

The anomalous magnetic moment of the muon: is the lattice spacing small enough?

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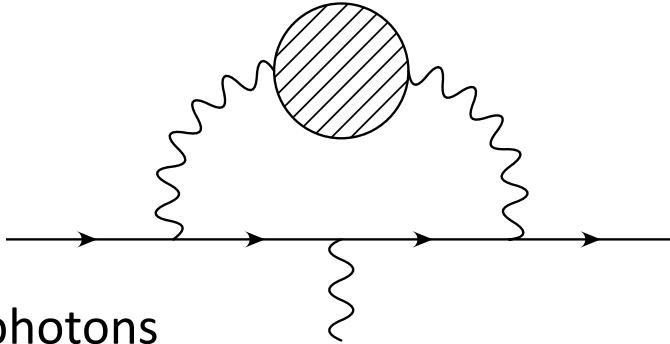
arXiv:2204:12256

Lattice 2022: Bonn, August 8-13, 2022

Outline

- The Hadronic Vacuum Polarization contribution and Chiral Perturbation Theory
 - New staggered results and the continuum limit
- NNLO ChPT works for a_μ^{HVP} if (all) pion masses are light enough (less than approx. 250 MeV)
(This corrects statements in Golterman, Maltman & Peris, arXiv:1701.08686)
It does **not** work for the “standard” (intermediate) window – no EFT method available
- Need smaller lattice spacings for staggered significantly smaller than 0.06 fm

a_μ^{HVP} and ChPT (Aubin *et al.* 2020)



a_μ^{HVP} is a low-energy quantity \rightarrow EFT of pions, muons and photons

$$\mathcal{L} = \bar{\mu} i \not{D} \mu + \frac{1}{4} f_\pi^2 \text{tr}(D_\alpha \Sigma D_\alpha \Sigma^\dagger) + \text{higher dim. operators}$$

$$a_\mu^{\text{HVP}} = \alpha^2 \int_0^\infty dQ^2 f(Q^2) \hat{\Pi}(Q^2) \quad f(Q^2) \sim m_\mu^4/Q^6 \quad (\text{Lautrup } \textit{et al.} 1972, \text{Blum 2003})$$

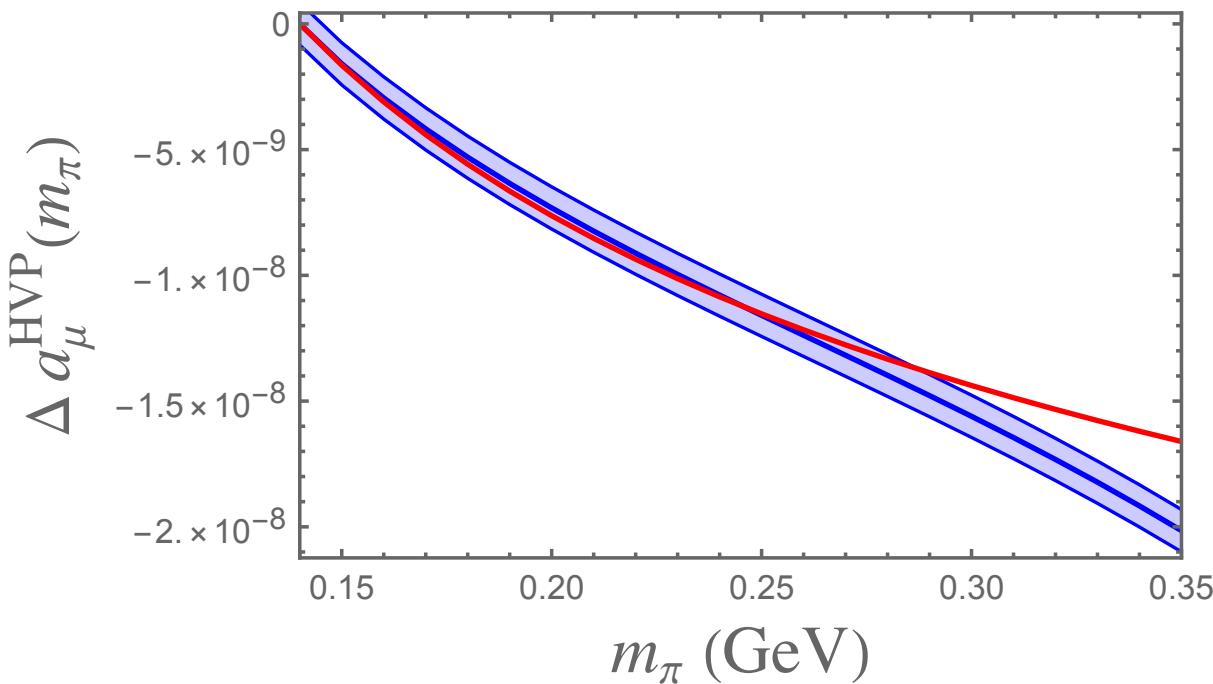
with, in ChPT, $\hat{\Pi}(Q^2) \sim (Q^2)^{k-1}$ for large Q^2 at N^kLO (modulo logs)

Hence,

- for $k \leq 2$, counter terms (LECs) are those of ChPT for pions
- for $k \geq 3$, need new counter terms, e.g., for $k = 3$, $\frac{\alpha^2 m_\mu^3}{(4\pi f_\pi)^4} (\bar{\mu} \sigma_{\alpha\beta} F_{\alpha\beta} \mu) \text{tr}(Q_L \Sigma Q_R \Sigma^\dagger)$
- $a_\mu^{\text{HVP}} = (660 \pm 160) \times 10^{-10}$ in NNLO ChPT – uncertainty from O(p⁶) LEC c_{56}

a_μ^{HVP} and ChPT: pion mass dependence and convergence

Study pion-mass dependence: $a_\mu^{\text{HVP}}(m_\pi) - a_\mu^{\text{HVP}}(m_\pi^{\text{phys}})$

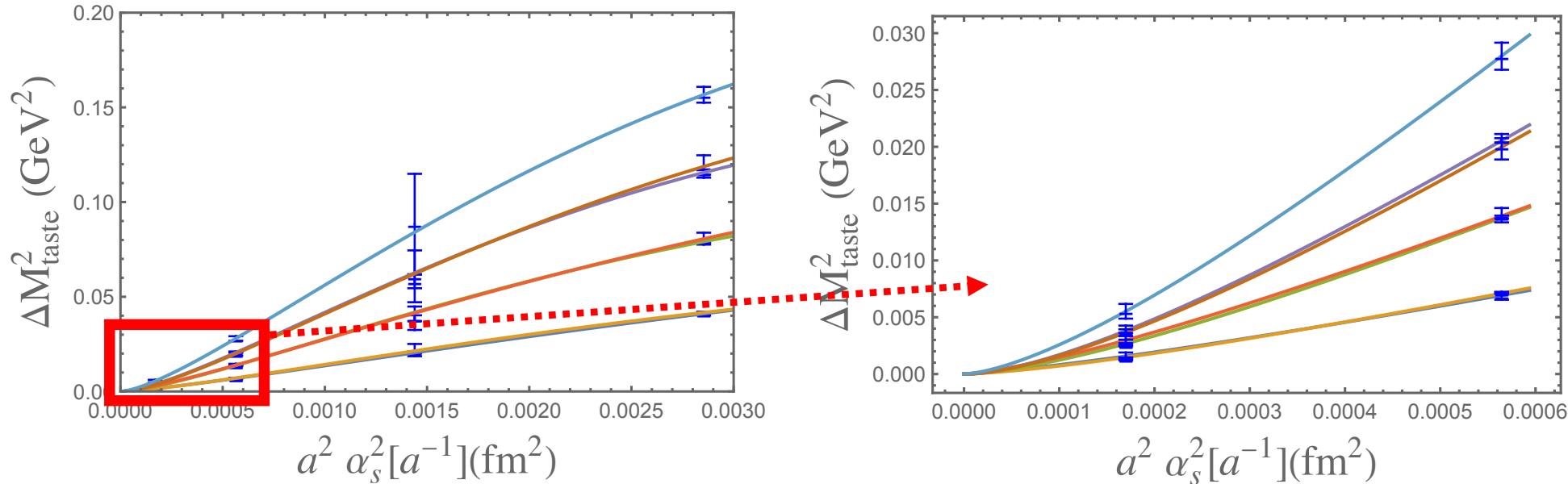


blue band: Colangelo *et al.* 2021
resums NNLO ChPT with IAM and Omnès relation
agrees well with experiment

red curve: NNLO ChPT
works well up to $m_\pi \lesssim 250$ MeV !
(note: c_{56} drops out of differences)

NNLO (staggered) ChPT useful for corrections

Taste splittings (on “HISQ” staggered ensembles – courtesy MILC collaboration)



Aubin *et al.* 2022

Taste splittings as a function of a^2 for $a = (0.057, 0.088, 0.12, 0.15) \text{ fm}$
 Heaviest pion has mass of $153, 212, 326, 418 \text{ MeV}$ (lightest pion physical)

- Serious lattice artifact, quite non-linear in a^2 -- makes continuum extrapolation difficult!
 Can correct for taste splittings using NNLO SChPT, if they are small enough: SChPT predicts a^2 behavior at leading order – not present in these fits (which include only $a^4 + a^6$)!

a_μ^{HVP} light-quark connected part with 2+1+1 HISQ staggered fermions

$$a_\mu^{\text{HVP}} = \alpha^2 \sum_{t=-T/2}^{T/2} w(t) C(t) \quad C(t) = \frac{1}{3} \sum_{\vec{x}, i} \langle j_i(\vec{x}, t) j_i(0) \rangle \quad (\text{Bernecker } et al., 2011)$$

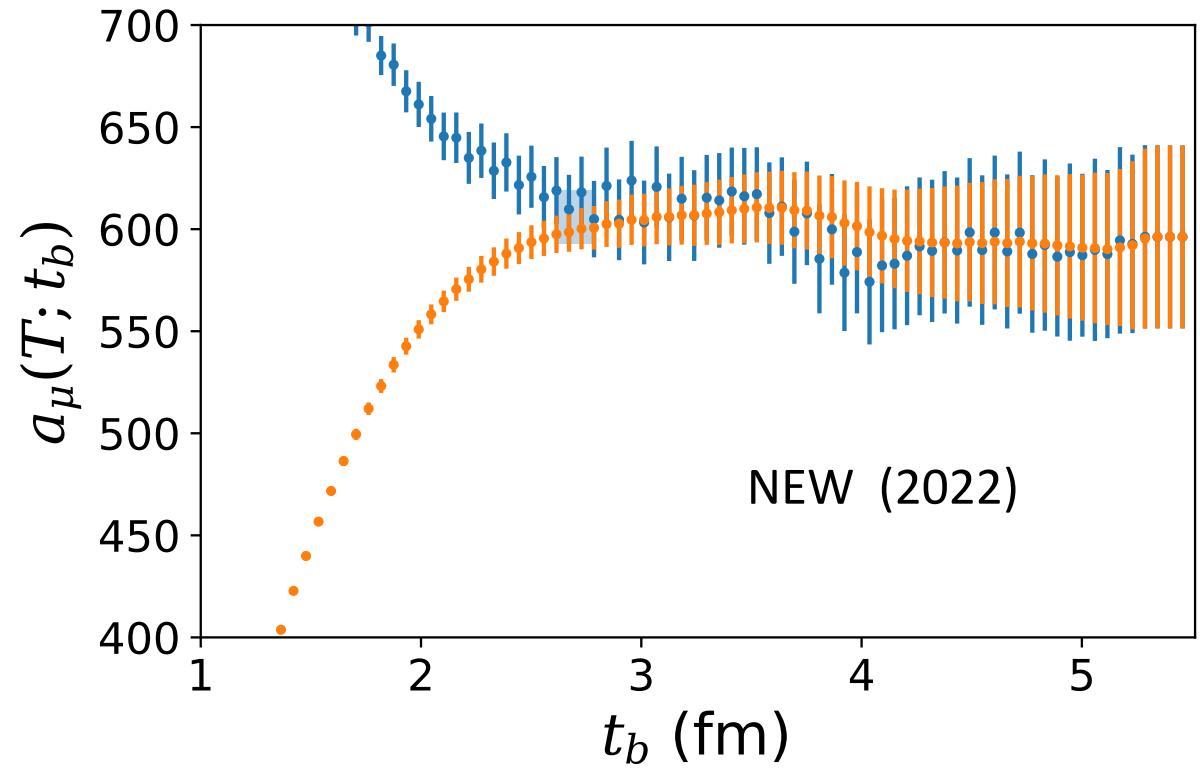
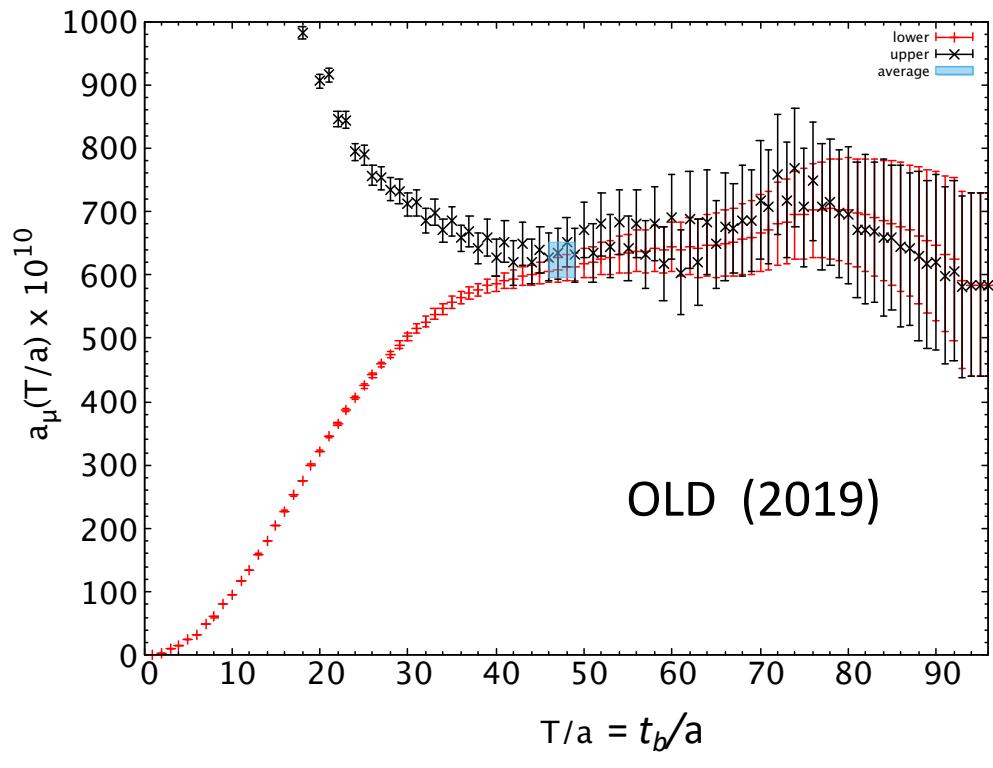
use bounding method (RBC/UKQCD 2018, BMW 2018) with transition to bounds around $t_b \approx 3$ fm and trapezoidal rule

label	a (fm)	$L^3 \times T$	m_π (MeV)	m_S (MeV)	$m_\pi L$	#configs	sep.	#low modes
96	0.05684	$96^3 \times 192$	134.3	153	3.71	77	60	8000
64	0.08787	$64^3 \times 96$	129.5	212	3.69	78	100	8000
48I	0.12121	$48^3 \times 64$	132.7	326	3.91	32	100	8000
32	0.15148	$32^3 \times 48$	133.0	418	3.27	48	40	8000
48II	0.15099	$48^3 \times 64$	134.3	418	4.93	40	100	8000

- more configs
- better separation
- more low modes

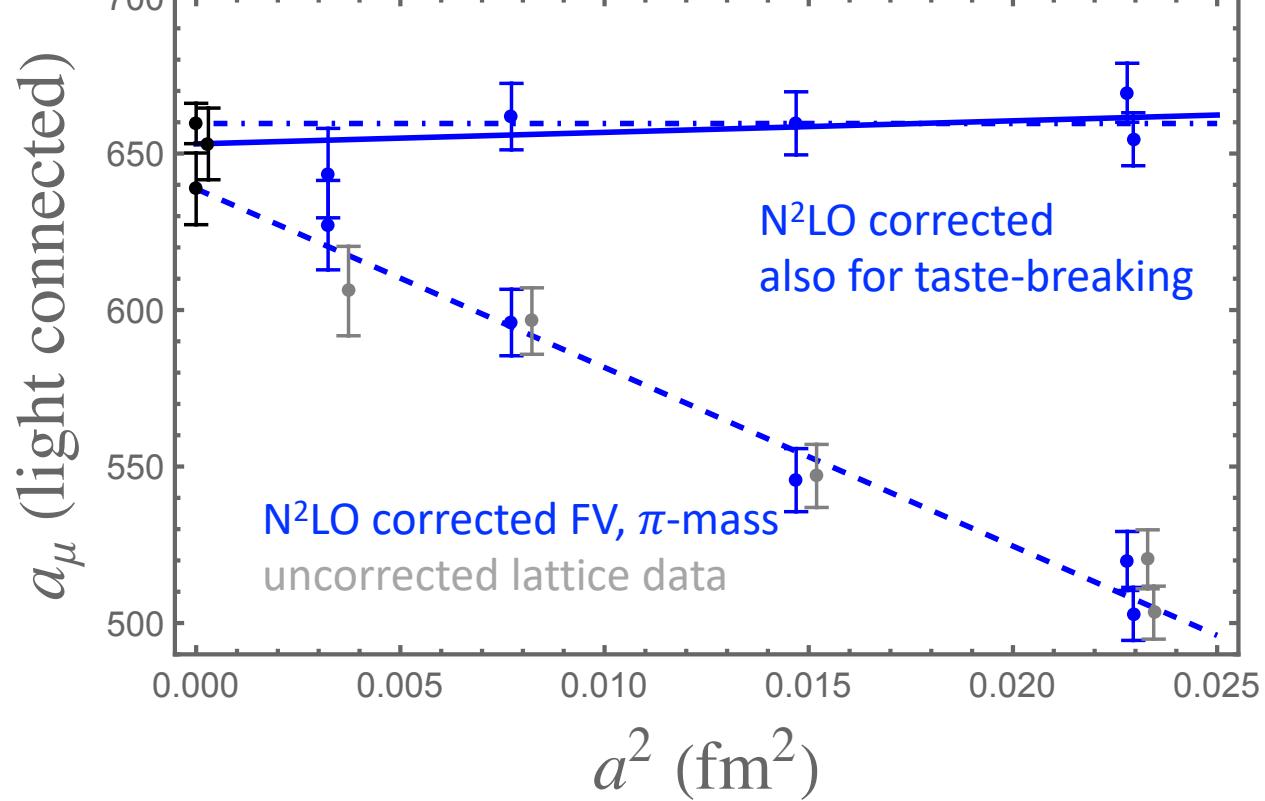
(m_S is mass of heaviest taste pion)

a_μ^{HVP} light-quark connected part with 2+1+1 HISQ staggered fermions



OLD vs. NEW on 96^3 lattice (note difference in vertical scale)

Results for a_μ^{HVP} light-quark connected



solid/dashed: linear fit
dot-dashed: constant fit

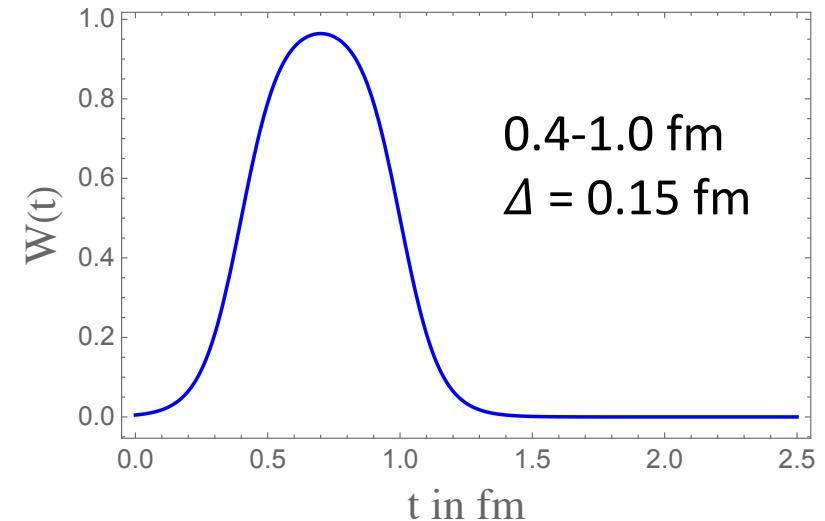
check differences: (BMW 2020)
SRHO model: NLO ChPT + ρ , γ (HPQCD 2016)

	$a_\mu(96) - a_\mu(64)$	$a_\mu(96) - a_\mu(48\text{I})$	$a_\mu(96) - a_\mu(32)$	$a_\mu(96) - a_\mu(48\text{II})$
lattice	10(16)	59(16)	103(15)	86(15)
NLO SChPT	11	28	38	37
NNLO SChPT	28	75	114	111
SRHO	35	89	129	128

Need smaller lattice spacing
and better statistics
Discard 0.12 and 0.15 fm lattice spacings?

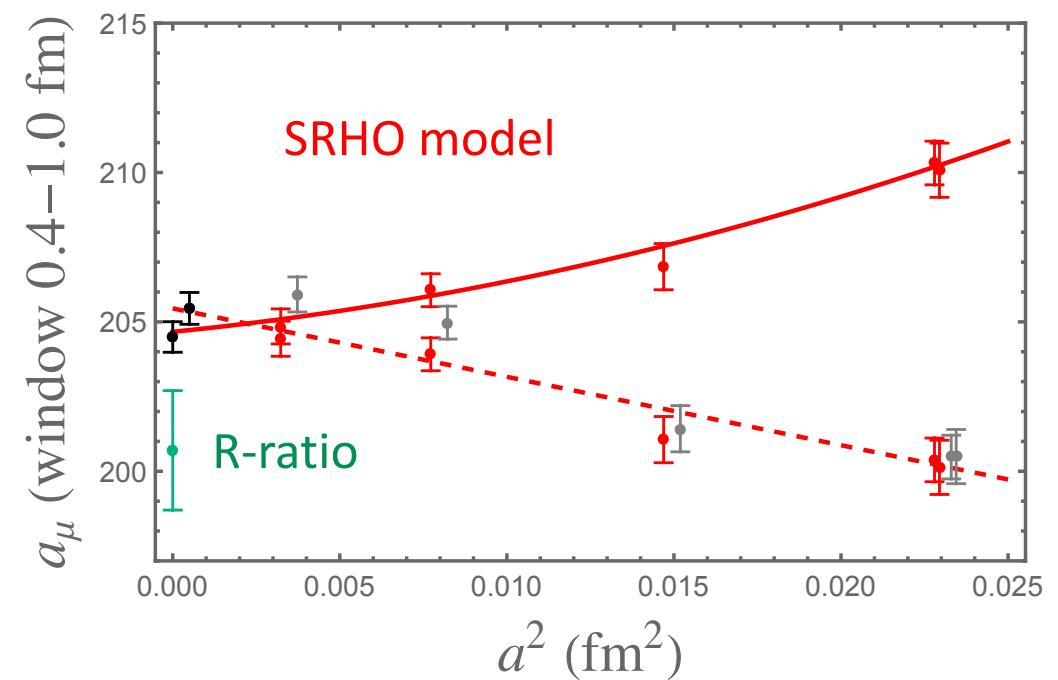
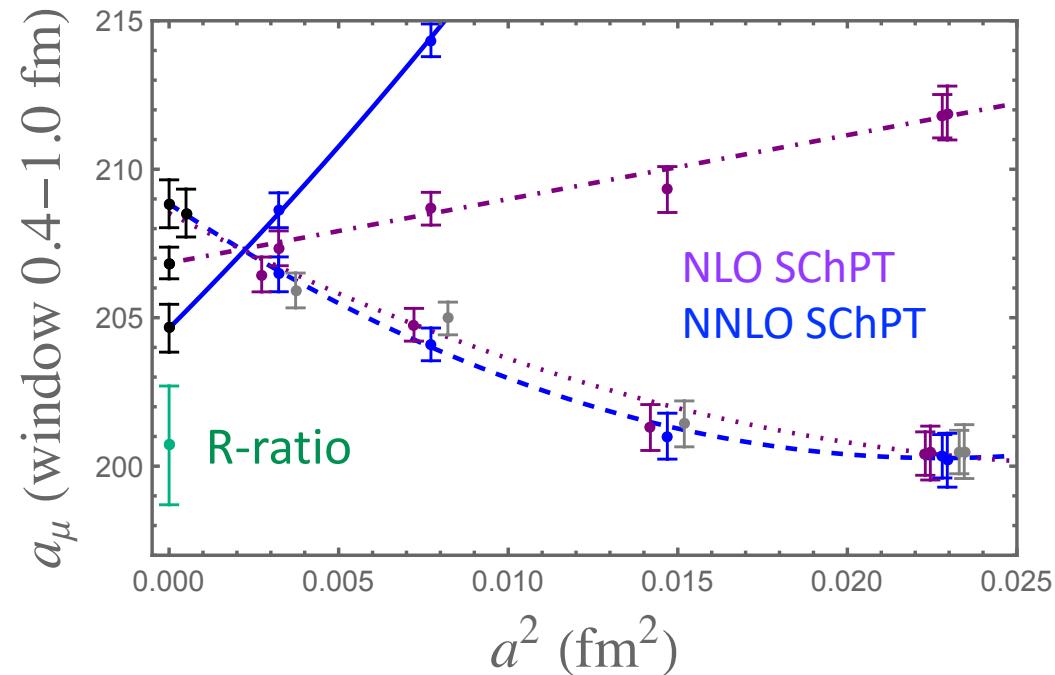
An intermediate, more precise quantity: the “window”

Introduce the “window” quantity $a_\mu^W = 2 \int_0^\infty dt W(t) w(t) C(t)$
(RBC/UKQCD 2018)



- advantages:
 - cuts out short distance (lattice spacing artifacts)
 - cuts out long distance (large-time tail, finite-volume effects)
 - can be computed very precisely on the lattice – **lattice computations have to agree!**
- **caveat:** systematic effects much smaller, but not negligible!
intermediate distance (0.4-1.0 fm) not accessible to ChPT to correct for finite-volume *etc.* effects
need to resort to models (at least at present) -- see next slide (Aubin *et al.* 2022, also BMW'20)

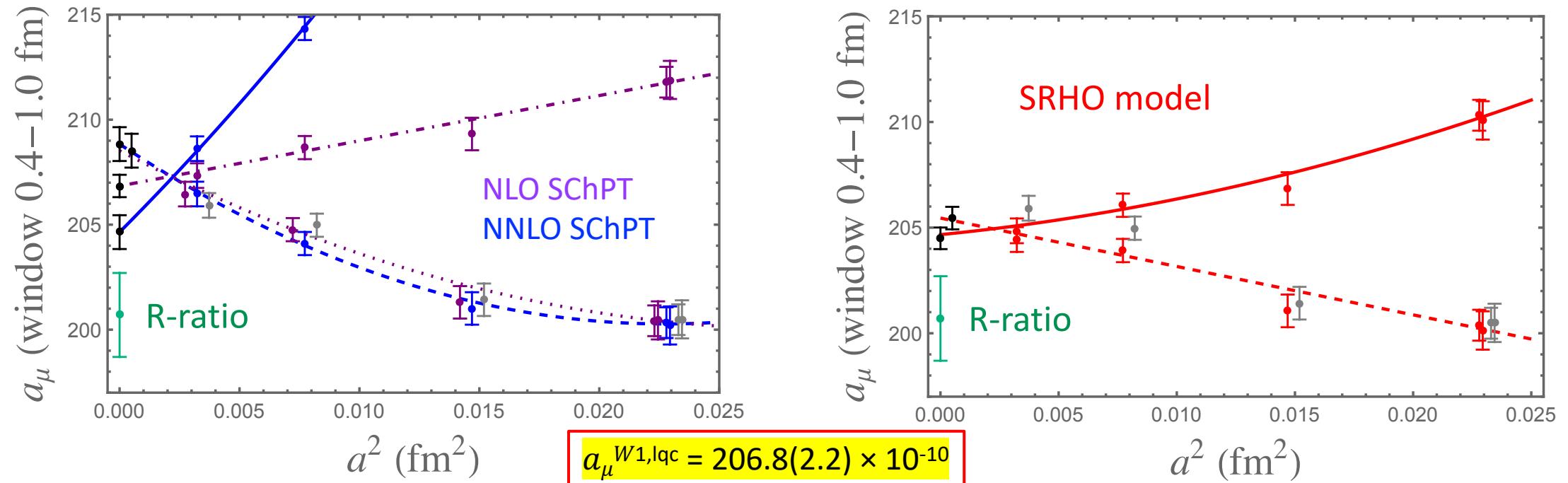
Results for window W1: 0.4–1 fm ($\Delta = 0.15$ fm), light-quark connected



	W1(96) – W1(64)	W1(96) – W1(48I)	W1(96) – W1(32)	W1(96) – W1(48II)
lattice	0.94(46)	4.49(66)	5.43(79)	5.44(58)
NLO SChPT	2.28	6.47	9.98	9.89
NNLO SChPT	6.67	21.08	35.88	35.87
SRHO	2.15	6.49	10.65	10.91

Compatible with BMW; NNLO SChPT no good
 SRHO model and NLO SChPT about the same
 but different values in continuum limit
 Need smaller lattice spacing
 Issue: no reliable calc. of corrections!

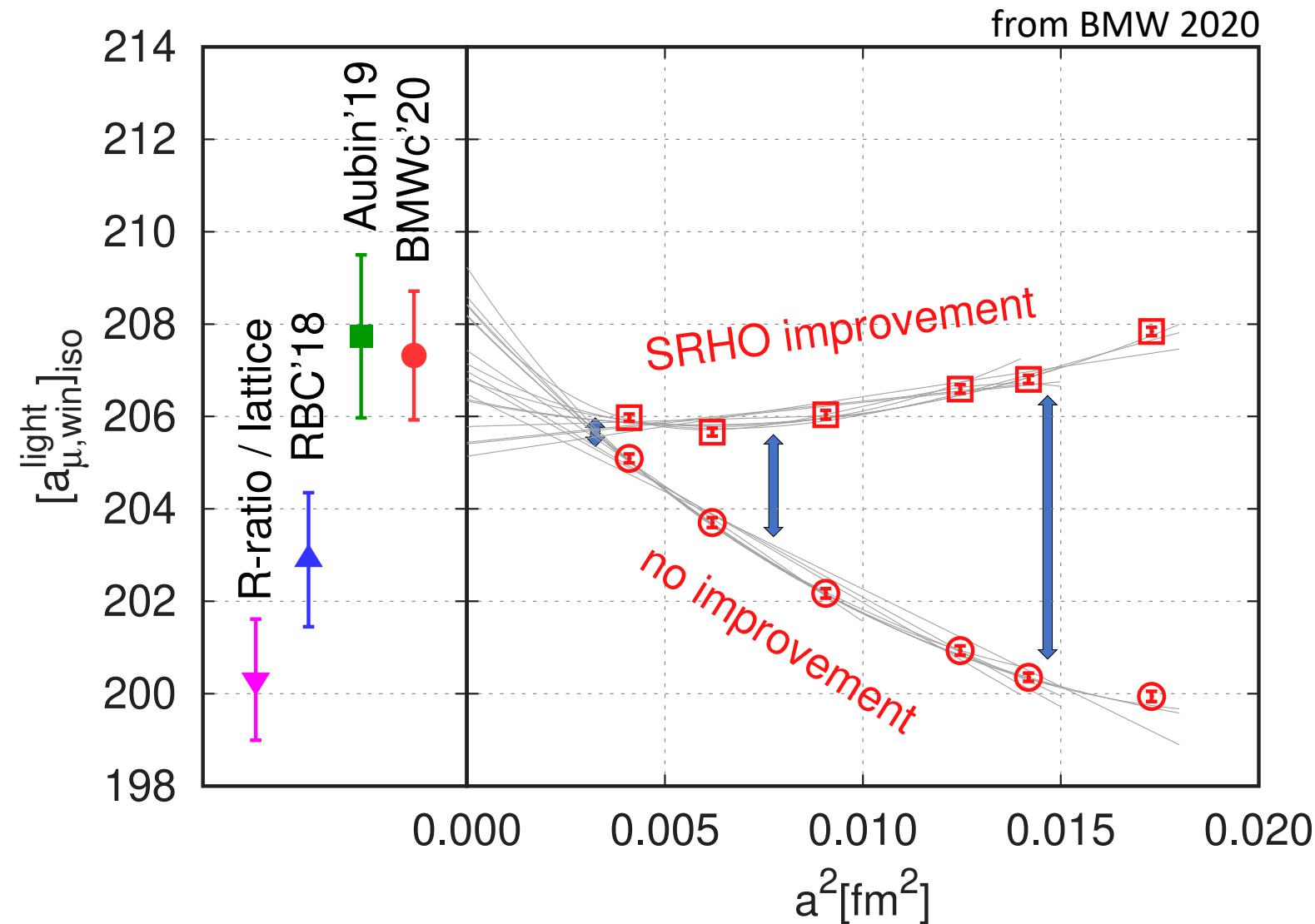
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Window with staggered fermions – continuum limit

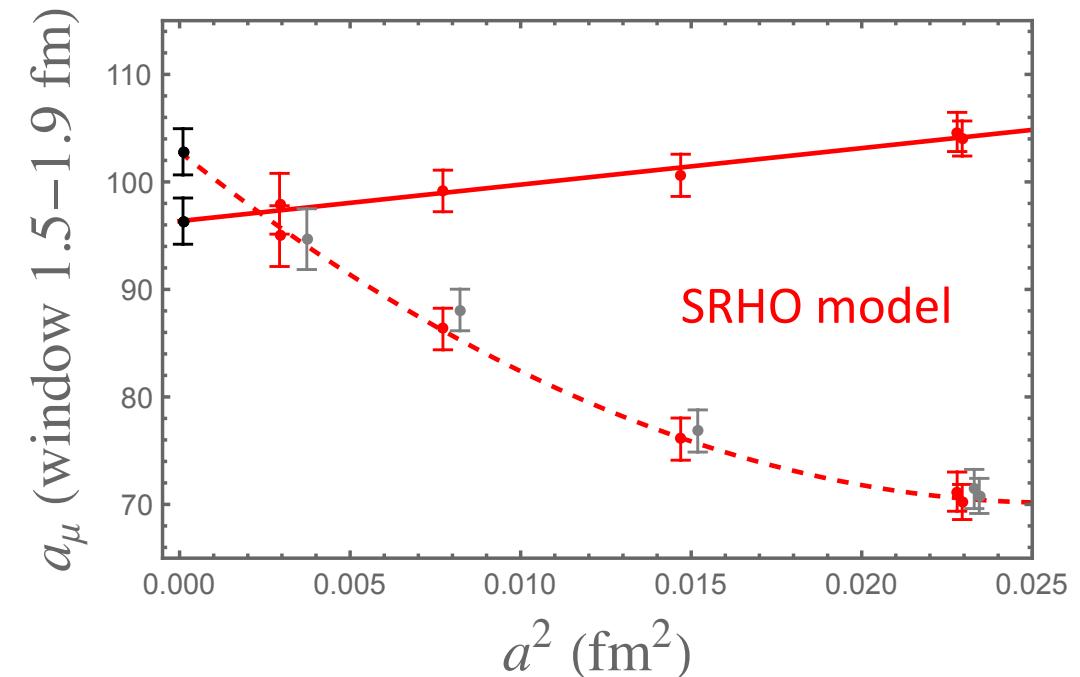
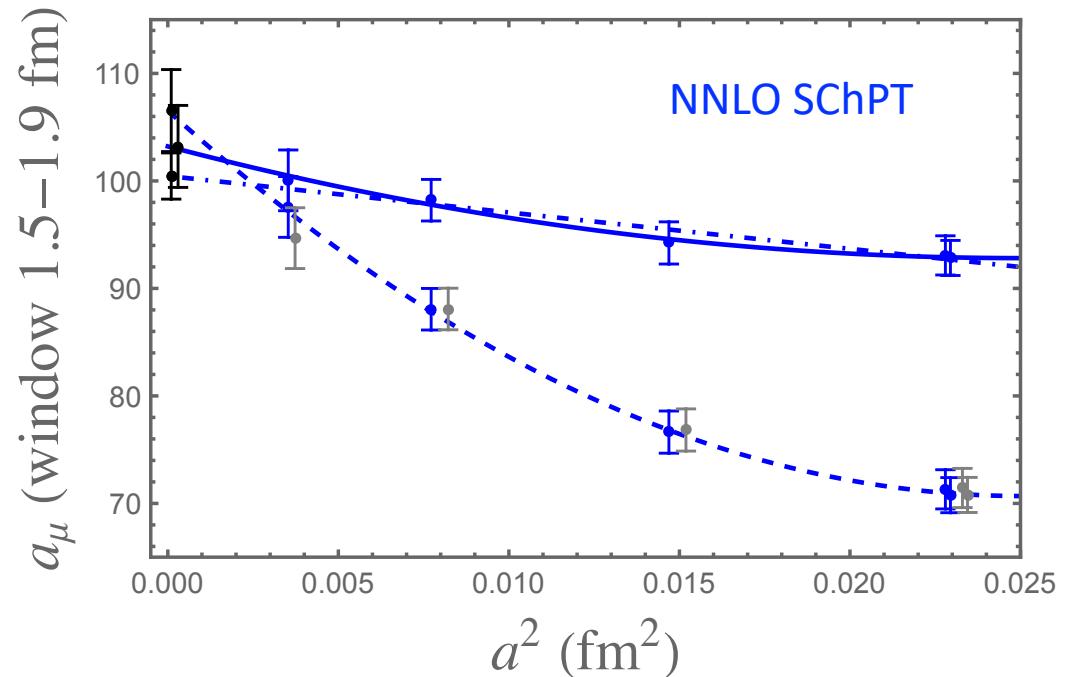


Taste-breaking effect as a function of a^2 (BMW 2020) computed with “SRHO” model (NLO ChPT plus ρ, γ)

Superimposed arrows:
Taste splittings for Aubin *et al.* '22

- Very similar taste-breaking effects, despite different lattice actions!
- Continuum extrapolation very non-linear (in a^2)

Results for a new window W2: 1.5-1.9 fm ($\Delta = 0.15$ fm), light-quark connected



	W2(96) – W2(64)	W2(96) – W2(48I)	W2(96) – W2(32)	W2(96) – W2(48II)
lattice	6.6(2.7)	17.8(2.8)	23.9(2.6)	23.2(2.7)
NLO SChPT	2.1	5.0	6.7	6.4
NNLO SChPT	4.7	12.0	16.7	16.3
SRHO	7.8	20.5	30.0	29.9

NNLO SChPT well behaved
 SRHO model and NNLO SChPT comparable
 Discard larger lattice spacings?
 Need smaller lattice spacing (& better statistics)

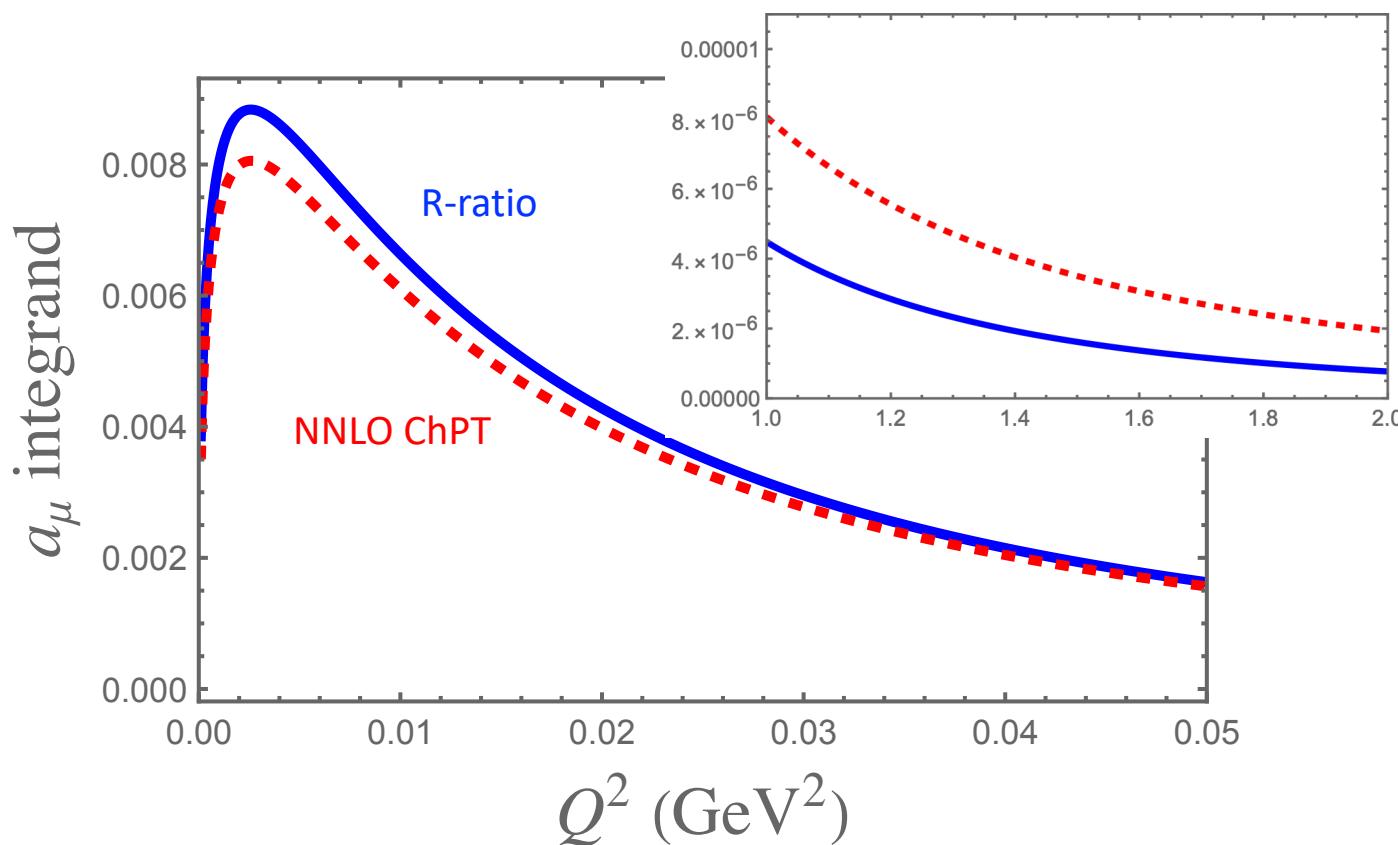
Conclusions

- a_μ^{HVP} is accessible to ChPT – if pion masses (including taste partners) are small enough
ChPT (EFT-like methods) do **not** work for short/intermediate windows – only models (at present)!
- Staggered computations need lattice spacings significantly smaller than 0.06 fm
to control the continuum limit; appear to be far from the linear a^2 regime
- Taste splittings need to be understood better (don't see a^2 regime?!)
- Consider longer-distance windows? Larger statistical errors (for now), but can use ChPT
- Improve scale setting – significant part of error

BACK-UP SLIDES

a_μ^{HVP} and ChPT: why it works

Compare integrand for a_μ^{HVP} from NNLO ChPT with $\hat{\Pi}(Q^2)$ from R-ratio (KNT data)



At NNLO 3 LECs:

f_π
 $\ell_6(m_\rho)$
 $c_{56}(m_\rho)$

$O(p^4)$
 $O(p^6)$ poorly known

Anticipates why short-distance windows are not accessible to ChPT!

a_μ^{HVP} in Staggered ChPT

(BMW 2020, Aubin *et al.* 2022)

$$a_\mu^{\text{HVP}} = \alpha^2 \int dt w(t) C(t) \quad C(t) = \frac{1}{3} \sum_{\vec{x}, i} \langle j_i(\vec{x}, t) j_i(0) \rangle \quad (\text{Bernecker } \textit{et al.} 2011)$$

$$C(t) = \frac{1}{48L^3} \sum_{\vec{p}, X} \frac{\vec{p}^2}{E_X^2(p)} e^{-2tE_X(p)} \quad \text{NLO}$$

$$\begin{aligned} & \times \left(1 - \frac{1}{4f_\pi^2 L^3} \sum_{\vec{k}, Y} \frac{1}{2E_Y(k)} - \frac{16\ell_6(\vec{p}^2 + m_X^2)}{f_\pi^2} \right. \\ & \left. + \frac{1}{24f_\pi^2 L^3} \sum_{\vec{k}, Y} \frac{\vec{k}^2}{E_Y(k)} \frac{1}{\vec{k}^2 - \vec{p}^2 + m_Y^2 - m_X^2} \right) + \text{term proportional to } c_{56} \delta^{\text{iv}}(t) \end{aligned} \quad \text{NNLO}$$

- X, Y label staggered pion tastes (16 tastes with partial degeneracies: P, A, T, V, S)
- used to compute finite-volume, taste-breaking and pion-mass-mistuning corrections
- contact terms, including c_{56} , drop out; at NNLO ℓ_6 dominates

Results for a_μ^{HVP} total

- Our value for light-quark connected: $(645.9 \pm 11.4 \pm 7.2 \pm 3.0 \pm 0.4) \times 10^{-10} = 646(14) \times 10^{-10}$
errors: (1) statistical+latt.spacing, (2) different continuum extrap., (3,4) higher orders in ChPT
 - Add data-driven strange plus light-quark disconnected from Boito *et al.* 2022: $39.4(2.1) \times 10^{-10}$
 - Add charm from RBC/UKQCD 2018, Mainz 2019, ETM 2018 (agree with BMW): $14.6(0.7) \times 10^{-10}$
 - Add QED and SIB corrections from BMW 2020, James *et al.* 2021 (SIB only) $1.4(1.4) \times 10^{-10}$
(avoid using staggered results, except for QED corrections, with BMW 2020 only complete comp.)
- Total HVP contribution: $701(14) \times 10^{-10}$ agrees with BMW 2020 and data-driven

Staggered fermions & taste breaking

- Lattice fermions with exact chiral symmetry have “species doublers”
(Karsten&Smit, Nielsen&Ninomiya, 1981): “naïve fermions” have 16 doublers
- Staggered fermions minimize the number of doublers by “spreading out spin” over the lattice:
1-component fermion \Rightarrow 16 components in continuum = (4 spin) \times (4 “tastes” = flavors),
16 components on corners of hypercube: hence all symmetries broken like rotational symmetry
- only discrete subgroup of $SU(4)$ “taste” $\times SO(4)_{\text{rot}}$ remains
 - ⇒ 16 charged pions made of 4 up and 4 down quarks split into 8 non-degenerate multiplets;
only one exact Nambu—Goldstone (NG) pion (one exact U(1) axial symmetry)
- Even if the NG pion is physical, the other 15 pions are heavier \Rightarrow “taste splittings”;
lattice spacing artifact: disappears in the continuum limit
- To reduce number of quarks on loops: take 4th root of determinant – another talk!

Data-driven strange-connected plus disconnected part

(Boito *et al.* 2022)

- Estimate strange-connected plus disconnected part:

In isospin symmetric world, equals $(I = 0) - \frac{1}{9}(I = 1)$

- Most exclusive modes: identify isospin from G-parity

(perturbation theory used above 1.937 GeV (KNT 2019) or 1.8 GeV (DHMZ), error includes DVs)

- $K\bar{K}$: use $\tau \rightarrow K\bar{K}\nu_\tau$ and CVC; $K\bar{K}\pi$ $I = 0/1$ split from BaBar 2007 Dalitz-plot analysis
- Remaining G-parity-ambiguous modes: use conservative split ($50\% \pm 50\%$)
- QED “un”correction in $I=0-(1/9)(I=1)$ small, with strong cancellation (confirmed by BMW-based estimate)
- Mixed-isospin “un”correction: ρ , ω -region 2pi, 3pi (SIB+EM) from data: 2pi (Colangelo *et al.* 2019),

3pi (BaBar 2021)

- All QED+SIB corrections small, of order 1% or less (despite $\rho-\omega$ interference)

➤ Analysis carried out for KNT and DHMZ, good agreement

➤ Good agreement with lattice (RBC/UKQCD 2018, BMW 2020), except Mainz 2019