Lattice calculation of Collins-Soper Kernel and Soft function

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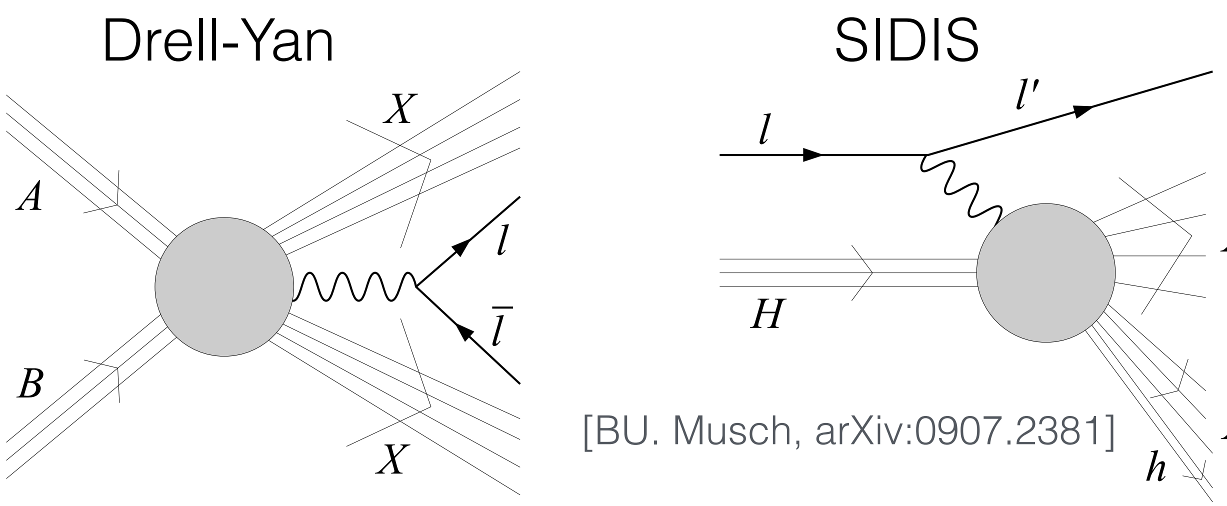
From TMDs to Collins-Soper kernel

- CS kernel encodes the rapidity scaling properties of transverse momentum dependent parton distribution functions (TMDPDFs) / wave functions (TMDWFs):

$$2\zeta \frac{\mathrm{d}}{\mathrm{d}\zeta} \ln f^{\mathrm{TMD}}(x, b_\perp, \mu, \zeta) = K(b_\perp, \mu)$$

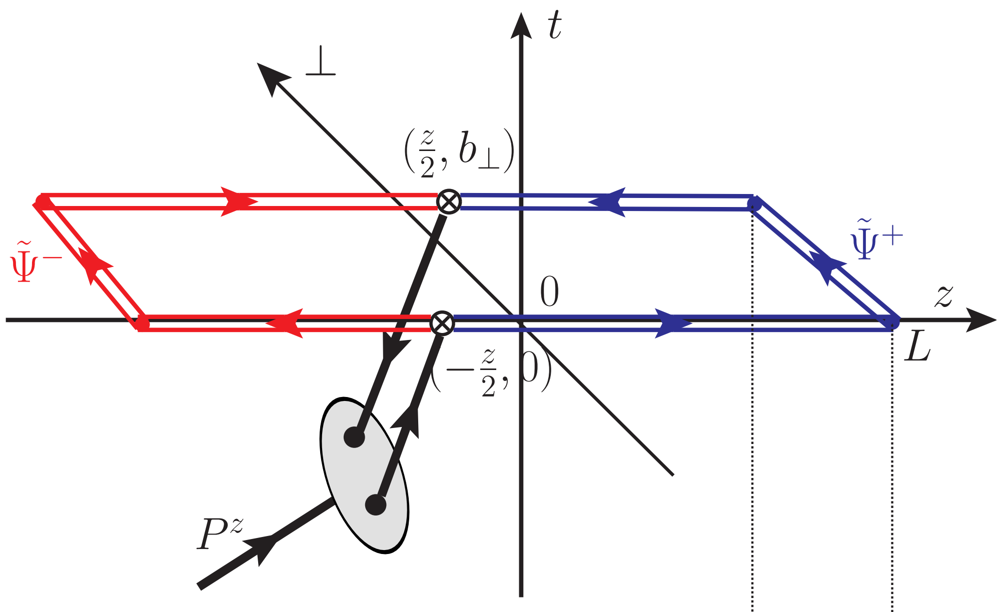
- Universal in Drell-Yan process and semi-inclusive deep inelastic scattering (SIDIS)



[BU. Musch, arXiv:0907.2381]

- TMD wave functions on the lattice:

$$\phi_l(z, b_\perp, P^z, l) = \langle 0 | \bar{\psi}(\vec{z}/2 + \vec{b}) \Gamma \mathcal{W}(\vec{b}, l) \psi(-\vec{z}/2) | \pi(\vec{P}) \rangle$$



[M.-H. Chu et al, arXiv:2204.00200]

- Factorization based on LaMET:

$$K(b_\perp, \mu) = \frac{1}{\ln(P_1^z/P_2^z)} \ln \left| \frac{C(xP_2^z, \mu)\Phi_{\overline{\mathrm{MS}}}(x, b_\perp, P_1^z, \mu)}{C(xP_1^z, \mu)\Phi_{\overline{\mathrm{MS}}}(x, b_\perp, P_2^z, \mu)} \right| = \frac{1}{\ln(P_1^z/P_2^z)} \ln \left| \frac{\phi(0, b_\perp, P_1^z)}{\phi(0, b_\perp, P_2^z)} \right| + \mathcal{O}(\alpha_s, \gamma^{-2})$$

From form factor to soft function

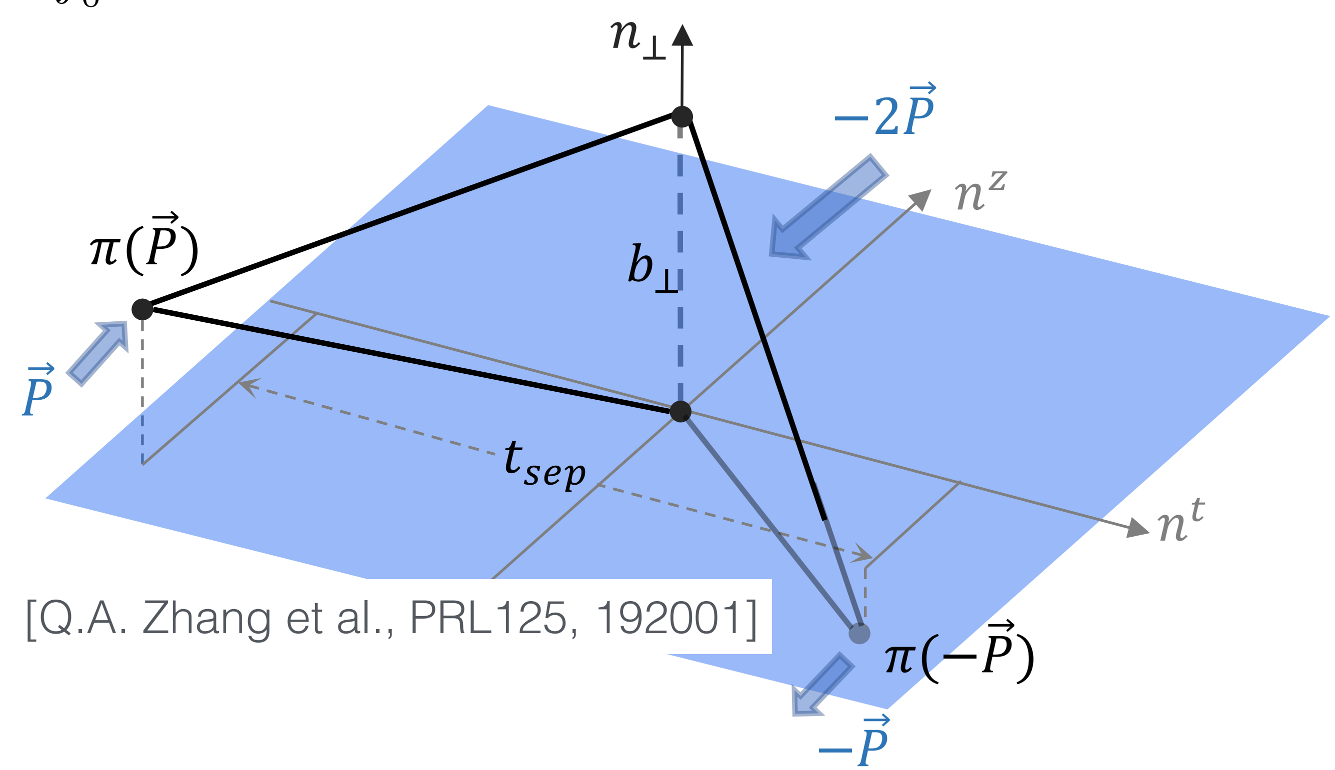
- TMD soft function is a cross section of a quark-antiquark pair (or gluon) traveling in the opposite light-cone directions which radiate soft gluons in DY:

$$S(b_\perp, \mu, Y+Y') = e^{(Y+Y')K(b_\perp,\mu)} S_I^{-1}(b_\perp, \mu)$$

- Factorization from LaMET: $F(b_\perp, P^z) = S_I(b_\perp)\int_0^1 dx\, dx' H(x, x', P^z)\Phi^\dagger(x', b_\perp, -P^z)\,\Phi(x, b_\perp, P^z)$

- Pseudo-scalar meson form factor:

$$F(b_\perp, P^z) = \langle \pi(-\vec{P})|(\bar{q}_1\Gamma q_1)(\vec{b})(\bar{q}_2\Gamma q_2)(0)|\pi(\vec{P})\rangle_c$$



[Q.A. Zhang et al., PRL125, 192001]

- At leading-order: $H(x, x', P^z) = 1/(2N_c)$
- Extract soft function from form factor **F** and wave function **phi**:

$$F(b_\perp, P^z) = S_I(b_\perp)|\phi(z=0, b_\perp, P^z)|^2/(2N_c)$$

- Light cone TMDWF from CS kernel and soft function

$$\tilde{\Psi}^+(x, b_\perp, \mu, \zeta_z) S_r^{1/2}(b_\perp, \mu) = H^+(\zeta_z, \bar{\zeta}_z, \mu^2)\exp\left[\frac{1}{2}K(b_\perp, \mu)\ln\frac{-\zeta_z - i\epsilon}{\zeta}\right]\Psi^+(x, b_\perp, \mu, \zeta) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{\zeta_z}, \cdots\right)$$

[X. Ji and Y. Liu, PRD105,076014]

Lattice setup

- We use newly generated 2+1 flavor clover fermion CLS ensemble X650

β	$L^3 \times T$	a	m_π^{sea}	m_π^v	#conf
3.34	$48^3 \times 48$	0.098 fm	333 MeV	662 MeV	1000

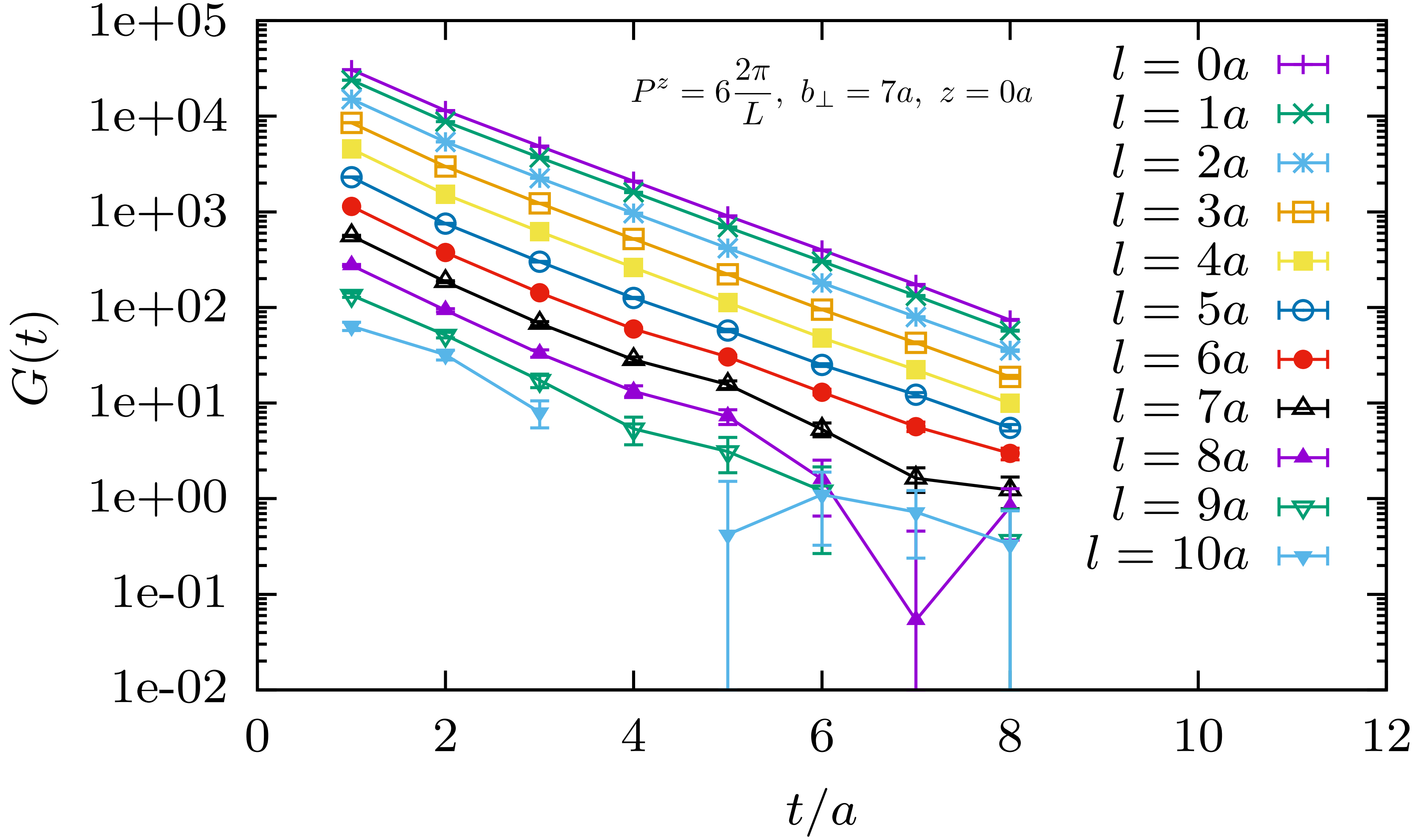
- HYP smearing + Wall source
- 2 sources per configuration
- Transverse direction: x and y
- Momenta: P^z={0, 6,8,10,12} x 2pi/L

  (i.e. 0,1.6, 2.1, 2.6, 3.2 GeV)
- 0<=l/a<=10, 0<=b/a<=7, 0<=t/a<=8

A glance at the wave functions on the lattice

- Wave function from 2pt correlation function: $\phi_l(z, b_\perp, P^z, l) = \langle 0|\bar{\psi}(\vec{z}/2+\vec{b})\Gamma\mathcal{W}(\vec{b}, l)\psi(-\vec{z}/2)|\pi(\vec{P})\rangle$

$$C^{2pt}_{\Gamma_\phi}(b_\perp, l, P^z, t) = \frac{Z_\phi}{L^3}\sum_{\vec{x}} e^{-iP^z x_z}\langle O_\phi(t, b_\perp, l)\, O^\dagger_\pi(0, P^z)\rangle$$
$$= \frac{A_w(p_z)A_p}{2E}e^{-Et}\phi_\ell(0, b_\perp, P^z, \ell)(1+c_0 e^{-\Delta E t})$$

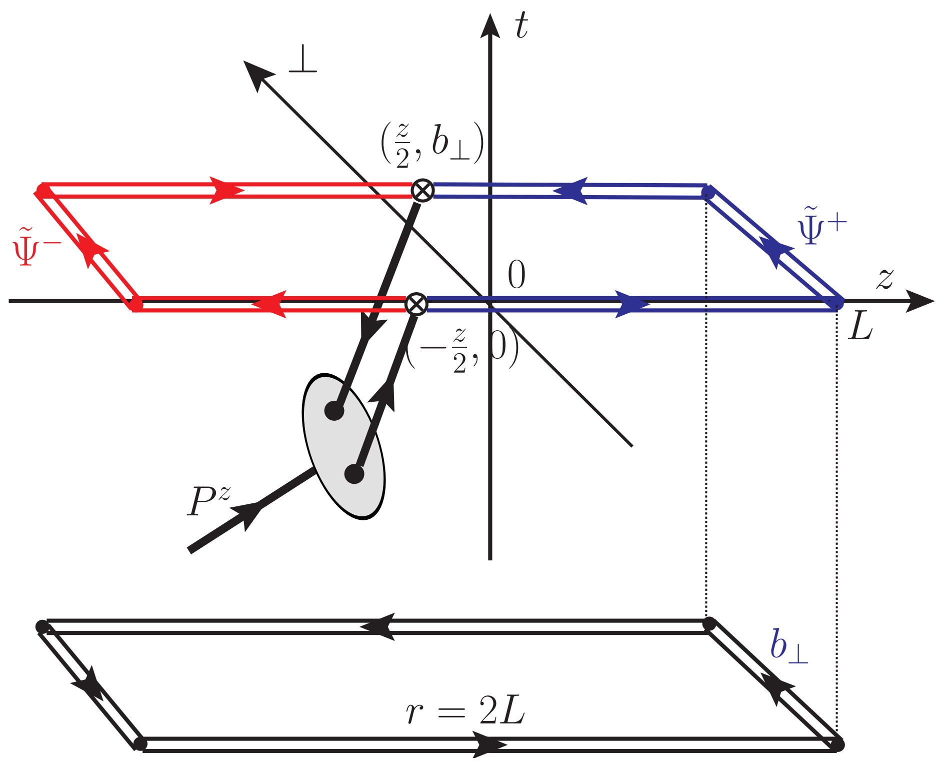


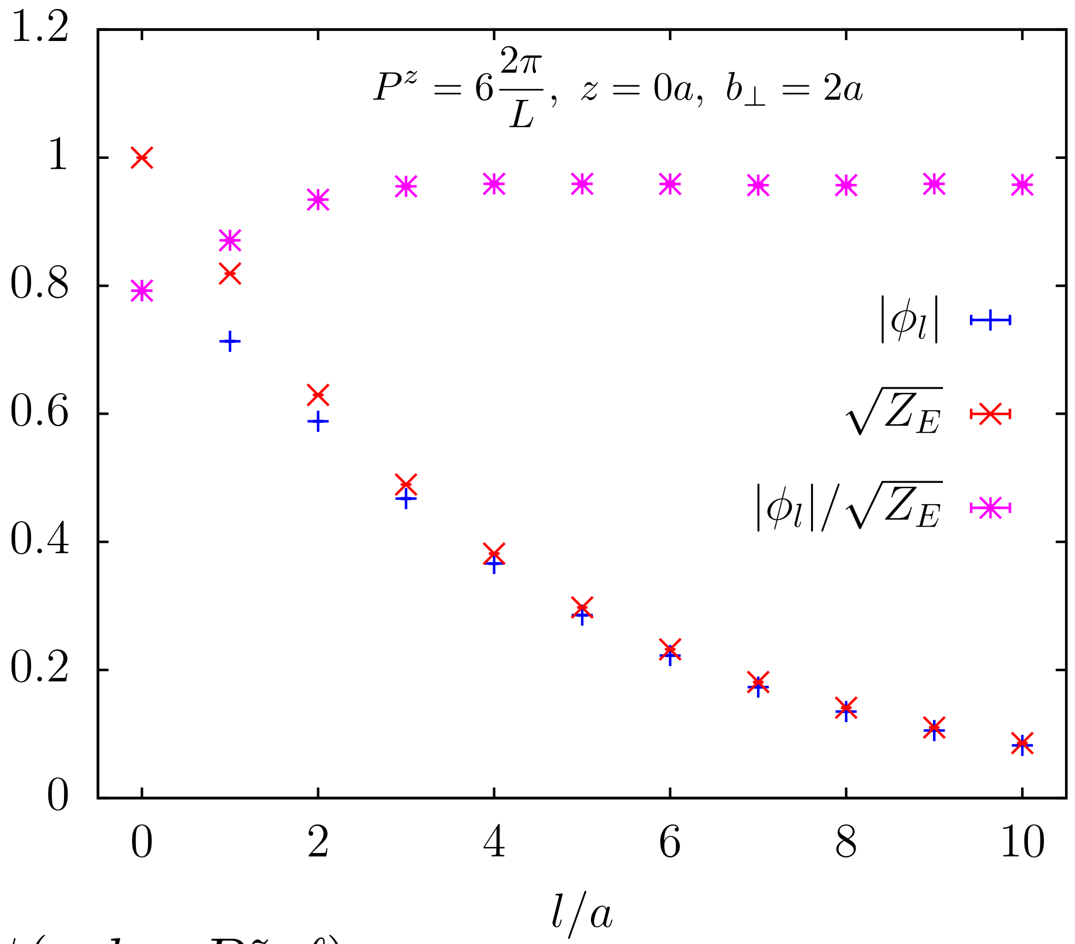
- Joint fit of all l&b (sharing energies) for each P^z

Renormalization of the TMDWFs

$$\Phi(z, b_\perp, L, a, P^z) \propto e^{-\delta\bar{m}(2L+b_\perp)} e^{-V(b_\perp)L} Z_O$$

- The linear divergence and heavy quark potential is removed by Wilson loop
- Residual logarithmic divergence is cancelled when taking ratio



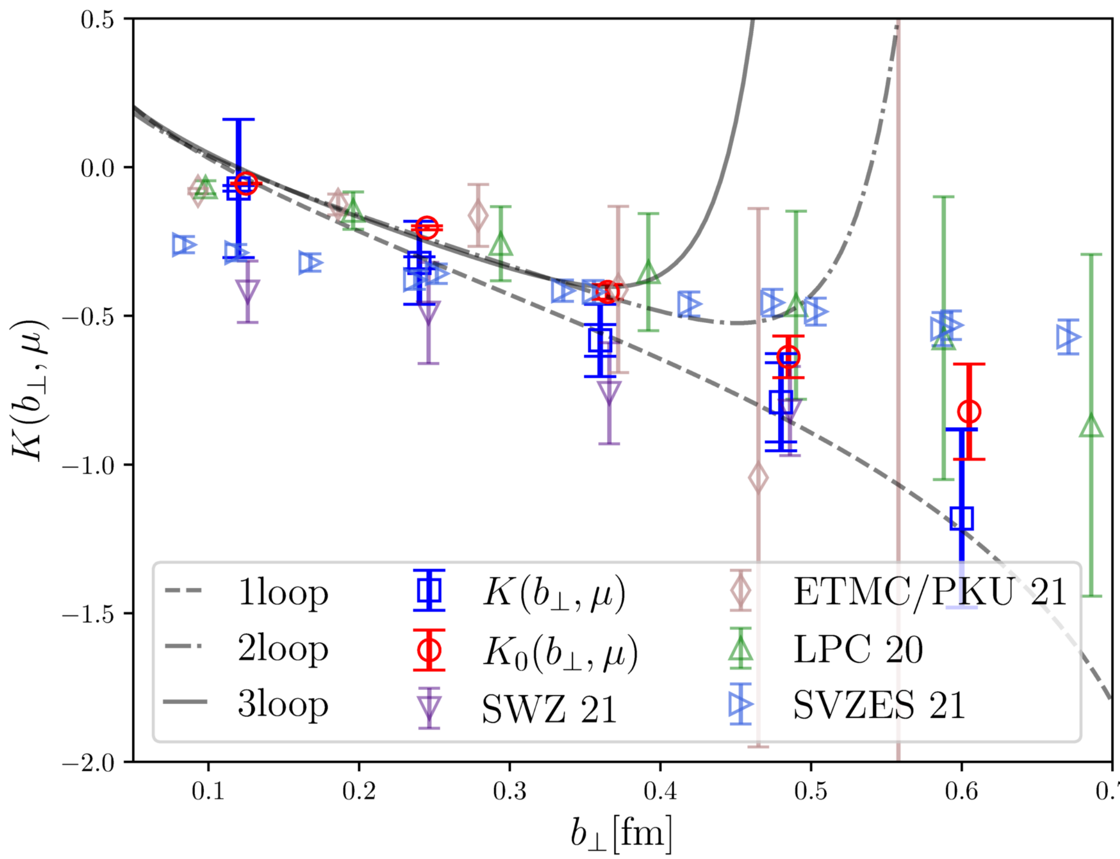


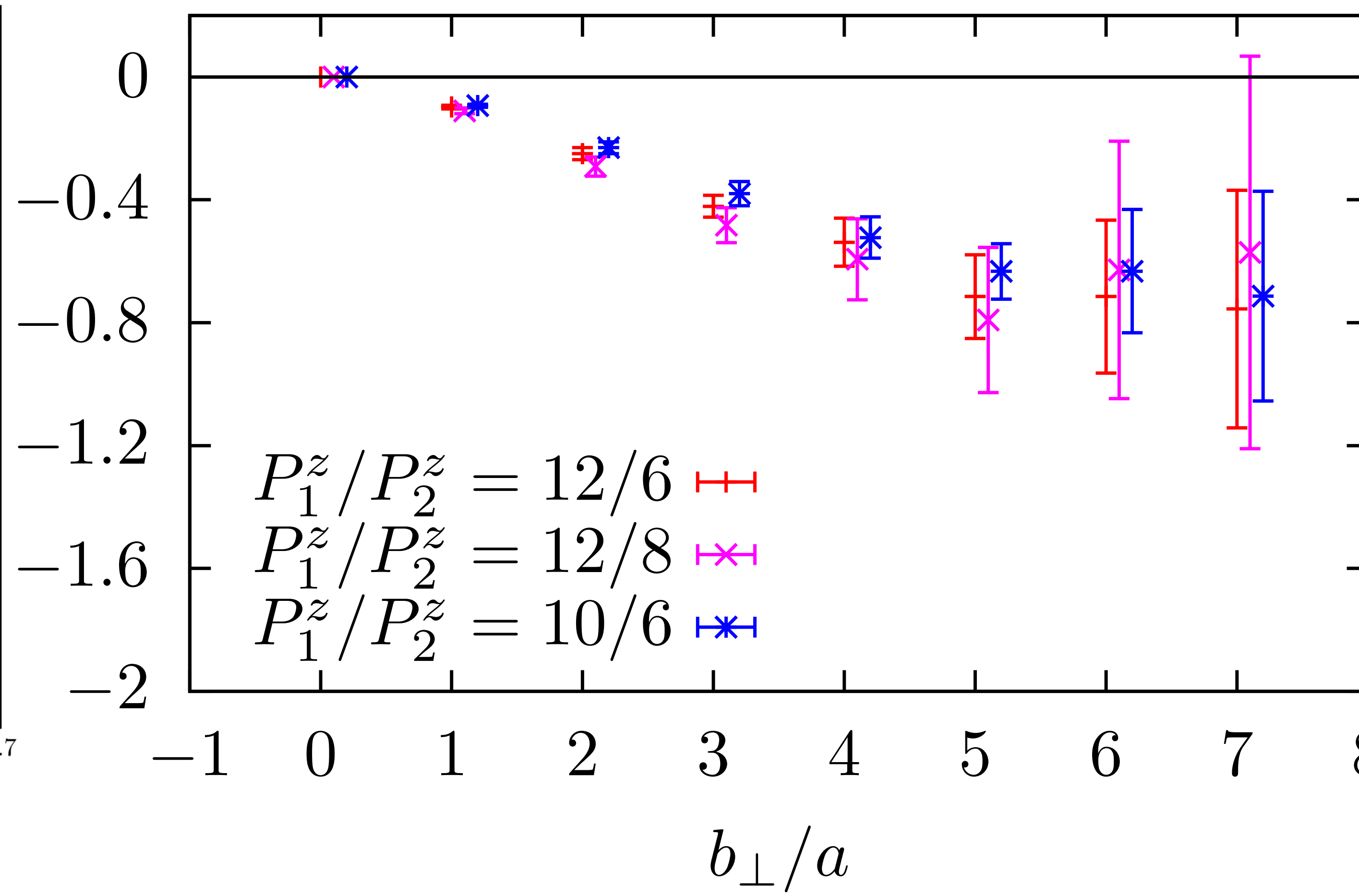
$$\Phi(z, b_\perp, P^z) = \lim_{\ell\to\infty} \frac{\phi(z, b_\perp, P^z, \ell)}{\sqrt{Z_E(2\ell, b_\perp)}}$$

Collins-Soper kernel

$$K(b_\perp, \mu) = \frac{1}{\ln(P_1^z/P_2^z)} \ln \left| \frac{\phi(0, b_\perp, P_1^z)}{\phi(0, b_\perp, P_2^z)} \right| + \mathcal{O}(\alpha_s, \gamma^{-2})$$

[M.H. Chu et al, 2204.00200]



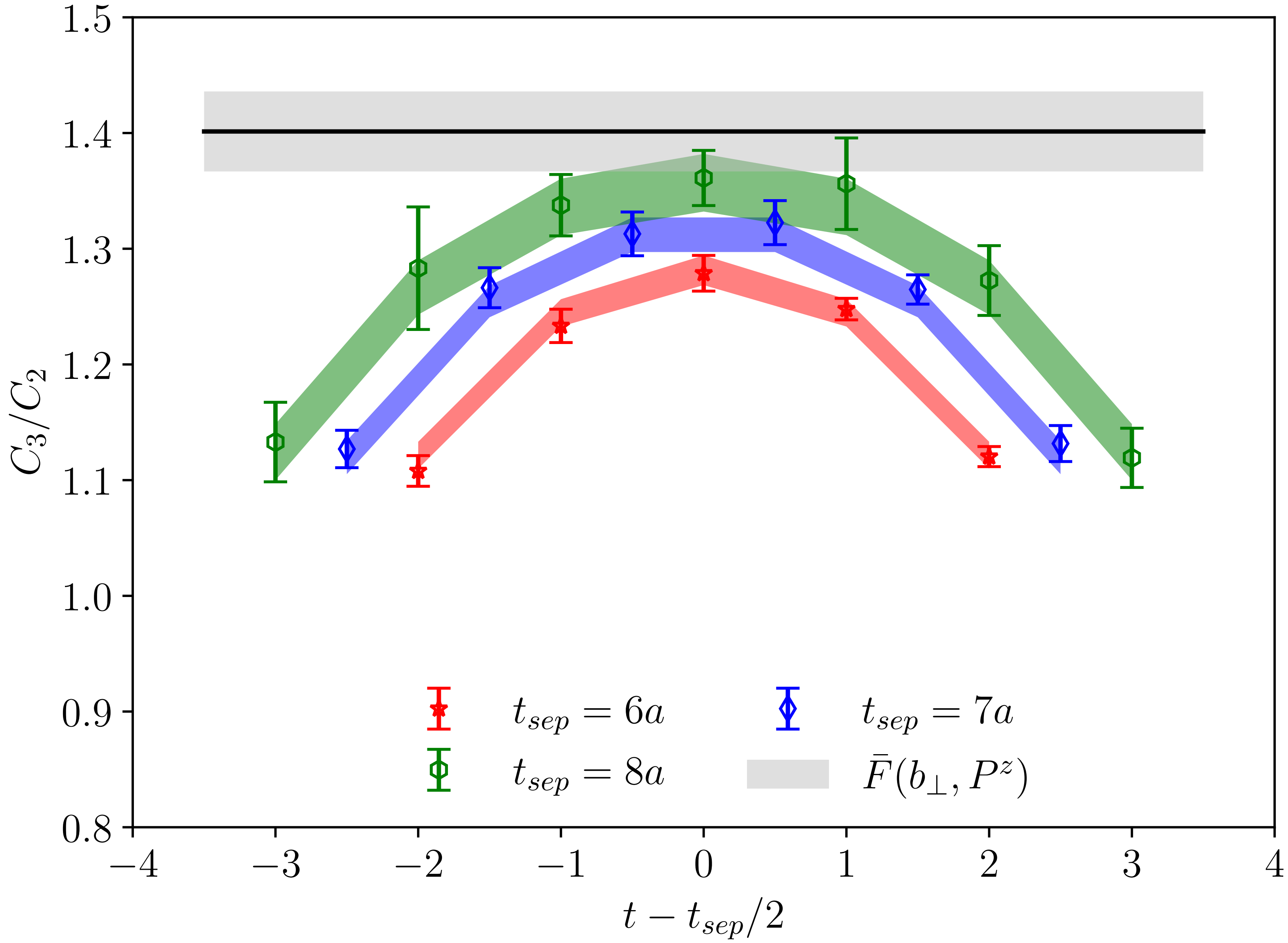


- Good agreement with previous lattice calculations

Form factor from 3pt

- Form factor available from 2-state fit of 3pt function:

$$C_\Gamma^{3pt}(b_\perp, P^z, t_{\rm sep}, t) = \frac{1}{L^3}\sum_{\vec{x}} e^{-2iP^z x_z} Z_\Gamma^2 \langle O_\pi(t_{\rm sep}, -P^z)\, \bar{u}\Gamma u(t, \vec{x}+b_\perp)\, \bar{d}\Gamma d(t, \vec{x})\, O_\pi^\dagger(0, P^z)\rangle$$
$$= \frac{A_w(p_z)^2}{(2E)^2} e^{-Et_{\rm sep}}\left[F(b_\perp, P^z) + c_1(e^{-\Delta E t} + e^{-\Delta E(t_{\rm sep}-t)}) + c_2 e^{-\Delta E t_{\rm sep}}\right]$$



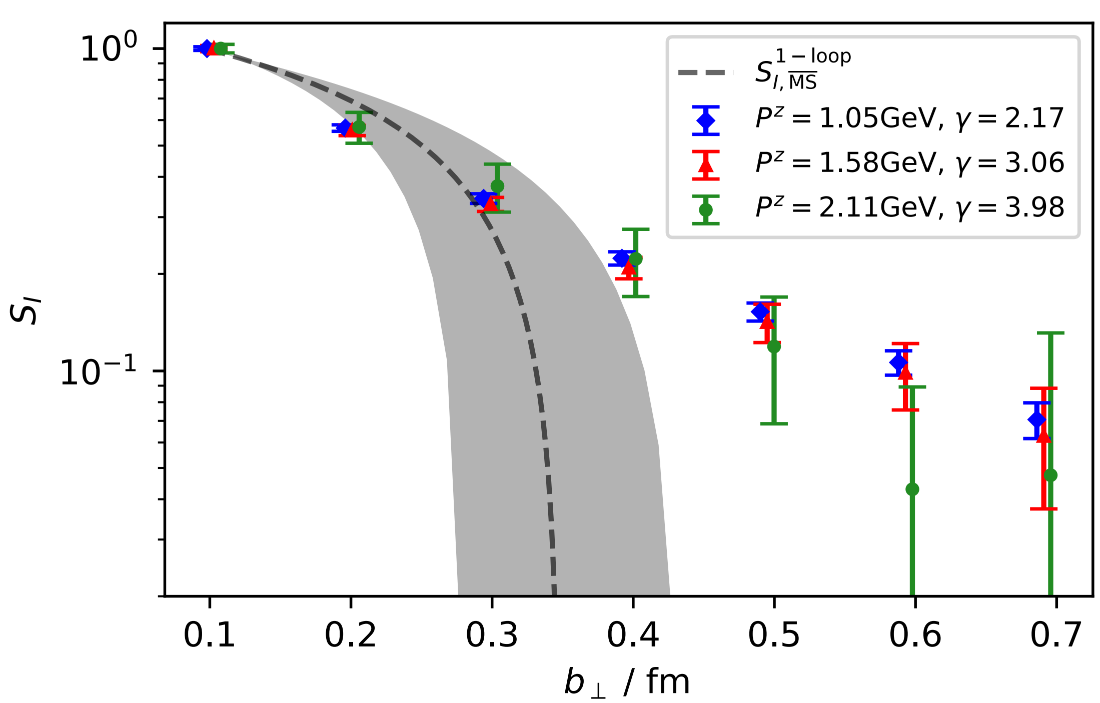
$$P^z = 6\frac{2\pi}{L}$$
$$b_\perp = 2a$$
$$\Gamma = I$$

Soft function from form factor

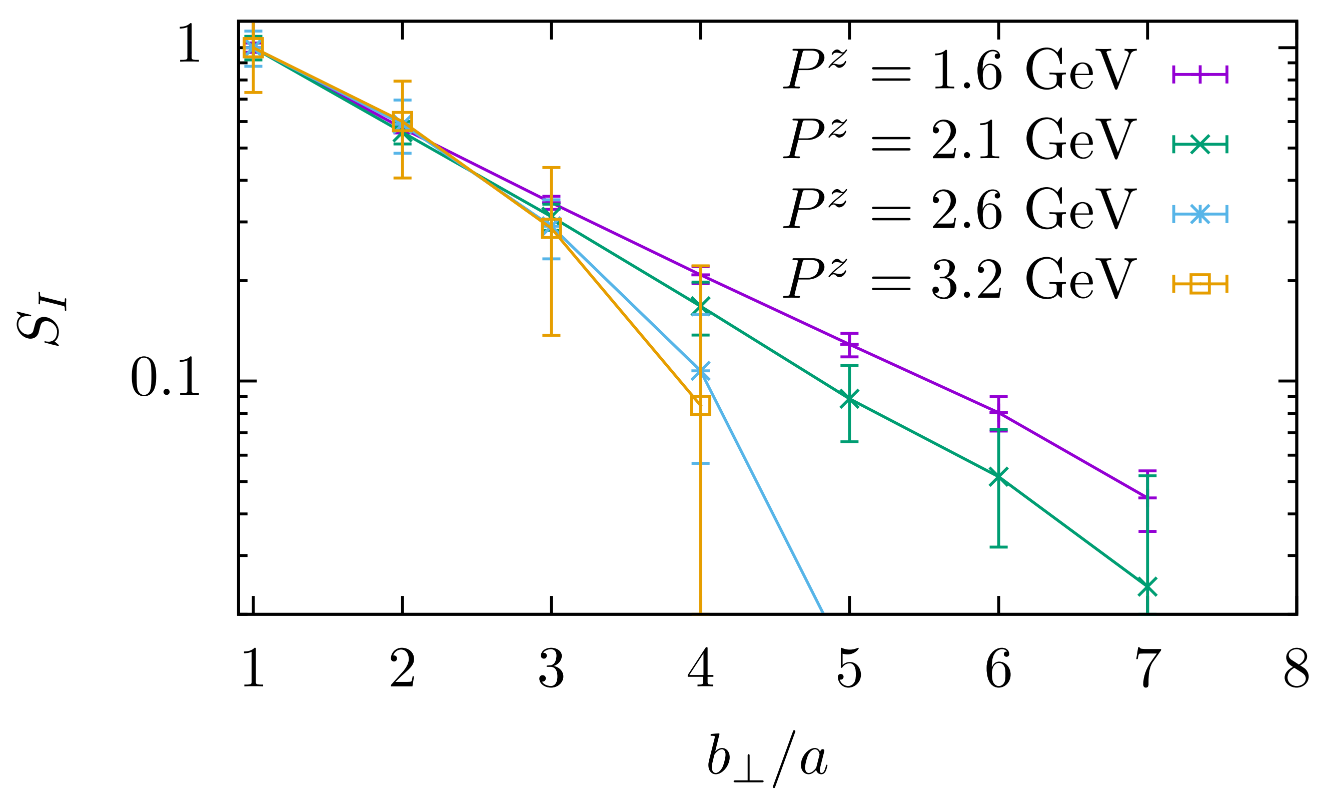
Intrinsic soft function from form factor and TMD wave function:

$$S_{I,\overline{\mathrm{MS}}}(b_\perp,\mu) = \frac{F(b_\perp,P^z)}{F(b_{\perp,0},P^z)}\frac{|\phi(0,b_{\perp,0},P^z)|^2}{|\phi(0,b_\perp,P^z)|^2} + \mathcal{O}(\alpha_s,\gamma^{-2})$$

[Q.A. Zhang et al, PRL125, 192001]



[this work]

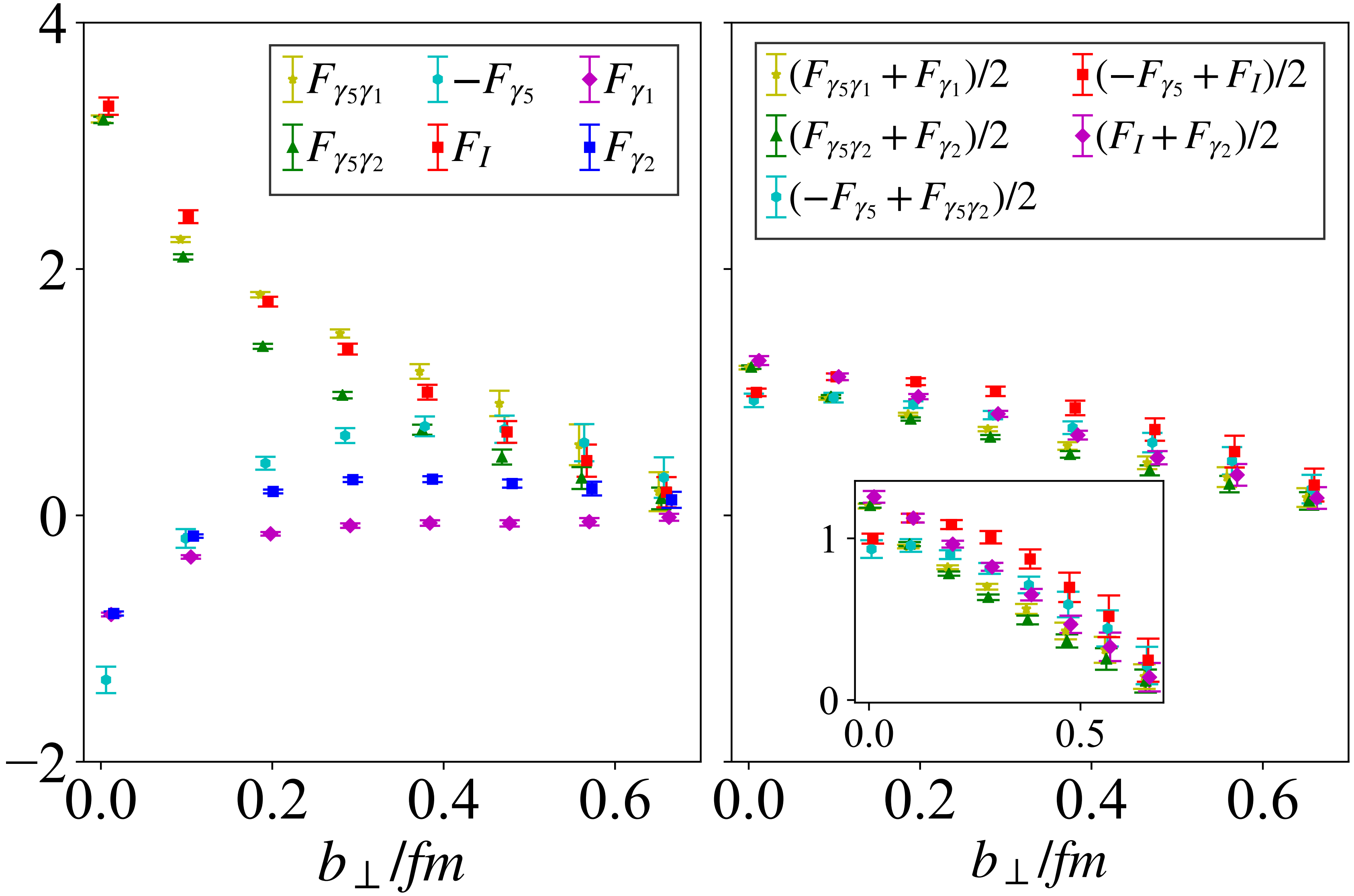


from form factor with unity gamma matrix insertion

Suppress high-twist contamination

- Reduction of higher-twist contamination from Fierz rearrangement

$$F_{\Gamma,P^z} = S(b_\perp)H^0_\Gamma|\phi(b_\perp,l,P^z)|^2 + \sum_{\Gamma'\neq\gamma_5\gamma_0,\gamma_5\gamma_3} S_{\Gamma'}(b_\perp)H^0_{\Gamma\Gamma'}|\phi_{\Gamma'}(b_\perp,l,P^z)|^2$$



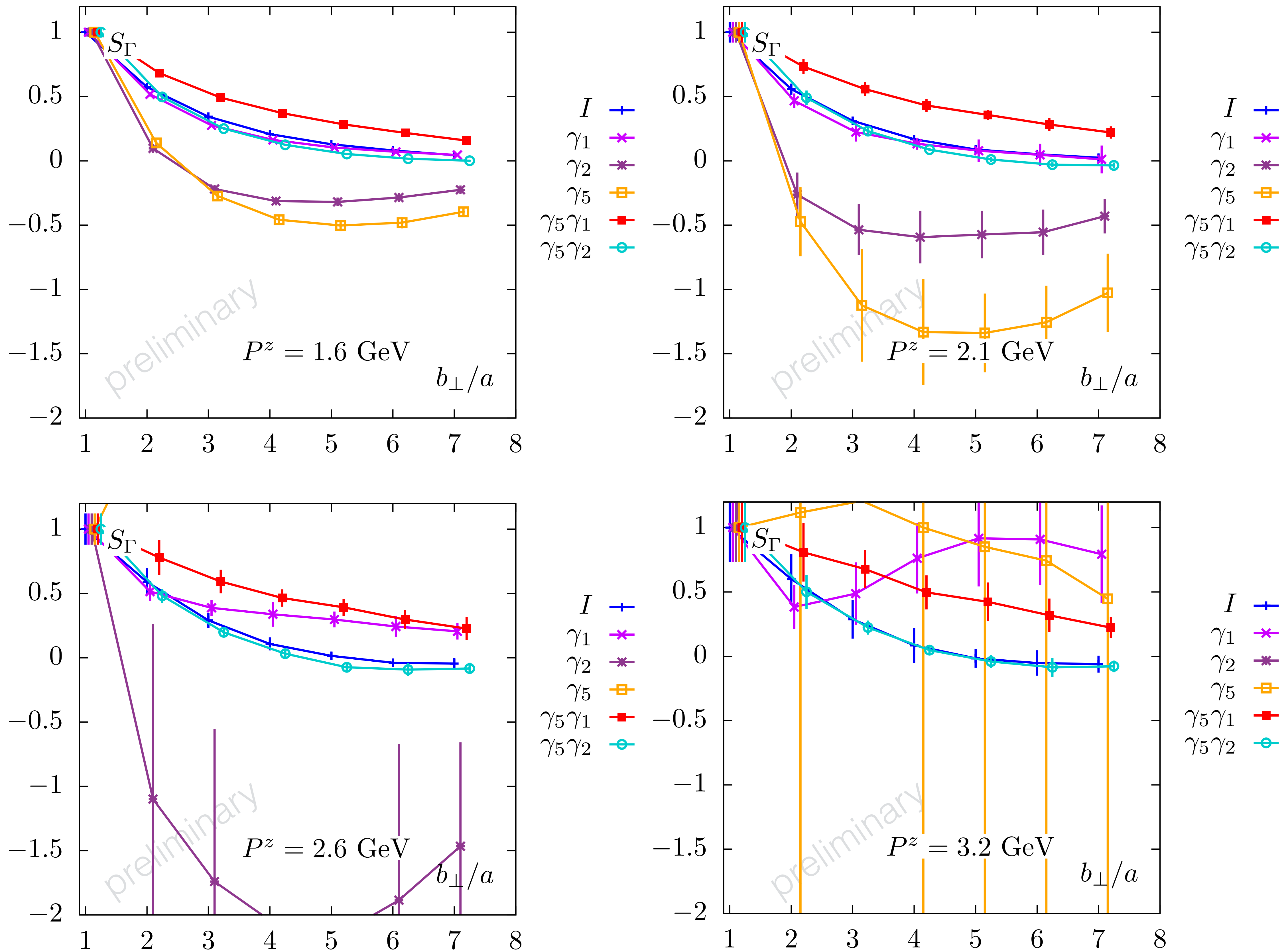
$P^z = 3.3$ GeV, $m_\pi = 827$ MeV

5 possible combinations:

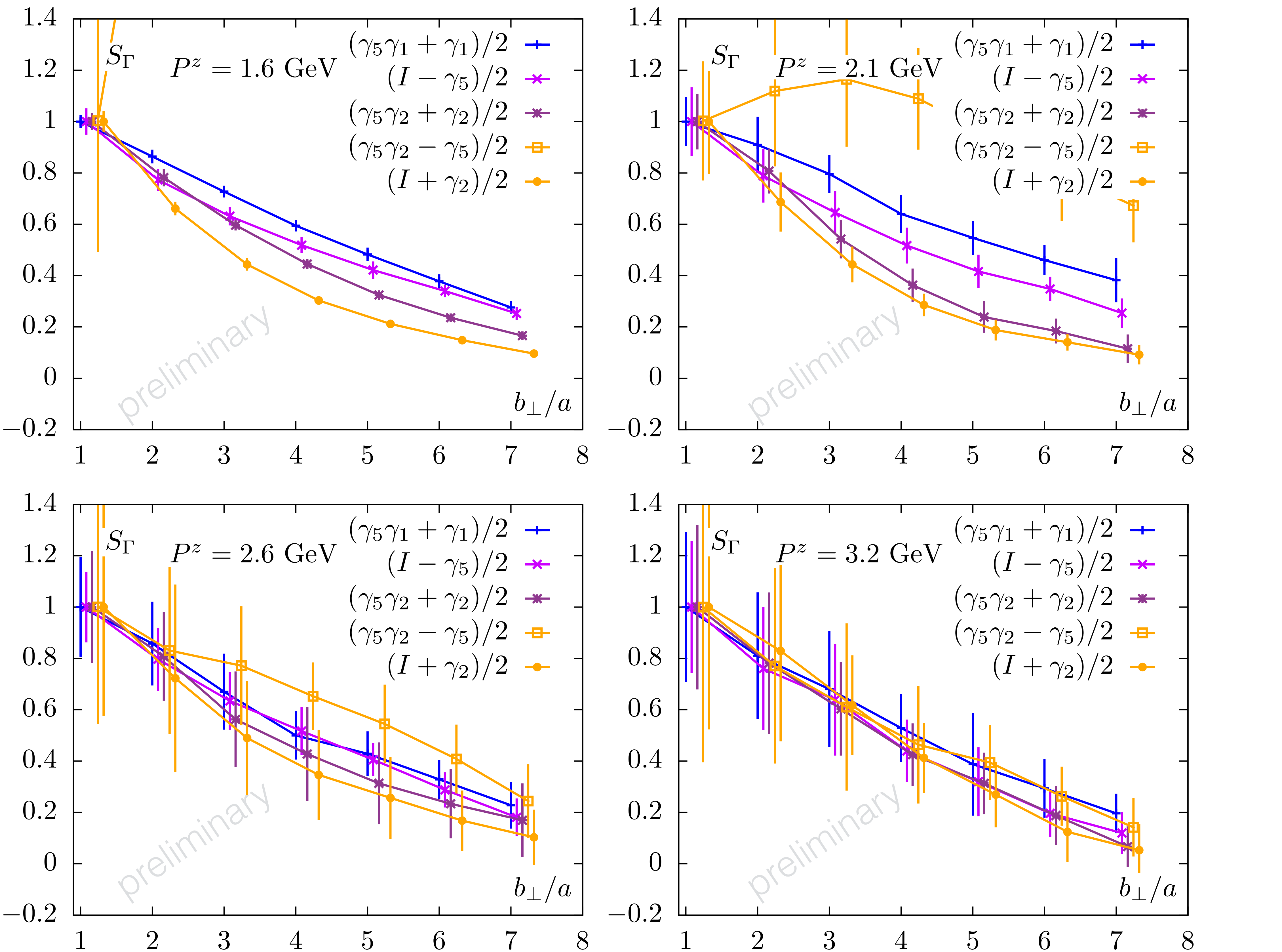
$$\frac{1}{2}(F_{\gamma5\gamma1}+F_{\gamma1}),$$
$$\frac{1}{2}(F_{\gamma5\gamma2}+F_{\gamma2}),$$
$$\frac{1}{2}(-F_{\gamma5}+F_{\gamma5\gamma2}),$$
$$\frac{1}{2}(-F_{\gamma5}+F_I),$$
$$\frac{1}{2}(F_I+F_{\gamma2})$$

[Yuan Li et al, PRL.128(2022)6,062002]

Soft function from separate channels



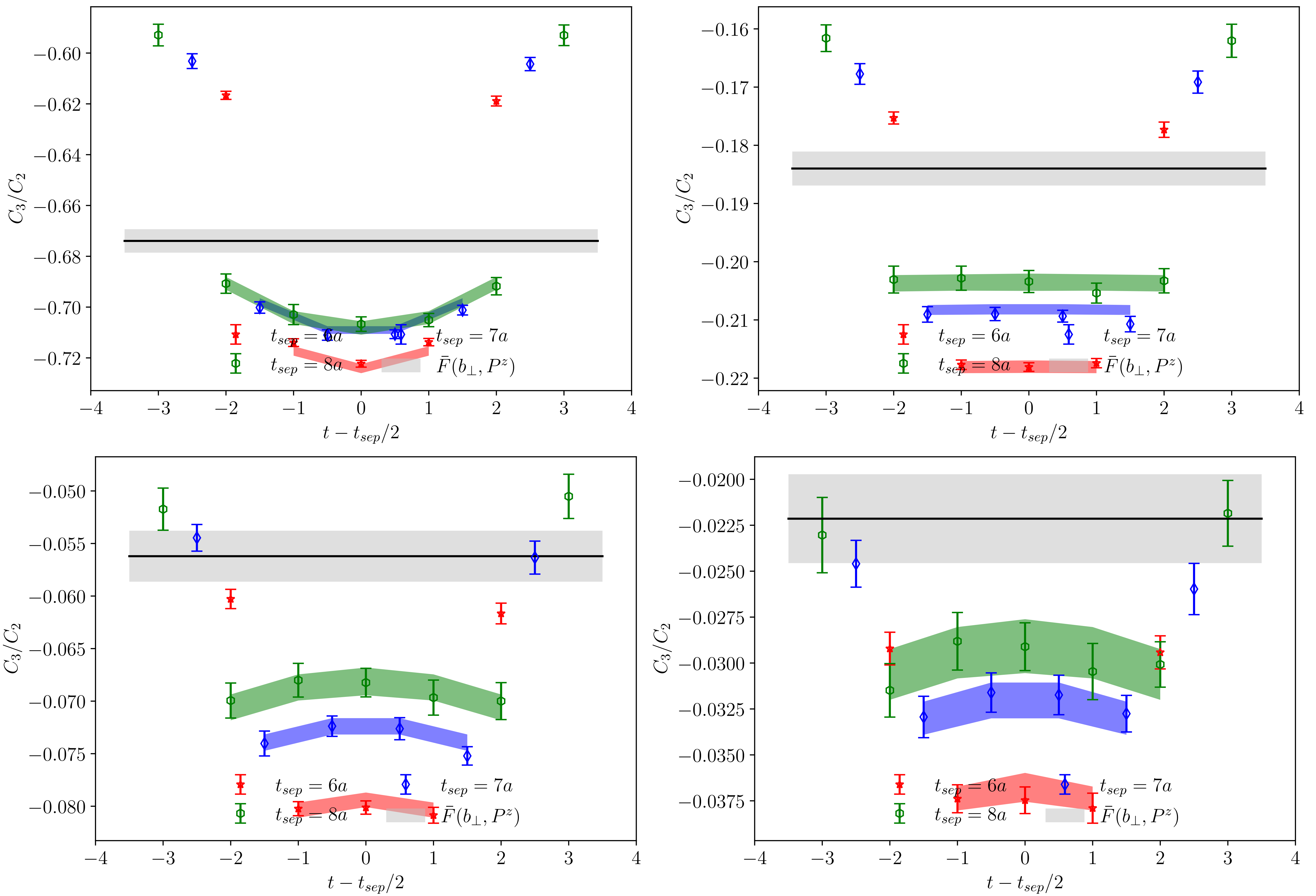
Soft function after Fierz rearrangement



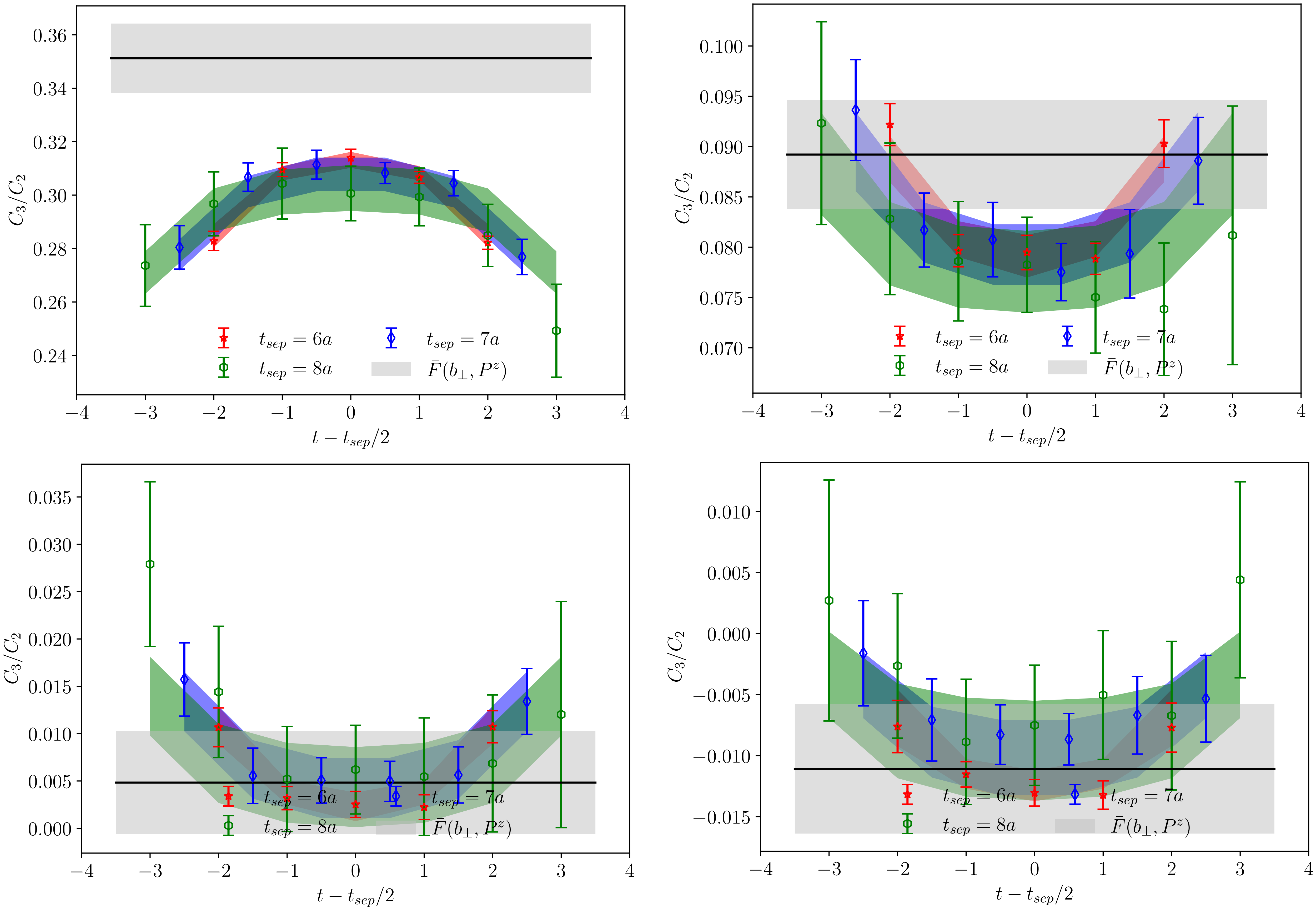
Summary & outlook

- New CLS ensemble X650 generated
- CS kernel and soft function measured on X650
- Good agreement with literature for CS kernel
- Confirmed the validity of Fierz rearrangement for soft function
- Determination of light cone pion TMDWF on the way

Backup: k=6, Gamma=g1, b=0,2,4,6



Backup: k=10, Gamma=g5g2, b=0,2,4,6



Backup: ETMC

$$C^{wf,r}_{\Gamma_\phi}(b_\perp, l, P^z, t) = \frac{C^{wf,b}_{\Gamma_\phi}(b_\perp, l, P^z, t)}{C^{wf,b}_{\Gamma_\phi}(b_\perp, l, 0, t)} C^{wf,\overline{\mathrm{MS}}}_{\Gamma_\phi}(0,0,0,t),$$

$$S(b_\perp) = \lim_{l\to\infty} \lim_{t_s\to\infty} \frac{C^{3pt}_{\Gamma}(b_\perp, P^z, t_s, t)}{H^0_\Gamma \left|C^{wf}_{\Gamma_\phi}(b_\perp, l, P^z, \frac{t_s}{2})\right|^2}.$$

Backup: renormalization

$$h^{\mathrm{SDR}}_{\chi,\gamma_t}(b,z,P_z;\frac{1}{b_0}) = \frac{h_{\chi,\gamma_t}(b,z,P_z;1/a)}{h_{\pi,\gamma_t}(b_0,z_0=0,0,1/a)},$$

lattice data/sqrt(Z)

reference point:
same on the lattice and in MSbar

$$h^{\overline{\mathrm{MS}}}_{\chi,\gamma_t}(b_0,z_0,0;\mu) = 1 + \frac{\alpha_s C_F}{2\pi}\left\{\frac{1}{2} + 3\gamma_E - 3\ln 2 + \frac{3}{2}\ln[\mu^2(b_0^2+z_0^2)] - 2\frac{z_0}{b_0}\arctan\frac{z_0}{b_0}\right\} + \mathcal{O}(\alpha_s^2).$$

$$h^{\overline{\mathrm{MS}}}_{\chi,\gamma_t}(b,z,P_z;\mu) = h^{\overline{\mathrm{MS}}}_{\gamma_t}(b_0,0,0;\mu)h^{\mathrm{SDR}}_{\chi,\gamma_t}(b,z,P_z;\frac{1}{b_0}),$$

for PDF, same for WF at p=0

LPC, arXiv:2205.13402

Backup: LaMET

$$\frac{P^-}{P^+} \ll 1, \qquad \frac{1}{|b|P^+} \ll 1, \qquad \frac{|b|}{L} \ll 1, \qquad \frac{\ell}{L} \ll 1, \qquad \ell\Lambda_{\mathrm{QCD}} \ll 1,$$

pi/a=6.4GeV