Double parton distributions in the nucleon from lattice simulations: Flavor interference effects

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Work done in collaboration with

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A. Schäfer *(U. Regensburg)*, **G. Bali** *(U. Regensburg)*

Content

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Introduction

Motivation:

- Double parton scattering processes (DPS), e.g. Double Drell-Yan (DDY), are important standard model contributions in LHC experiments, especially after high-luminosity upgrade
- Approximation via Pocket formula:

$$\sigma_{\mathrm{DPS},i_1 i_2,j_1 j_2} \approx \frac{1}{C} \frac{\sigma_{\mathrm{SPS},i_1 j_1} \sigma_{\mathrm{SPS},i_2 j_2}}{\sigma_{\mathrm{eff}}}$$

- More fundamental description by double parton distributions (DPDs) :

$$\frac{\mathrm{d}\sigma_{\mathrm{DPS},i_1 i_2,j_1 j_2}}{\mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_1' \mathrm{d}x_2'} \propto \int \mathrm{d}^2 \boldsymbol{y}\, F_{i_1 i_2}(x_1, x_2, \boldsymbol{y}) F_{j_1 j_2}(x_1', x_2', \boldsymbol{y})$$

- So far, DPDs unknown from experiments, non-perturbative objects, access via lattice simulations
- Lattice results for the pion *[arXiv:1807.03073]*, *[arXiv:2006.14826]*
- Two involved quarks $\Rightarrow$ interferences which are in general considered to be suppressed; possible interferences w.r.t. flavor, color, fermion number;

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This talk: Mellin Moments of DPDs for the nucleon from Lattice QCD. Consider flavor interference in order to get an estimate of their size
Flavor diagonal results published in *[arXiv:2106.03451]*

Content

Double Parton Distributions

- Light cone coordinates for a given 4-vector x^μ: $x^\pm = (x^0 \pm x^3)/\sqrt{2}$, $\boldsymbol{x} = (x^1, x^2)$
- Consider a proton rapidly moving in 3-direction, i.e. $p^+ \sim Q \gg \Lambda \sim m$, $\boldsymbol{p} = \boldsymbol{0}$, $p^- \sim \Lambda^2/Q$

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Definition of proton DPDs for quarks *[arXiv:1111.0910]*

$$F_{ab}(x_1, x_2, \boldsymbol{y}) := 2p^+ \int \mathrm{d}y^- \left[\prod_{j=1,2} \int \frac{\mathrm{d}z_j^-}{2\pi} e^{i x_j p^+ z_j^-} \right] \times \\ \times \tfrac{1}{2} \sum_\lambda \langle p, \lambda | \, \mathcal{O}_a(y, z_1^-) \mathcal{O}_b(0, z_2^-) \, | p, \lambda \rangle \Big|_{y^+=0}$$

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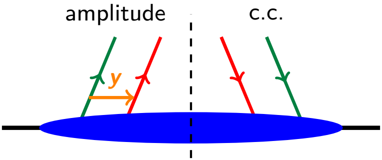

Joint probability to find quark *a* with momentum $x_1 p^+$ and quark *b* with momentum $x_2 p^+$ at transverse distance $\boldsymbol{y}$ ($|x_1| + |x_2| \leq 1$)

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Light cone operators

$$\mathcal{O}_a(y, z^-) = \bar{q}(y - \tfrac{z}{2}) \Gamma_a q(y + \tfrac{z}{2}) \Big|_{\boldsymbol{z}=\boldsymbol{0}, z^+=0}$$

- $\bar{q}$, q quark operators for certain flavor (light-like distance z^-)
- Γ_a quark polarization

Double Parton Distributions

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Twist-2 components: Quark polarizations

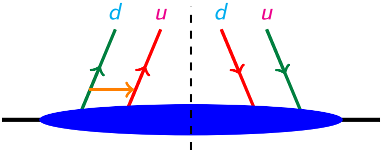
operators	twist-2 comp.	polarization
$V_q^\mu = \bar{q}\gamma^\mu q$	$V_q^+ = \mathcal{O}_q$	$q : q^\uparrow + q^\downarrow$ (unpolarized)
$A_q^\mu = \bar{q}\gamma^\mu\gamma_5 q$	$A_q^+ = \mathcal{O}_{\Delta q}$	$\Delta q : q^\uparrow - q^\downarrow$ (longitudinal)
$T_q^{\mu\nu} = \bar{q} i \sigma^{\mu\nu}\gamma_5 q$	$T_q^{+j} = \mathcal{O}_{\delta q}^j$	$\delta q^j : q^{\uparrow,j} - q^{\downarrow,j}$ (transverse)

Double Parton Distributions

Interference distributions

$$F_{duud}(x_1, x_2, \boldsymbol{y}) := 2p^+ \int \mathrm{d}y^- \left[\prod_{j=1,2} \int \frac{\mathrm{d}z_j^-}{2\pi} e^{ix_j p^+ z_j^-} \right] \times$$

$$\times \tfrac{1}{2} \textstyle\sum_\lambda \langle p, \lambda | \, \mathcal{O}_{du}(y, z_1^-) \mathcal{O}_{ud}(0, z_2^-) \, | p, \lambda \rangle \Big|_{y^+=0}$$



Flavor changing operators

$$\mathcal{O}_{ud}(y, z^-) = \bar{u}(y - \tfrac{z}{2}) \Gamma_a d(y + \tfrac{z}{2}) \Big|_{\boldsymbol{z}=\boldsymbol{0}, z^+=0}$$

$$\mathcal{O}_{du}(y, z^-) = d(y - \tfrac{z}{2}) \Gamma_a \bar{u}(y + \tfrac{z}{2}) \Big|_{\boldsymbol{z}=\boldsymbol{0}, z^+=0}$$

Double Parton Distributions: Factorization

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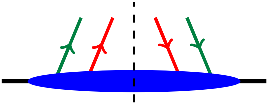
Factorization assumption I

$$\langle p | \, \mathcal{O}_a(y, z_1) \mathcal{O}_b(0, z_2) \, | p \rangle \approx \int \frac{\mathrm{d}^2 \boldsymbol{p}' \mathrm{d}p'^+}{(2\pi)^3 2p'^+} \langle p | \, \mathcal{O}_a(y, z_1) \, | p' \rangle \, \langle p' | \, \mathcal{O}_b(0, z_2) \, | p \rangle$$
$$\Rightarrow \quad F_{ab}(x_1, x_2, \boldsymbol{y}) \approx \int \mathrm{d}^2 \boldsymbol{b} \; f_a(x_1, \boldsymbol{b} + \boldsymbol{y}) \; f_b(x_2, \boldsymbol{b})$$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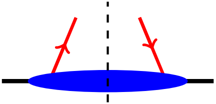
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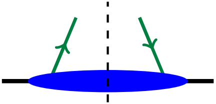

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Factorization assumption I (interference)

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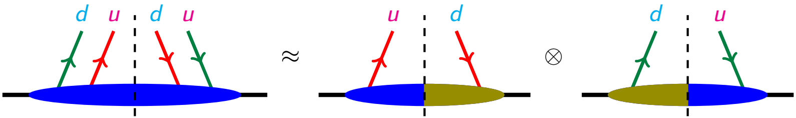

Double parton distributions on the lattice

Accessible quantities

$$\langle p|\, \mathcal{O}_a(y, z_1^-)\mathcal{O}_b(0, z_2^-)\,|p\rangle \xrightarrow[y^+=0,\ \text{twist-2}]{p^+\int \mathrm{d}y^-\,\mathrm{d}z_i^-\, e^{-iz_i x_i p^+}} F_{ab}(x_i, \boldsymbol{y})$$

(*) into basis tensors and scalar functions

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not accessible on the lattice
if $z_i^- > 0$

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Double parton distributions on the lattice

Accessible quantities

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\begin{tikzcd}
& F_{ab}(x_i, \boldsymbol{y}) \arrow[dd, "\int \mathrm{d}x_i"] \\
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& M_{ab}(\boldsymbol{y}) \\
\langle p|\, \mathcal{O}_a(y)\mathcal{O}_b(0)\, |p\rangle \arrow[ur, "p^+ \int \mathrm{d}y^-", "y^+=0,\ \text{twist-2}"'] &
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accessible if $y^0 = 0$

(*) into basis tensors and scalar functions

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Accessible quantities

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\begin{tikzcd}[column sep=large, row sep=large]
 & F_{ab}(x_i, \boldsymbol{y}) \arrow[rr, leftrightarrow, "\text{decomposition(*)}", "\text{trans. rot. symmetry}"'] \arrow[d, "\int \mathrm{d}x_i"] & & f_{ab}(x_i, \boldsymbol{y}^2) \cdots \arrow[d, "\int \mathrm{d}x_i"'] \\
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\end{tikzcd}
$$

accessible for
$-y^2 > 0$, $py < |\vec{p}||\vec{y}|$

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\end{tikzcd}
$$

$y^+ = 0,\ \boldsymbol{p} = \boldsymbol{0}$:
$\Rightarrow p^+ y^- = py$
$\Rightarrow -y^2 = \boldsymbol{y}^2 > 0$

$py < |\vec{p}||\vec{y}|$:
Have to extrapolate in py
$\Rightarrow$ **need data for large $|\vec{p}|$**

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\langle p|\,\mathcal{O}_a(y,z_1^-)\mathcal{O}_b(0,z_2^-)\,|p\rangle \arrow[ur, "p^+\int \mathrm{d}y^-\,\mathrm{d}z_i^-\,e^{-iz_ix_ip^+}", "y^+=0,\text{ twist-2}"'] \arrow[dd, "z_i^-=0"'] & & & \\
& M_{ab}(\boldsymbol{y}) \arrow[rr, leftrightarrow, "\text{decomposition}(*)", "\text{trans. rot. symmetry}"'] & & \boxed{I_{ab}(\boldsymbol{y}^2)\cdots} \\
\langle p|\,\mathcal{O}_a(y)\mathcal{O}_b(0)\,|p\rangle \arrow[ur, "p^+\int \mathrm{d}y^-", "y^+=0,\text{ twist-2}"'] \arrow[rr, leftrightarrow, "\text{decomposition}(*)", "\text{Lorentz symmetry}"'] & & \boxed{A_{ab}(py,y^2)\cdots} \arrow[ur, "\int \mathrm{d}(py)", "y^+=0,\text{ twist-2}"'] &
\end{tikzcd}
$$

Results for these quantities

(*) into basis tensors and scalar functions

Content

Two-current matrix elements on the lattice

In Euclidean spacetime:

Access via 4-point functions

$$\frac{1}{2}\sum_{\lambda} \langle p, \lambda| \mathcal{O}_{a_1}(y)\mathcal{O}_{a_2}(0) |p, \lambda\rangle \,\Big|_{y^0=0} = 2V\sqrt{m^2+\vec{p}^2}\; \frac{C_{\mathrm{4pt}}^{\vec{p},ij}(t,\tau,\vec{y})}{C_{\mathrm{2pt}}^{\vec{p}}(t)}\Bigg|_{0\ll\tau\ll t}$$

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with 4-point / 2-point function ($P_+ = \frac{1}{2}(\mathbb{1}+\gamma_4)$):

$$C^{\vec{p},ij}_{\mathrm{4pt}}(t,\tau,\vec{y}) = \sum_{\vec{x}} e^{-i\vec{p}(\vec{z}'-\vec{z})}\;\left\langle \mathrm{tr}\left\{P_+\mathcal{P}^{\vec{p}}(\vec{z}',t)\;\mathcal{O}^{q_1q_2}_i(\vec{0},\tau)\;\mathcal{O}^{q_3q_4}_j(\vec{y},\tau)\;\overline{\mathcal{P}}^{\vec{p}}(\vec{z},0)\right\}\right\rangle$$

$$C^{\vec{p}}_{\mathrm{2pt}}(t) = \sum_{\vec{x}} e^{-i\vec{p}(\vec{z}'-\vec{z})}\;\left\langle \mathrm{tr}\left\{P_+\mathcal{P}^{\vec{p}}(\vec{z}',t)\;\overline{\mathcal{P}}^{\vec{p}}(\vec{z},0)\right\}\right\rangle$$

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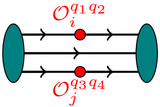
and Proton interpolators:

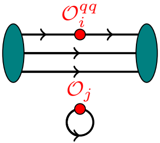
$$\mathcal{P}^{\vec{p}}(\vec{x},t) = \epsilon_{abc}\, u_a(x)\left[u_b^T(x)\, C\gamma_5\, d_c(x)\right]\Big|_{x^4=t}$$

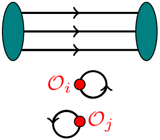
$$\overline{\mathcal{P}}^{\vec{P}}(\vec{x},t) = \epsilon_{abc}\, \left[\bar{u}_a(x)\, C\gamma_5\, \bar{d}_b^T(x)\right] \bar{u}_c(x)\Big|_{x^4=t}$$

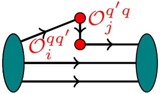
Two-current matrix elements on the lattice

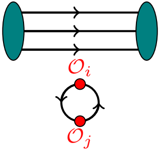
Wick contractions

$C^{ij}_{1,q_1\ldots q_4} =$ 

$S^{ij}_{1,q} =$ 

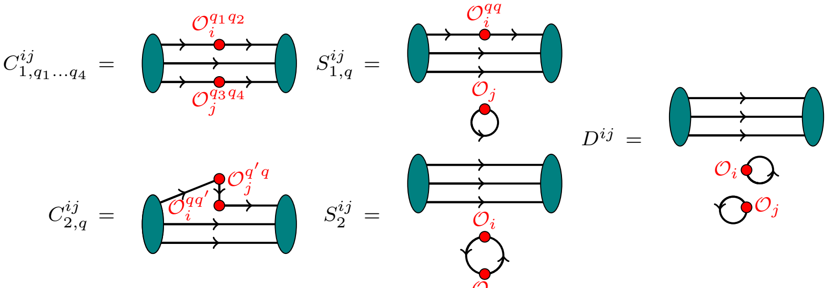
$D^{ij} =$ 

$C^{ij}_{2,q} =$ 

$S^{ij}_{2} =$ 

Two-current matrix elements on the lattice

Wick contractions



Physical matrix elements

$$\langle p|\,\mathcal{O}_i^{uu}(\vec{0})\mathcal{O}_j^{dd}(\vec{y})\,|p\rangle = C^{ij,\vec{p}}_{1,uudd}(\vec{y}) + S^{ij,\vec{p}}_{1,u}(\vec{y}) + S^{ji,\vec{p}}_{1,d}(-\vec{y}) + D^{ij,\vec{p}}(\vec{y})$$

$$\begin{aligned}\langle p|\,\mathcal{O}_i^{uu}(\vec{0})\mathcal{O}_j^{uu}(\vec{y})\,|p\rangle &= C^{ij,\vec{p}}_{1,uuuu}(\vec{y}) + C^{ij,\vec{p}}_{2,u}(\vec{y}) + C^{ji,\vec{p}}_{2,u}(-\vec{y}) + S^{ij,\vec{p}}_{1,u}(\vec{y}) + S^{ji,\vec{p}}_{1,u}(-\vec{y})\\ &\quad + S^{ij,\vec{p}}_{2}(\vec{y}) + D^{ij,\vec{p}}(\vec{y})\end{aligned}$$

$$\langle p|\,\mathcal{O}_i^{dd}(\vec{0})\mathcal{O}_j^{dd}(\vec{y})\,|p\rangle = C^{ij,\vec{p}}_{2,d}(\vec{y}) + C^{ji,\vec{p}}_{2,d}(-\vec{y}) + S^{ij,\vec{p}}_{1,d}(\vec{y}) + S^{ji,\vec{p}}_{1,d}(-\vec{y}) + S^{ij,\vec{p}}_{2}(\vec{y}) + D^{ij,\vec{p}}(\vec{y})$$

$$\langle p|\,\mathcal{O}_i^{du}(\vec{0})\mathcal{O}_j^{ud}(\vec{y})\,|p\rangle = C^{ij,\vec{p}}_{1,duud}(\vec{y}) + C^{ij,\vec{p}}_{2,u}(\vec{y}) + C^{ji,\vec{p}}_{2,d}(-\vec{y}) + S^{ij,\vec{p}}_{2}(\vec{y})$$

Two-current matrix elements on the lattice

Wick contractions

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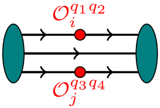

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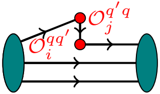

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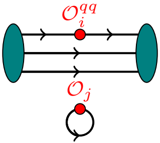
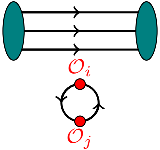
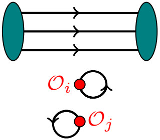


$S^{ij}_{2} =$



$D^{ij} =$



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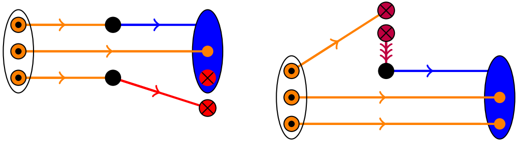
Technical Details



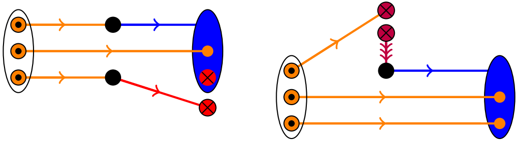
point source / propagator

stochastic source / propagator / with HPE

sequential source / propagator with constituents

- APE smearing *[Nucl. Phys. B251 (1985)]*
- Boosted sources (momentum smearing) *[arXiv:1602.05525]*
- Sequential source technique *[Nucl. Phys. B316 (1989)]*
- Stochastic wall sources: $\eta^{\ell}_{\alpha a \vec{x}} = (\pm 1 \pm i)/\sqrt{2}$ on requested time slice
  Stochastic propagator: $\mathcal{D}\psi^{\ell} = \eta^{\ell}$, $N_{\mathrm{stoch}} = 2$ (C_1), or $N_{\mathrm{stoch}} = 96$ (C_2)
- Remove trivial terms from stoch. propagators by applying hopping parameter expansion :
  C_2: apply $n(\vec{y}) = \sum_{i=1}^{3} \min(|y_i|, L - |y_i|)$ hopping terms

Technical Details



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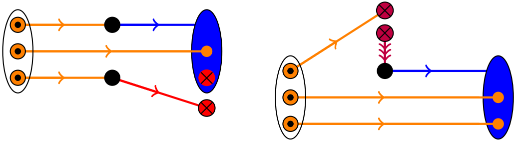

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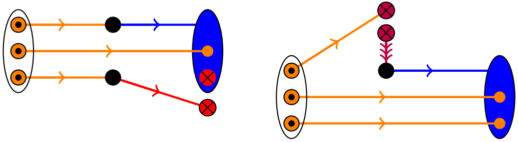

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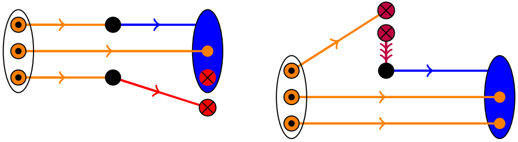

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Content

Lattice Setup

CLS ensemble H102 ($n_f = 2+1$, Wilson fermions, order-a improved *[arXiv:1411.3982]*), 990 configs used:

id	β	a[fm]	$L^3 \times T$	$\kappa_{l/s}$	$m_{\pi/K}$[MeV]	$m_\pi L$	conf.
H102	3.4	0.0856	$32^3 \times 96$	0.136865	355	4.9	2037
				0.136549339	441		

- $t_{\mathrm{src}} = 48a$ (point sources at random spatial position)
- $t = t_{\mathrm{snk}} - t_{\mathrm{src}} = \begin{cases} 12a & \vec{p} = \vec{0} \\ 10a & \vec{p} \neq \vec{0} \end{cases}$
- Insertion time $\tau \in [t_{\mathrm{src}} + 3a, t_{\mathrm{snk}} - 3a]$ for C_1 (fit), else fix $\tau = t_{\mathrm{src}} + t/2$
- 6 Momenta up to $|\vec{p}| = \sqrt{12}\frac{2\pi}{La} \approx 1.57$ GeV

Lattice Setup

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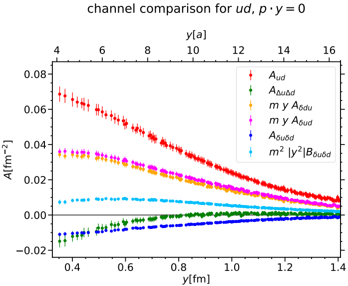
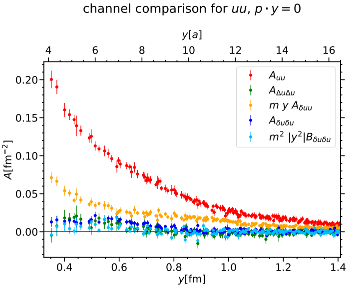
Renormalization for $\beta = 3.4$, including conversion to $\overline{\mathrm{MS}}$ at $\mu = 2\mathrm{GeV}$ *[arXiv:2012.06284]*:

	V	A	T
Z	0.7128	0.7525	0.8335

Results for $A(py = 0, y^2)$: Polarization dependence

Invariant functions $A(py = 0, y^2)$, connected graphs only (notation $y = \sqrt{-y^2}$, $y^2 = y^\mu y_\mu$):





- Signal of good quality for most channels
- ud: Clear contributions from all polarized channels (large for δud, δdu)
- uu: Polarization effects suppressed, but visible for δuu

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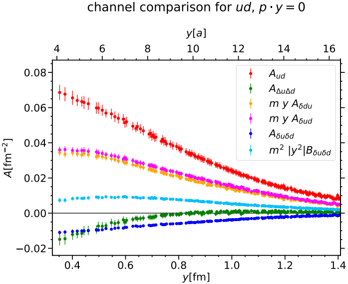

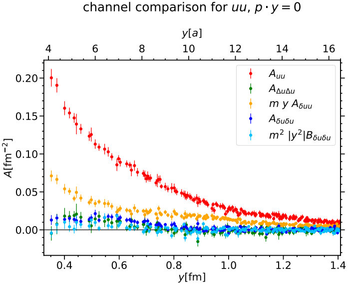

- Signal of good quality for most channels
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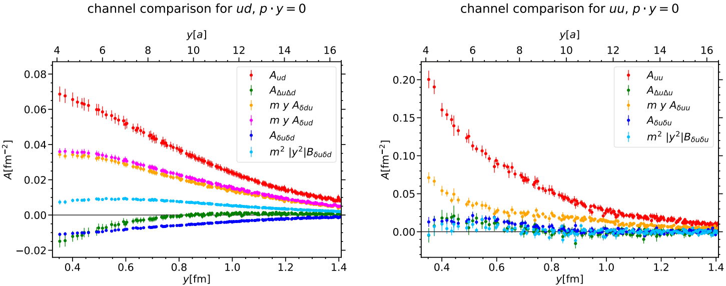
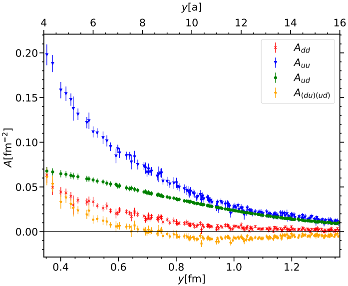
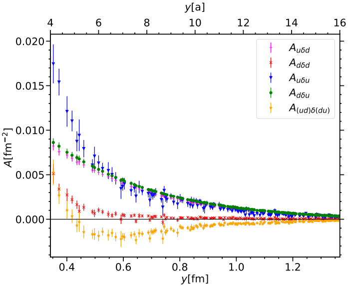


- Signal of good quality for most channels
- *ud*: Clear contributions from all polarized channels (large for δud, δdu)
- *uu*: Polarization effects suppressed, but visible for δuu

Results for $A(py = 0, y^2)$: Flavor dependence





- Clear flavor dependence visible, behavior of *uu* and *dd* different from *du*
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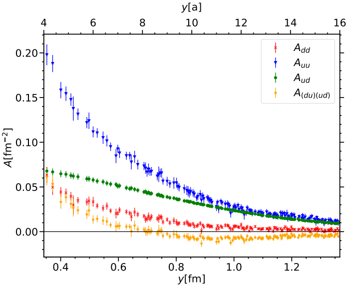

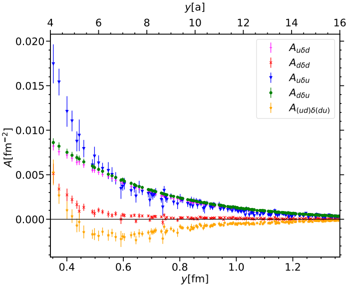

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Results for $A(py = 0, y^2)$: Comparison with $SU(6)$-model

$SU(6)$-symmetric proton wave-function:

$$|p^{\uparrow}\rangle = \frac{1}{3\sqrt{2}} \Big[|u^{\uparrow} u^{\downarrow} d^{\uparrow}\rangle + |u^{\downarrow} u^{\uparrow} d^{\uparrow}\rangle - 2\, |u^{\uparrow} u^{\uparrow} d^{\downarrow}\rangle + |u^{\uparrow} d^{\uparrow} u^{\downarrow}\rangle + |u^{\downarrow} d^{\uparrow} u^{\uparrow}\rangle - \\ -2\, |u^{\uparrow} d^{\downarrow} u^{\uparrow}\rangle + |d^{\uparrow} u^{\uparrow} u^{\downarrow}\rangle + |d^{\uparrow} u^{\downarrow} u^{\uparrow}\rangle - 2\, |d^{\downarrow} u^{\uparrow} u^{\uparrow}\rangle \Big] \, .$$

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Ratios

$$\frac{f_{duud}}{f_{ud}} = -\frac{1}{2}\,, \qquad \frac{f_{duud}}{f_{uu}} = -\frac{1}{2}\,, \qquad \frac{f_{ud}}{f_{uu}} = +1\,,$$

$$\frac{f_{\Delta du\Delta ud}}{f_{\Delta u\Delta d}} = -\frac{5}{4}\,, \qquad \frac{f_{\Delta du\Delta ud}}{f_{\Delta u\Delta u}} = +\frac{5}{2}\,, \qquad \frac{f_{\Delta u\Delta d}}{f_{\Delta u\Delta u}} = -2\,,$$

$$\frac{f_{\Delta u\Delta d}}{f_{ud}} = -\frac{2}{3}\,, \qquad \frac{f_{\Delta u\Delta u}}{f_{uu}} = +\frac{1}{3}\,, \qquad \frac{f_{\Delta du\Delta ud}}{f_{duud}} = -\frac{5}{3}\,.$$

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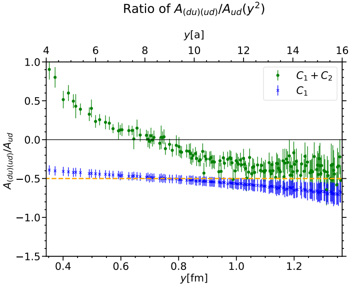

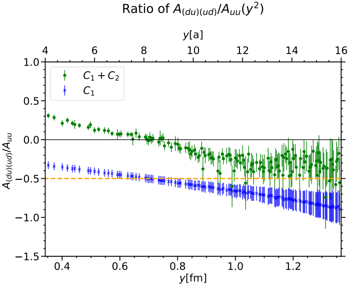

- C_1 data for unpolarized quarks roughly coincides with $SU(6)$ prediction (orange line)
- Large deviations in particular for small y when considering all leading contractions
- No agreement for polarized channels

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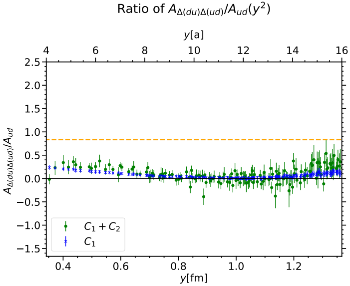

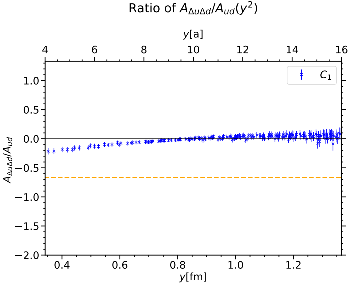

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Factorization tests

Factorization in terms of impact parameter distributions $f_q(x, \boldsymbol{b})$:

$$F_{qq'}(x_1, x_2, \boldsymbol{y}) \approx \int \mathrm{d}^2\boldsymbol{b}\; f_q(x_1, \boldsymbol{b} + \boldsymbol{y})\; f_{q'}(x_2, \boldsymbol{b})$$

Factorization tests

At the level of invariant functions $A(py, y^2)$ have expression in terms of nucleon form factors $F_1(t)$ and $F_2(t)$:

$$A_{qq'}(py = 0, y^2) \approx \frac{1}{2\pi} \int_{-1}^{1} \mathrm{d}\zeta \frac{(1-\zeta/2)^2}{1-\zeta} \int \mathrm{d}r\ r\ J_0(yr) \times \\ \times \left[\left(1 - \frac{\zeta^2}{(2-\zeta)^2}\right) F_1^q(t)\ F_1^{q'}(t) + \dots \right]$$

$\Rightarrow$ Obtain form factors F_1, F_2 from the lattice *[T. Wurm, priv. comm.]*

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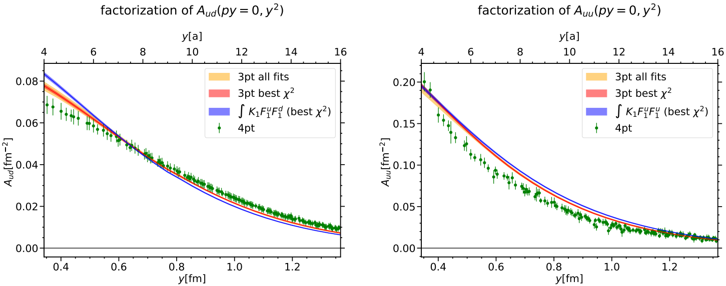

Factorization tests

For interference distributions use isospin symmetry to replace "transition distributions" f_{ud}, f_{du} :

$$F_{uddu}(x_1, x_2, \boldsymbol{y}) \approx \int \mathrm{d}^2\boldsymbol{b}\ f_{ud}(x_1, \boldsymbol{b}+\boldsymbol{y})\ f_{du}(x_2, \boldsymbol{b})$$
$$= \int \mathrm{d}^2\boldsymbol{b}\ [f_u(x_1, \boldsymbol{b}+\boldsymbol{y})\ f_u(x_2, \boldsymbol{b}) - f_u(x_1, \boldsymbol{b}+\boldsymbol{y})\ f_d(x_2, \boldsymbol{b})$$
$$-f_d(x_1, \boldsymbol{b}+\boldsymbol{y})\ f_u(x_2, \boldsymbol{b}) + f_d(x_1, \boldsymbol{b}+\boldsymbol{y})\ f_d(x_2, \boldsymbol{b})]$$

Factorization tests

At the level of invariant functions $A(py, y^2)$:

$$A_{qq'}(py = 0, y^2) \approx \frac{1}{2\pi} \int_{-1}^{1} \mathrm{d}\zeta \frac{(1-\zeta/2)^2}{1-\zeta} \int \mathrm{d}r \; r \; J_0(yr) \left[\left(1 - \frac{\zeta^2}{(2-\zeta)^2} \right) \right.$$
$$\left. \big(F_1^u(t) \; F_1^u(t) - 2 F_1^u(t) \; F_1^d(t) + F_1^d(t) \; F_1^d(t) \big) + \dots \right]$$

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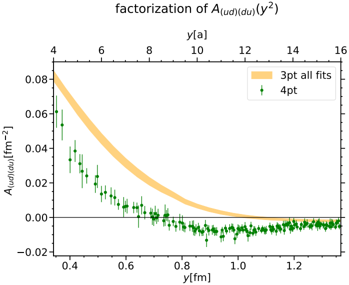

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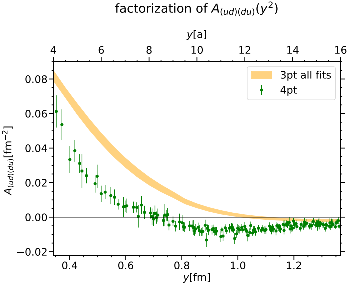

Deviations larger for interference contributions

Content

Summary and Outlook

Achieved/Observed:

- Calculated two-current matrix elements on the lattice
- Extracted Lorentz invariant functions for specific quark polarizations / flavors; these can be related to the first DPD Mellin moment (results for Mellin moments available, not discussed here)
- Polarization effects visible for ud, suppressed for uu
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- Consider derivatives (higher Mellin moments)
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Thank you for your attention!

Backup slides

Parameterization of DPDs

Decomposition in terms of rotational invariant functions $f(x_1, x_2, \boldsymbol{y}^2)$:

$$
\begin{aligned}
F_{qq'}(x_1, x_2, \boldsymbol{y}) &= f_{qq'}(x_1, x_2, y^2) \\
F_{\Delta q \Delta q}(x_1, x_2, \boldsymbol{y}) &= f_{\Delta q \Delta q'}(x_1, x_2, y^2) \\
F_{q \Delta q'}(x_1, x_2, \boldsymbol{y}) &= F_{\Delta q q'}(x_1, x_2, \boldsymbol{y}) = 0 \\
F^{j}_{q \delta q'}(x_1, x_2, \boldsymbol{y}) &= \epsilon^{j\ell} y^{\ell} m \, f_{q \delta q'}(x_1, x_2, y^2) \\
F_{\Delta q \delta q'}(x_1, x_2, \boldsymbol{y}) &= F_{\delta q \Delta q'}(x_1, x_2, \boldsymbol{y}) = 0 \\
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F^{jk}_{\delta q \delta q'}(x_1, x_2, \boldsymbol{y}) &= \delta^{jk} f_{\delta q \delta q'}(x_1, x_2, y^2) + \left(2 y^j y^k - \delta^{jk} \boldsymbol{y}^2\right) m^2 f^{t}_{\delta q \delta q'}(x_1, x_2, y^2)
\end{aligned}
$$

Parameterization of DPDs

Decomposition in terms of rotational invariant functions $I(\boldsymbol{y}^2)$:

$$
\begin{aligned}
M_{qq'}(\boldsymbol{y}) &= I_{qq'}(\boldsymbol{y}^2) \\
M_{\Delta q\Delta q}(\boldsymbol{y}) &= I_{\Delta q\Delta q'}(\boldsymbol{y}^2) \\
M_{q\Delta q'}(\boldsymbol{y}) &= M_{\Delta qq'}(\boldsymbol{y}) = 0 \\
M^{j}_{q\delta q'}(\boldsymbol{y}) &= \epsilon^{j\ell} y^{\ell} m \; I_{q\delta q'}(\boldsymbol{y}^2) \\
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\end{aligned}
$$

Parameterization of two-current matrix elements

Decomposition in terms of Lorentz invariant functions $A(py, y^2), B(py, y^2), \ldots$, blue: twist-2 contributions:

$$\langle p|\, V_q^{\{\mu}(0) V_{q'}^{\nu\}}(y)\, |p\rangle =$$

$$= (2p^\mu p^\nu - \tfrac{m^2}{2} g^{\mu\nu}) A_{q'q}(py, y^2) + (2p^{\{\mu} y^{\nu\}} - \tfrac{py}{2} g^{\mu\nu}) m^2 B_{q'q}(py, y^2)$$

$$+ (2y^\mu y^\nu - \tfrac{y^2}{2} g^{\mu\nu}) m^4 C_{q'q}(py, y^2) + g^{\mu\nu} D_{q'q}(py, y^2)$$

$$\langle p|\, A_q^{\{\mu}(0) A_{q'}^{\nu\}}(y)\, |p\rangle =$$

$$= (2p^\mu p^\nu - \tfrac{m^2}{2} g^{\mu\nu}) A_{\Delta q'\Delta q}(py, y^2) + (2p^{\{\mu} y^{\nu\}} - \tfrac{py}{2} g^{\mu\nu}) m^2 B_{\Delta q'\Delta q}(py, y^2)$$

$$+ (2y^\mu y^\nu - \tfrac{y^2}{2} g^{\mu\nu}) m^4 C_{\Delta q'\Delta q}(py, y^2) + g^{\mu\nu} D_{\Delta q'\Delta q}(py, y^2)$$

$$\langle p|\, T_q^{\mu\nu}(0) V_{q'}^{\rho}(y)\, |p\rangle + \tfrac{2}{3} g_{\lambda\sigma} g^{\rho[\mu} \langle p|\, T_q^{\nu]\lambda}(0) V_{q'}^{\sigma}(y)\, |p\rangle =$$

$$= (4y^{[\mu} p^{\nu]} p^\rho + \tfrac{4m^2}{3} g^{\rho[\mu} y^{\nu]} - \tfrac{4py}{3} g^{\rho[\mu} p^{\nu]}) m\, A_{q'\delta q}(py, y^2)$$

$$+ (4y^{[\mu} p^{\nu]} y^\rho + \tfrac{4py}{3} g^{\rho[\mu} y^{\nu]} - \tfrac{4y^2}{3} g^{\rho[\mu} p^{\nu]}) m^3 B_{q'\delta q}(py, y^2)$$

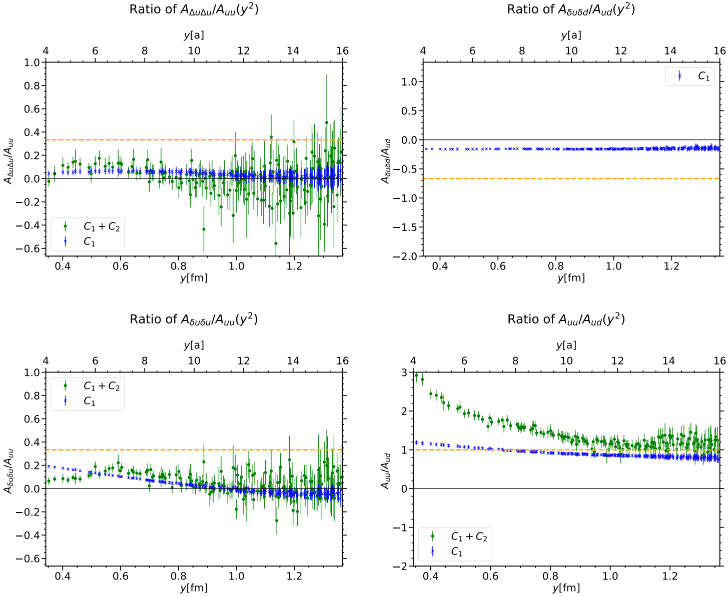
$$\tfrac{1}{2} \langle p|\, T_q^{\mu\nu}(0) T_{q'}^{\rho\sigma}(y)\, |p\rangle + \tfrac{1}{2} \langle p|\, T_q^{\rho\sigma}(0) T_{q'}^{\mu\nu}(y)\, |p\rangle =$$

$$= -8p^{[\nu} g^{\mu][\rho} p^{\sigma]} A_{\delta q'\delta q}(py, y^2) - (16y^{[\mu} p^{\nu]} y^{[\rho} p^{\sigma]} - 8y^2\, p^{[\nu} g^{\mu][\rho} p^{\sigma]}) m^2 B_{\delta q'\delta q}(py, y^2)$$

$$- (4p^{[\nu} g^{\mu][\rho} y^{\sigma]} + 4y^{[\nu} g^{\mu][\rho} p^{\sigma]}) m^2 C_{\delta q'\delta q}(py, y^2) - 8y^{[\nu} g^{\mu][\rho} y^{\sigma]} m^4 D_{\delta q'\delta q}(py, y^2)$$

$$+ 2g^{\mu[\rho} g^{\sigma]\nu} m^2 E_{\delta q'\delta q}(py, y^2)$$

More on $SU(6)$ comparison



Fit ansatz for invariant functions

Skewed DPDs (additional phase with skewness ζ):

$$F_{ab}(x_1, x_2, \zeta, \boldsymbol{y}) := 2p^+ \int \mathrm{d}y^- \, e^{-i\zeta p^+ y^-} \left[\prod_{i=1,2} \frac{\mathrm{d}z_i^-}{2\pi} e^{i x_i p^+ z_i} \right] \langle p | \, \mathcal{O}_a(y, z_1) \mathcal{O}_b(0, z_2) \, | p \rangle$$

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Symmetries and support region:

$$F(x_1, x_2, \zeta, \boldsymbol{y}) = F(x_1, x_2, -\zeta, \boldsymbol{y}) \qquad |x_i \pm \zeta/2| \leq 1 \qquad |\zeta| \leq 1$$

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Ansatz:

$$I(\zeta, \boldsymbol{y}^2) \propto \sum_n \zeta^{2n} \Theta(1 - \zeta^2) \quad \Rightarrow \quad A(py, y^2) = A(0, y^2) \sum_n a_n(y^2) h_n(py)$$

$$h_n(x) := \frac{1}{2} \int_{-1}^{1} \mathrm{d}\zeta e^{ix\zeta} \zeta^{2n} = \sin(x) s_n(x) + \cos(x) c_n(x)$$

$$s_n(x) := \sum_{m=0}^{n} \frac{(2n)!(-1)^m}{(2n-2m)! x^{1+2m}} \qquad c_n(x) := \sum_{m=0}^{n-1} \frac{(2n)!(-1)^m}{(2n-2m-1)! x^{2+2m}}$$

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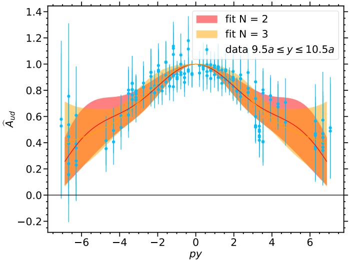
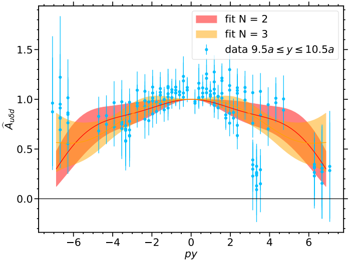
local fit on $\widehat{A}_{ud}$ at $y = 10a$, $K = 0$



local fit on $\widehat{A}_{u\delta d}$ at $y = 10a$, $K = 0$



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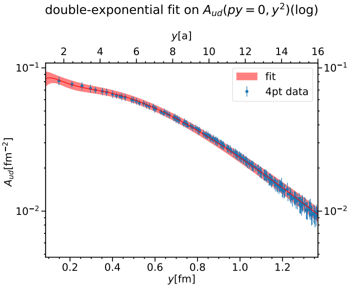
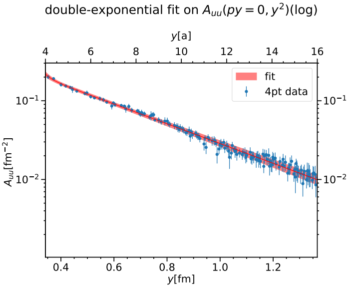
y^2-dependence:

$$A(0, y^2) = \sum_{i=1,2} A_i (\eta_i y)^{\delta} e^{-\eta_i (y - y_0)}$$





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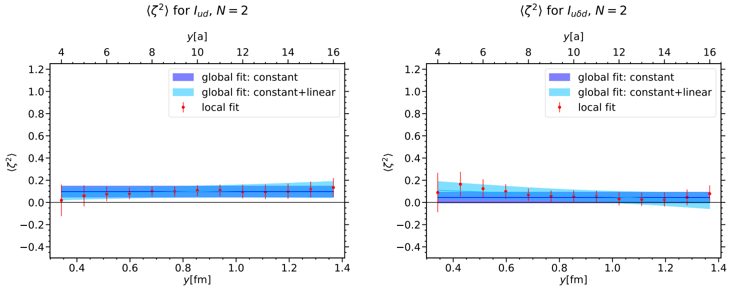
Ansatz $a_m(y^2)$:

$$a_m(y^2) = \sum_k c'_{mk} \sqrt{-y^2}^{\,k} \qquad (Tc')_{nk} = c_{nk} \qquad \left. \frac{\partial^{2n} A(py, y^2)}{\partial (py)^{2n}} \right|_{py=0} = A(0, y^2) \sum_k c_{nk} \sqrt{-y^2}^{\,k}$$



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$$F(x_1, x_2, \zeta, \boldsymbol{y}) = F(x_1, x_2, -\zeta, \boldsymbol{y}) \qquad |x_i \pm \zeta/2| \le 1 \qquad |\zeta| \le 1$$

Total ansatz (red: fit parameters)

$$A(py, y^2) = \sum_{i=1,2} A_i e^{-\eta_i (y - y_0)} \sum_{n,m=0}^{N} \sum_{k=0}^{K} c'_{nk} \, \sqrt{-y^2}^{\,k+\delta} \, {\eta_i}^{\delta} \, h_n(py)$$

$$I(\boldsymbol{y}^2) = \pi \sum_{i=1,2} A_i e^{-\eta_i (y - y_0)} \sum_{k=0}^{K} c'_{0k} \sqrt{-y^2}^{\,k+\delta} {\eta_i}^{\delta}$$

Fit ansatz for invariant functions

Skewed DPDs (additional phase with skewness ζ):

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- $c'_{nk} = T^{-1}_{nm} c_{mk}$
- Notice: $c_{00} \equiv 1$ and $c_{01} \equiv 0$ (only influence $A(0, y^2)$)
- In this work: $(N, K) = (2, 0), (2, 1), (3, 0)$

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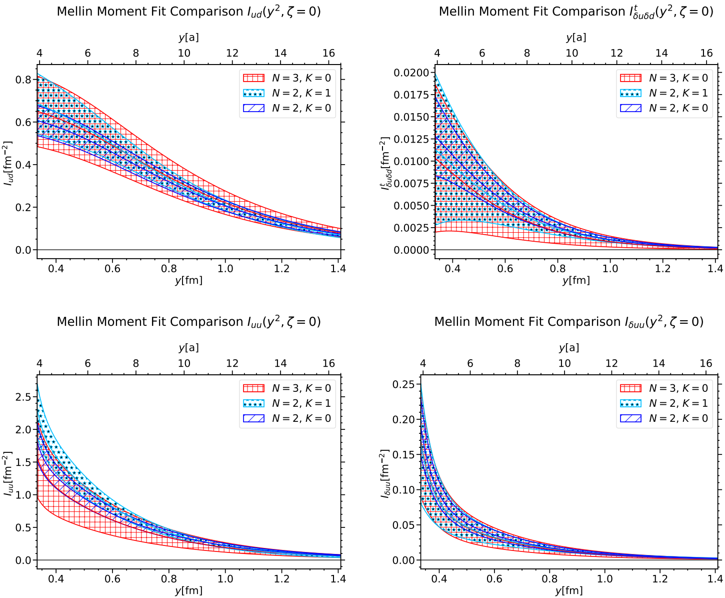
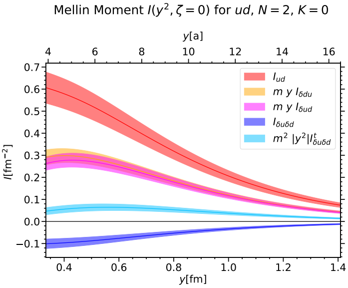
Caution: Preliminary ansatz! We are currently exploring more sophisticated models based on parton splitting at small y

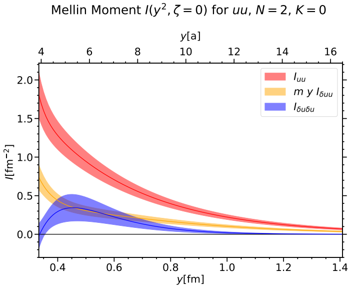
Mellin moments: Fit ansatz dependence



Results for $I(\boldsymbol{y}^2)$: Polarization dependence

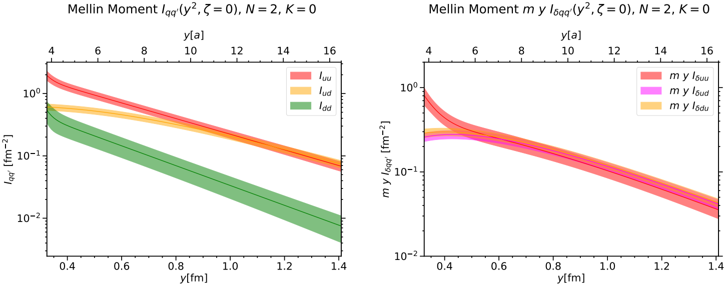
DPD moments $I(\boldsymbol{y}^2)$ (notation $y = |\boldsymbol{y}|$):





- Moments: Similar conclusions as for invariant functions

Results for $I(\boldsymbol{y}^2)$: Flavor dependence



- Clear flavor dependence observable
- Reminder: Assumption for the pocket formula:

$$F_{ab}(x_1, x_2, \boldsymbol{y}) = f_a(x_1) f_b(x_2) T(\boldsymbol{y}) \quad \Rightarrow \quad I_{ab}(\boldsymbol{y}^2) = C_{ab} T(\boldsymbol{y}^2)$$

  with **unique** $T(\boldsymbol{y})$
- **Clearly not fulfilled**

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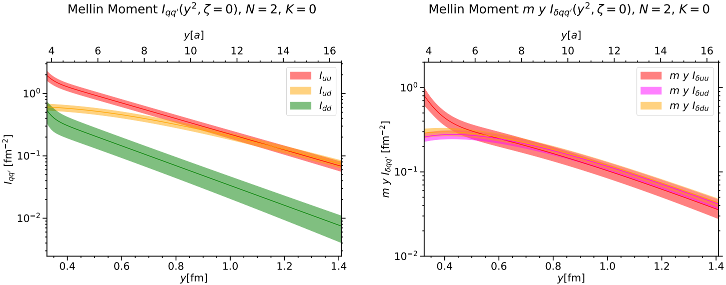

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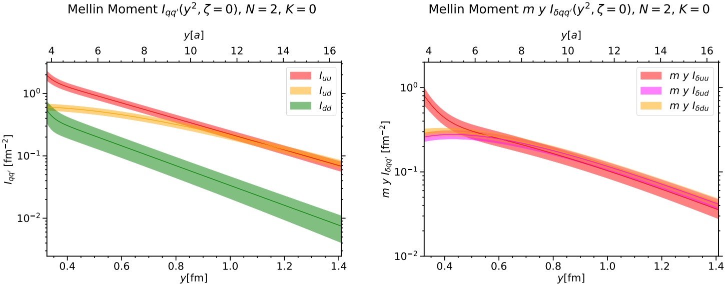

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DPD number sum rule

For $x_1 > 0$ (otherwise $F_{qq'}(x_1, ...) \to -F_{\bar{q}q'}(-x_1, ...)$)

The number sum rule *[Gaunt, Stirling '10; Diehl, Plößl, Schäfer '19]*

$$\int_{-1}^{1} \mathrm{d}x_2 \int_{b_0/\mu} \mathrm{d}^2\boldsymbol{y} F_{qq'}(x_1, x_2, \boldsymbol{y}; \mu) = \\ = (N_{q'} + \delta_{q\bar{q}'} - \delta_{qq'}) f_q(x_1; \mu) + \mathcal{O}(\alpha_s(\mu)) + \mathcal{O}((b_0 \Lambda/\mu)^2)$$

with $b_0 = 2e^{-\gamma}$ and $\mu = 2\ \mathrm{GeV}$ ($\gamma \approx 0.577$, splitting singularity $\sim \alpha_s/\boldsymbol{y}^2$)

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Implies for I_{ud}

$$\int_{b_0/\mu} \mathrm{d}^2\boldsymbol{y}\, I_{ud}(\boldsymbol{y}^2) = 2 + \mathcal{O}(\alpha_s^2(\mu)) + \mathcal{O}((b_0\Lambda/\mu)^2)$$

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From our data:

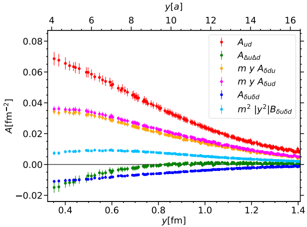
N	K	χ^2/dof	integral
2	0	0.47	1.93(23)
3	0	0.46	2.07(51)
2	1	0.46	1.98(24)

Results for the pion

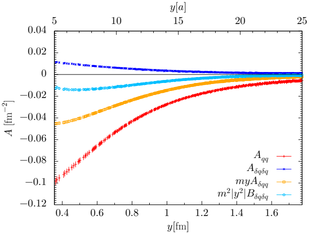
Comparison of A_{ab} and I_{ab} for ud:

proton (p)	pion π^+

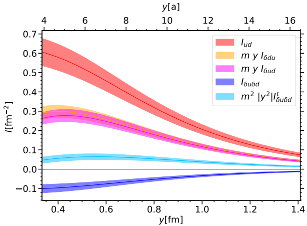
channel comparison for ud, $p \cdot y = 0$



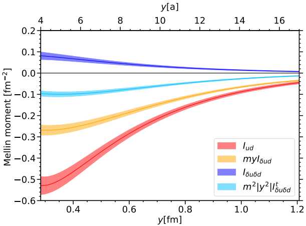
A_{qq} vs $A_{\delta q \delta q}$ vs $A_{\delta qq}$ vs $B_{\delta q \delta q}$, C_1, $p^2 = 0$ ($L = 40$)



Mellin Moment $I(y^2, \zeta = 0)$ for ud, $N = 2$, $K = 0$



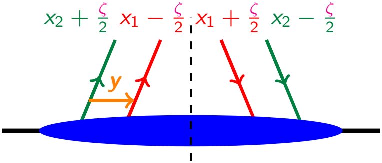
Mellin Moment $I_{aa'}(y^2, \zeta = 0)$ for ud, $N = 1$, $M = 1$



Non-forward DPDs

- Introduce difference of the momentum fractions of the emitted/absorbed quarks:

$$F_{ab}(x_1, x_2, \zeta, y) := 2p^+ \int \mathrm{d}y^- \, e^{-i\zeta p^+ y^-} \left[\prod_{j=1,2} \int \frac{\mathrm{d}z_j^-}{2\pi} e^{ix_j p^+ z_j^-} \right] \times$$
$$\times \, \frac{1}{2} \sum_\lambda \langle p, \lambda | \, \mathcal{O}_a(y, z_1^-) \mathcal{O}_b(0, z_2^-) \, | p, \lambda \rangle \Big|_{y^+=0}$$



- Parton content
- Factorization
- Support mismatch

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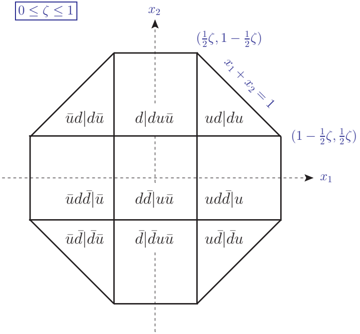

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Non-forward DPDs

- Introduce difference of the momentum fractions of the emitted/absorbed quarks:
- Parton content
- Factorization

$$F_{ab}(x_1, x_2, \zeta, \boldsymbol{y}) := \frac{1}{2(1-\zeta)} \int \frac{\mathrm{d}^2}{(2\pi)^2} e^{-i\boldsymbol{r}\boldsymbol{y}} \sum_{\lambda\lambda'} \times$$
$$\times\, f_a^{\lambda\lambda'}(\bar{x}(x_1,\zeta), -\xi(\zeta), \mathbf{0}, -\boldsymbol{r}) f_a^{\lambda\lambda'}(\bar{x}(x_2,\zeta), \xi(\zeta), -\boldsymbol{r}, \mathbf{0})$$

- Support mismatch

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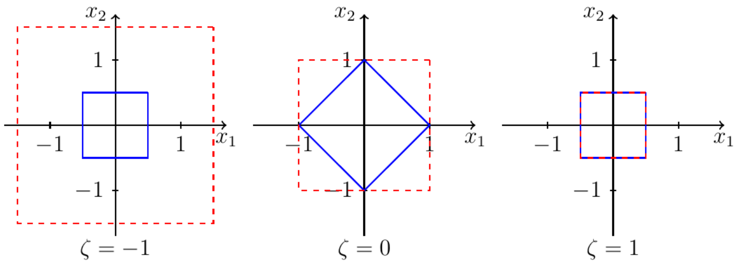

Factorization tests for Mellin moments

Factorization in terms of impact parameter distributions $f_q(x, \boldsymbol{b})$:

$$F_{qq'}(x_1, x_2, \boldsymbol{y}) \approx \int \mathrm{d}^2\boldsymbol{b}\; f_q(x_1, \boldsymbol{b} + \boldsymbol{y})\; f_{q'}(x_2, \boldsymbol{b})$$

Factorization tests for Mellin moments

For the Mellin moments

$$I_{qq'}(\boldsymbol{y}) \approx \int \frac{\mathrm{d}r}{2\pi}\ rJ_0(ry)\left[F_1^q(-\boldsymbol{r}^2)F_1^{q'}(-\boldsymbol{r}^2) + \frac{\boldsymbol{r}^2}{4m^2}F_2^q(-\boldsymbol{r}^2)F_2^{q'}(-\boldsymbol{r}^2)\right]$$

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$\Rightarrow$ Obtain form factors F_1, F_2 from the lattice *[T. Wurm, priv. comm.]*

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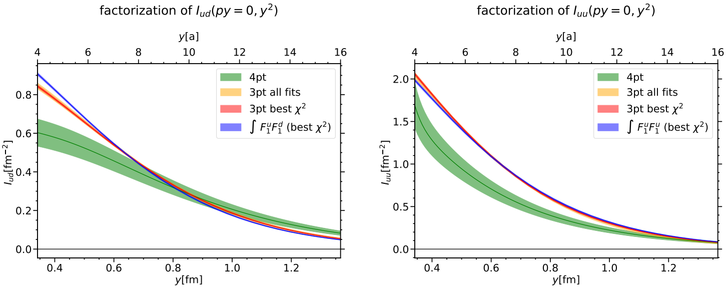
Results for I_{ud} and I_{uu}:



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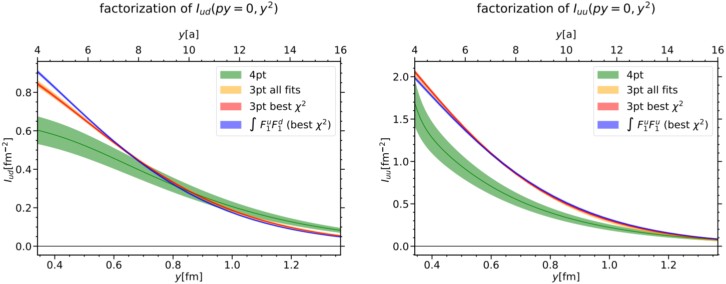

Comparable size but deviations are visible