Towards determining polarized gluon distribution in the nucleon from Lattice QCD

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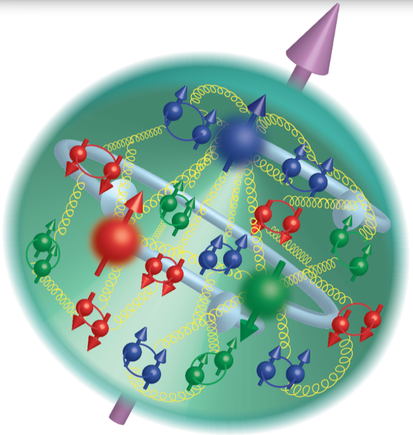
Where does the proton spin come from and how?

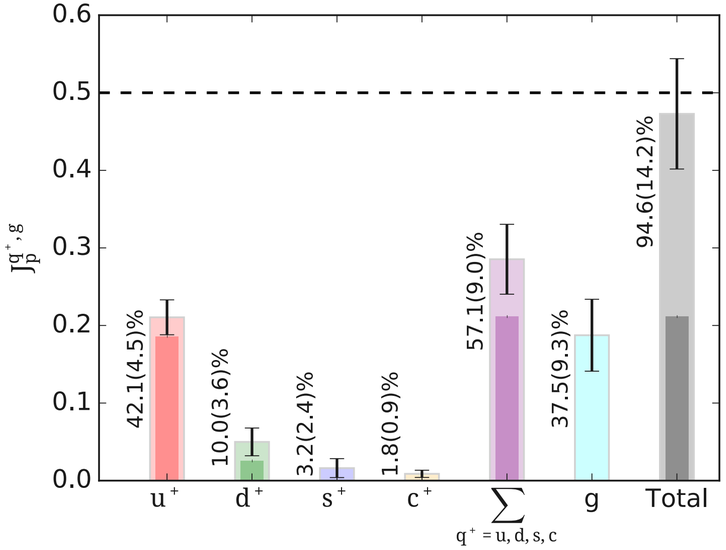


$$\frac{1}{2} = \boxed{\text{spin of all quarks } \sim 30\%} + \boxed{\text{glue spin}} + \boxed{\text{quark orbital angular momentum}} + \boxed{\text{gluon orbital angular momentum}}$$

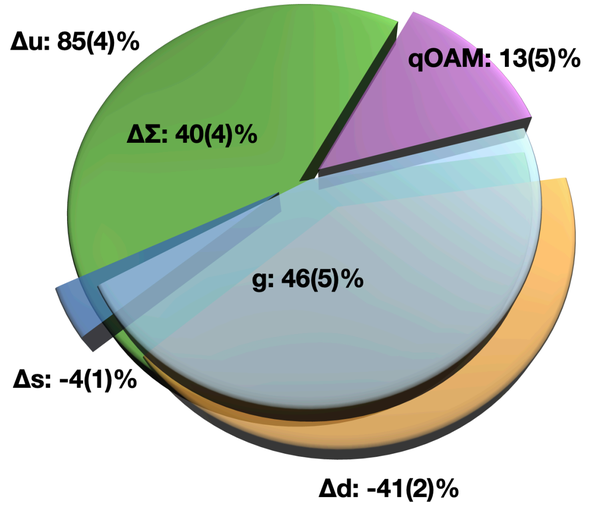
- Proton spin decomposition & Lattice QCD

$$J = J_q + J_G = \frac{1}{2}\Delta\Sigma + L_q^{Ji} + J_G$$

Ji [PRL 1997]



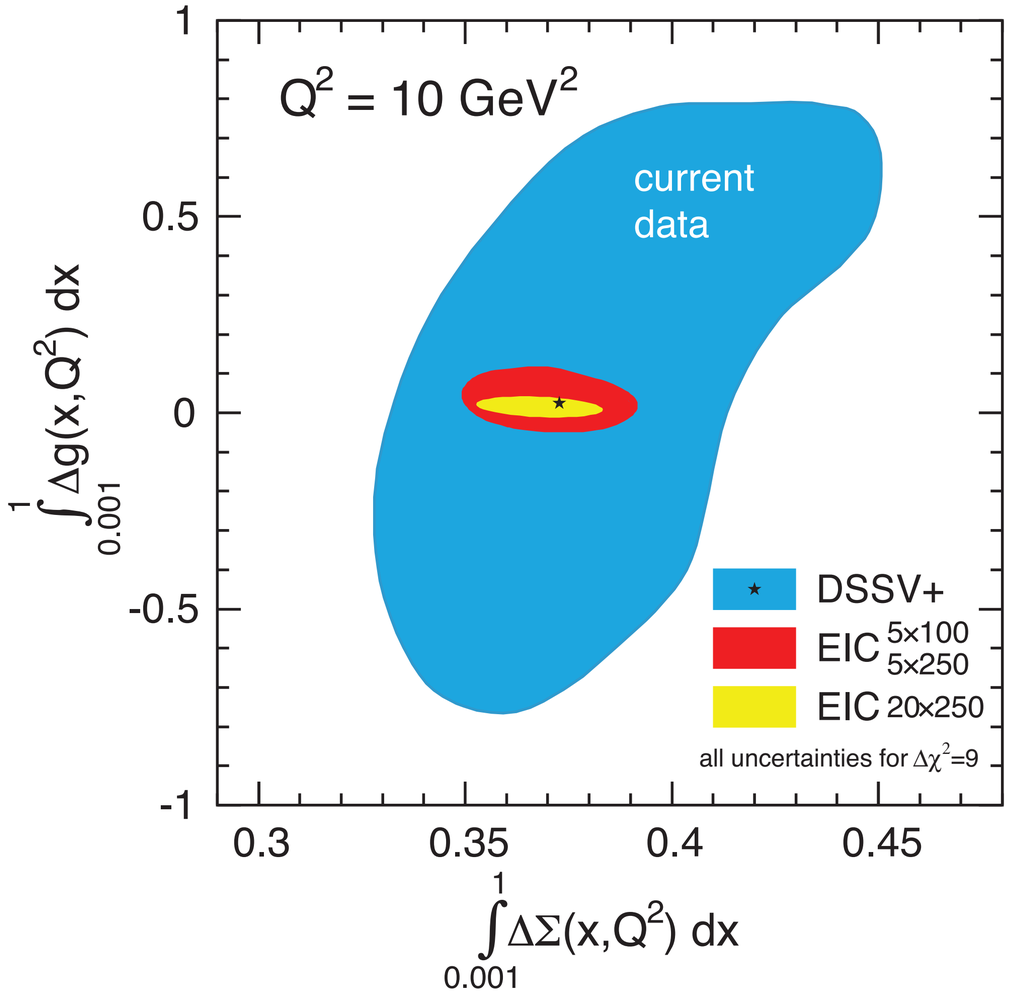
Alexandrou, et al (ETMC, PRD 2020)



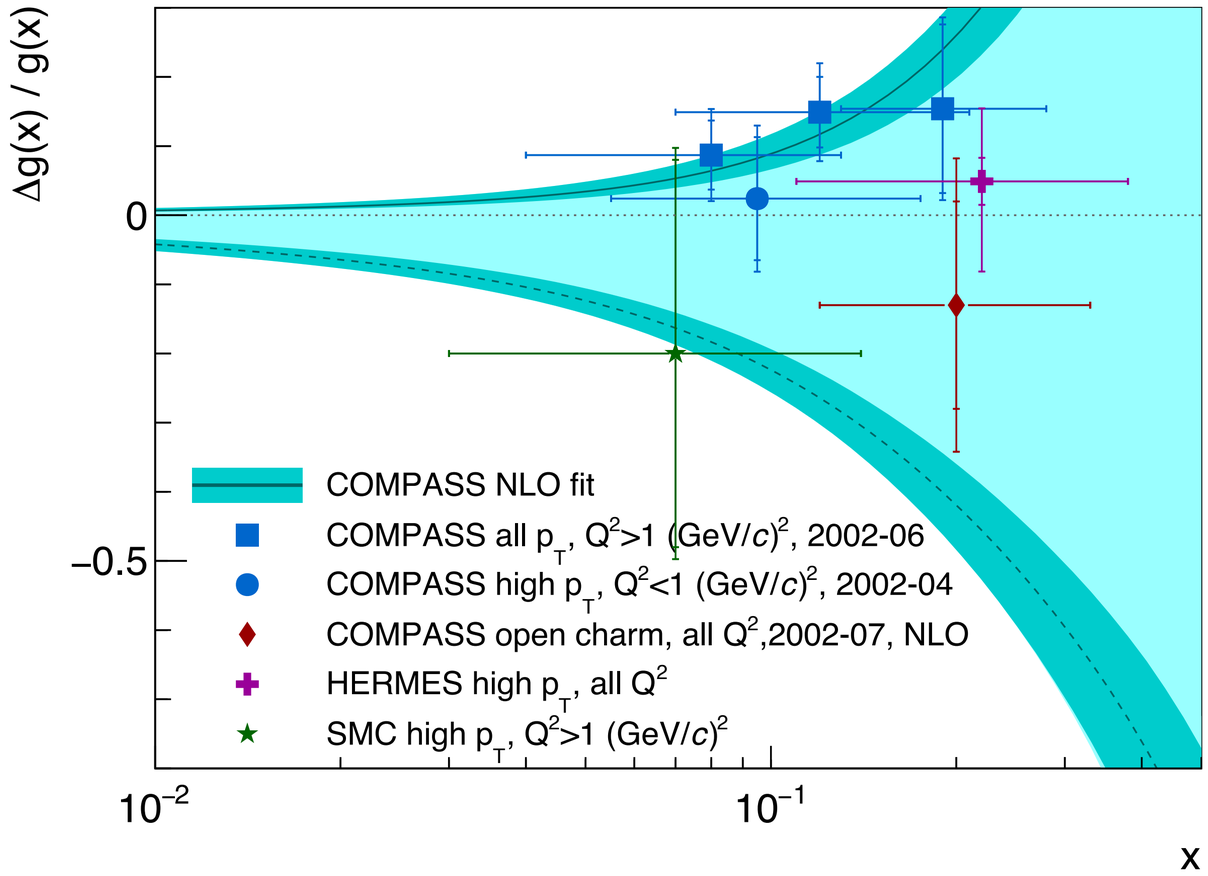
Wang, et al (χQCD, PRD 2022)

Status of gluon helicity distribution from experiments

- Gluon contribution to proton spin (ΔG) is not well-constrained from experimental data



EIC white paper: EPJA (2016)



COMPASS Collaboration: PRD (2018)

Lattice QCD determination of gluon spin contribution

- LQCD determination of gluon spin from local matrix element:

$$\vec{S}_g \to \vec{E} \times \vec{A}_{\text{phys}}$$

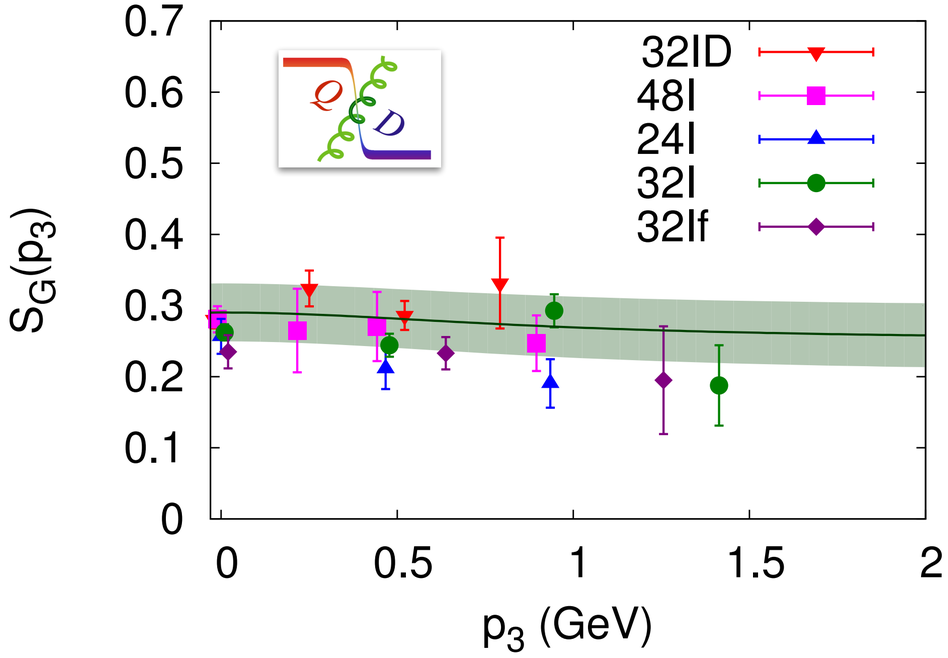
↓

LaMET matching
[Ji, Sci. China Phys 2014]

↓

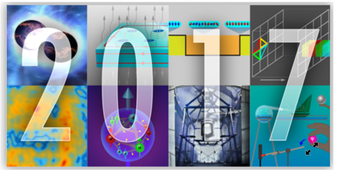
$$\Delta G$$

Ji, Zhang, Zhao [PRL 2013]



RSS, Glatzmaier, Yang, Liu, et al
PoS LATTICE(2014)
Yang, *RSS*, et al (PRL 2017)

Highlights of the Year

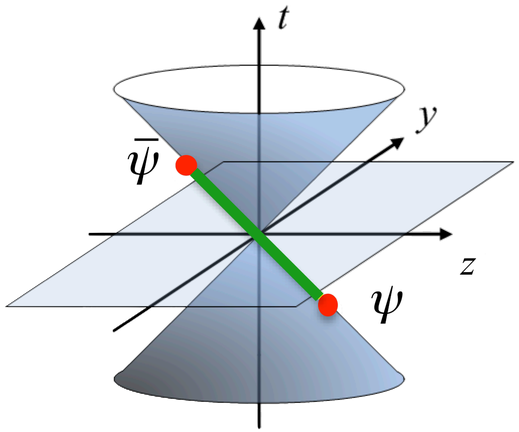


- After 1-loop matching

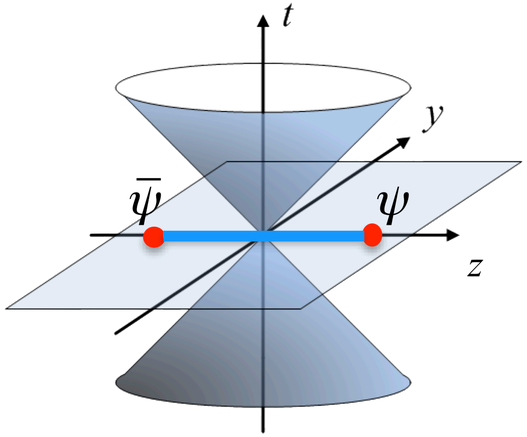
$$\Delta G(\mu^2 = 10\,\text{GeV}^2) = 0.251(47)(16)$$

LQCD formalism for calculating gluon PDFs

Light cone



Lattice



- On the lattice, calculate spatial correlation in coordinate space
  - **For polarized gluon PDF:**

X. Ji [PRL 2013]

$$\widetilde{M}_{\mu\alpha;\lambda\beta}(z,p) \equiv \langle p,s|\, G_{\mu\alpha}(z)\, W[z,0]\, \widetilde{G}_{\lambda\beta}(0)\, |p,s\rangle$$

- Appropriate combination for LQCD calculation

$$\widetilde{M}_{00}(z,p_z) \equiv [\widetilde{M}_{ti;ti}(z,p_z) + \widetilde{M}_{ij;ij}(z,p_z)] \qquad (i,j=x,y)$$

Balitsky et al [JHEP 2022]

LQCD formalism for calculating gluon PDFs

- Multiplicative renormalizability

Zhang, et al [PRL 2019], Li, et al [PRL 2019]

- Renormalization:

$$\widetilde{\mathfrak{M}}(z,p_z) \equiv i\frac{[\widetilde{M}_{00}(z,p_z)/p_z p_0]/Z_{\rm L}(z/a_L)}{M_{00}(z,p_z=0)/m_p^2}$$

"Pseudo-PDF" - Radyushkin [PRD 2017]
Balitsky et al [JHEP 2022]

- $\widetilde{\mathfrak{M}}(\nu,z^2) \xrightarrow[\text{matching}]{\text{perturbative}} \widetilde{\mathcal{I}}_p(\nu,\mu^2) = \frac{i}{2}\int_{-1}^{1} \mathrm{d}x\, e^{-ix\nu}\, x\, \Delta g(x,\mu^2)$

Lattice distribution

Light-cone distribution

Gluon helicity distribution

(Ioffe time, $\nu = p_z z$)

LQCD matrix elements for polarized gluon distribution

- What we want is the light-cone Ioffe-time distribution:

$$\widetilde{\mathcal{I}}_p(\nu) \equiv i\,[\widetilde{\mathcal{M}}^{(+)}_{ps}(\nu) - \nu\widetilde{\mathcal{M}}_{pp}(\nu)]$$

  - Gluon helicity: $\Delta G(\mu^2) = \int_0^\infty \mathrm{d}\nu\ \widetilde{\mathcal{I}}_p(\nu,\mu^2) = \int_0^1 \mathrm{d}x\ \Delta g(x,\mu^2)$ Braun, et al [PRD 1995]

- What we get from the lattice calculation:

$$\widetilde{\mathfrak{M}}(\nu,z^2) = \left[\widetilde{\mathcal{M}}^{(+)}_{sp}(\nu,z^2) - (1+m_p^2/p_z^2)\nu\widetilde{\mathcal{M}}_{pp}(\nu,z^2)\right]$$

$$= \left[\widetilde{\mathcal{M}}^{(+)}_{sp}(\nu,z^2) - \nu\widetilde{\mathcal{M}}_{pp}(\nu,z^2)\right] - \frac{m_p^2}{p_z^2}\nu\widetilde{\mathcal{M}}_{pp}(\nu,z^2)$$

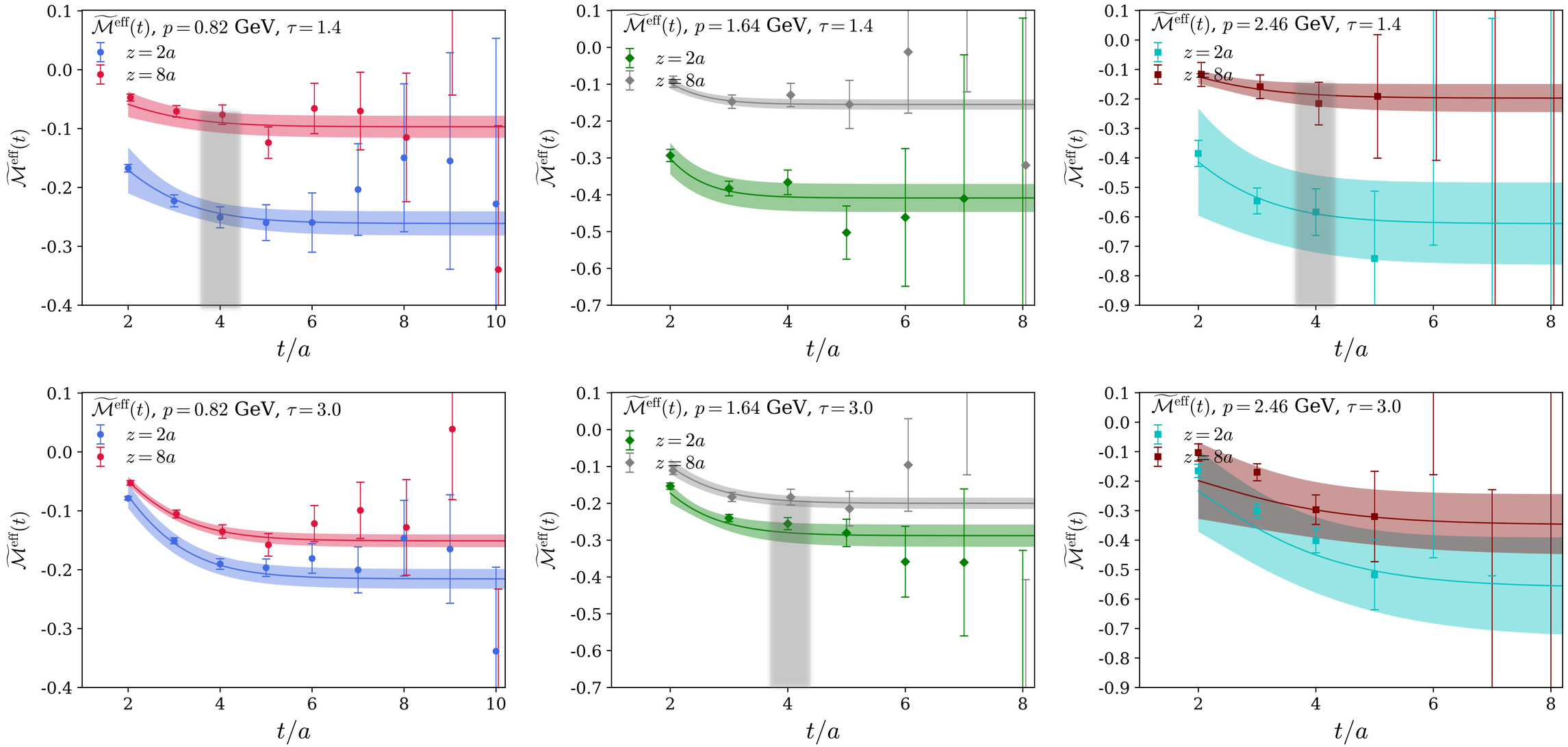
Lattice QCD calculation

- Lattice details:
  - $L \times T = 32^3 \times 64 \qquad a \approx 0.094\,\text{fm} \qquad m_\pi = 358\,\text{MeV}$
- Nucleon correlation function using Distillation — Peardon, et al [PRD 2009]
- Gluonic operator using Wilson flow — Luscher, JHEP 2010
- Solutions of summed generalized eigenvalue problem (sGEVP) for estimators of matrix elements — Bulava, et al, JHEP 2012
  - $C \exp(-\Delta E t/2)$ (GEVP)
  - $D\, t \exp(-\Delta E t)$ (sGEVP)
- Similar numerical technique as in **Khan, *RSS*, et al (HadStruc Collaboration) : PRD 2021**

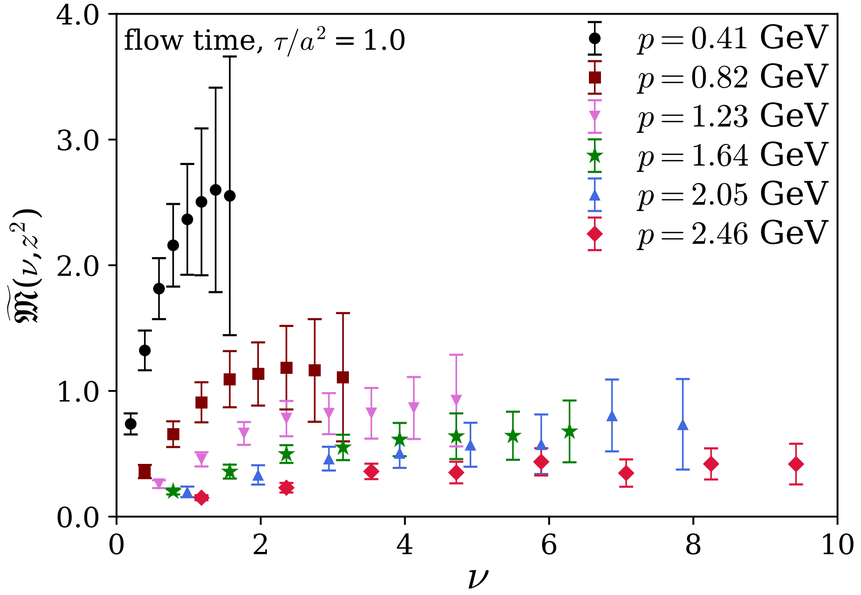
Lattice QCD matrix elements

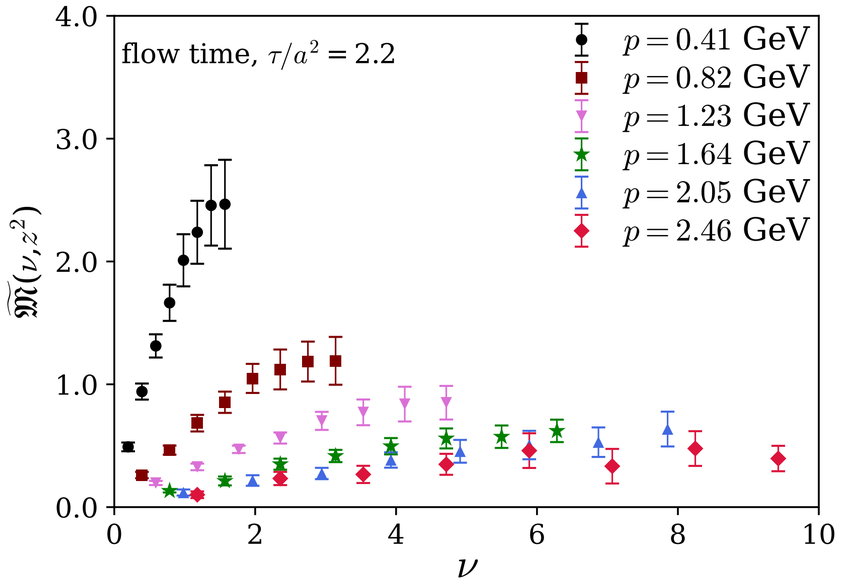
- Simultaneous correlated fit to matrix elements for all z (fixed momentum & flow time)

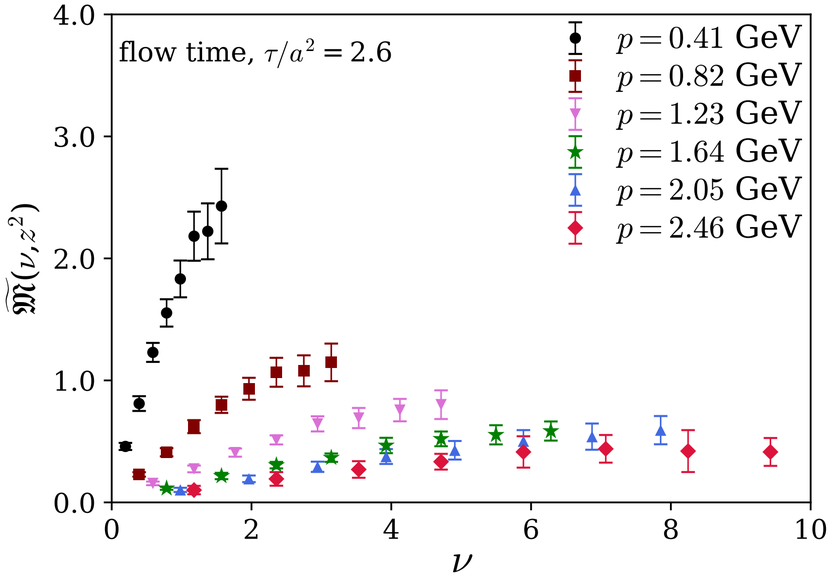
$$\widetilde{\mathcal{M}}_i^{\text{eff}}(t) = A_i + B_i\, t \exp(-\Delta E\, t)$$

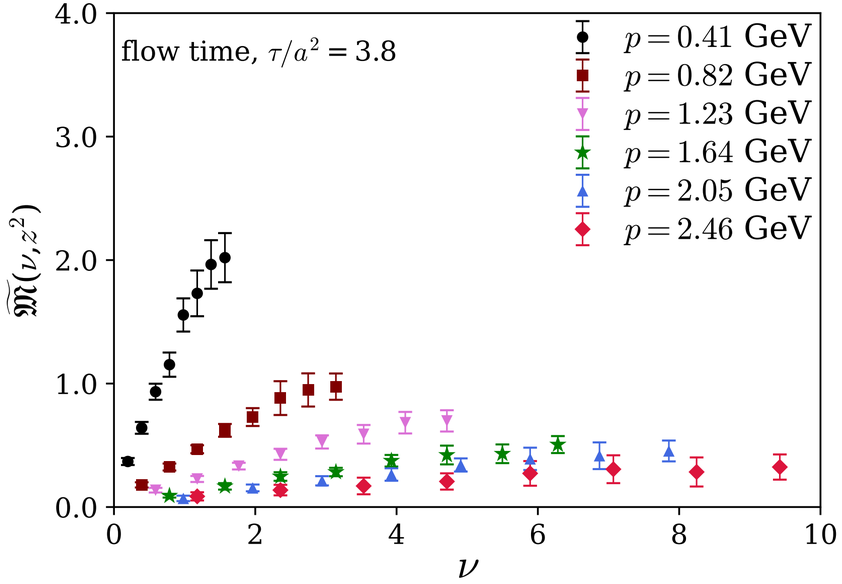


LQCD calculated pseudo-distribution as a function of flow-time



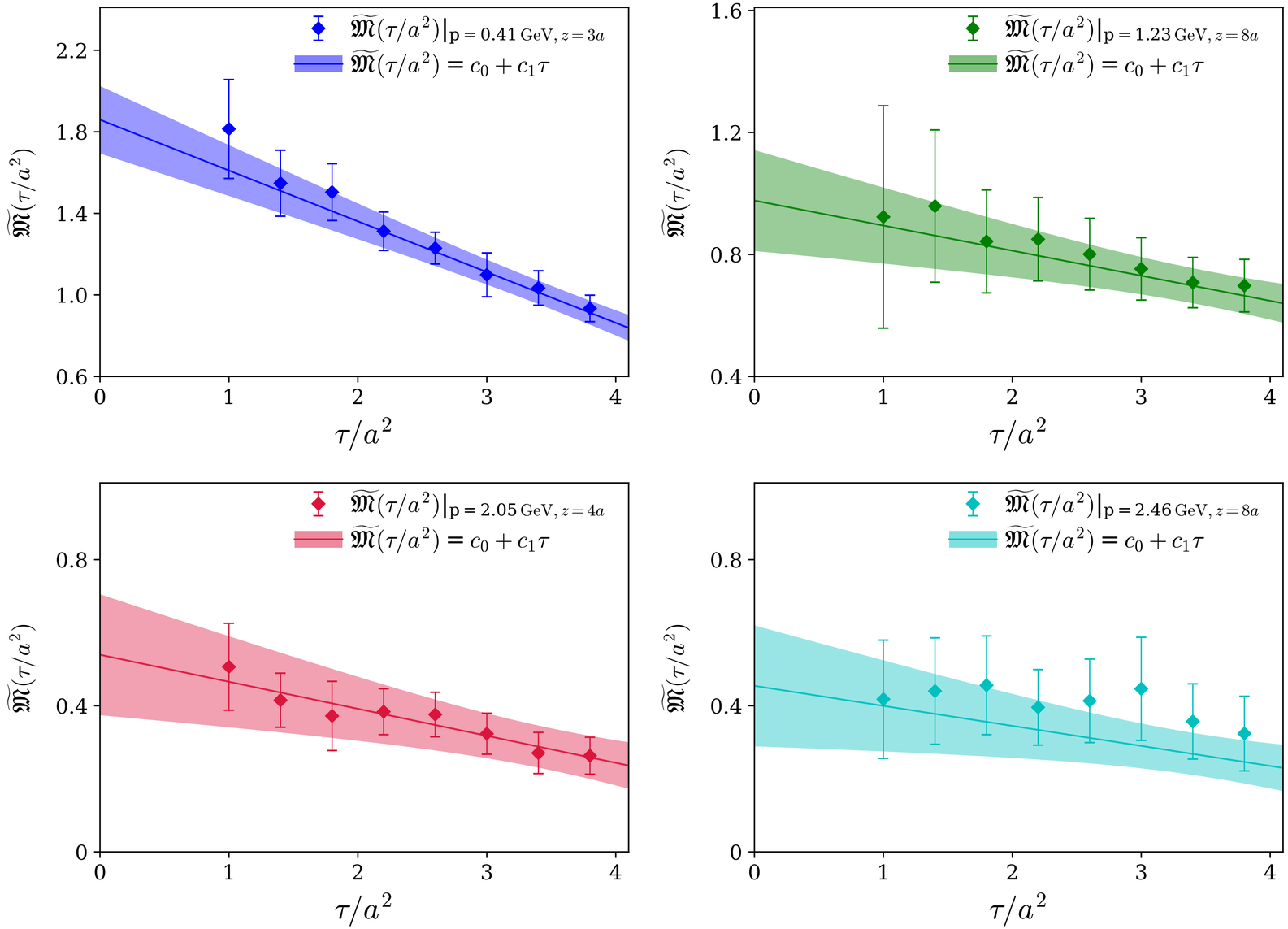






Zero flow time extrapolation of pseudo-distribution

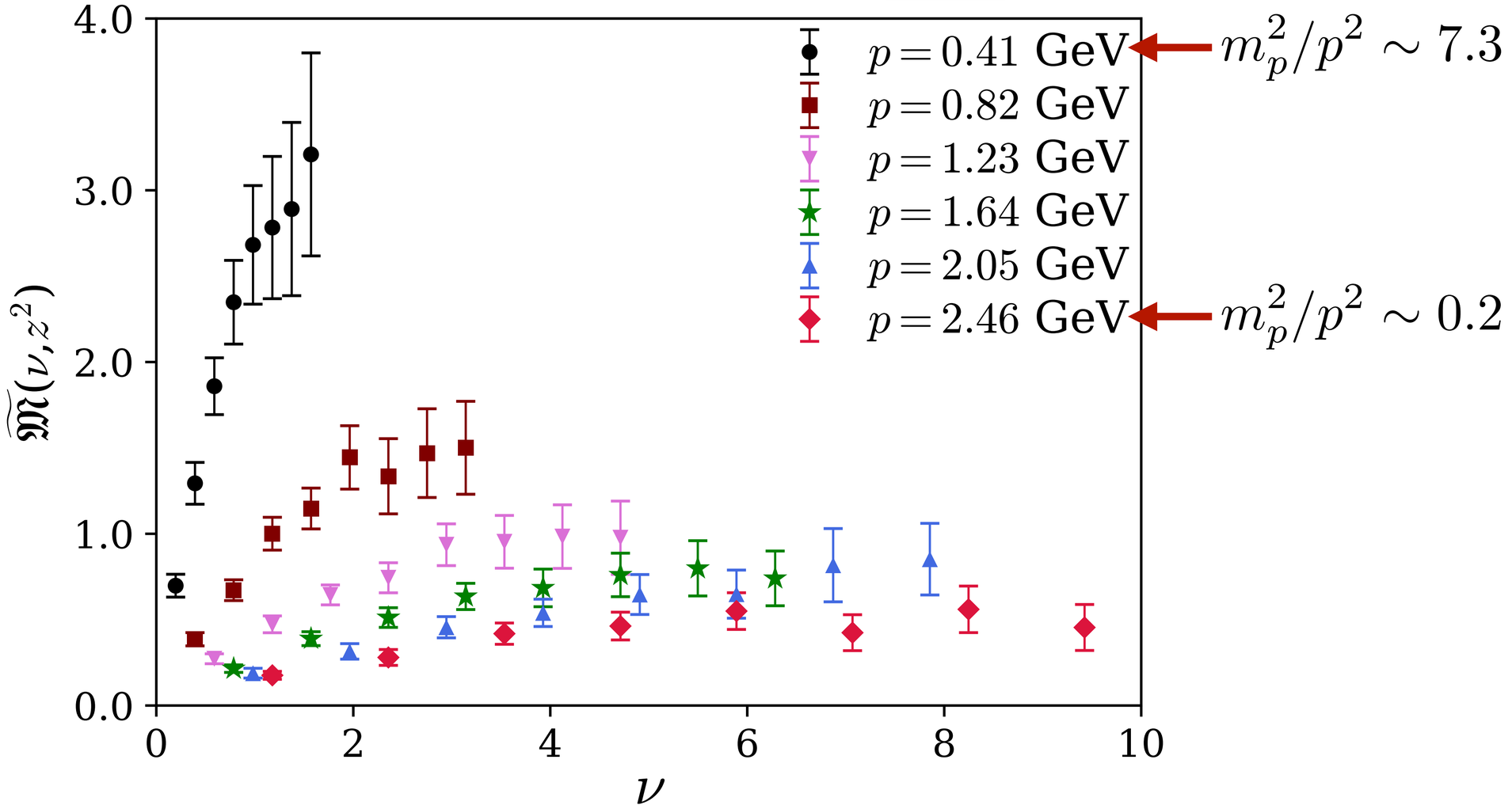
- For fixed p & z, fit forms: $c_0 + c_1\tau$, $c_0 + c_1\tau + c_2\tau^2$, etc.



Ioffe time pseudo-distribution in the zero flow time limit

- Contamination term present in LQCD matrix element dominates

$$\widetilde{\mathfrak{M}}(\nu, z^2) = \left[\widetilde{\mathcal{M}}^{(+)}_{sp}(\nu, z^2) - (1 + m_p^2/p_z^2)\nu\widetilde{\mathcal{M}}_{pp}(\nu, z^2)\right]$$



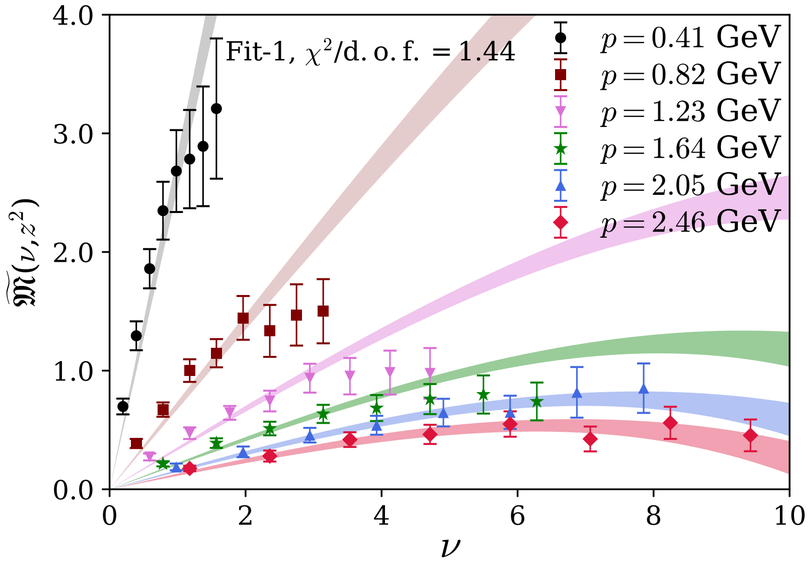
Isolating gluon helicity Ioffe-time distribution from LQCD data

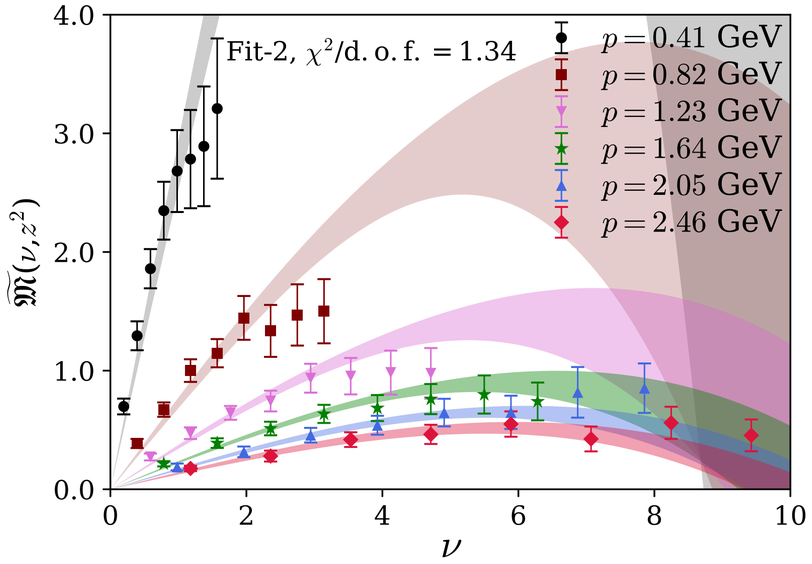
- Correction through fits using moments

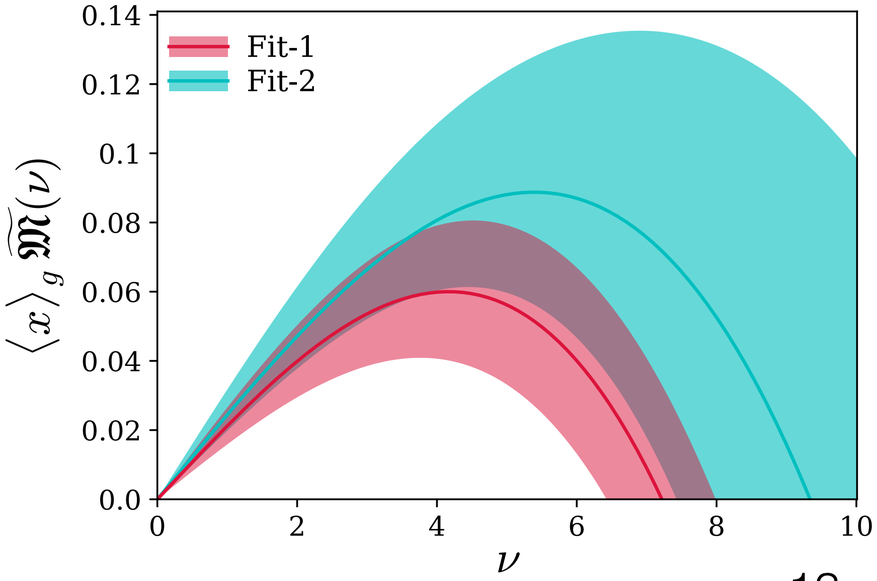
$$\widetilde{\mathfrak{M}}(\nu, z^2) = \left[\widetilde{\mathcal{M}}^{(+)}_{sp}(\nu, z^2) - \nu \widetilde{\mathcal{M}}_{pp}(\nu, z^2)\right] - \frac{m_p^2}{p_z^2}\nu \widetilde{\mathcal{M}}_{pp}(\nu, z^2)$$

- $\widetilde{\mathcal{M}}^{(+)}_{sp}$: odd in ν
- $\widetilde{\mathcal{M}}_{pp}$: even in ν

$$\widetilde{\mathfrak{M}}(\nu) = \sum_{i=0} \frac{(-1)^i}{(2i+1)!} a_i \nu^{2i+1} + \nu \frac{m_p^2}{p^2} \sum_{j=0} \frac{(-1)^j}{(2j)!} b_j \nu^{2j}$$







Isolating gluon helicity loffe-time distribution from LQCD data

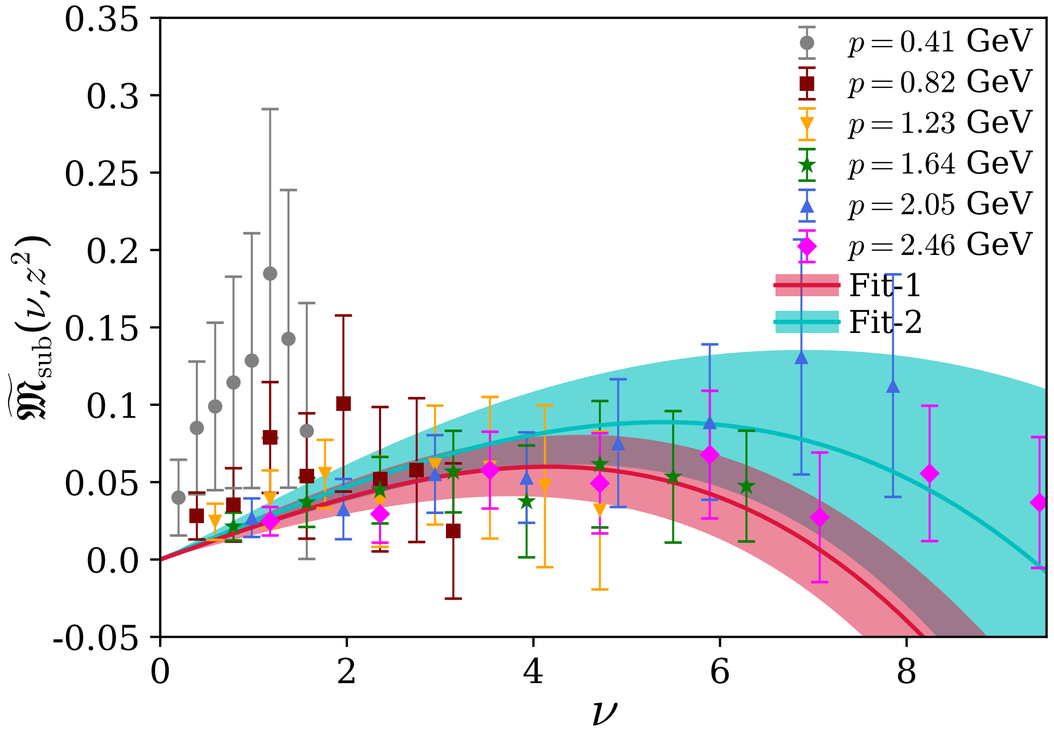
- Correction by subtracting zero momentum matrix elements

$$\widetilde{M}_{0i;0i}(z,p_z)+\widetilde{M}_{ij;ij}(z,p_z)=-2p_zp_0\widetilde{\mathcal{M}}^{(+)}_{sp}(\nu,z^2)+2p_0^3z\widetilde{\mathcal{M}}_{pp}(\nu,z^2)$$

non-vanishing at $p_z=0$

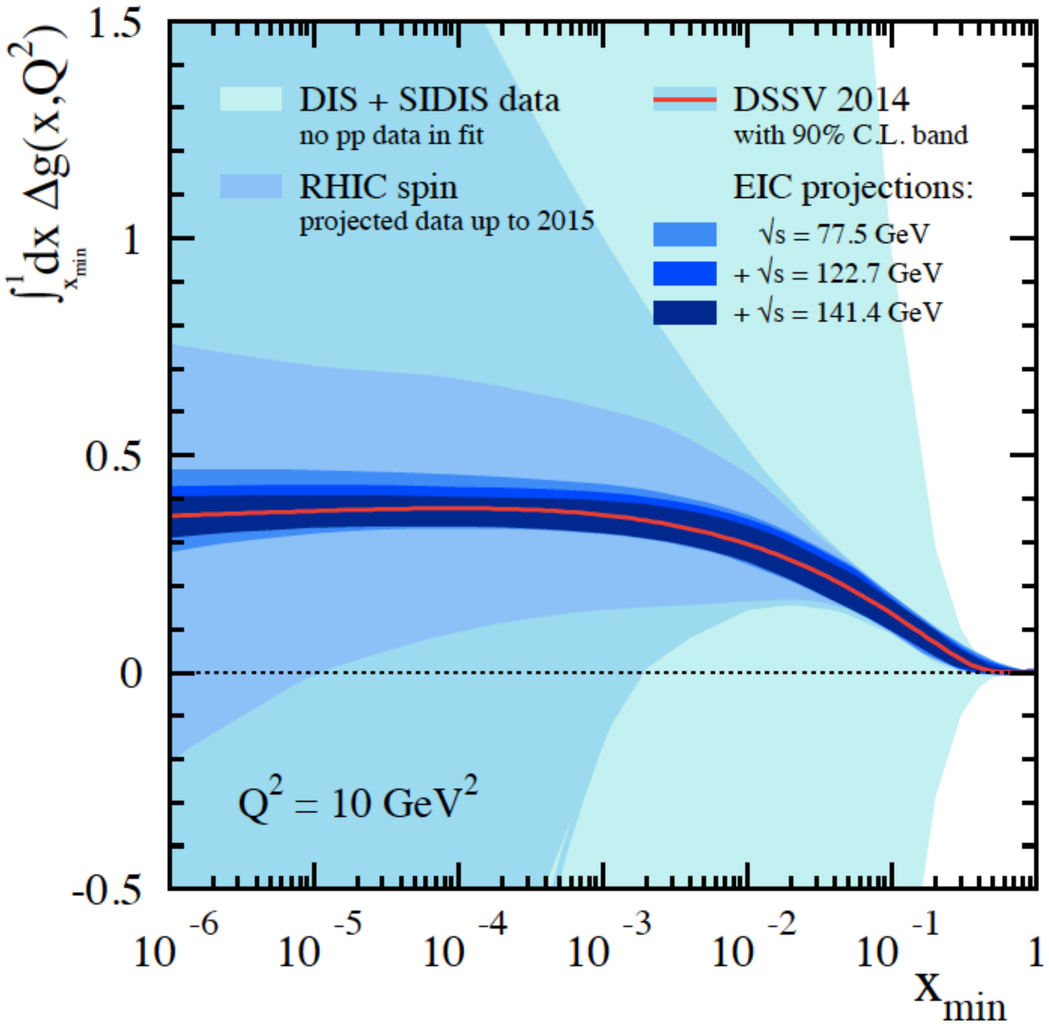
- Proposed subtraction :

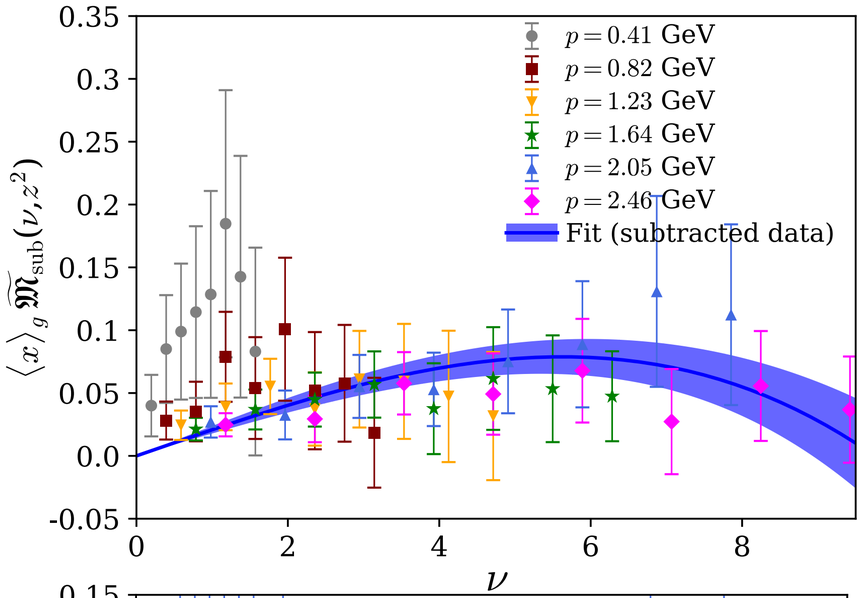
$$\widetilde{\mathcal{M}}_{\rm sub}(z,p_z)=\widetilde{\mathcal{M}}^{(+)}_{sp}(\nu,z^2)-\nu\widetilde{\mathcal{M}}_{pp}(\nu,z^2)-\nu\frac{m_p^2}{p_z^2}\left[\widetilde{\mathcal{M}}_{pp}(\nu,z^2)-\widetilde{\mathcal{M}}_{pp}(\nu=0,z^2)\right]$$

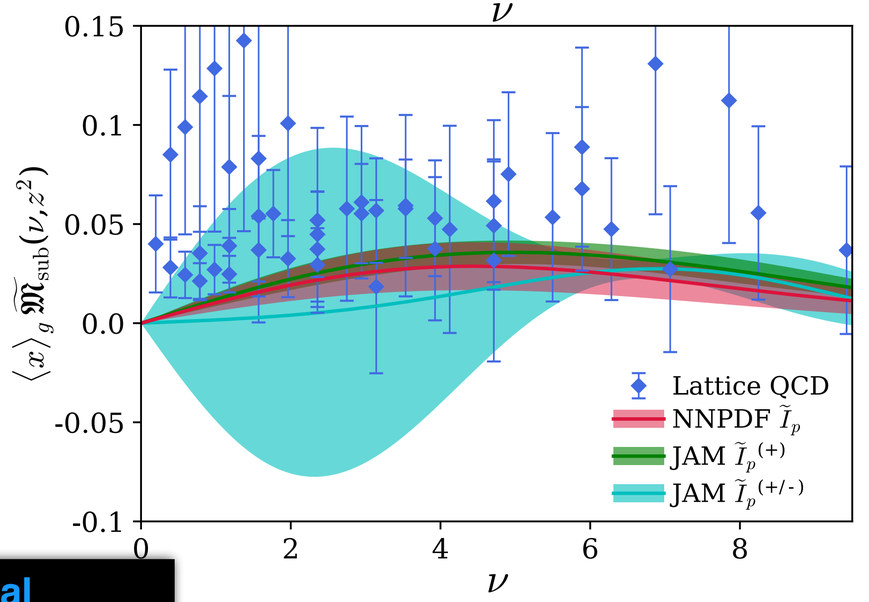


Comparison with global fits

- Sign of gluon helicity distribution is unsettled from global analyses of experimental data





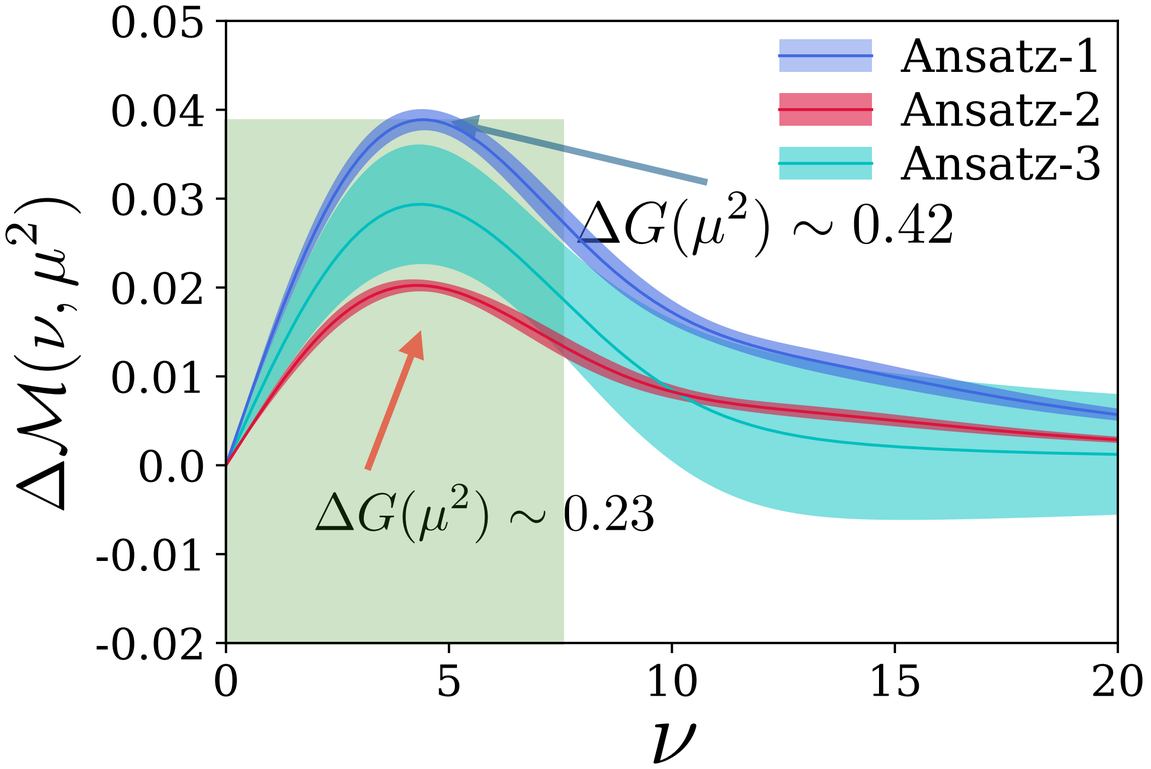


***RSS*, Khan, Karthik, et al (HadStruc Collaboration) : 2207.08733**

Impact of LQCD distribution on the magnitude of gluon helicity

- Gluon helicity from light cone Ioff-time distribution

$$\Delta G(\mu^2) = \int_0^\infty d\nu \; \Delta\mathcal{M}_{\text{light-cone}}(\nu, \mu^2)$$



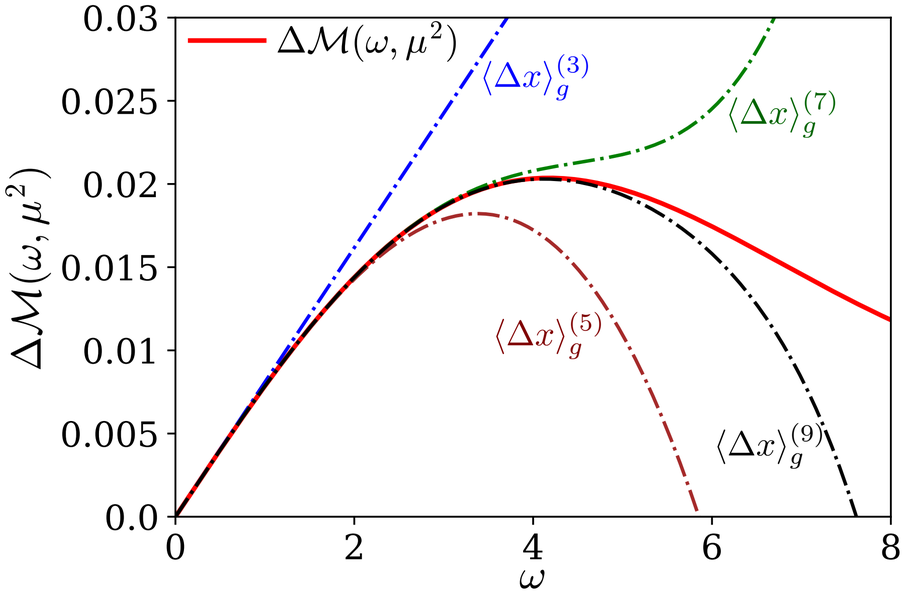
RSS, Liu, Paul [PRD 2021]

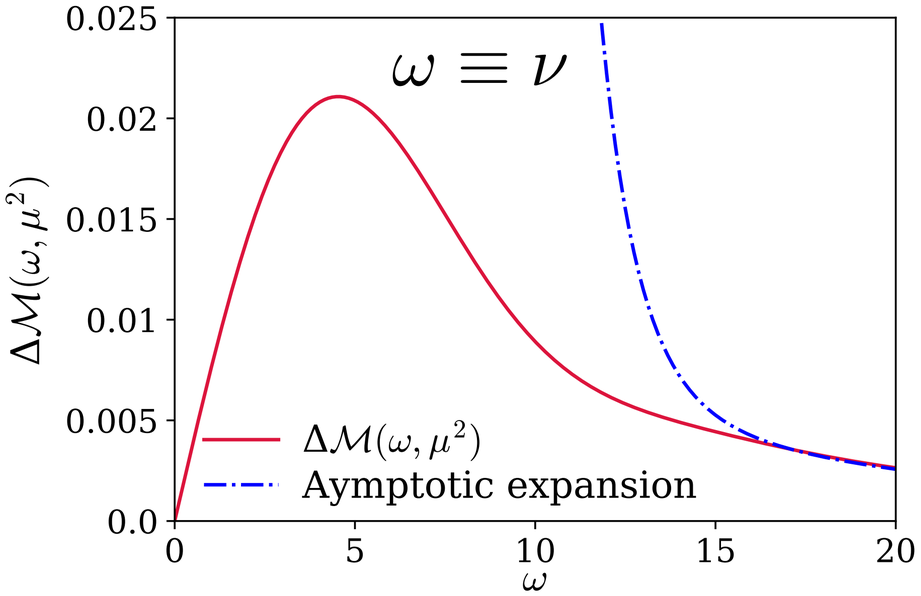
- Precise LQCD determination of polarized gluon ITD, even in $\nu \leq 6$ can have mportant impact

Challenge for Lattice QCD calculation of x-dependent gluon helicity distribution

- Lattice data in a limited range of ν

- Available lattice data is sensitive up to first few moments





- Asymptotic form:

$$\mathcal{M}(\omega,\mu^2) = A\,\Big[\,\Big(\,C_R(\alpha,4+\beta;\,\omega) + \gamma\,C_R(\alpha+1/2,4+\beta;\,\omega) + \delta\,C_R(\alpha+1,4+\beta;\,\omega)\,\Big) + \big(\,\beta\to\beta+2\,\big)\,\Big] + B\,\big[\,\beta\to\beta+1\,\big] + \mathcal{O}\big(1/\omega^{a+R+1}\big)$$

RSS, Liu, Paul
PRD 2021

Summary & Outlook

- First LQCD determination of polarized gluon Ioffe-time distribution

- Future calculation:

  - With precise LQCD matrix elements, perform pQCD matching to obtain light-cone Ioffe-time distribution

  - Consider mixing with singlet quark distribution

- Goal: determination of gluon contribution to proton spin & x-dependent helicity distribution

Thank you!