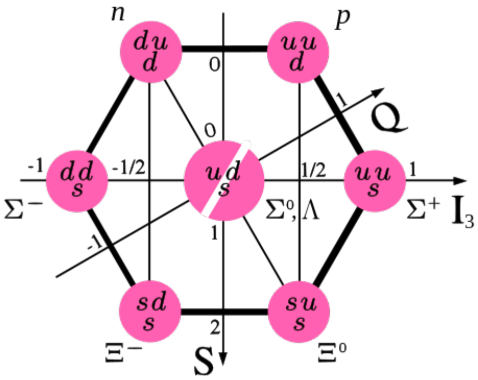
Sigma terms of the baryon octet in $N_\mathrm{f} = 2 + 1$ QCD with Wilson quarks

How do the other octet baryons compare to the nucleon
and do we control excited states sufficiently ?

Pia Leonie Jones Petrak
Gunnar Bali, Sara Collins, Jochen Heitger, Daniel Jenkins, Simon Weishäupl

Why determine the sigma terms?

- **decomposition of the hadron mass**
- investigate **flavour symmetry breaking** in the **baryon octet**
  $\Rightarrow$ nucleon N, lambda Λ, sigma Σ and xi Ξ (in our setup $m_{\mathrm{u}} = m_{\mathrm{d}}$)
- **WIMP-nucleon scattering cross-sections** (e.g. XENON1T)
- **discrepancies** between results for the **nucleon pion sigma term from LQCD and phenomenology** still to be resolved



How are the sigma terms defined?

$$\boxed{\sigma_{qB} = m_q \langle B | J | B \rangle}$$

- with the quark mass m_q and a current J
- In the matrix element B refers to the ground state of a baryon B.

We're interested in:

- baryon at rest
- the **scalar current** $J = \bar{q}\,\mathbf{1}\,q,\ \ q \in \{u, d, s\}$
- **strange sigma terms** σ_{sB}
- **pion sigma terms** $\sigma_{\pi B} = \sigma_{uB} + \sigma_{dB}$

How are the sigma terms defined?

$$\boxed{\sigma_{qB} = m_q \langle B | J | B \rangle}$$

- with the quark mass m_q and a current J
- In the matrix element B refers to the ground state of a baryon B.

We're interested in:

- baryon at rest
- the **scalar current** $J = \bar{q}\,\mathbf{1}\,q,\ \ q \in \{u, d, s\}$
- **strange sigma terms** σ_{sB}
- **pion sigma terms** $\sigma_{\pi B} = \sigma_{uB} + \sigma_{dB}$

- **renormalisation** via normalisation factor $\mathbf{r_m}$ (determined by ALPHA [2101.10969], RQCD), the ratio of flavour non-singlet and singlet scalar density renormalisation parameters
  $\rightarrow$ accounts for the **mixing of quark flavours under renormalisation** for Wilson fermions

How to access the matrix element

$\rightarrow$ **spectral decompositions**

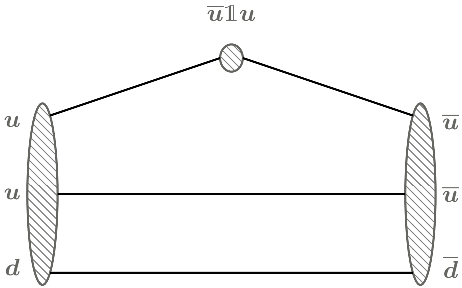
$$C_{2\text{pt}}(t_\text{f}) = \sum_{\vec{x}} \left\langle \mathcal{O}_\text{snk}(\vec{x}, t_\text{f}) \bar{\mathcal{O}}_\text{src}(\vec{0}, 0) \right\rangle = \sum_n |Z_n|^2 e^{-E_n t_\text{f}}$$

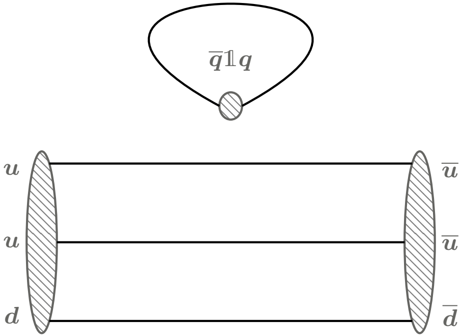
where $Z_n = \langle \Omega | \mathcal{O}_\text{snk} | n \rangle$ (vacuum state Ω) is the overlap of the interpolator $\mathcal{O}_\text{snk}$ onto the state n

$$\begin{aligned} C_{3\text{pt}}(t_\text{f}, t) &= \sum_{\vec{x},\vec{y}} \left\langle \mathcal{O}_\text{snk}(\vec{x}, t_\text{f}) J(\vec{y}, t) \bar{\mathcal{O}}_\text{src}(\vec{0}, 0) \right\rangle - \sum_{\vec{x},\vec{y}} \langle J(\vec{y}, t) \rangle \left\langle \mathcal{O}_\text{snk}(\vec{x}, t_\text{f}) \bar{\mathcal{O}}_\text{src}(\vec{0}, 0) \right\rangle \\ &= \sum_{n,n'} Z_{n'} Z_n^* \langle \mathbf{n}' | \mathbf{J} | \mathbf{n} \rangle e^{-E_n t} e^{-E_{n'}(t_\text{f} - t)} \end{aligned}$$

t_f is the source-sink separation & t is the insertion time of the current

Connected and disconnected contributions





How to access the scalar matrix element

ratio method

Combining the two spectral decompositions leads to the **ratio**

$$R(t_\mathrm{f}, t) = \frac{C_{3\mathrm{pt}}(t_\mathrm{f}, t)}{C_{2\mathrm{pt}}(t_\mathrm{f})} = g_S^q + c_{01}\mathrm{e}^{-\Delta \cdot t} + c_{10}\mathrm{e}^{-\Delta \cdot (t_\mathrm{f} - t)} + c_{11}\mathrm{e}^{-\Delta \cdot t_\mathrm{f}} + \ldots$$

where $g_S^q = \langle B|J|B\rangle = \langle B|\bar{q}\,\mathbf{1}\,q|B\rangle$ is the ground-state matrix element of interest.

- energy gap between the ground state and the first excited state, $\Delta = E_1 - E_0$
- baryon at rest, $\boldsymbol{c_{01} = c_{10} \equiv c_{0\leftrightarrow 1}}$ holds in this case, **cannot resolve $\mathbf{c_{11}}$** so far
- c_{01}, c_{10}, c_{11} made up of matrix elements of different transitions such as $N_1 \to N$, $N \to N_1$ and $N_1 \to N_1$ for the nucleon
- N_1 can be a multi-particle state

How to access the scalar matrix element

ratio method

Combining the two spectral decompositions leads to the **ratio**

$$R(t_\mathrm{f}, t) = \frac{C_{3\mathrm{pt}}(t_\mathrm{f}, t)}{C_{2\mathrm{pt}}(t_\mathrm{f})} = g_S^q + c_{0\leftrightarrow 1}\left(\mathrm{e}^{-\Delta\cdot t} + \mathrm{e}^{-\Delta\cdot(t_\mathrm{f}-t)}\right) + \cancel{c_{11}}\,\mathrm{e}^{-\Delta\cdot t_\mathrm{f}} + \ldots$$

where $g_S^q = \langle B|J|B\rangle = \langle B|\bar{q}\,\mathbf{1}\,q|B\rangle$ is the ground-state matrix element of interest.

- energy gap between the ground state and the first excited state, $\Delta = E_1 - E_0$
- baryon at rest, $c_{01} = c_{10} \equiv c_{0\leftrightarrow 1}$ holds in this case, **cannot resolve $\mathbf{c}_{11}$** so far
- c_{01}, c_{10}, c_{11} made up of matrix elements of different transitions such as $N_1 \to N_0$, $N_0 \to N_1$ and $N_1 \to N_1$ for the nucleon
- N_1 can be a multi-particle state

How to access the scalar matrix element

summation method

sum over insertion times t

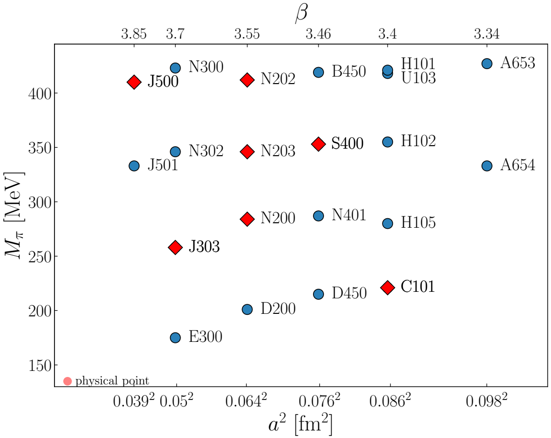
$$\sum_{t=c}^{t_\mathrm{f}-c} R(t_\mathrm{f}, t) = g_S^q (t_\mathrm{f} - 2c + 1) + \frac{2c_{0\leftrightarrow 1}}{1 - \mathrm{e}^{\Delta}} \left(\mathrm{e}^{\Delta(c-t_\mathrm{f})} - \mathrm{e}^{\Delta(1-c)}\right) + c_{11}(t_\mathrm{f} - 2c + 1)\mathrm{e}^{-\Delta t_\mathrm{f}} + \dots$$

- BUT: only have access to a large number of insertion times for R^{dis}
- Summed ratio is approximately linear for large source-sink separations:

$$\sum_{t=c}^{t_\mathrm{f}-c} R(t, t_\mathrm{f}) \to g_S^q (t_\mathrm{f} - 2c + 1) + \mathrm{constant}$$

- $c > 0$ to preserve reflection positivity, we set c to 2

Numerical setup



- *CLS* gauge field ensembles employing the Lüscher-Weisz gluon action and the Sheikholeslami-Wohlert fermion action with $N_\mathrm{f} = 2 + 1$
- $\mathrm{Tr} M = \mathrm{const}$
- five different lattice spacings
- High statistics: error estimation in the analysis via the Γ-method

[Wolff: arXiv:hep-lat/0306017]

[Ramos: arXiv:1809.01289]

Numerical setup

Connected three-point functions

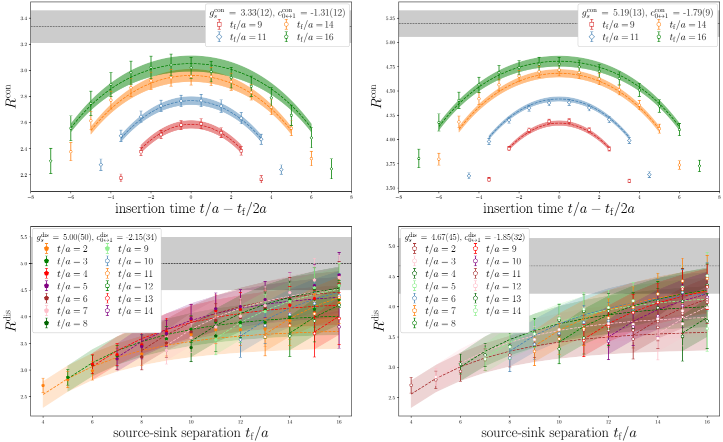
- $m_l = m_s$ ensembles, the **standard sequential source method**,
  e.g. one measurement at $t_\mathrm{f}/a = 11$, two at $t_\mathrm{f}/a = [14, 16]$ and four at $t_\mathrm{f}/a = 19$
- **stochastic method estimating a timeslice-to-all propagator** [G. Bali et. al.: arXiv:1711.02384]
  $\rightarrow$ enables us to obtain measurements for all baryons of interest as multiple source and insertion positions can be estimated simultaneously
- four different source-sink separations typically corresponding to
  $t_\mathrm{f} \approx [0.7\,\mathrm{fm}, 0.9\,\mathrm{fm}, 1\,\mathrm{fm}, 1.2\,\mathrm{fm}]$
- two measurements (forward and backward direction) for each t_f on every configuration

Numerical setup

Disconnected three-point functions

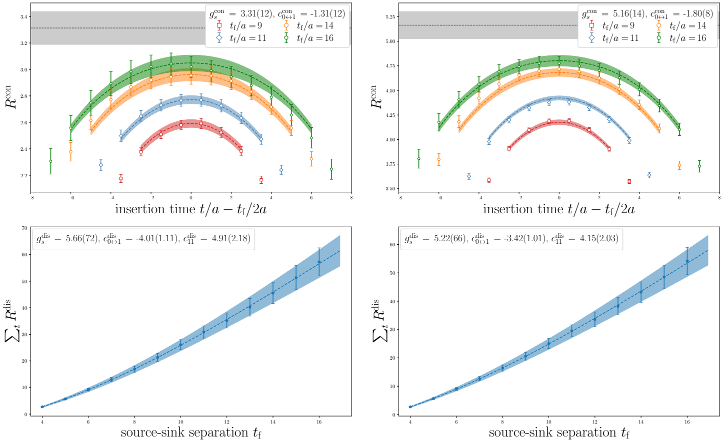
- correlate a quark loop with a baryon two-point function
- **stochastic estimation of loop**, to reduce the additional noise:
  - the truncated solver method [G. Bali et. al.: arXiv:0910.3970]
  - the hopping parameter expansion technique [C. Thron et. al.: arXiv:hep-lat/9707001]
  - time partitioning [S. Bernardson et. al.: Comput. Phys. Commun. 78, 1993]
- $\mathbf{20}$ different spatial source positions on every configuration of the two-point function (different for $m_l = m_s$ ensembles e.g. N202: 26, J500: 27)
- A reasonable signal is obtained for t_f up to around $\mathbf{1.22\,fm}$.

Simultaneous fits to connected & disconnected ratios



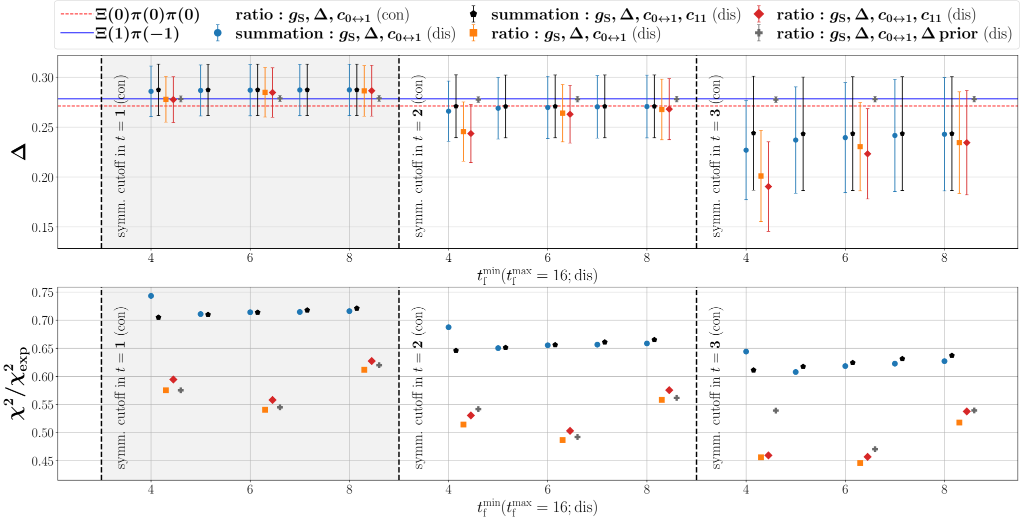
- **Ξ baryon** at $a = 0.076\,\mathrm{fm}$
- $m_\pi = 352\,\mathrm{MeV}$ (S400)
- **simultaneous fit:**
  $\rightarrow \chi^2/\chi^2_{\mathrm{exp}} \approx 0.6$
  $\rightarrow \Delta \approx 720\,\mathrm{MeV}$
- **top:**
  $\rightarrow \bar{u}u$ current (left)
  $\rightarrow \bar{s}s$ current (right)
- **bottom:**
  $\rightarrow \bar{l}l$ current (left)
  $\rightarrow \bar{s}s$ current (right)

Simultaneous fits to connected & summed disconnected ratios

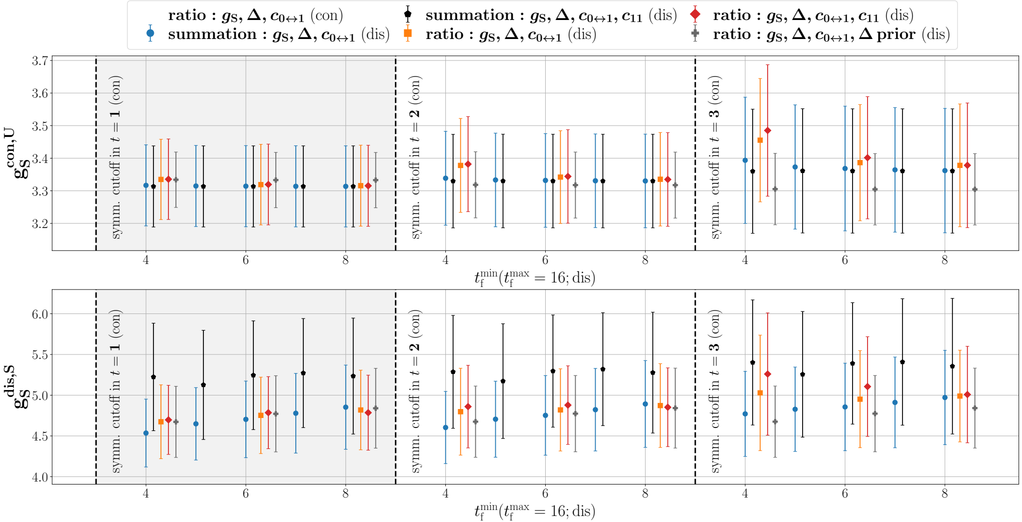


- **Ξ baryon** at $a = 0.076\,\mathrm{fm}$
- $m_\pi = 352\,\mathrm{MeV}$ (S400)
- **simultaneous fit:**
  $\rightarrow \chi^2/\chi^2_{\mathrm{exp}} \approx 0.7$
  $\rightarrow \Delta \approx 745\,\mathrm{MeV}$
- **top:**
  $\rightarrow \bar{u}u$ current (left)
  $\rightarrow \bar{s}s$ current (right)
- **bottom:**
  $\rightarrow \bar{l}l$ current (left)
  $\rightarrow \bar{s}s$ current (right)

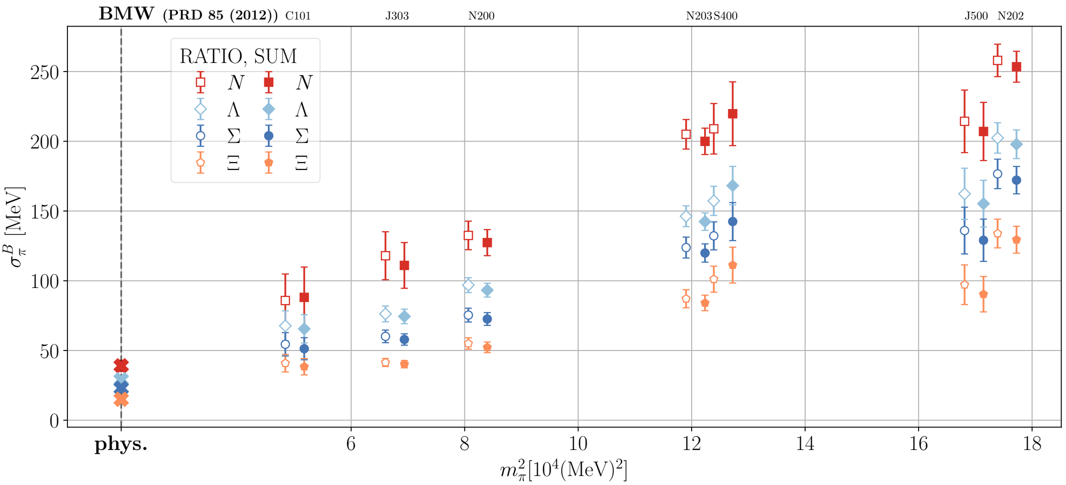
Fit form and fit range variation - the Ξ baryon $(a = 0.076\,\mathrm{fm},\ m_\pi = 352\,\mathrm{MeV})$



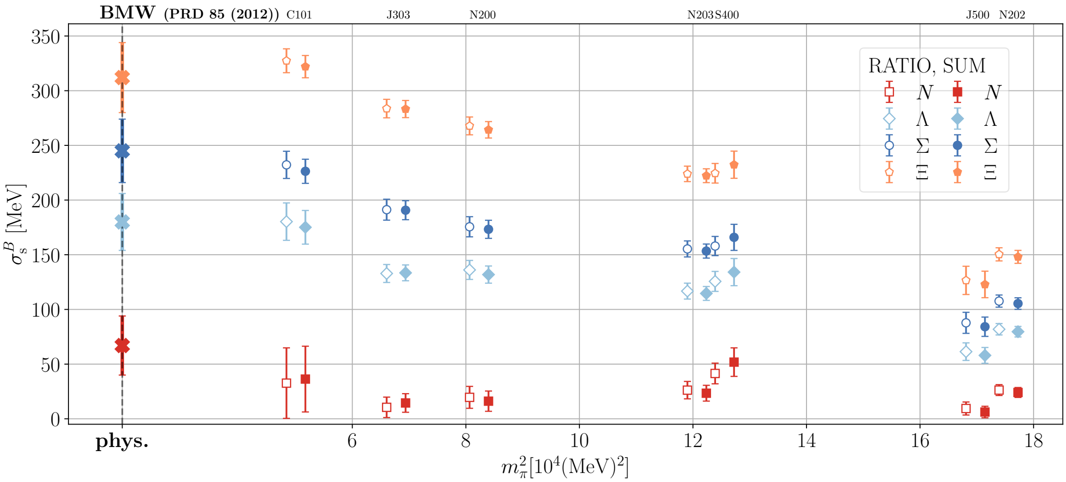
Fit form and fit range variation - the Ξ baryon ($a = 0.076\,\mathrm{fm}$, $m_\pi = 352\,\mathrm{MeV}$)



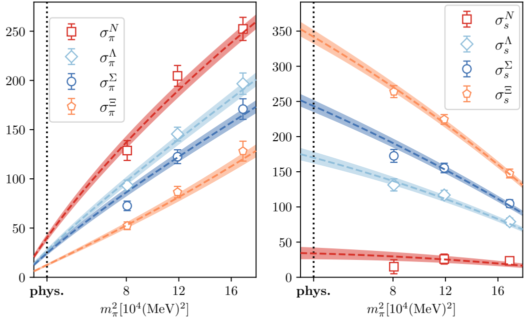
Preliminary results



Preliminary results



Preliminary chiral extrapolation at $\beta = 3.55$

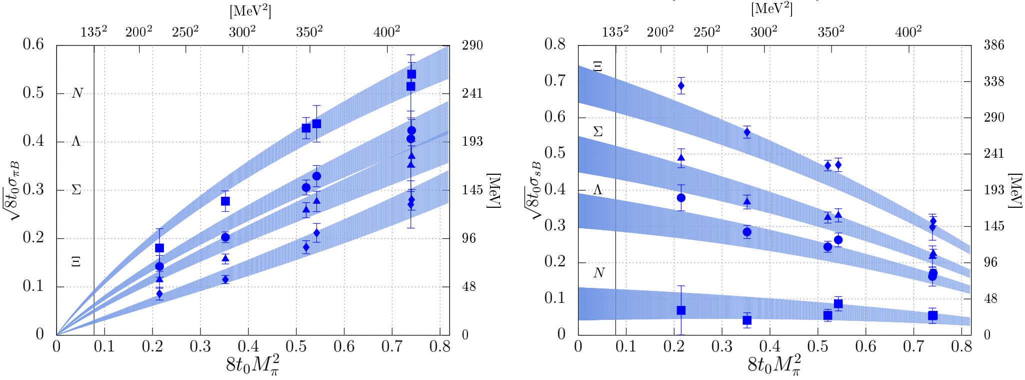


NNLO BChPT simultaneous fit to all baryons $\to \chi^2/\mathrm{d.o.f} = 1.29$ [PLJP et. al.: arXiv: 2112.00586]

- **fixed** $F = 0.446(7)$, $D = 0.731(12)$ & $m_0 = 729(42)\,\mathrm{MeV}$ from a preliminary analysis of the nucleon mass and the axial charges in the chiral limit
- **fit results for LECs:** $\bar{b} = 0.00317(29)$, $b_F = -0.000335(27)$, $b_D = 0.0000493(21)$
- $F_0 = 119.9(9.8)\,\mathrm{MeV}$ **different** from $F_0 = 71(2)\,\mathrm{MeV}$, the preliminary value from a combined fit to the pion decay constant and the pion mass

Consistency with indirect determinations

- quark mass dependence of $\sigma_{\pi B}$ and σ_{sB} via the Feynman-Hellmann theorem and a NNLO BChPT, FV and continuum limit fit of 47 ensembles (error bands)
  RQCD, see talk by S. Collins and poster of W. Söldner
- preliminary direct determinations from the ratio method (data points)



Summary and outlook

- progress in the determination of the pion-baryon and strange sigma terms for the octet baryons ✓
- used variations of summation and ratio methods to cross-check whether we **control excited state contributions** sufficiently also including priors

to do:

- investigate discrepancies between energy gap and $B(1)\pi(-1)$ and/or $B(0)\pi(0)\pi(0)$ via correlated fits, other fit forms
- **additional ensembles**
- **chiral extrapolation** to the physical pion mass and an investigation of **cut-off and finite-volume effects**

Chiral extrapolation

From **Baryon Chiral Perturbation Theory** (BChPT) we can derive the pion mass dependence expected from SU(3) flavour symmetry ; we apply the **Feynman-Hellmann theorem** that relates sigma terms to derivatives of the baryon mass with respect to quark masses, resulting in

$$\sigma_{\pi B} = M_\pi^2 \left\{ \frac{2}{3}\bar{b} - \delta b_B + \frac{m_0^2}{(4\pi F_0)^2} \left[\frac{g_{B,\pi}}{2M_\pi} f'\left(\frac{M_\pi}{m_0}\right) + \frac{g_{B,K}}{4M_K} f'\left(\frac{M_K}{m_0}\right) + \frac{g_{B,\eta}}{6M_\eta} f'\left(\frac{M_\eta}{m_0}\right) \right] \right\},$$

Chiral extrapolation

From **Baryon Chiral Perturbation Theory** (BChPT) we can derive the pion mass dependence expected from SU(3) flavour symmetry ; we apply the **Feynman-Hellmann theorem** that relates sigma terms to derivatives of the baryon mass with respect to quark masses, resulting in

$$\sigma_{\pi B} = M_\pi^2 \left\{ \frac{2}{3}\bar{b} - \delta b_B + \frac{m_0^2}{(4\pi F_0)^2} \left[\frac{g_{B,\pi}}{2M_\pi} f'\left(\frac{M_\pi}{m_0}\right) + \frac{g_{B,K}}{4M_K} f'\left(\frac{M_K}{m_0}\right) + \frac{g_{B,\eta}}{6M_\eta} f'\left(\frac{M_\eta}{m_0}\right) \right] \right\},$$

$$\sigma_s = \left(2M_K^2 - M_\pi^2\right) \left\{ \frac{1}{3}\bar{b} + \delta b_B + \frac{m_0^2}{(4\pi F_0)^2} \left[\frac{g_{B,K}}{4M_K} f'\left(\frac{M_K}{m_0}\right) + \frac{g_{B,\eta}}{3M_\eta} f'\left(\frac{M_\eta}{m_0}\right) \right] \right\},$$

where m_0 and F_0 are the **octet baryon mass** and **pion decay constant** in the chiral limit.

δb_B is a combination of two of the three BChPT next-to-leading order **(NLO) low energy constants (LECs)** $\boldsymbol{b_D, b_F, \bar{b} = -6b_0 - 4b_D}$ and depends on the baryon,

$$\delta b_N = \frac{2}{3}(3b_F - b_D), \quad \delta b_\Lambda = -\frac{4}{3}b_D, \quad \delta b_\Sigma = \frac{4}{3}b_D, \quad \delta b_\Xi = -\frac{2}{3}(3b_F + b_D). \tag{1}$$

The couplings $g_{B,\pi}, g_{B,K}$ and g_{B,η_8} are made up of different combinations of the leading order **(LO) LECs** $\mathbf{F}$ **and** $\mathbf{D}$ that **also appear in the ChPT expressions** for the **axial charges**.

δb_B is a combination of two of the three BChPT next-to-leading order **(NLO) low energy constants (LECs)** $\boldsymbol{b_D, b_F, \bar{b} = -6b_0 - 4b_D}$ and depends on the baryon,

$$\delta b_N = \frac{2}{3}(3b_F - b_D), \quad \delta b_\Lambda = -\frac{4}{3}b_D, \quad \delta b_\Sigma = \frac{4}{3}b_D, \quad \delta b_\Xi = -\frac{2}{3}(3b_F + b_D). \tag{1}$$

The couplings $g_{B,\pi}, g_{B,K}$ and g_{B,η_8} are made up of different combinations of the leading order **(LO) LECs $\mathbf{F}$ and $\mathbf{D}$** that **also appear in the ChPT expressions** for the **axial charges**.
f' is the derivative of the loop function f that is set to $f(x) = -\pi x^3$ in Heavy BChPT or

$$f(x) = -2x^3 \left[\sqrt{1 - \frac{x^2}{4}} \arccos\left(\frac{x}{2}\right) + \frac{x}{2}\ln(x)\right] \tag{2}$$

in covariant BChPT in the extended on-mass-shell (EOMS) scheme

Preliminary results - pion sigma terms

m_π [MeV]	a [fm]	$\sigma_{\pi N}$ [MeV]	$\sigma_{\pi\Lambda}$ [MeV]	$\sigma_{\pi\Sigma}$ [MeV]	$\sigma_{\pi\Xi}$ [MeV]
411	0.064	258.0(11.6)	202.4(10.9)	176.6(10.5)	133.9(10.3)
410	0.039	214.3(22.4)	162.3(18.3)	136.0(16.7)	97.2(14.2)
352	0.076	209.0(18.1)	157.3(10.5)	132.2(10.1)	101.1(9.3)
345	0.064	205.0(10.6)	146.3(7.5)	123.8(7.5)	87.1(6.5)
284	0.064	132.5(10.2)	96.9(5.3)	75.4(4.9)	55.0(4.1)
220	0.086	85.8(19.0)	67.7(10.7)	54.5(8.4)	40.9(6.3)

Preliminary results - strange sigma terms

m_π [MeV]	a [fm]	σ_{sN} [MeV]	$\sigma_{s\Lambda}$ [MeV]	$\sigma_{s\Sigma}$ [MeV]	$\sigma_{s\Xi}$ [MeV]
411	0.064	26.3(5.0)	81.9(5.2)	107.6(5.6)	150.4(6.0)
410	0.039	9.4(6.0)	61.4(8.0)	87.8(9.6)	126.6(12.9)
352	0.076	41.4(9.5)	125.6(9.1)	158.0(8.8)	224.5(9.0)
345	0.064	26.2(8.0)	116.8(7.3)	155.3(7.3)	223.9(7.1)
284	0.064	19.6(10.0)	136.1(8.7)	175.6(9.2)	267.8(8.1)
220	0.086	32.6(32.2)	180.3(17.1)	232.2(12.4)	327.4(10.9)