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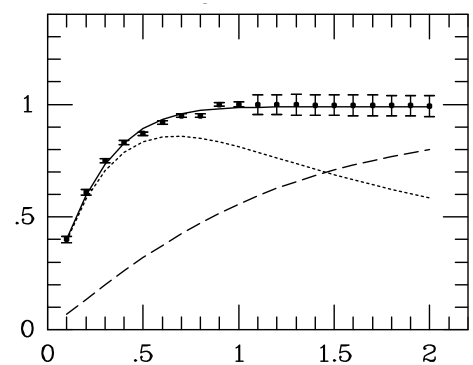
$\sigma_{\pi N}$ with $N_f = 2 + 1$ O(a)-improved Wilson fermions

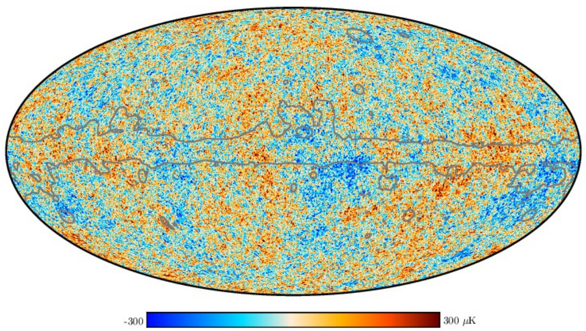
Andria Agadjanov, Dalibor Djukanovic, Georg von Hippel, Konstantin Ottnad, Harvey B. Meyer, Hartmut Wittig

Motivation-Dark Matter

Rotation Curves of Galaxies
Plot from *arXiv:2104.1148*

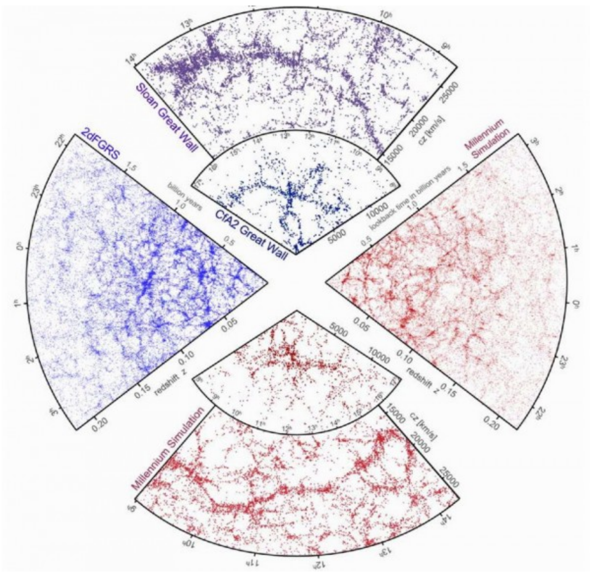


Evidence for Dark Matter



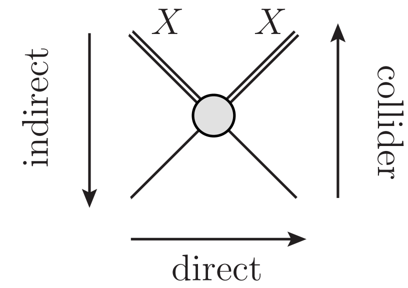
Cosmic Microwave Background
Plot from Y. Akrami, et al., Planck 2018 results. I. Overview and the cosmological legacy of Planck, arXiv:1807.06205.

Gravitational Lensing
Credit: NASA/CXC/CfA/M. Markevitch et al.; NASA/STScI; Magel- lan/U.Arizona/D. Clowe et al.; NASA/STScI; ESO WFI

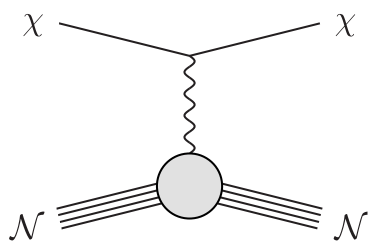


Simulation of Galaxy Structures
Plot from V. Springel, C. S. Frenk, S. D. White, The large-scale structure of the Universe, Nature, 440 (2006) 1137. arXiv:astro-ph/0604561

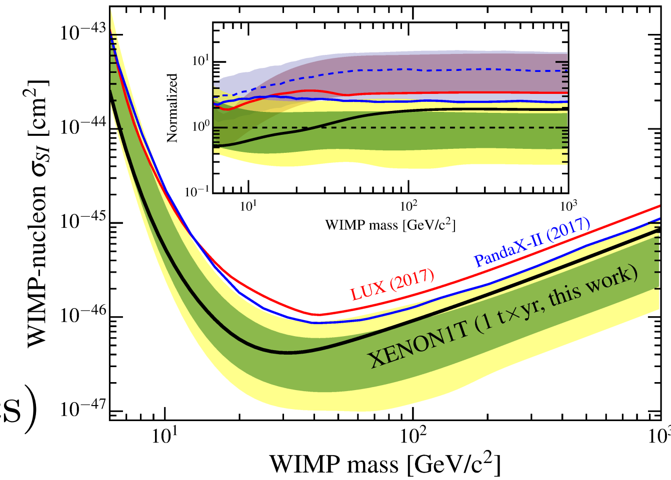
DM Direct Searches



- DM Candidate: WIMP
- WIMP-Nucleus-Scattering



Plot from E. Aprile et al,
Dark Matter Search Results from a One Tonne×Year Exposure of XENON1T
arXiV:1805.12562



- Rate WIMP-Nucleus Scattering

$$R \sim \frac{\sigma^{\mathrm{SI}}}{M_\chi \mu_N^2} \times (\text{Nuclear Physics}) \times (\text{Astrophysics})$$

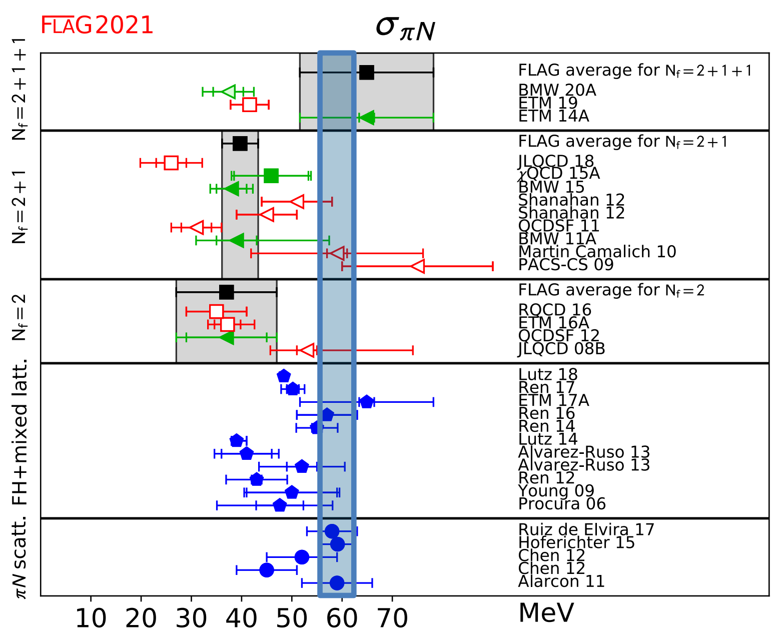
- σ^{SI} depends on Sigma-Term
- Crucial input for interpretation of experiments

Sigma-Term

- No scalar probe!
- Phenomenologically via Pion-Nucleon-Scattering (Chang-Dashen-Theorem + extrap.)
- Lattice calculation

$$\sigma_{\pi N} = m_l \langle N | \bar{u}u + \bar{d}d | N \rangle = m_l \frac{\partial m_N}{\partial m_l}$$

Directly or via Mass



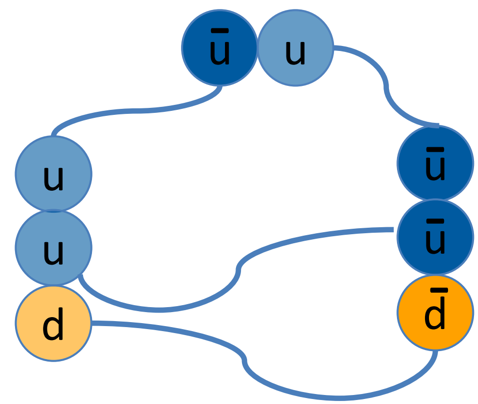
- Some tension between Roy-Steiner based estimate and Lattice

Direct Determination

- Connected part
  - Sequential Source
  - Zero Momentum at sink

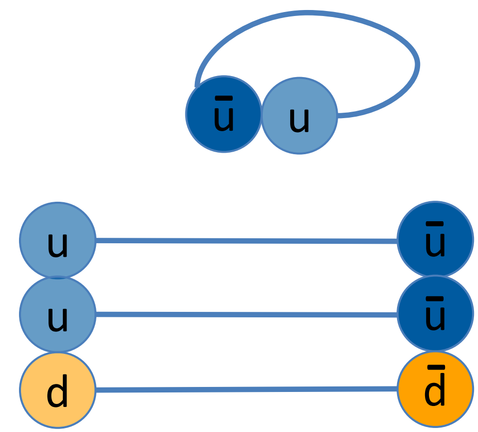
$$C_2(t;\mathbf{p}) = \Gamma_{\alpha\beta} \sum_{\mathbf{x}} e^{-i\mathbf{px}} \left\langle \Psi_\beta(\mathbf{x},t)\overline{\Psi}_\alpha(0) \right\rangle,$$

$$C_3(t,t_s;\mathbf{q}) = \Gamma'_{\alpha\beta} \sum_{\mathbf{x},\mathbf{y}} e^{i\mathbf{qy}} \left\langle \Psi_\beta(\mathbf{x},t_s)\mathcal{O}_{\mathrm{S}}(\mathbf{y},t)\overline{\Psi}_\alpha(0) \right\rangle,$$



- Disconnected part
  - Loops All-to-All: OET+HPE+HP
  - Noisy:

    → Additional two-point functions



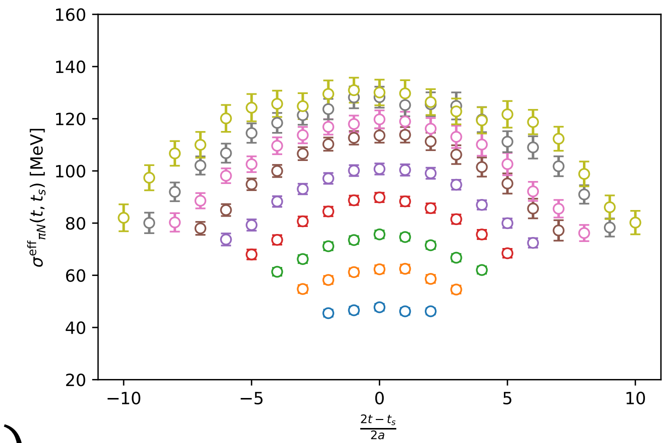
$$C_3^{\mathrm{disc}}(t,t_s;\mathbf{0}) = \left\langle L_S(\mathbf{0},z_0)\cdot C_2(\mathbf{p}',y_0,x;\Gamma') \right\rangle - \left\langle L_S(\mathbf{0},z_0) \right\rangle \cdot \left\langle C_2(\mathbf{p}',y_0,x;\Gamma') \right\rangle$$

Excited States – Summation

- Usual Ratio (forward limit):

$$R(t,t_s) = \frac{C_3(t,t_s)}{C_2(t_s)} \qquad \mathrm{Re}\,R(t,t_s) \xrightarrow{t,(t_s-t)\gg 0} G_\mathrm{S}$$

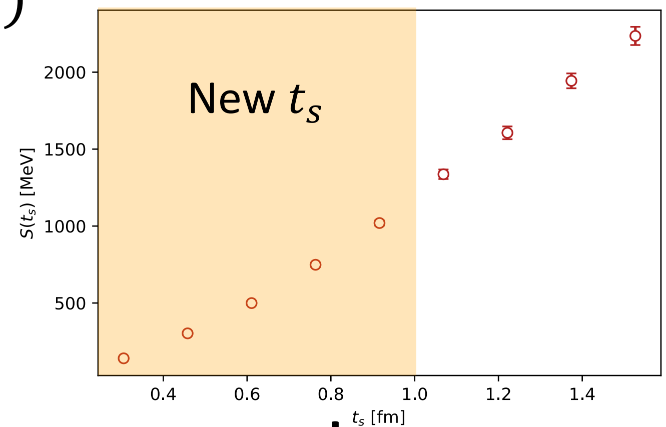
$$G_\mathrm{S}^{\mathrm{eff}}(t,t_s) = \mathrm{Re}\,R(t,t_s)$$



Excited states $\sim e^{-\Delta t}, e^{-\Delta(t_s-t)}$

- Summed correlator:

$$S(t_s) = \sum_{t=t_c}^{t_s-t_c} \sigma^{\mathrm{eff}}_{\pi N}(t,t_s)$$



Excited states parametrically suppressed

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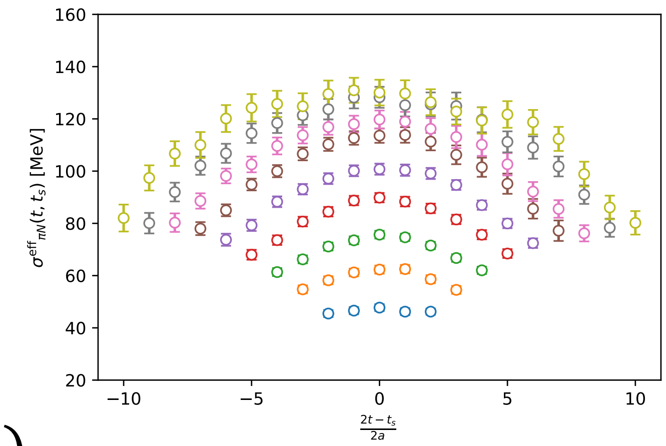

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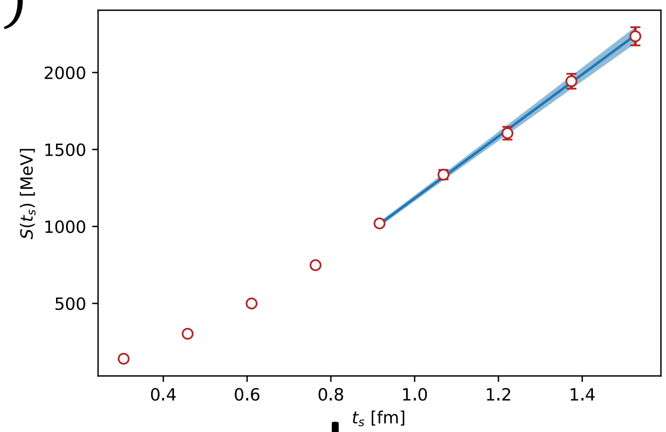

Excited states parametrically suppressed

$$S(t_s) = (\sigma_{\pi N} \qquad\qquad) \, (1 + t_s - 2t_c) \qquad\qquad + \ldots$$

Excited States – Summation

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$$R(t,t_s) = \frac{C_3(t,t_s)}{C_2(t_s)} \qquad \mathrm{Re}\,R(t,t_s) \xrightarrow{t,(t_s-t)\gg 0} G_\mathrm{S}$$

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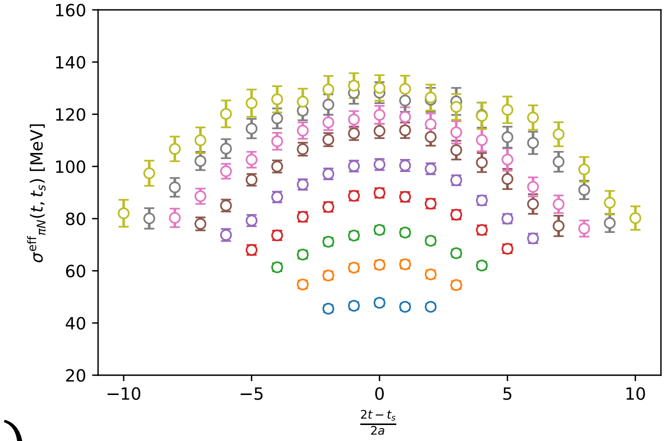

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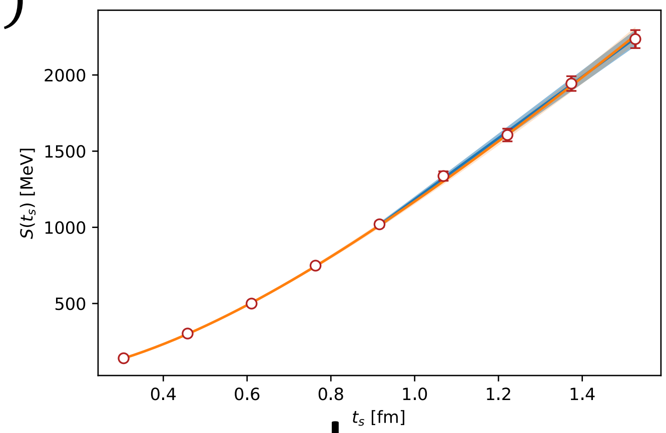
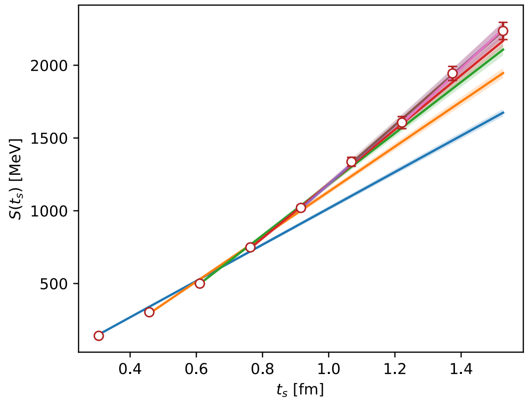
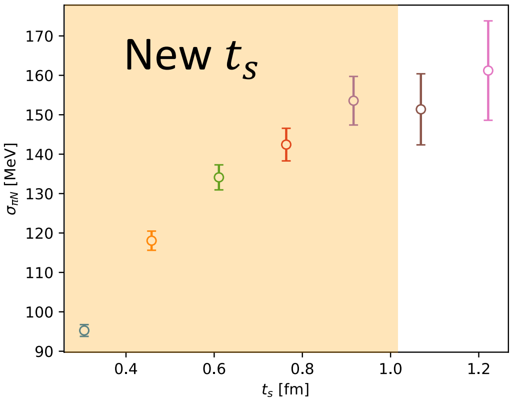


Excited states parametrically suppressed

$$S(t_s) = \left(\sigma_{\pi N} \boxed{+\, m_{11} e^{-\Delta t_s}}\right)(1 + t_s - 2t_c) + e^{-\Delta t_s} \frac{2m_{10}\left(e^{\Delta(1-t_c+t_s)} - e^{\Delta t_c}\right)}{e^\Delta - 1} + \ldots$$

Summation

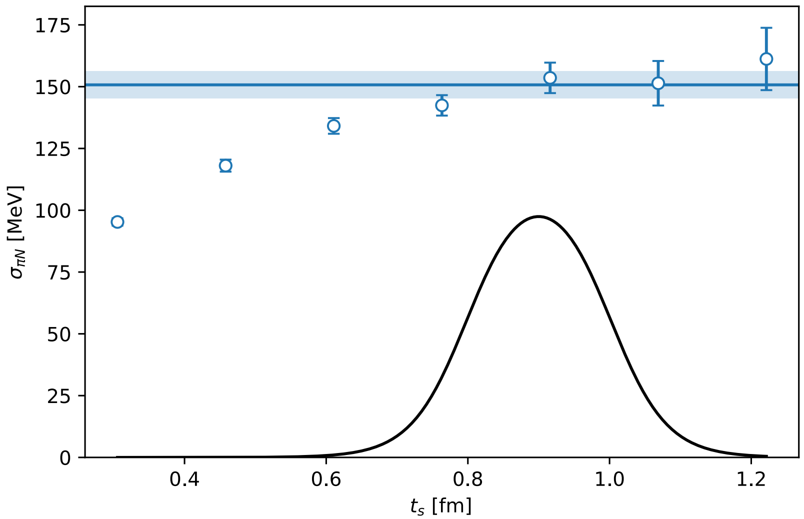




- Excited State Fits need priors for gap Δ (like explicit 2-state-Fit)
- Linear Fits:
  - Not trustworthy for small t_s
  - Error increases with larger starting t_s
  - Several possibilities
    - Choose one, use weights according to AIC, p-values, …
    - Define a fit-range in physical units (see Talks by J. Koponen, M. Salg) → Window

$$w_i = \frac{1}{2}\tanh\frac{t_s - t_{\text{lo}}}{\Delta t} - \frac{1}{2}\tanh\frac{t_s - t_{\text{up}}}{\Delta t}$$

Summation

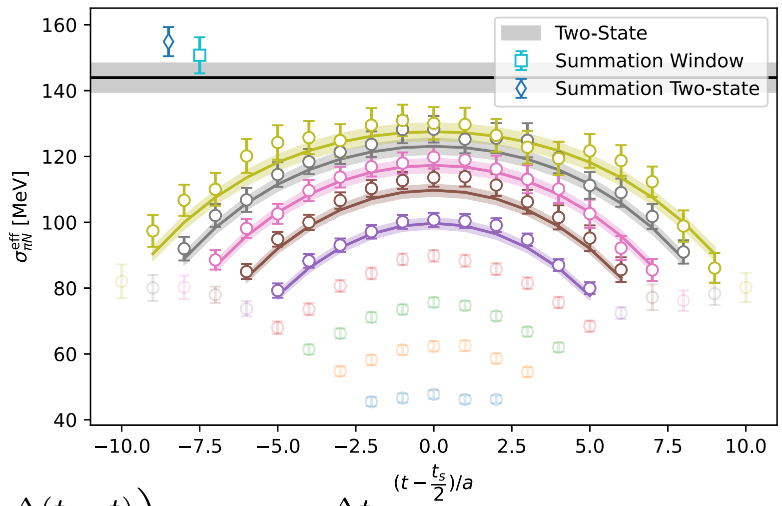


- Instead of one particular starting t_s use window function as weights with

$$t_{\mathrm{lo}} = 0.8\,\mathrm{fm}, \quad t_{\mathrm{up}} = 1.0\,\mathrm{fm} \quad \text{and} \quad \Delta t = 0.08\,\mathrm{fm}$$

  for all ensembles
- Close to „plateau“ average for every ensemble
- Less affected by single-point-estimate-fluctuation

Comparison of Methods



- Two-State Direct Fit uses:

$$\sigma_{\pi N}^{\text{eff}} = \sigma_{\pi N} + m_{10}\left(e^{-\Delta t} + e^{-\Delta(t_s - t)}\right) + m_{11} e^{-\Delta t_s}$$

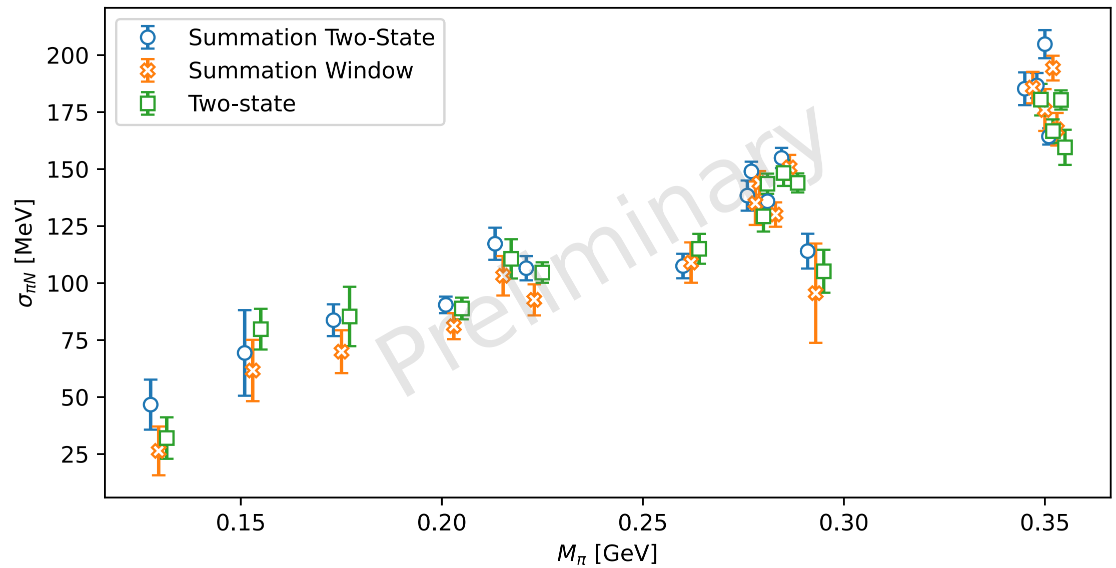
  Needs gaussian priors $\Delta = 2m_\pi$ (5% width)
- Summation Two-State:
  - Fits to summed correlator including first excited state (excluding m_{11})
  - Needs gaussian priors $\Delta = 2m_\pi$ (5% width)
  - Single starting t_s (no average)
- Summation Window:
  - Fits to summed correlator no excited states
  - Window average
  - No priors
- For all ensembles **Summation Window** compatible within **2 $\boldsymbol{\sigma}$**

Lattice Setup

ID	a [fm]	T/a	L/a	M_π[MeV]	$M_\pi L$	t_{sep}[fm]	N_{cfg}
H102	0.086	96	32	354	4.96	0.35, 0.43, 0.52, 0.6, 0.69, 0.78, 0.86, 0.95, 1.04, 1.12, 1.21, 1.3, 1.38, 1.47	2005
H105		96	32	280	3.93		1027
C101		96	48	225	4.73		2000
N101		128	48	281	5.91		1596
S400	0.076	128	32	350	4.33	0.31, 0.46, 0.61, 0.76, 0.92, 1.07, 1.22, 1.37, 1.53	2873
N451		128	48	286	5.31		1011
D450		128	64	216	5.35		500
D452		128	64	153	3.79		1000
N203	0.064	128	48	346	5.41	0.26, 0.39, 0.51, 0.64, 0.77, 0.9, 1.03, 1.16, 1.29, 1.41	1543
N200		128	48	281	4.39		1712
D200		128	64	203	4.22		2000
E250		192	96	129	4.04		400
S201		128	32	293	3.05		2093
N302	0.050	128	48	348	4.22	0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1., 1.1, 1.2, 1.3, 1.39	2201
J303		192	64	260	4.19		1073
E300		192	64	174	4.21		570

- Enlarged range in t_{sep}
  → Monitor excited state contribution
- Roughly same statistics at every t_{sep} (setup see talk K. Ottnad)
  → Number of sources adapted to t_{sep}
- Chiral/Continuum/Finite-Size extrapolation possible

Results



- For ensembles < 250 MeV
  Two-State analysis (including priors for Δ) generally slightly above
- Two-State analysis very sensitive to priors

CCF-extrapolation

- Chiral Expansion

$$\sigma_{\pi N} = (k_1 + k_a a^2)M^2 + k_2 M^3 + 2k_3 M^4 \log\frac{M}{\mu} + k_4 M^4 + k_L M^2\left(\frac{1}{L} - \frac{M}{2}\right)e^{-ML}$$

- k_1, k_2, k_3 depend on (known) LECs
- Challenging fits, leaving all parameters free
  - Fits not stable, especially with cuts in pion mass
  - Only fit up to M^3
    $\rightarrow$ Coefficients inconsistent with ChPT
  - k_3 and k_4 competing (cancellations)
    $\rightarrow$ drop k_3
  - Or take ChPT information into account via priors

CCF-extrapolation

- Chiral Expansion

$$\sigma_{\pi N} = (k_1 + k_a a^2)M^2 + k_2 M^3 + 2k_3 M^4 \log\frac{M}{\mu} + k_4 M^4 + k_L M^2 \left(\frac{1}{L} - \frac{M}{2}\right) e^{-ML}$$

- k_1, k_2, k_3 depend on (known) LECs
- Challenging fits, leaving all parameters free
  - Fits not stable, especially with cuts in pion mass
  - Restrict $k_1, k_2(, k_3)$via gaussian priors (still fitted)

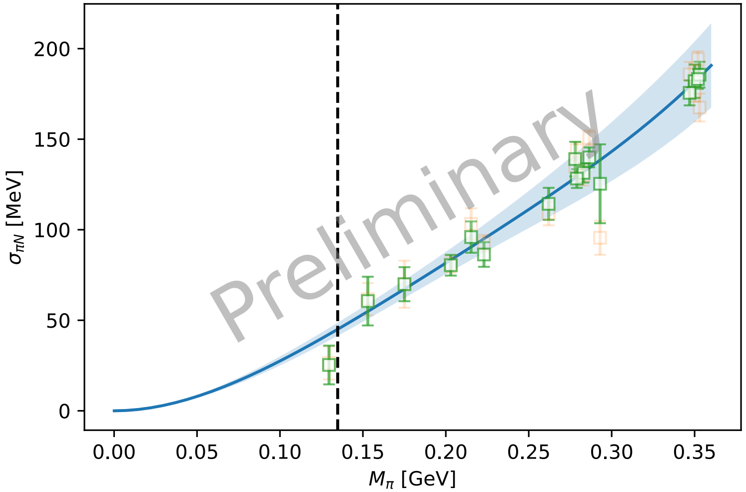
$$k_1 = -4c_1 = (4.44 \pm 0.12)\,\mathrm{GeV}^{-1}$$

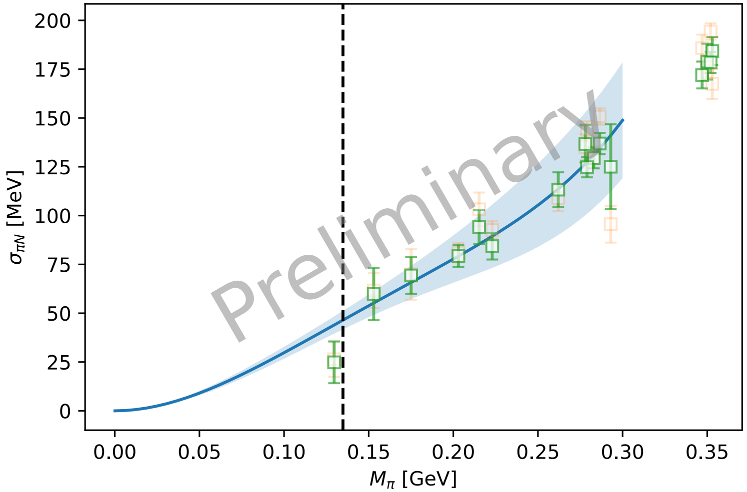
$$k_2 = -\frac{9g_A^2}{64\pi F_\pi^2} = (-8.52 \pm 0.04)\,\mathrm{GeV}^{-2}$$

$$\begin{aligned} k_3 &= -\frac{3}{32\pi^2 F_\pi^2}\left(\frac{g_A^2}{m_N} - 8c_1 + c_2 + 4c_3\right) - \frac{c_1}{8\pi^2 F_\pi^2} \\ &= (-11.38 \pm 0.35)\ \mathrm{GeV}^{-3} \end{aligned}$$

  - Width 5x the error for $k_1, k_2(, k_3)$

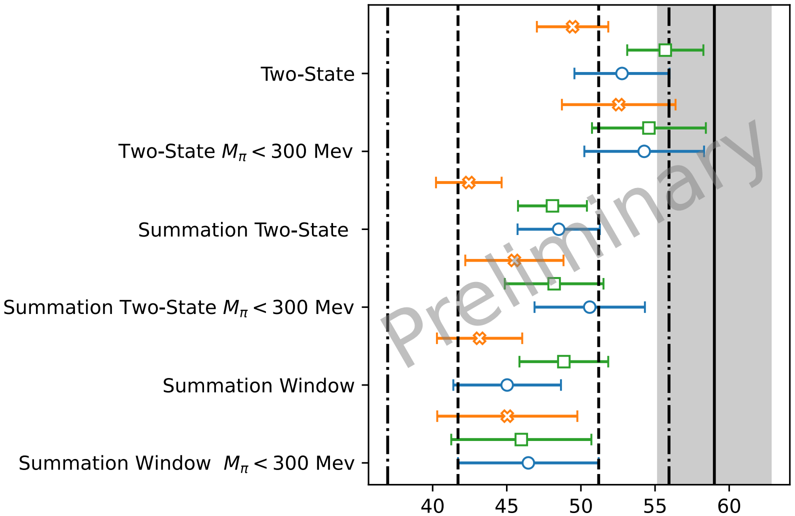
CCF-extrapolation





- Data from window average of summation data
- Green points:
  After correction of lattice artefacts (only central value) of original data (orange points)
- Left: No Pion mass cut
  Right: Pion mass cut 300 MeV

CCF-extrapolation



- With Restrictions from ChPT: Result stable w.r.t
  - Cuts in pion mass
  - Window vs Two-State
- Two-State result higher but within 2 σ of Summation Window result
- Grey Area: Result using Roy-Steiner
- CCF result depends on priors

- Fits: blue, green, orange = (k1,k2),(k1,k2,k3),(k1,k2,no log)
- Further Improvements:
  - Increase statistics for the disconnected part
  - Simultaneous fit with nucleon mass

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Thanks!