

# A lattice QCD calculation of the off-forward Compton amplitude and generalised parton distributions

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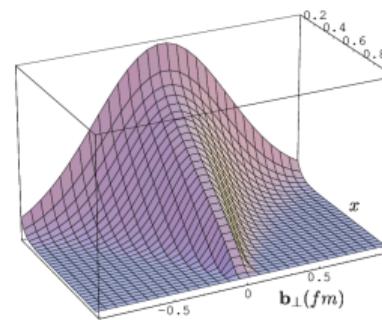
# Outline of the problem

Generalised parton distributions are

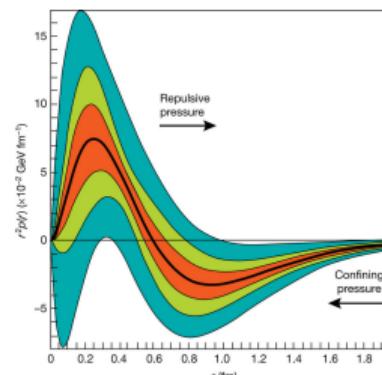
- extensions of PDFs
- related to elastic FFs

Contain a **staggering amount of physical information**:

- a solution to proton spin puzzle
- the spatial distributions of hadron constituents
- mechanical properties



Quark spatial distribution (Burkardt, 2002)



Proton pressure distribution (Burkert et al., 2018)

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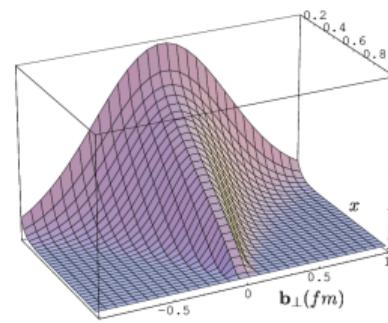
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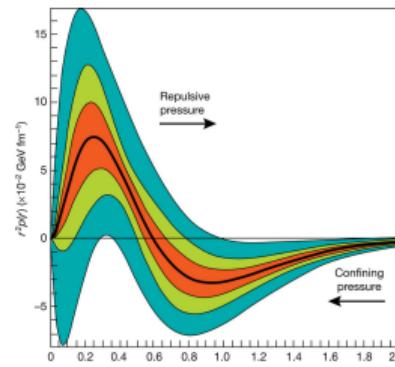
**However...**

- Difficult to measure experimentally
- and difficult to calculate on the lattice

**In this talk:** a new lattice method to calculate GPDs (Feynman-Hellmann), with strong parallels to experiment → electron-ion collider



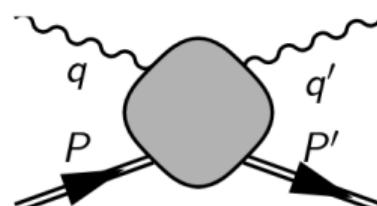
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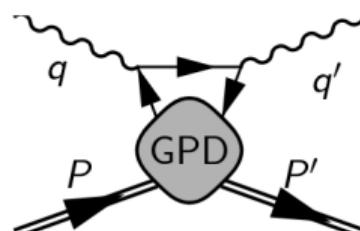
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# What are generalised parton distributions?

## Compton amplitude

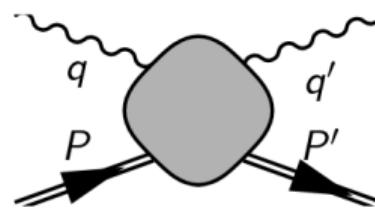


Factorisation for large  
 $-q^2$  or  $-q'^2$ :

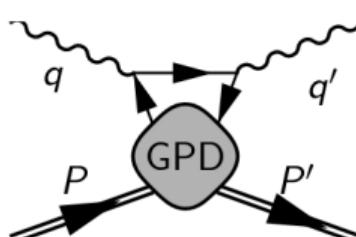


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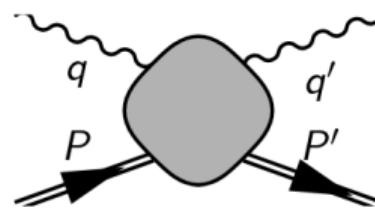
## Formal definition

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \overbrace{\langle P' | \bar{\psi}_q(-\lambda n/2) \not{p} \psi_q(\lambda n/2) | P \rangle}^{\text{light-cone matrix elem}} = H^q(x, \xi, t) \bar{u}(P') \not{p} u(P)$$
$$+ E^q(x, \xi, t) \bar{u}(P') \frac{i\sigma^{\mu\nu} n_\mu (P' - P)_\nu}{2M} u(P).$$

- $H^q$  and  $E^q$  helicity-conserving and -flipping GPDs
- $t = (P' - P)^2$  is momentum transfer → how 'off-forward'

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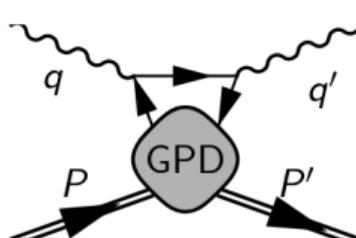
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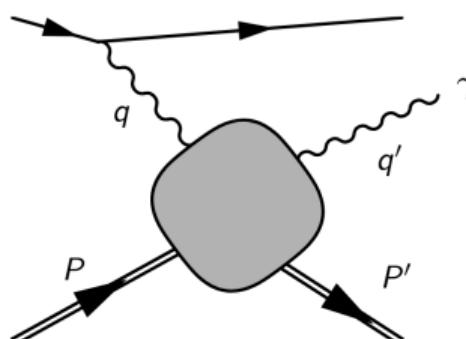


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## Forward limit

$H^q(x, \xi, t) \xrightarrow{t \rightarrow 0} q(x)$ , the regular parton distribution function (PDF).

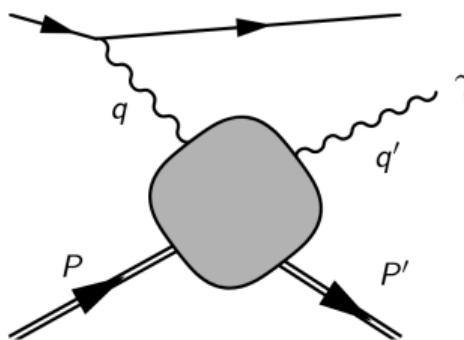
# Measurement of GPDs



- Deeply virtual Compton scattering:  
 $e^- + p \rightarrow e^- + p + \gamma$ .
- Measure the off-forward Compton amplitude
- Compton form factors at large  $-q^2$

$$\text{CFF} = \int_{-1}^1 dx \left( \frac{1}{x - \xi + i\varepsilon} \pm \frac{1}{x + \xi + i\varepsilon} \right) \text{GPD}$$

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## Difficulties:

- deconvolution problem,
- spanning kinematics,
- lack of theoretical constraints.

## Lattice calculations:

- provide theoretical constraints,
- access unphysical kinematics ( $\xi = 0$ ),
- exclude models.

**Our aim:** calculate this OFCA with lattice QCD for  $\xi = 0$ .

**Other lattice calculations:** focus on leading-order.

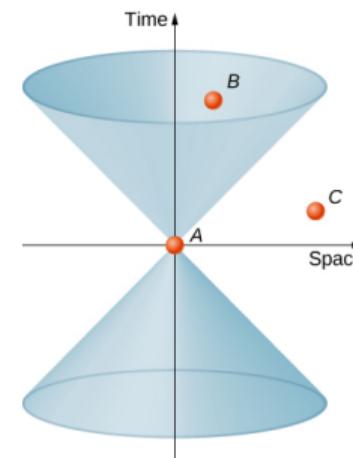
# Why can't we calculate parton distributions directly?

On lattice, we need to Wick rotate to make calculations feasible:  $t \rightarrow -i\tau$ .

## Wick rotated separation:

$$x^2 = (-i\tau)^2 - |\vec{x}|^2 = -\tau^2 - |\vec{x}|^2 < 0.$$

Separations are **spacelike**, but PDs require **lightlike**.  
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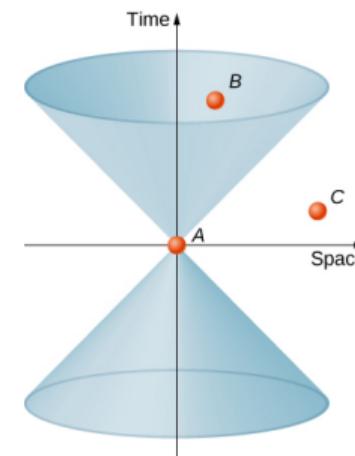
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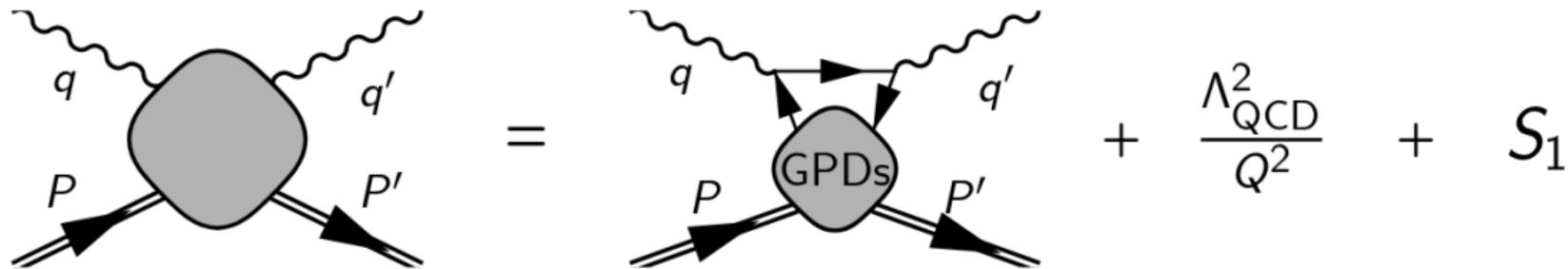
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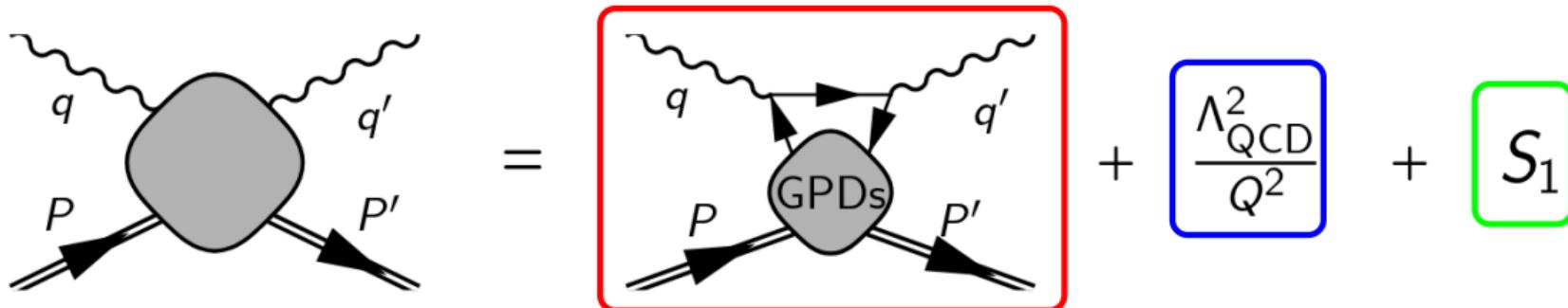
## Related quantities we can calculate

- ① moments from 3-pt functions—highest are  $n = 3$  for GPDs
- ② quasi- and pseudo-distributions
- ③ scattering amplitude in **unphysical kinematic region**; more generally two-current matrix elements

# Why the Compton amplitude?

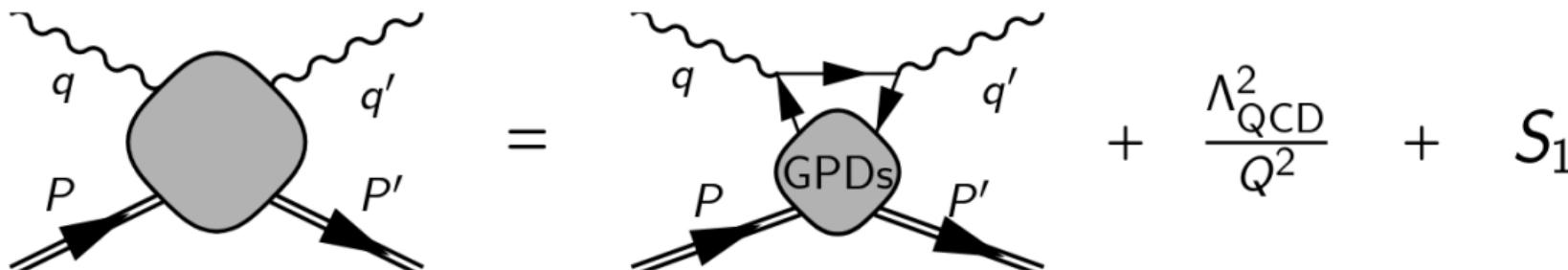


# Why the Compton amplitude?



- 3-pt moments and quasi **leading-order**
- In DVCS expt., hard scale isn't huge:  
 $Q^2 \leq 12 \text{ GeV}^2 \Rightarrow$  **corrections**
- Unknown **subtraction function**,  $S_1$

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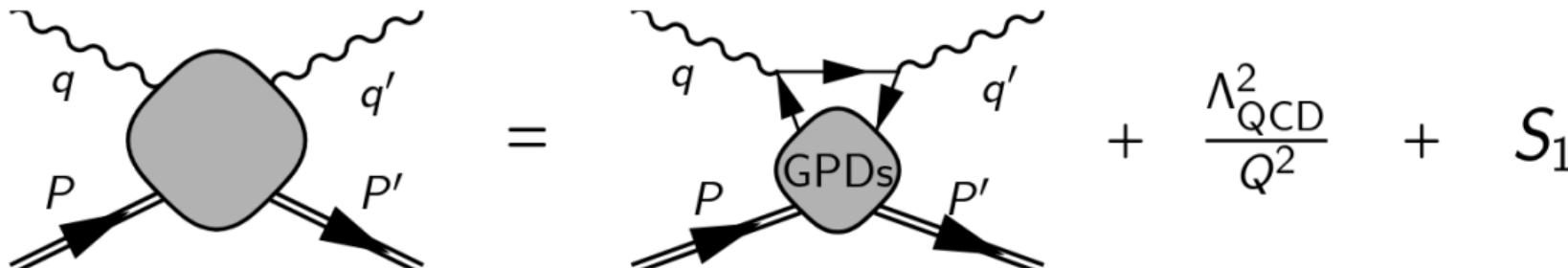


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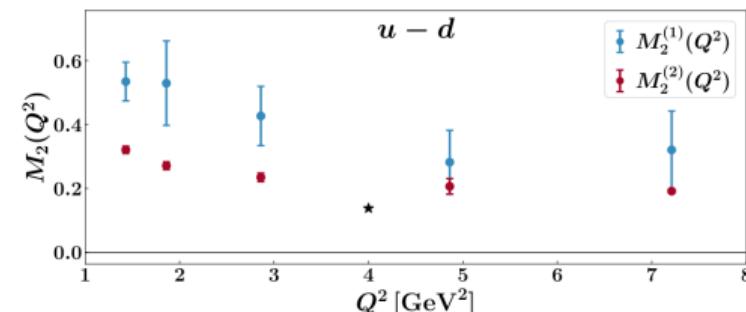


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For the *forward* ( $P = P'$ ) Compton amplitude, we have calculated these properties with Feynman-Hellmann.

## Calculate OFCA on lattice more parallels to experiment

- $Q^2$  dependence [Latt. 2021 PoS 324]
- higher-order terms [Latt. 2019 PoS 278]
- subtraction function [Latt. 2021 PoS 028]



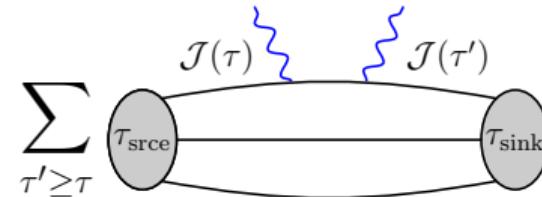
# Why Feynman-Hellmann?

## Lattice OFCA:

$$T_{\mu\nu} = \sum_{z_\mu} e^{\frac{i}{2}(q+q')\cdot z} \langle P' | T\{j_\mu(z) j_\nu(0)\} | P \rangle.$$

Requires **4-pt function**:

$$\langle \chi(z_4) j_\mu(z_3) j_\nu(z_2) \chi^\dagger(z_1) \rangle$$



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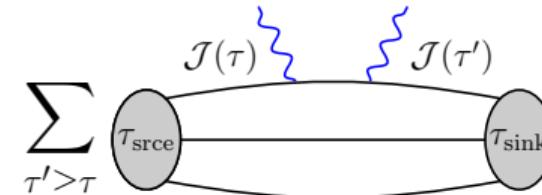
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**Instead, Feynman-Hellmann:** Calculate perturbed quark propagator:

$$S_{(\lambda_1, \lambda_2)}(x_n - x_m) = [M - \lambda_1 \cos(\vec{q}_1 \cdot \vec{x}) \gamma_3 - \lambda_2 \cos(\vec{q}_2 \cdot \vec{x}) \gamma_3]_{n,m}^{-1}.$$

Two couplings,  $\lambda_1, \lambda_2$ ; two momenta,  $\vec{q}_1$  and  $\vec{q}_2$ ; choose  $\gamma_3$ , which gives  $T_{33}$  component.

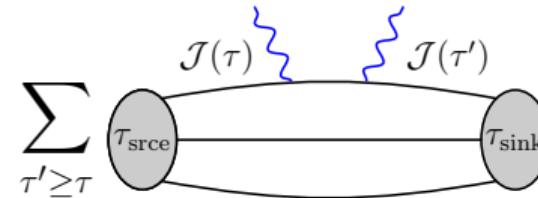
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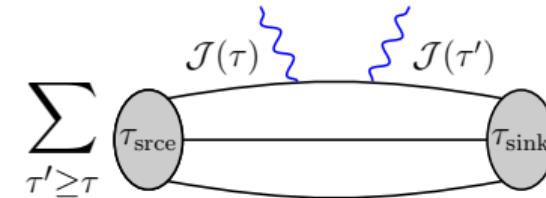
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$$S_{\vec{\lambda}} = \underbrace{S}_{\text{unperturbed}} + \sum_i \lambda_i \underbrace{S \mathcal{J}_3(\vec{q}_i) S}_{\text{three-point}} + \sum_{i,j} \lambda_i \lambda_j \underbrace{S \mathcal{J}_3(\vec{q}_i) S \mathcal{J}_3(\vec{q}_j) S}_{\text{four-point}} + \mathcal{O}(\lambda^3)$$

# Nucleon propagator

Put into nucleon propagator:  $\mathcal{G}_{\vec{\lambda}}^{dd} \simeq \langle S^u S^u S_{\vec{\lambda}}^d \rangle$ , or  $\mathcal{G}_{\vec{\lambda}}^{uu} \simeq \langle S_{\vec{\lambda}}^u S_{\vec{\lambda}}^u S^d \rangle$ .

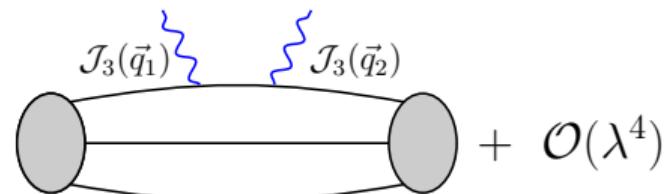
$$\mathcal{G}_{(\lambda_1, \lambda_2)}^{dd} = \text{Diagram with two horizontal lines and two shaded circles} + \sum_n \lambda_n \sum_{\tau'} \text{Diagram with two horizontal lines and two shaded circles, with a blue curly brace labeled } \mathcal{J}_3(\vec{q}_n, \tau') + \sum_{n,m} \lambda_n \lambda_m \sum_{\tau'' \geq \tau' \geq 0} \text{Diagram with two horizontal lines and two shaded circles, with two blue curly braces labeled } \mathcal{J}_3(\vec{q}_n, \tau') \text{ and } \mathcal{J}_3(\vec{q}_m, \tau'') + \mathcal{O}(\lambda^3)$$

The  $(\lambda_1)^2$  and  $(\lambda_2)^2$  terms give forward Compton amplitudes. The  $\lambda_1 \lambda_2$  term gives **OFCA**.

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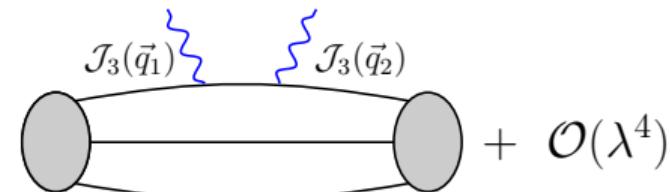


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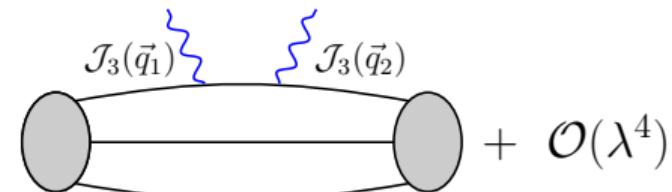
## Feynman-Hellmann relation

$$R_\lambda \equiv \frac{\mathcal{G}_{(\lambda,\lambda)} + \mathcal{G}_{(-\lambda,-\lambda)} - \mathcal{G}_{(-\lambda,\lambda)} - \mathcal{G}_{(\lambda,-\lambda)}}{\mathcal{G}_0} \simeq 2\lambda^2 \tau \frac{T_{33}}{E_N}.$$

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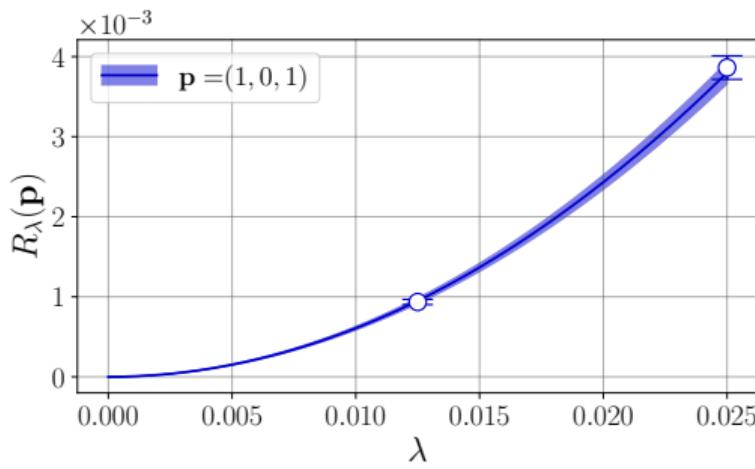
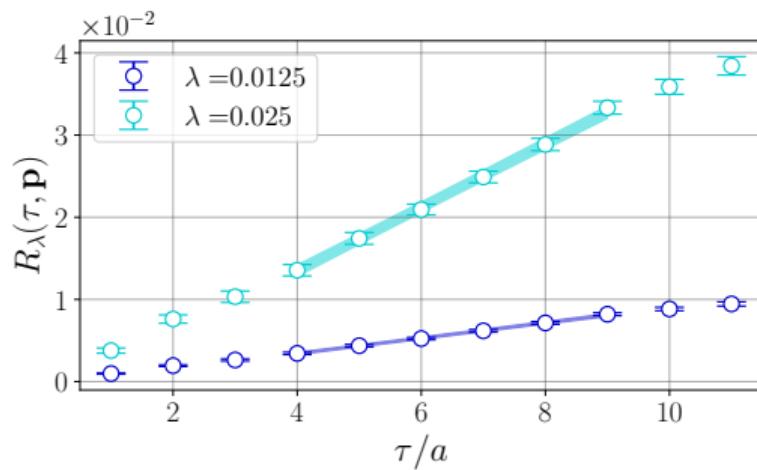
- $T_{33}$  is OFCA for  $\mu = \nu = 3$ .
- Linear in  $\tau$ , Euclidean time; quadratic in  $\lambda$ .

# Fits and signal quality

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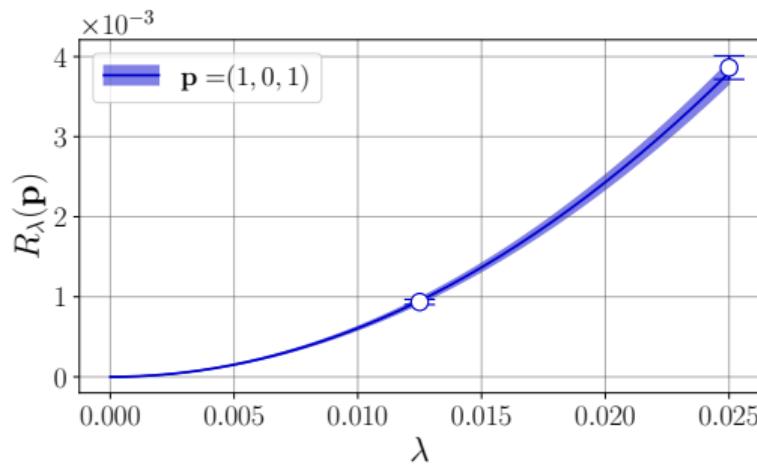
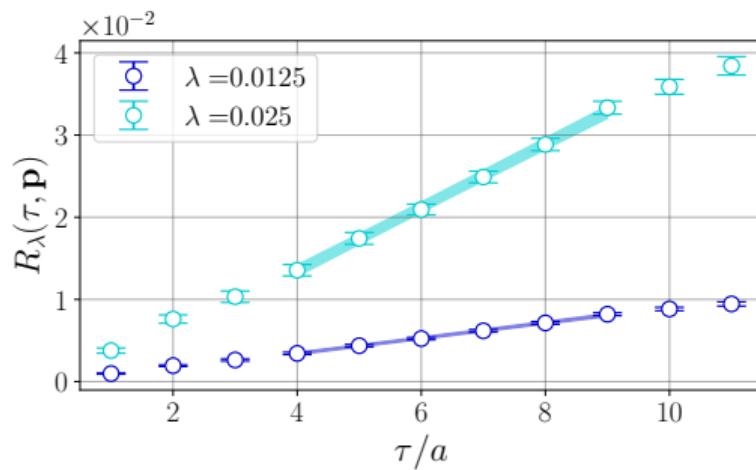
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Extract  $T_{33}$  for a given sink momentum,  $\mathbf{p}$ . Now, what do we do with it?

# Lattice Compton amplitude

OFCA parameterised in terms of Compton form factors:

$$T_{\mu\nu} = \frac{1}{2\bar{P} \cdot \bar{q}} \left[ - (h \cdot \bar{q} \mathcal{H}_1 + e \cdot \bar{q} \mathcal{E}_1) g_{\mu\nu} + \frac{1}{\bar{P} \cdot \bar{q}} (h \cdot \bar{q} \mathcal{H}_2 + e \cdot \bar{q} \mathcal{E}_2) \bar{P}_\mu \bar{P}_\nu + \mathcal{H}_3 h_{\{\mu} \bar{P}_{\nu\}} \right] + \dots$$

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In unphysical lattice kinematics,  $|\bar{\omega}| \leq 1$

$$\text{CFF}(\bar{\omega}, t, \bar{Q}^2) = 2 \sum_n \bar{\omega}^n M_n(t, \bar{Q}^2)$$

- uses  $\xi = 0$  kinematics,
- power series in

$$\bar{\omega} = \frac{2(P + P') \cdot (q + q')}{(q + q')^2}$$

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Perform OPE to match each of these CFF to their GPD contributions (see AHG et al. PRD, 2022)

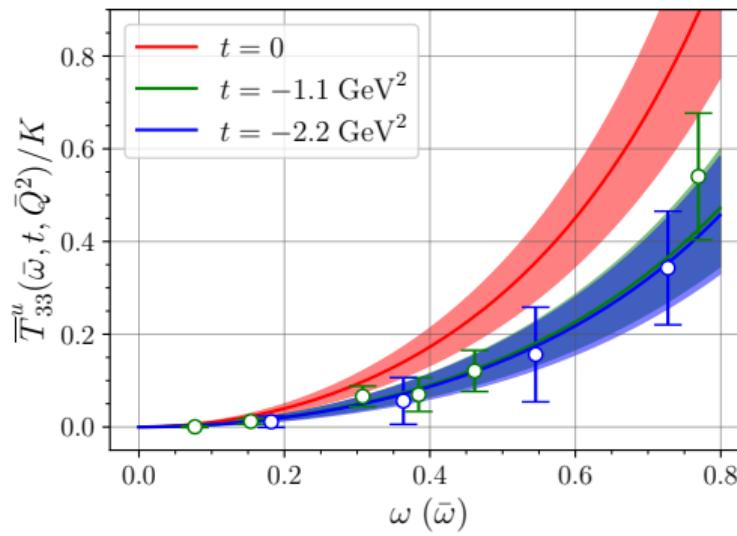
What we determine then are **Mellin moments**:

$$M_n(t, \bar{Q}^2) \xrightarrow{\bar{Q}^2 \rightarrow \infty} \int_{-1}^1 dx x^{n-1} \text{GPD}$$

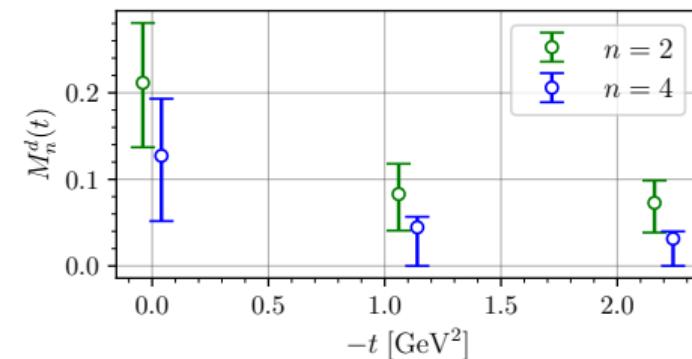
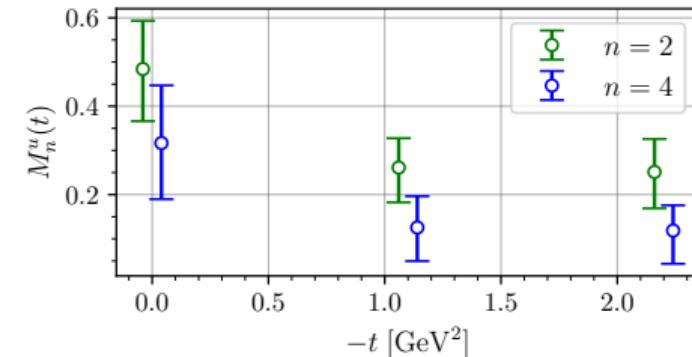
For us, this is just a guide. Like experiment, we are at finite  $\bar{Q}^2$ , so can match measurements and predict scaling

## Recap previous results

AHG et al. PRD, 2022: OFCA from Feynman-Hellmann



Fit in  $\bar{\omega}$  to extract GPD moments:



# New results

Set	$t$ [GeV $^2$ ]	$\bar{Q}^2$ [GeV $^2$ ]	$N_{\text{meas}}$
	0	4.86	10000
#1	-0.29	4.79	1000
#2	-0.57	4.86	1000
#3	-1.14	4.86	1000

- Lattice size:  $L^3 \times T = 48^3 \times 96$
- Pion mass  $m_\pi = 410$  MeV.
- For each set, calculate two couplings:  
 $\lambda = 0.0125, 0.025$

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## Improvements in new calculation:

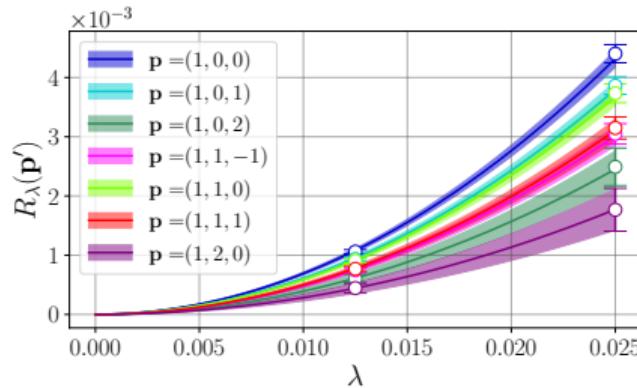
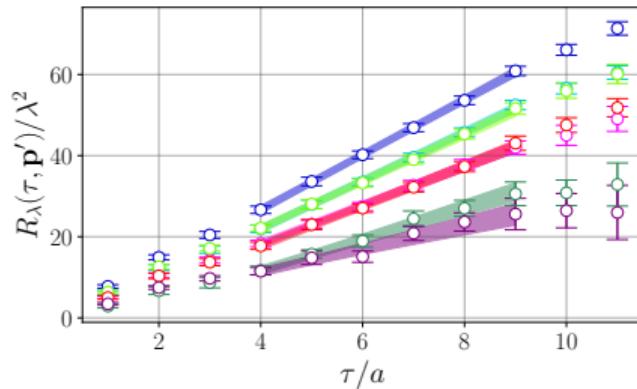
- Previously extracted linear combination of Compton form factors:

$$\mathcal{H} + \frac{t}{8m^2}\mathcal{E}.$$

Now calculate  $\mathcal{H}$  and  $\mathcal{E}$  separately

- More  $t$  values (variable of interest for GPDs)
- More  $\bar{\omega}$  values (fit parameter)
- Improved moment fitting method (see later slide)

# Applying Feynman-Hellmann



**Recall:**

$$R_\lambda \equiv \frac{\mathcal{G}_{(\lambda, \lambda)} + \mathcal{G}_{(-\lambda, -\lambda)} - \mathcal{G}_{(-\lambda, \lambda)} - \mathcal{G}_{(\lambda, -\lambda)}}{\mathcal{G}_0}$$
$$\simeq 2\lambda^2 \tau \frac{T_{33}}{E_N(\vec{p})}$$

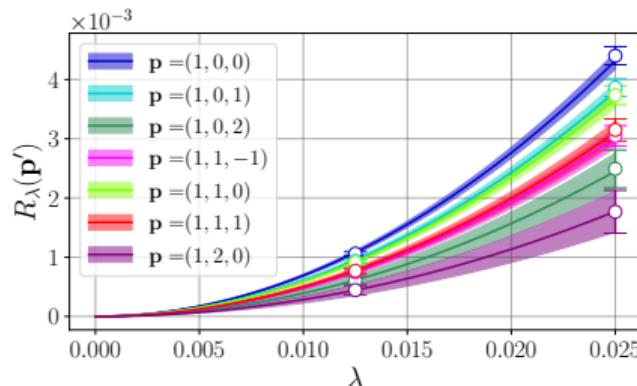
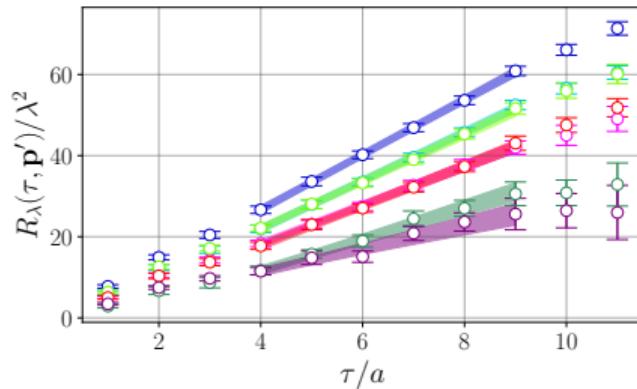
**Fits**

①  $f(\tau) = a\tau + b$

②  $g(\lambda) = c\lambda^2$

Final result is  $T_{33}(\bar{\omega})$ .

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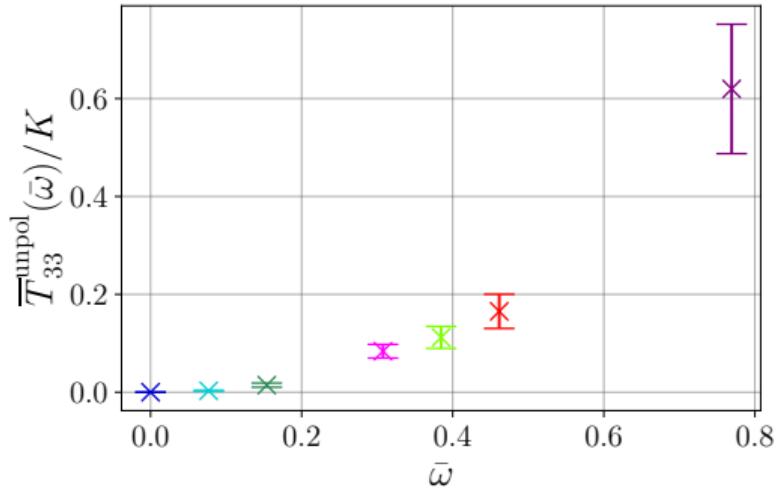
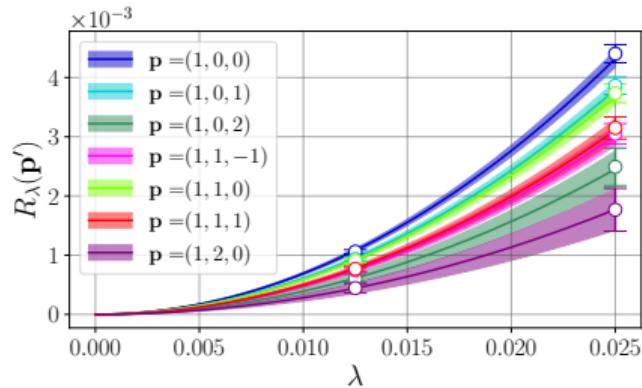
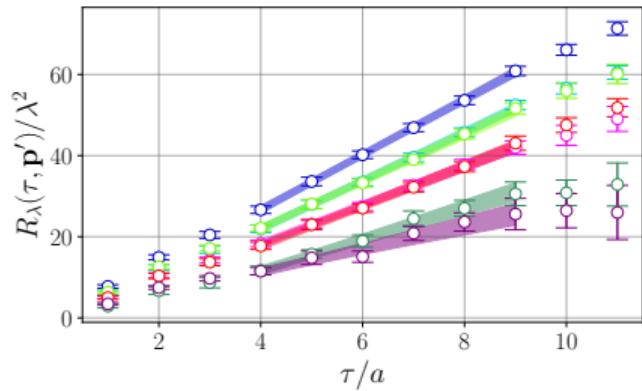
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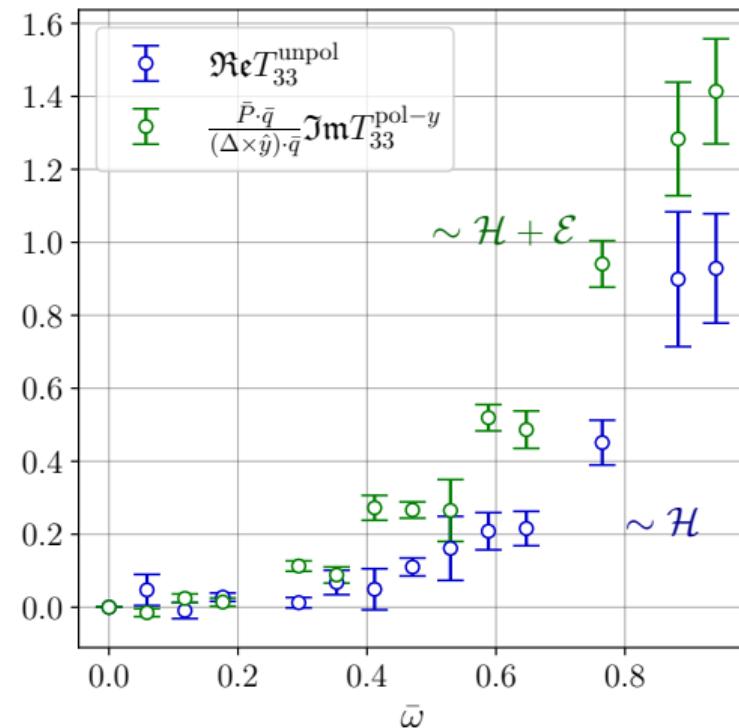


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# Compton form factors

- Can calculate unpolarised Compton amplitude ( $T_{\uparrow} + T_{\downarrow}$ )/2 or polarised ( $T_{\uparrow} - T_{\downarrow}$ )/2.
- Each is a different linear combination of  $\mathcal{H}_1$  and  $\mathcal{E}_1$ .



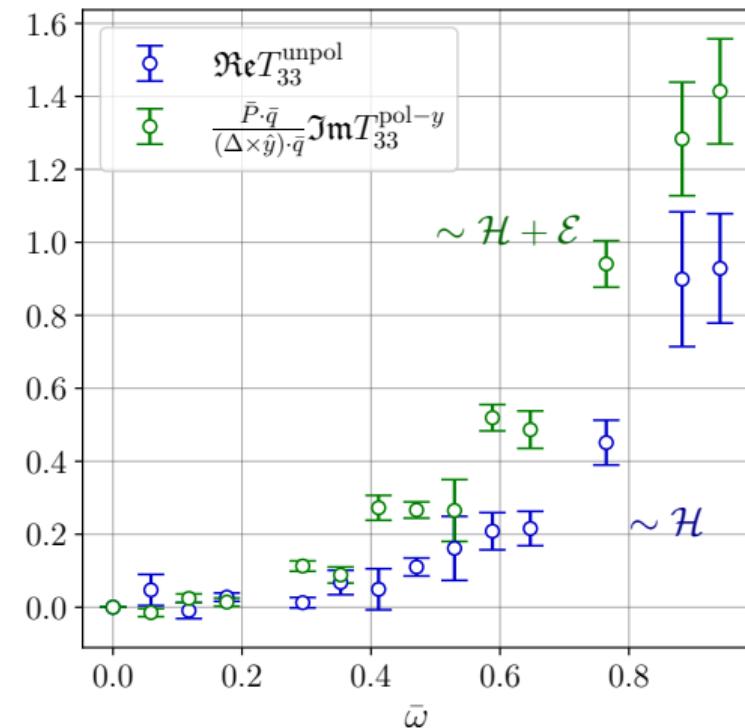
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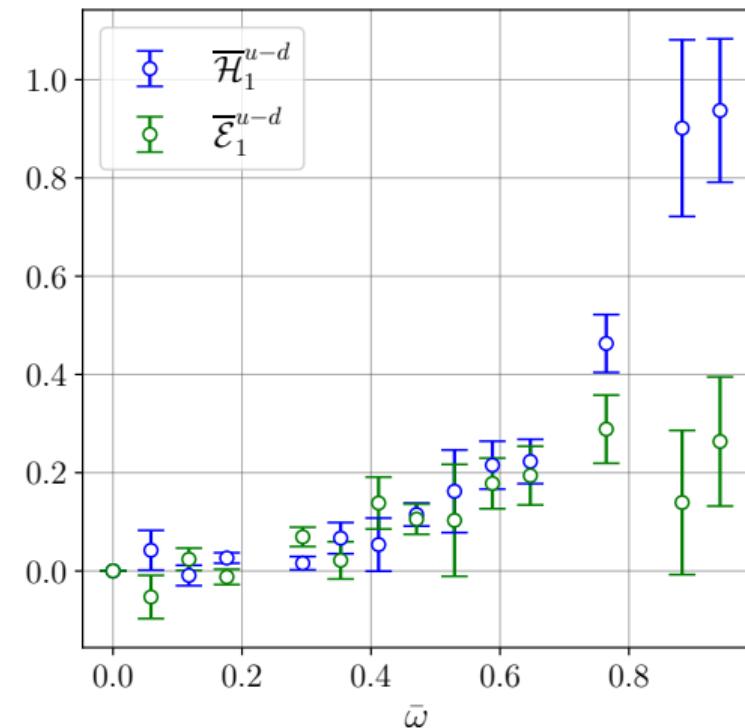
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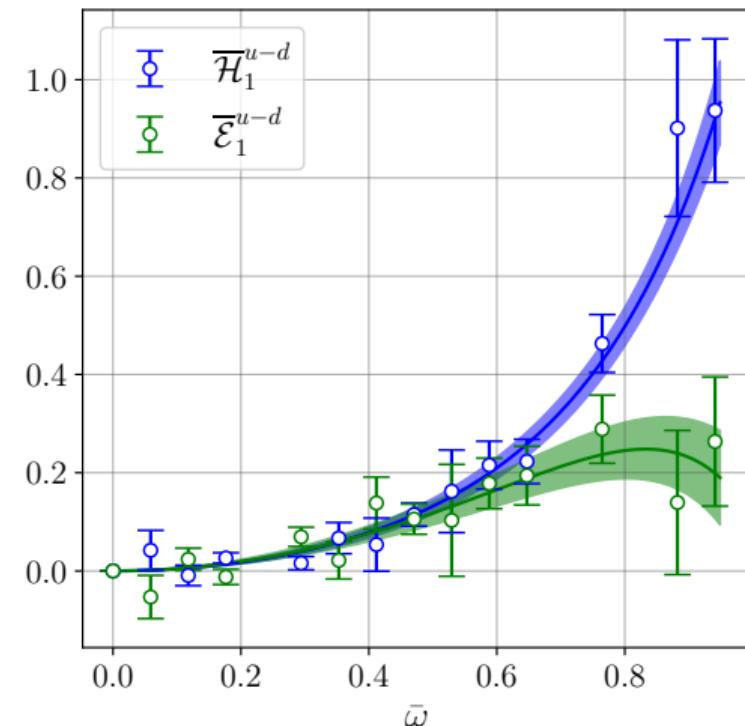
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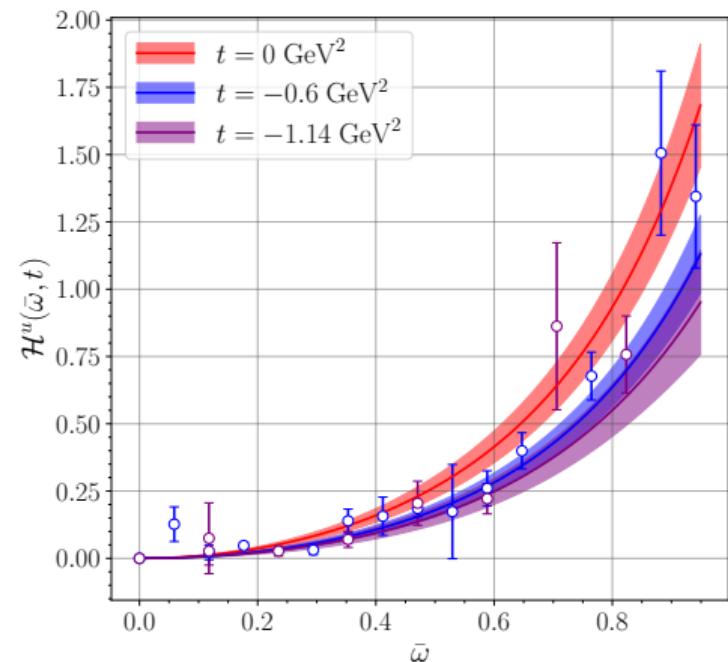
# GPD moments

Recall

$$\mathcal{H}_1(\bar{\omega}, t, \bar{Q}^2) = 2 \sum_{n=2,4,6}^{\infty} \bar{\omega}^n M_n^H(t, \bar{Q}^2)$$

As  $\bar{Q}^2 \rightarrow \infty$

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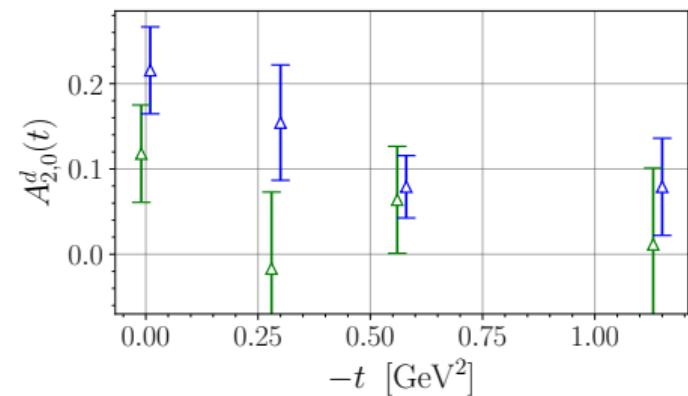
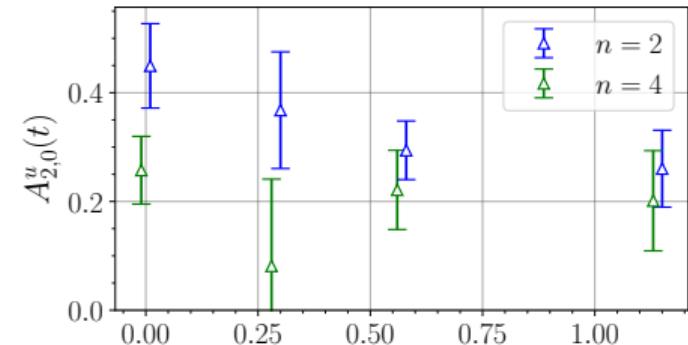
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Fit our data to

$$f_{N_{\max}}(\bar{\omega}) = M_2 \bar{\omega}^2 + M_4 \bar{\omega}^4 + \dots + M_{2N_{\max}} \bar{\omega}^{2N_{\max}}$$

Use MCMC with model-independent GPD constraints:

$$|A_{n,0}(t)| \leq A_{n,0}(0), \quad |B_{n,0}(t)| \leq \frac{2m}{\sqrt{-t}} A_{n,0}(0)$$



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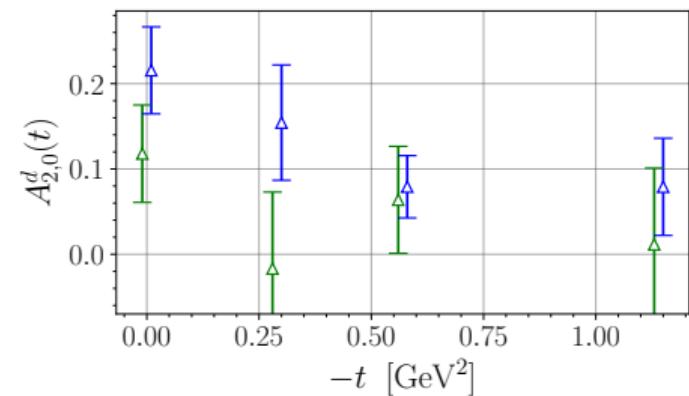
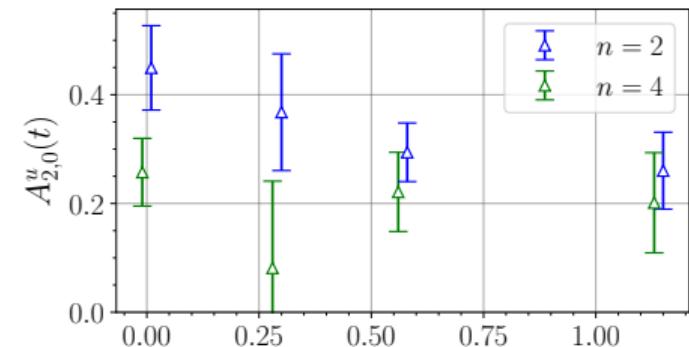
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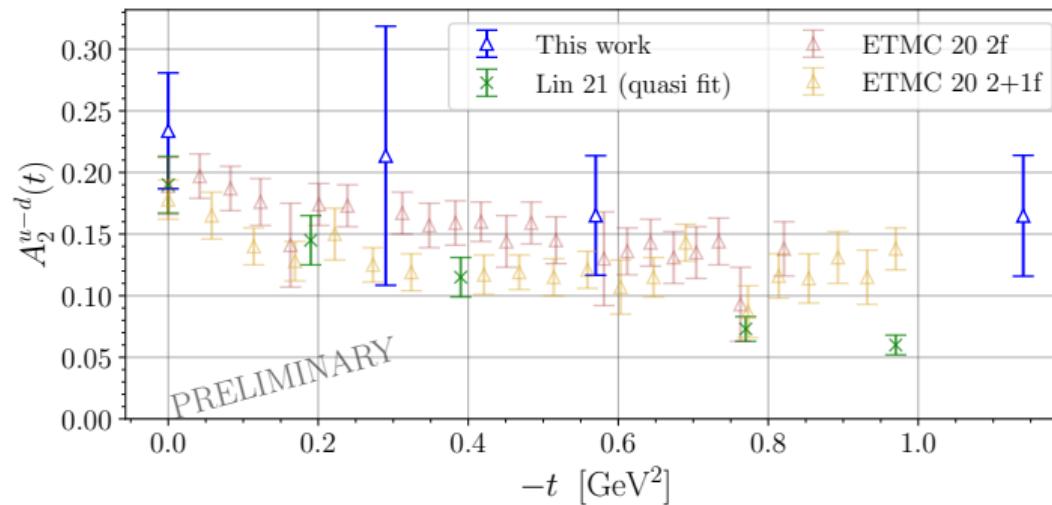


## Comparison to other moments

GPD moments also calculated with quasi [Lin '22] and three-point methods [ETMC '20, and many others]

**Note:** not a rigorous comparison; just demonstrating we're in the ball-park

- $m_\pi \approx 420$  MeV and  $\bar{Q}^2 \approx 5$  GeV $^2$
- Different systematics between all three
- Still consistent, especially for smaller  $-t$  values.

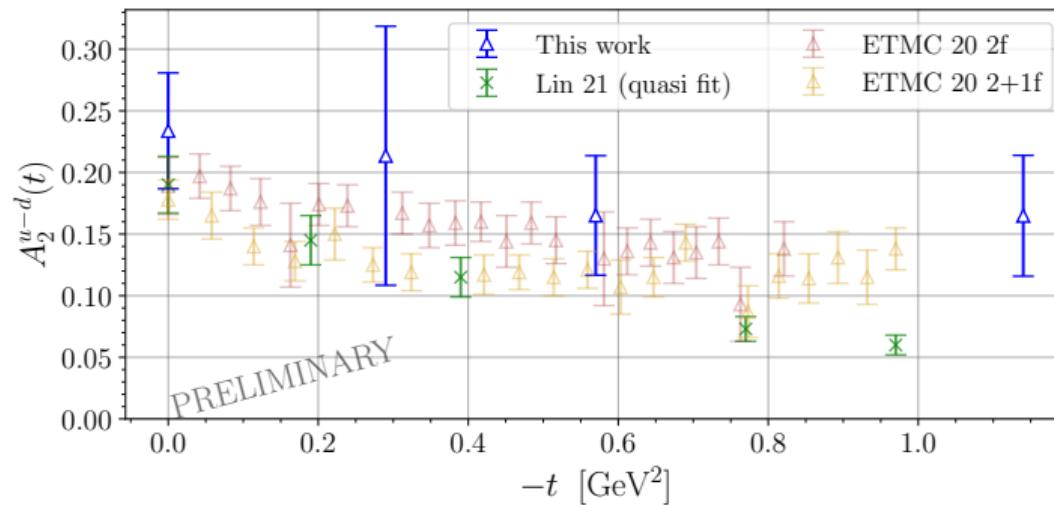


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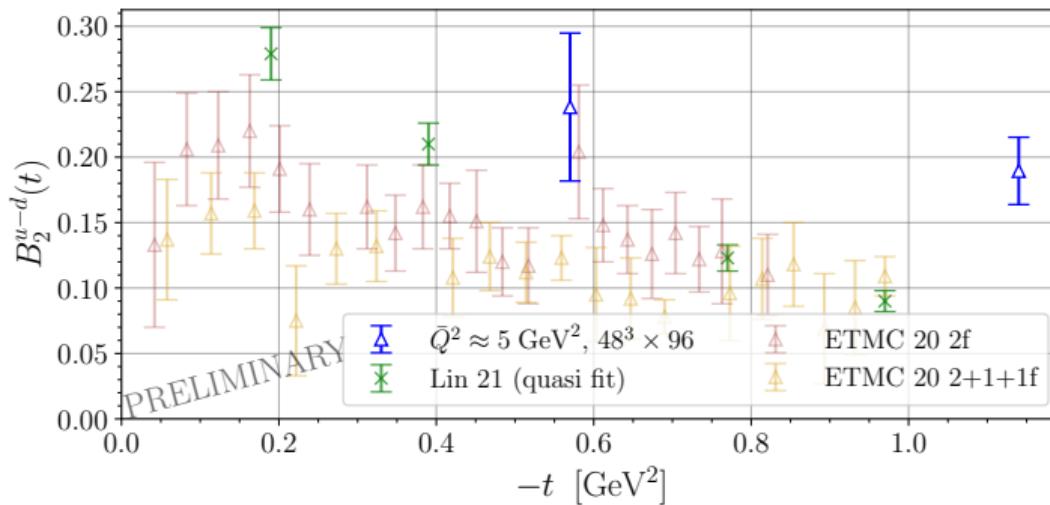


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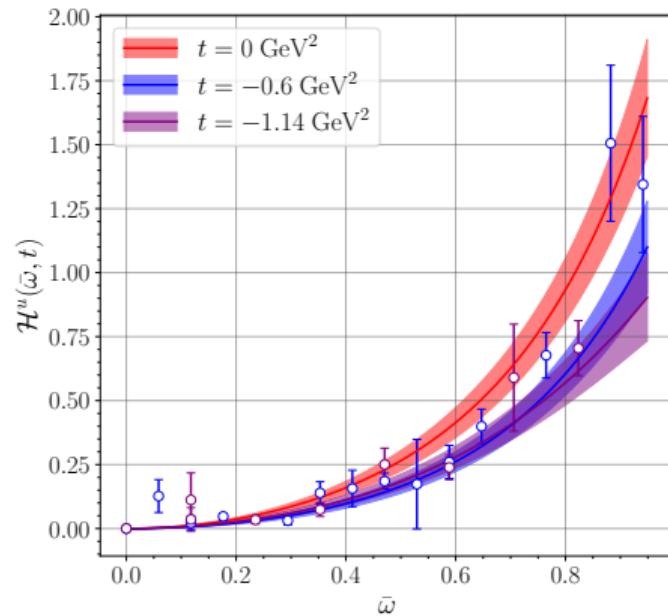
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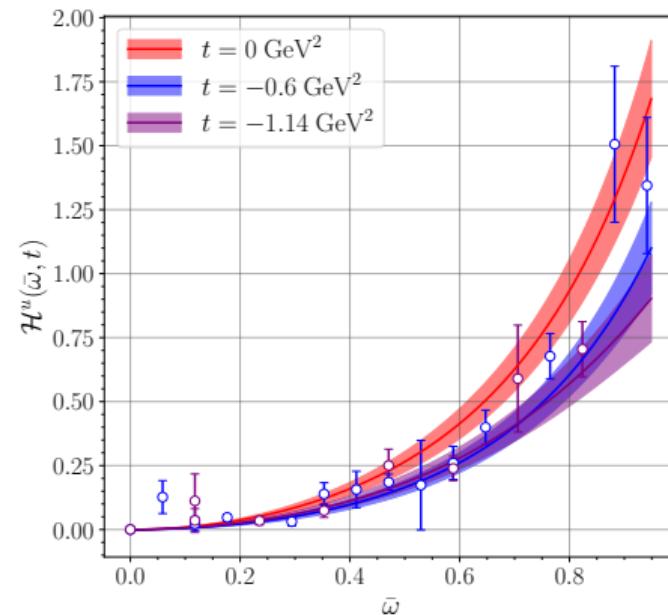


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## Outlook—connection to experiment

- vary  $\bar{Q}^2$  to get scaling—see Utku's talk this morning
- Off-forward analogue of  $\mathcal{F}_2$
- Subtraction function
- Beyond leading moments: GPD model fits, inversion methods

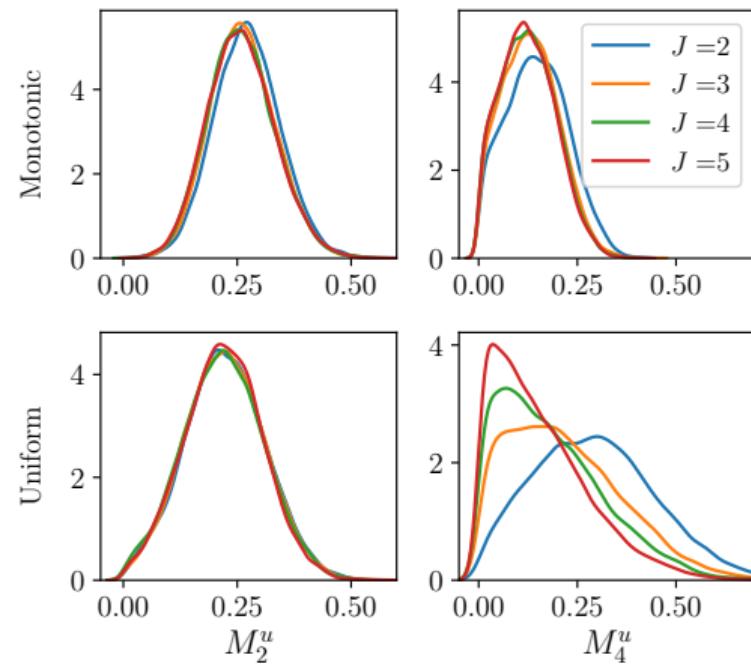


Thank you for listening—questions?

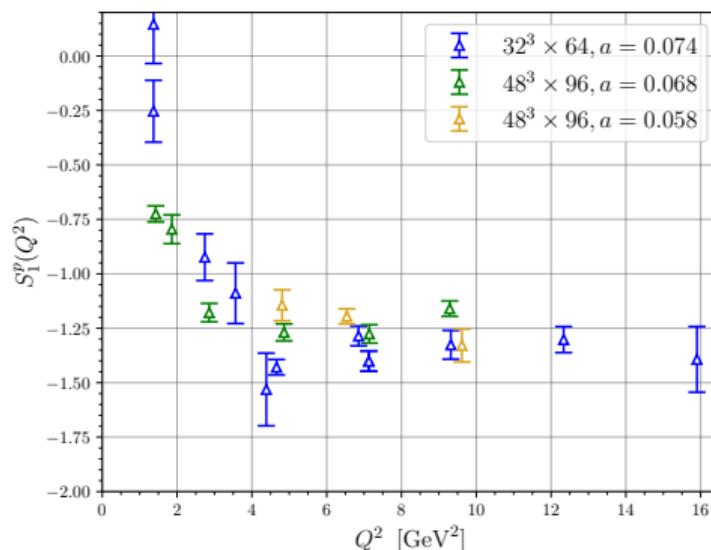
# Markov chain Monte Carlo fits

Compare fits:

- $J$  is number of moments.
- Uniform priors are  $[0, 100]$
- For  $32^3 \times 64$  dataset—few  $\bar{\omega}$  values.
- Yet to repeat with new data set or new priors.

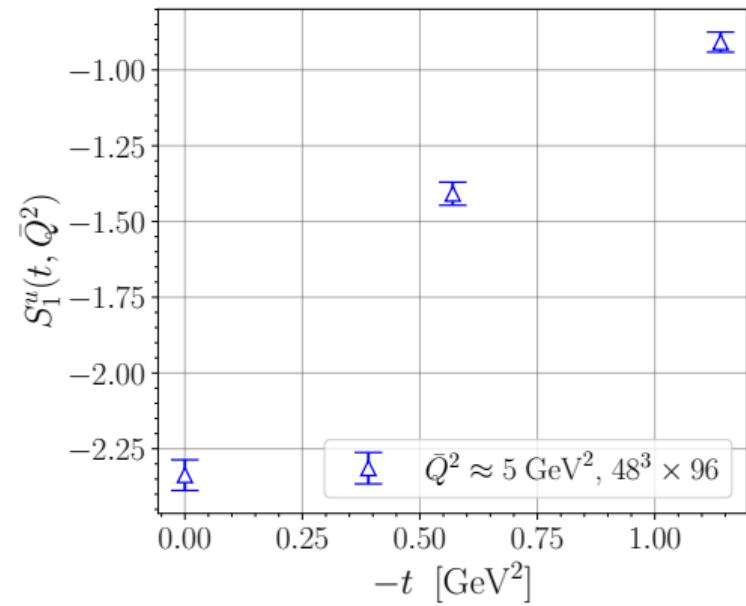
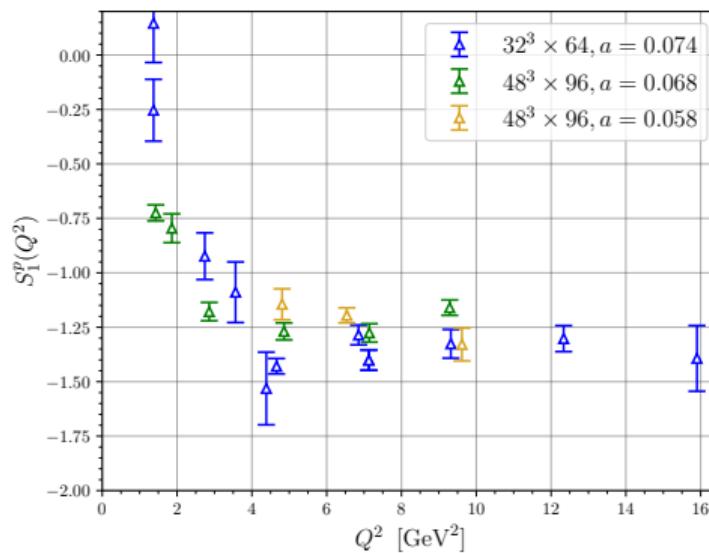


# Subtraction Term



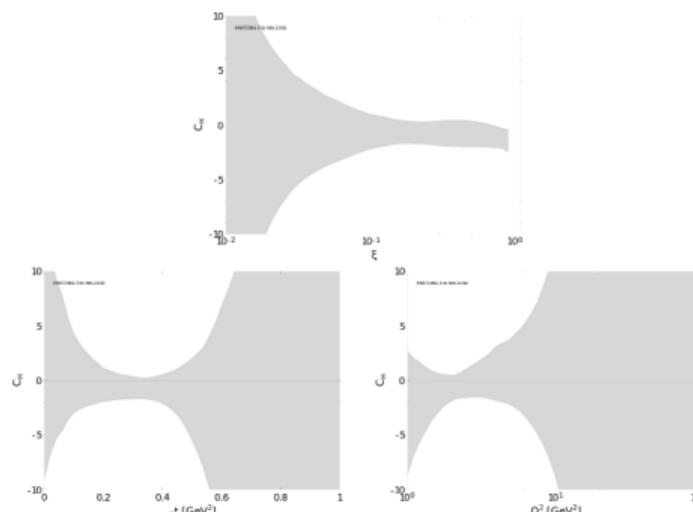
OPE predicts  $S_1(Q^2) \rightarrow 0$  with  $Q^2 \rightarrow \infty$ .

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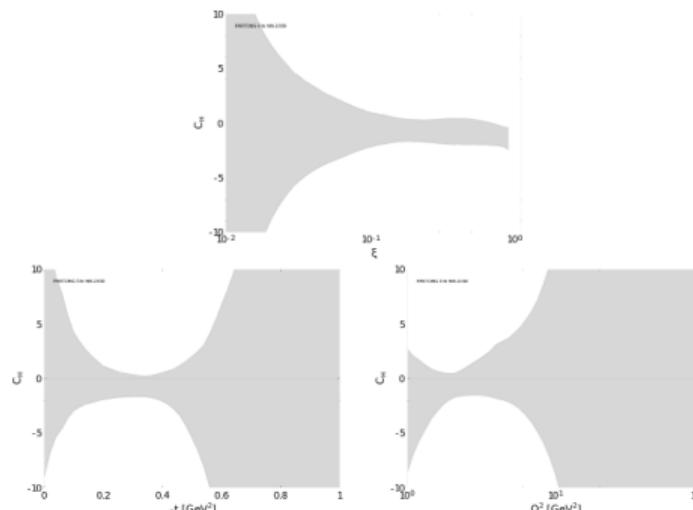
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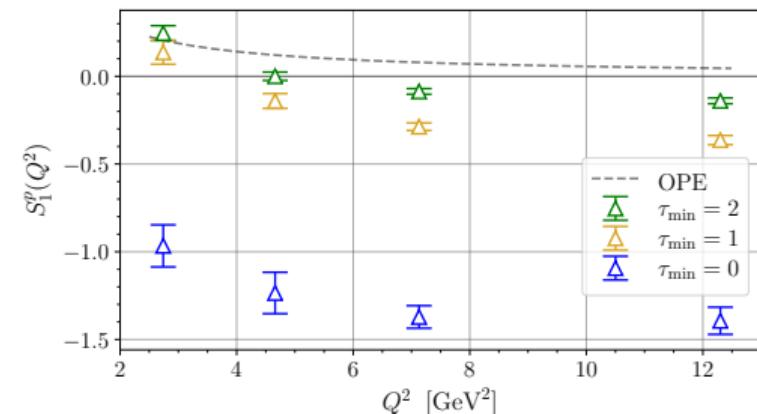


Subtraction function from DVCS; input for calculation of proton pressure distribution.

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Subtraction function from DVCS; input for calculation of proton pressure distribution.



Varies with discretisation: see forthcoming proceedings.