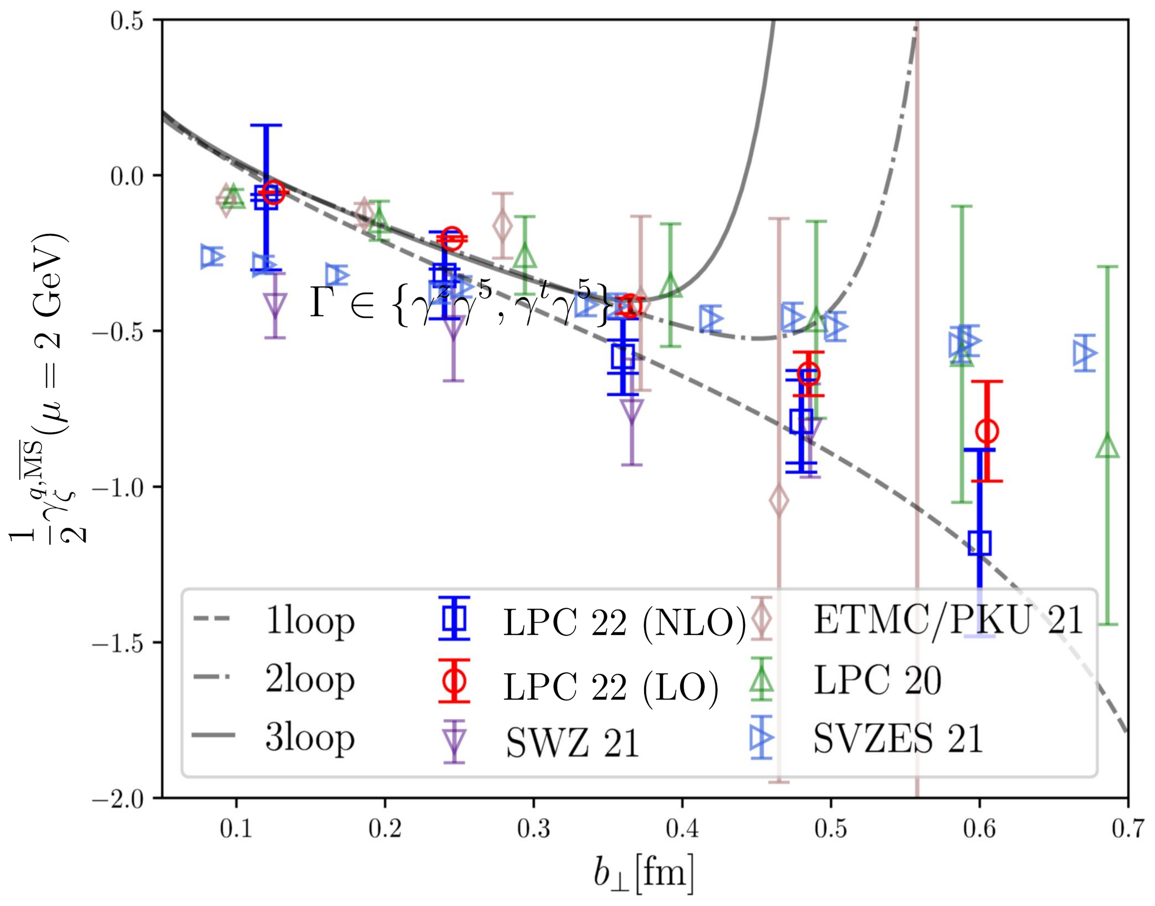
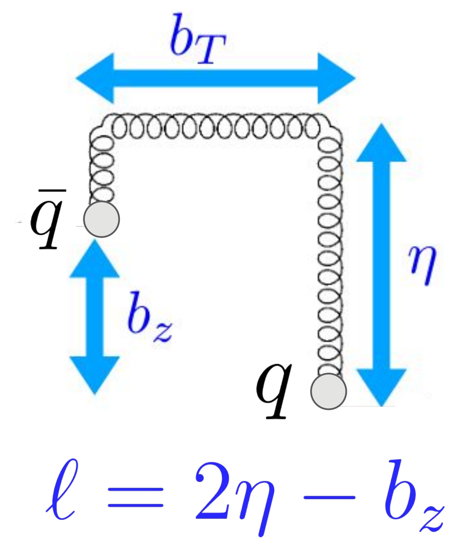


Figure from LPC 22, 2204.00200 [axes and legend labels modified]

The CS Kernel with Lattice QCD

Using Large-Momentum Effective Theory (LaMET), the CS kernel may be computed with Lattice QCD via space-like matrix elements of **staple-shaped operators**:

$$\mathcal{O}_\Gamma(b^\mu, z^\mu, \ell) = \bar{q}(z^\mu + b^\mu)\frac{\Gamma}{2}\widetilde{W}(\ell; b^\mu; z^\mu)q(z^\mu)$$



One possible observable is a **quasi-TMD wavefunction (WF)** with $\Gamma \in \{\gamma^z\gamma^5, \gamma^t\gamma^5\}$ from two-point correlation functions:

$$\tilde{\psi}(b^z, \mathbf{b}_\mathrm{T}, \ell, P^z) \propto \langle 0|\mathcal{O}_\Gamma(b^\mu, -P^z, \ell)|\pi(P^z)\rangle.$$

With bare matrix elements computed, the ratio yields the CS kernel after:

1. Renormalization;
2. $\ell \to \infty$ extrapolation & asymmetry corrections;
3. Fourier Transform;
4. Perturbative Matching.

$$\gamma_\zeta^q(\mu, b_\mathrm{T}) = \frac{1}{\ln(P_1^z/P_2^z)} \ln \left[\frac{C_q^{\mathrm{TMD\ WF}}(\mu, x, P_2^z)}{C_q^{\mathrm{TMD\ WF}}(\mu, x, P_1^z)} \times \frac{\int \mathrm{d}b^z e^{ib^z x P_1^z} P_1^z \lim_{\ell\to\infty} \tilde{\psi}_\pi^{\mathrm{ren.}}(\mu, b^z, \mathbf{b}_\mathrm{T}, \ell, P_1^z)}{\int \mathrm{d}b^z e^{ib^z x P_2^z} P_2^z \lim_{\ell\to\infty} \tilde{\psi}_\pi^{\mathrm{ren.}}(\mu, b^z, \mathbf{b}_\mathrm{T}, \ell, P_2^z)} \right] \boxed{+\mathcal{O}\left(\frac{1}{(xP^z b_\mathrm{T})^2}, \frac{m_\pi^2}{(xP^z)^2}\right)}$$

LQCD calculations are broadly consistent – but different treatments of matching, Fourier transform and renormalization lead to significant systematic effects, and power corrections can be sensitive to valence quark masses.

Observable	Collaboration	LQCD Setup	Matching	Fourier Transform	Operator Mixing
Beam Functions	**SWZ 20** PRD 102 (2020)	$\frac{(m_\pi^{\mathrm{val}})^2}{(P_{\max}^z)^2} = 0.219$ quenched	LO	**Yes**	✔
	SWZ 21 PRD 104 (2021)	$\frac{(m_\pi^{\mathrm{val}})^2}{(P_{\max}^z)^2} = 0.129$	**NLO**	**Yes**	✔
TMD WFs	**LPC 20** PRL 125 (2020)	$\frac{(m_\pi^{\mathrm{val}})^2}{(P_{\max}^z)^2} = 0.067$	LO	N/A	✔
	ETMC/PKU 21 PRL 128 (2022)	$\frac{(m_\pi^{\mathrm{val}})^2}{(P_{\max}^z)^2} = 0.063$	LO	N/A	✗
	LPC 22 2204.00200	$\frac{(m_\pi^{\mathrm{val}})^2}{(P_{\max}^z)^2} = 0.067$	**NLO**	**Yes**	✗
	This work (in progress)	$\frac{(m_\pi^{\mathrm{val}})^2}{(P_{\max}^z)^2} = 0.007$	**NLO**	**Yes**	✔
Mellin Moments	**SVZES 21** JHEP 08 (2021)	$\frac{(m_\pi^{\mathrm{val}})^2}{(P_{\max}^+)^2} = 0.035$	**NLO**	N/A	✗

Projected Improvements

Physical Pion Mass and Reduced Systematics from the Fourier Transform

Recent formal and code developments increase computational efficiency, which enables calculations at ~ physical m_π^{val} to remove partial quenching and suppress power corrections.

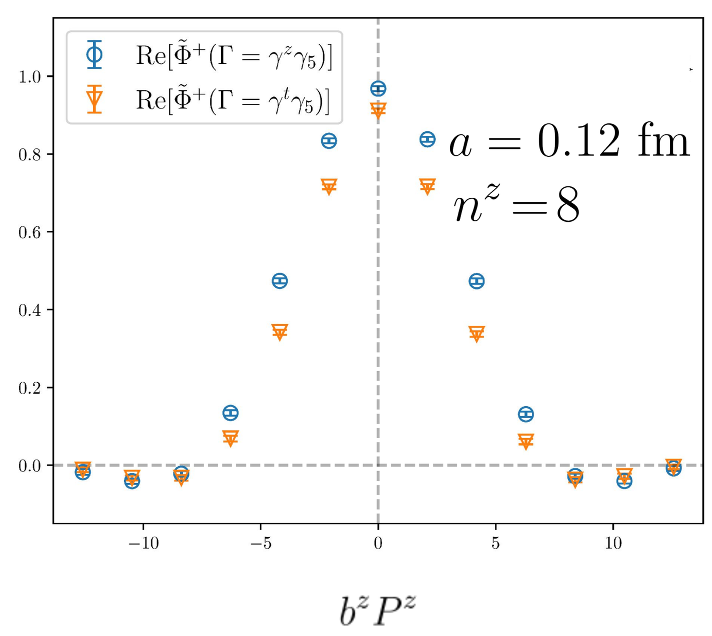
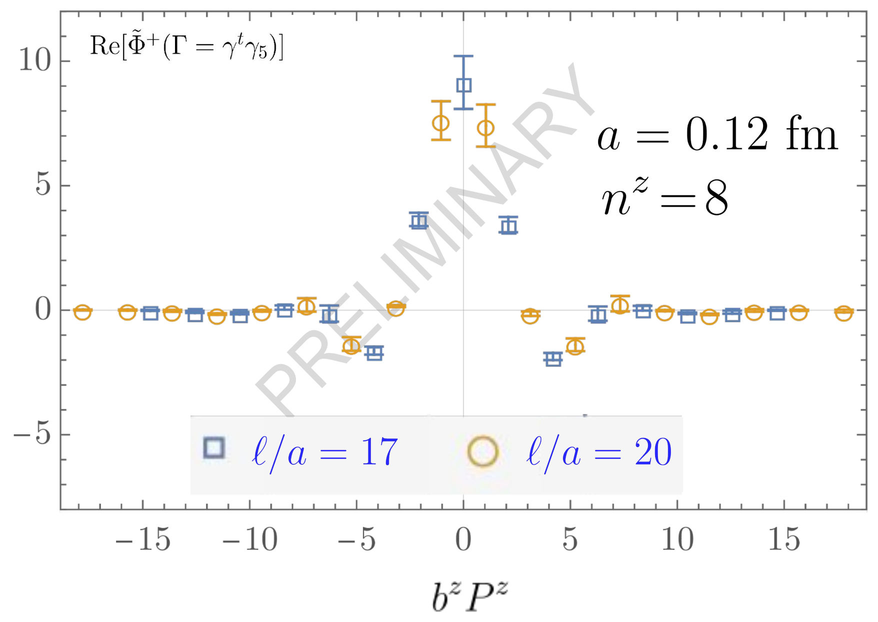


Figure from LPC 22, 2204.00200 [modified for clarity]



Preliminary figure from this work (different normalization)

Greater computational efficiency also allows to reduce systematic uncertainties from the Fourier transform by increasing data frequency and range in $|P^z b^z|$ (right panel) – which requires a greater number of and longer **staple configurations**, respectively.

Robust Non-Local Operator Renormalization

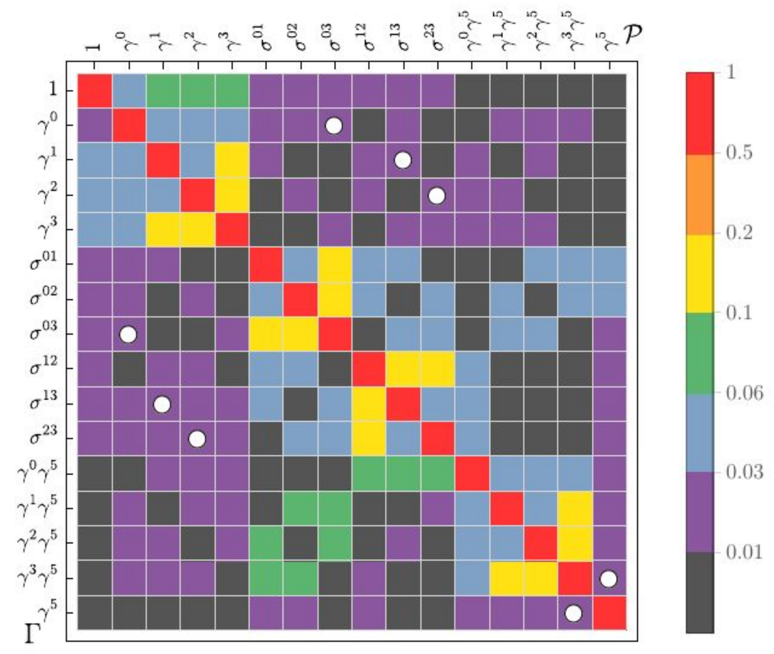
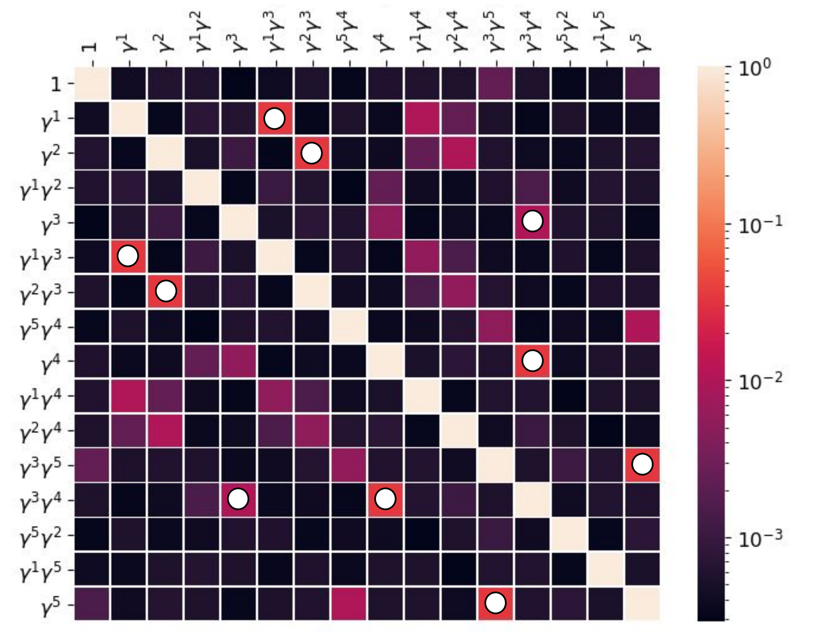


Figure from Shanahan, Wagman, and Zhao, PRD 101 (2020) quenched



Preliminary figure from this work (different ensemble and renormalization scale)

With the auxiliary-field approach, the renormalization of extended **staple-shaped operators** is simplified to that of point-like objects. In the RI$\underline{\text{x}}$-MOM scheme[1] (right panel, log scale), compared to the usual RI'-MOM scheme (left panel, linear scale), mixing effects are both reduced and in better agreement with one-loop lattice perturbation theory[2] (white circles).

[1] Green, Jansen, and Steffens, PRL 121 (2018) and PRD 101(2020).
[2] Constantinou, Panagopoulos, and Spanoudes, PRD 99 (2019) and PRD 96 (2017).