Study of quasi-beam function in twisted mass lattice QCD

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Introduction

- the aim is the non-perturbative calculation of transverse momentum dependent parton distribution functions (TMDPDFs) from first principle in lattice QCD
- an object of interest for future electron-ion colliders
- in the framework of large momentum effective theory (LaMET) (PhysRevLett.110.262002), it is obtained from a quasi-beam function and a soft function
- a study of the soft function has been already performed by the collaborators (PhysRevLett.128.062002) and previously by LPC (PhysRevLett.125.192001)
- calculations of quasi-beam function with pions have also been done (PhysRevD.104.114502)
- this work shows preliminary results for the calculation of the quasi-beam function with nucleon final states

Quasi-TMDPDF

- the relation between the TMDPDF and quasi-TMDPDF is as follows (RevModPhys.93.035005)

$$f^{TMD}(x, \vec{b}_T, \mu, \zeta) = H\left(\frac{\zeta_z}{\mu^2}\right) e^{-ln(\frac{\zeta_z}{\zeta})K(b_T,\mu)} \tilde{f}(x, \vec{b}_T, \mu, \zeta_z)\, S_r^{1/2}(b_T, \mu)$$

- $H(\zeta_z/\mu^2)$ is the perturbative matching kernel and $K(b_T, \mu)$ is the Collins-Soper kernel, with $\zeta_z = xP^z$ being the natural Collins-Soper scale
- $S_r^{1/2}$ is the reduced soft function, which, under LaMET, is obtained through a ratio of a meson form factor and the TMD wave function
- The quasi-TMDPDF $\tilde{f}$ is defined as

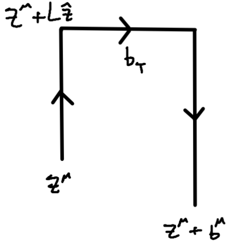
$$\tilde{f}(x, \vec{b}_T, \mu, \zeta_z) = \lim_{L\to\infty} \int \frac{db^z}{2\pi} e^{-ib^z\zeta_z} Z^{\overline{MS}}(\mu, b^z) \frac{P^z}{E_P} B(b^z, \vec{b}_T, L, P^z)$$

Quasi-beam function

- the quasi-beam function $B(b^z, \vec{b}_T, L, P^z)$ is defined in terms of the bare matrix element

$$B(b^z, \vec{b}_T, L, P^z) = \langle N(P^z)|\bar{\psi}(z^\mu + b^\mu)\gamma^z \mathcal{W}_z(z^\mu + b^\mu; L)\psi(z^\mu)|N(P^z)\rangle$$

- here $b^\mu = (0, \vec{b}_T, b^z)$ and $N(P^z)$ is the nucleon with momentum $P^\mu = (E_P, \vec{0}_T, P^z)$
- $\mathcal{W}_z(z^\mu + b^\mu; L)$ is the asymmetric staple with length L along $\hat{z}$ for the symmetric part and width b_T along the transverse direction



Quasi-beam function

- the bare matrix element has an intrinsic e^{-L} divergence
- this can be eliminated by using a rectangular Wilson loop
- we define the bare quasi-beam function as

$$B_{bare}(b^z, \vec{b}_T, P^z) = \frac{B(b^z, \vec{b}_T, L, P^z)}{\sqrt{Z_E(2(L-b^z), b_T)}}$$

- Z_E is the vacuum expectation value of the rectangular Wilson loop with length $2(L-b^z)$ along $\hat{z}$ and width b_T

Lattice simulation

- the bare matrix element can be obtained on the lattice through the ratio of 3-point and 2-point functions

$$C^{2pt}(\vec{P},t,0) = \sum_{\vec{x}} e^{-i\vec{P}\cdot\vec{x}} \langle 0|N(\vec{x},t)\bar{N}(\vec{0},0)|0\rangle$$

$$C^{3pt}(b^z,b_T,\vec{P},t_s,\tau,0,L) = \sum_{\vec{x},\vec{y}} e^{-i\vec{P}\cdot\vec{x}} \langle 0|N(\vec{x},t_s)\bar{\psi}(\vec{y}+b^z+b_T,\tau) \gamma^z W(\vec{y}+b^z+b_T;L)\psi(\vec{y},\tau)\bar{N}(\vec{0},0)|0\rangle$$

- where N is the standard 3-quark nucleon interpolator and $t(t_s)$ is the source - sink separation along time in the 2-point (3-point) function.

Lattice simulation

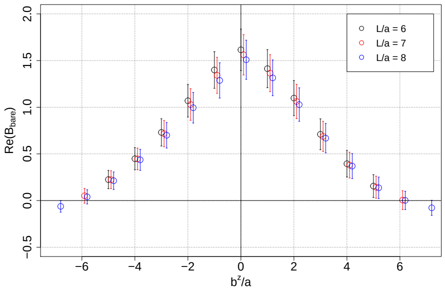
- calculations are done on a $N_f = 2+1+1$ twisted mass lattice ensemble generated by the ETMC collaboration

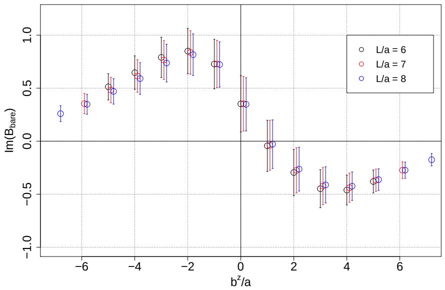
$L^3 \times T$	$a[fm]$	$a\mu_{sea}$	$m^{\pi}_{sea}[MeV]$	N_{conf}	N_{src}
$24^3 \times 48$	0.093	0.00530	350	100	8

- the gauge links are APE smeared and momentum smearing is applied on the propagators
- for the asymmetric staple, 5 steps of stout smearing is used

Bare quasi-beam function

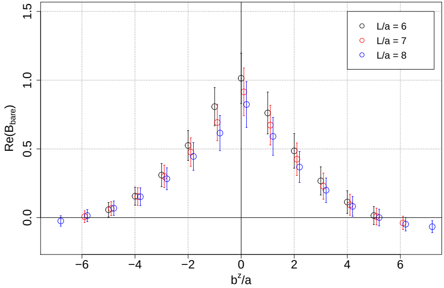
- $P^z = 1.7$ GeV
- $b_T = 0.09$ fm

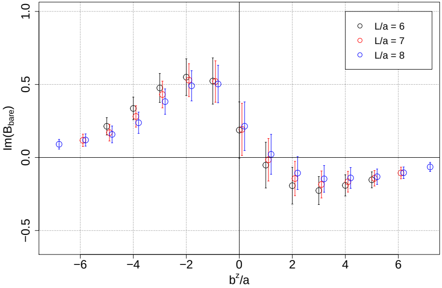




Bare quasi-beam function

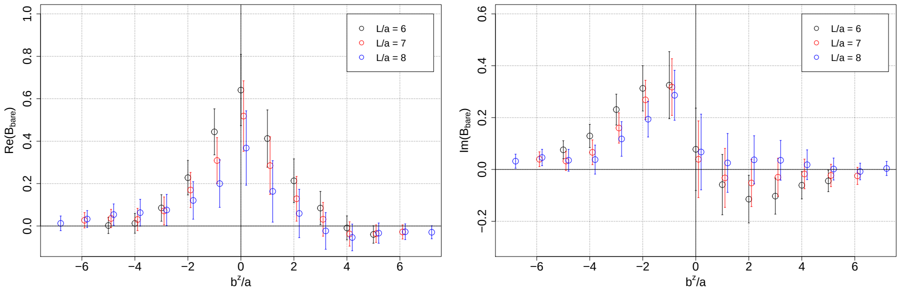
- $P^z = 1.7$ GeV
- $b_T = 0.18$ fm





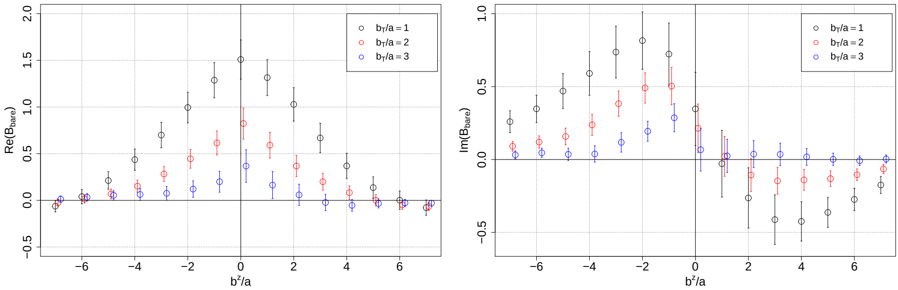
Bare quasi-beam function

- $P^z = 1.7$ GeV
- $b_T = 0.28$ fm



Bare quasi-beam function

- $P^z = 1.7$ GeV
- $L = 0.74$ fm



Renormalization using RI/MOM

- RI/MOM scheme (Nucl.Phys. B445, 81) is used for the renormalization of the bare quasi-beam function
- the renormalization factor $Z_{\Gamma\Gamma'}^{MOM}$ is obtained from the inversion of the projected vertex functions

$$Z_{\Gamma\Gamma'}^{MOM}(p) = Tr[\Lambda_{tree}(p)\Gamma'][Tr[\Lambda(p)\Gamma']]^{-1}$$

where

$$\Lambda(p) = S_u^{-1}(p)G(p)S_d^{-1}(p)$$

- $S(p)$ is the quark propagator and $G(p)$ is the quark-quark Green's function with an insertion of the staple shaped operator

Renormalization using RI/MOM

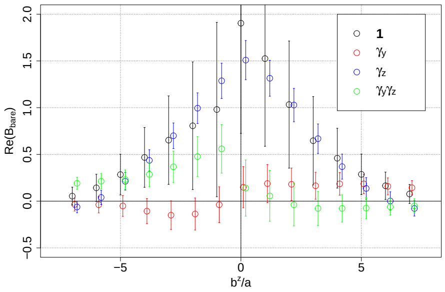
- the RI/MOM renormalized quasi-beam function is then calculated as

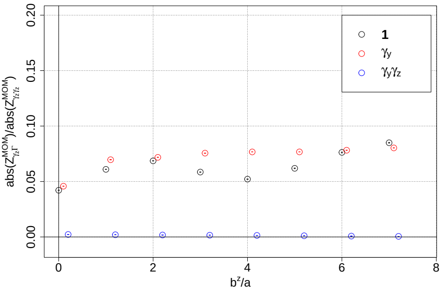
$$B^{MOM}(b^z, \vec{b}_T, L, P^z) = Z_{\Gamma\Gamma'}^{MOM}(p, b^z, \vec{b}_T, L) B_{\Gamma'}(b^z, \vec{b}_T, L, P^z)$$

- in case of the unpolarized quasi-TMDPDF ($\Gamma = \gamma_z$), it is observed through symmetry arguments that mixing is allowed for the structures $1, \gamma_y$ and $\gamma_y\gamma_z$
- Z^{MOM} is calculated considering the set of four operators $\{1, \gamma_y, \gamma_z, \gamma_y\gamma_z\}$ at a momentum $p = (4, 4, 4, 4)$

Operator mixing (preliminary)

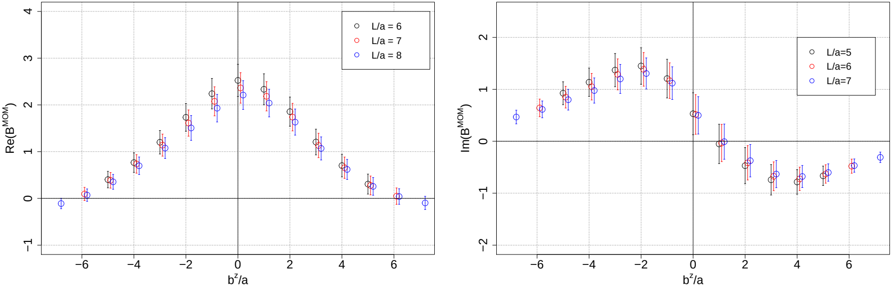
- $P^z = 1.7$ GeV
- $b_T = 0.09$ fm
- $L = 0.74$ fm





Renormalized quasi-beam function (preliminary)

- $P^z = 1.7$ GeV
- $b_T = 0.09$ fm



Outlook

- the preliminary results seem promising
- next step is to match the results to $\overline{MS}$
- work is currently underway on 1-loop calculation of the asymmetric staple
- this needs to be combined with the soft function results for an estimate of TMDPDF from first principle
- finally, take this calculation to physical point ensemble