



# Towards the continuum limit of nucleon form factors at the physical point using lattice QCD

Ryutaro TSUJI\* (Tohoku U., RIKEN R-CCS)

In collaboration with: Y. Aoki, K.-I. Ishikawa, Y. Kuramashi,  
S. Sasaki, E. Shintani and T. Yamazaki  
for PACS Collaboration

\* Present address is RIKEN R-CCS, Kobe, Japan.

---

# Introduction

- Nucleon as a transdisciplinary target
- Proton puzzles and tensions
- The conventional studies and this work

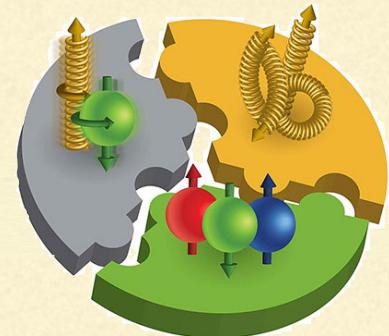
# Nucleon structure is non-trivial

## ① Particle physics (Intensity frontier)

Beyond S.M. = Experiment - S.M. of particle physics

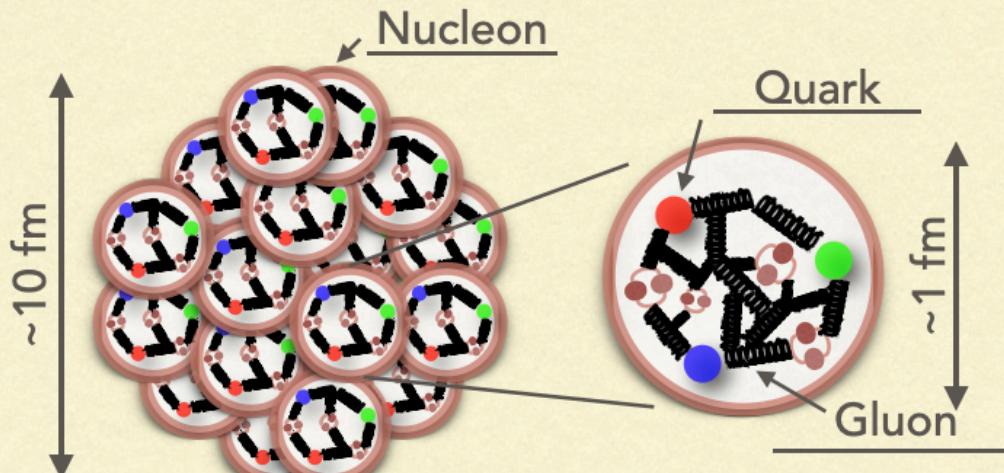
- eg) • Quark EDM  
• Muon g-2  
• Proton decay

### Nucleon structure



- eg) • Proton radius/spin problem  
• Short range correlation in nuclei  
• SSA

## ② Nuclear physics (Many body problem)



Nucleon structure is  
Quantum Many Body Prob.  
Blocks  $\rightarrow$  Quarks & Gluons  
Int.  $\rightarrow$  QCD  
Perturbation does NOT work

# Proton radius puzzle (2010~) (exp. vs exp.)

CODATA'06 (2008)

Bernauer (2010)

Pohl (2010)

Zhan (2011)

CODATA'10 (2012)

Antognini (2013)

Beyer (2017)

Fleurbaey (2018)

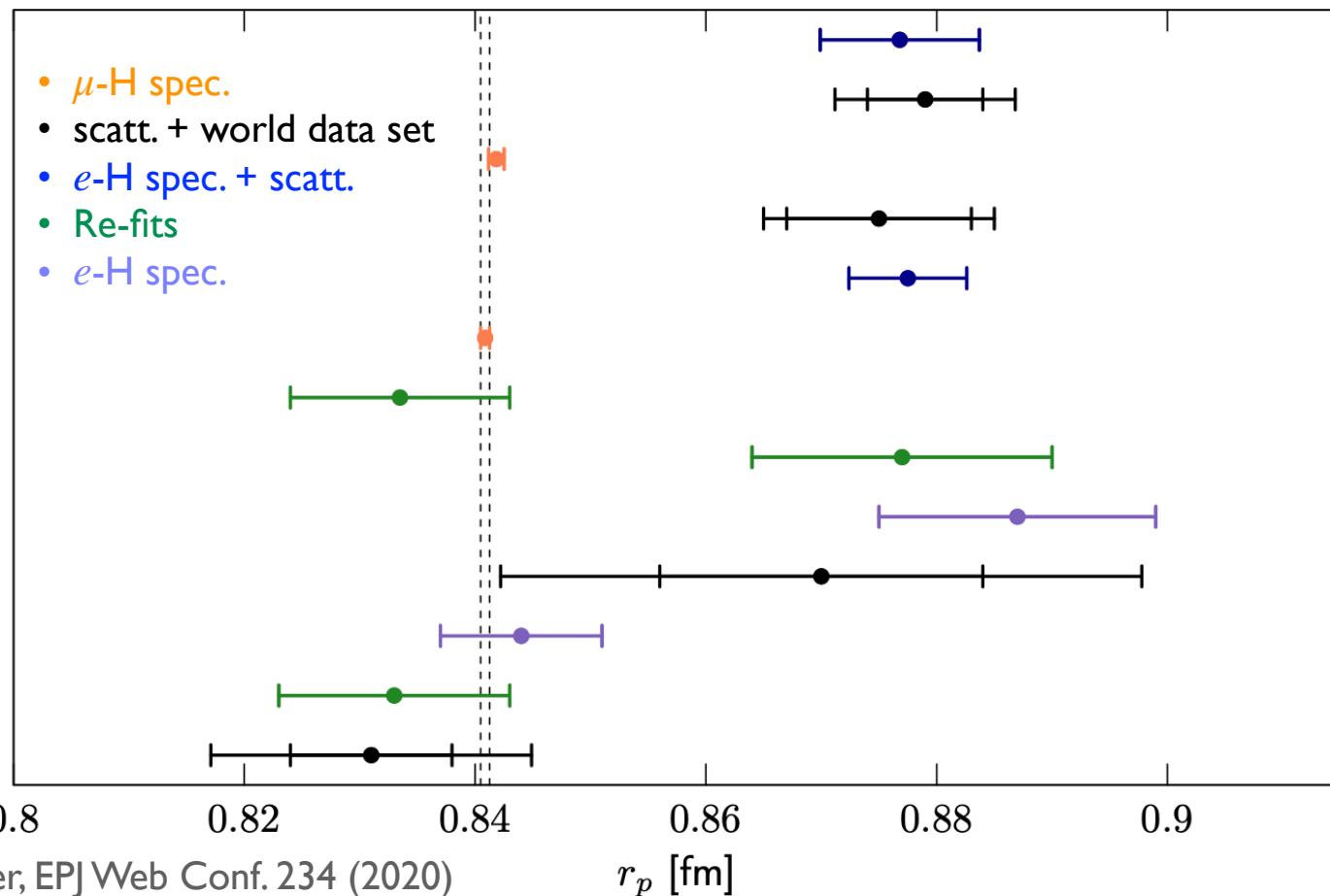
Sick (2018)

Mihovilović (2019)

Alarćon (2019)

Bezignov (2019)

Xiong (2019)



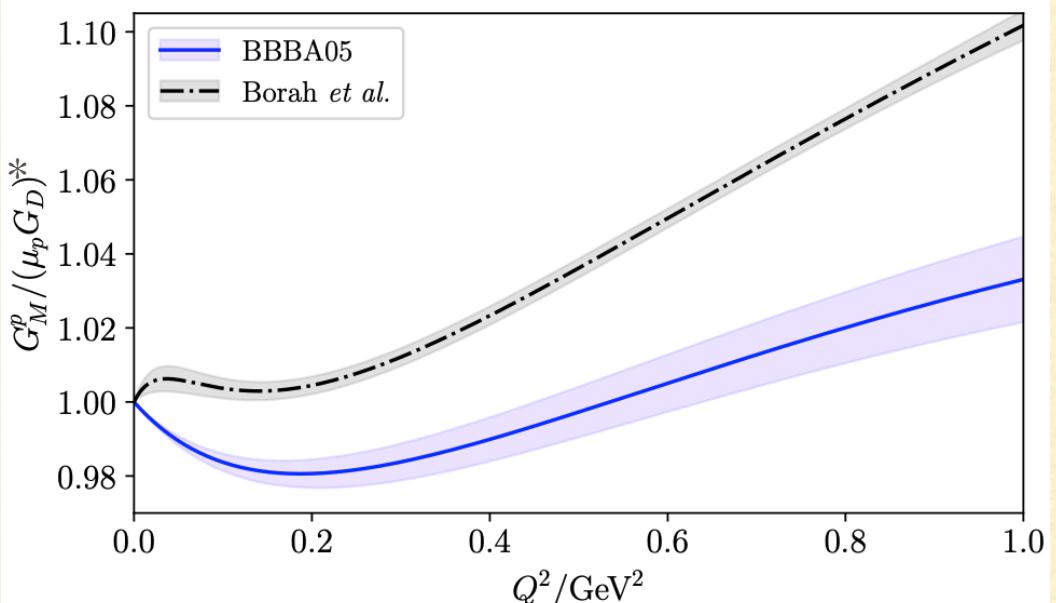
[1] Taken from Bernauer, EPJ Web Conf. 234 (2020)

Recent  
perspectives

- Experiments with similar kinematics as the earlier ones
- Muon v.s. Electron puzzle → Scatt. v.s. Spec.
- Our understandings are lacking something and more?

LQCD: NOT enough precision → A percent level is needed

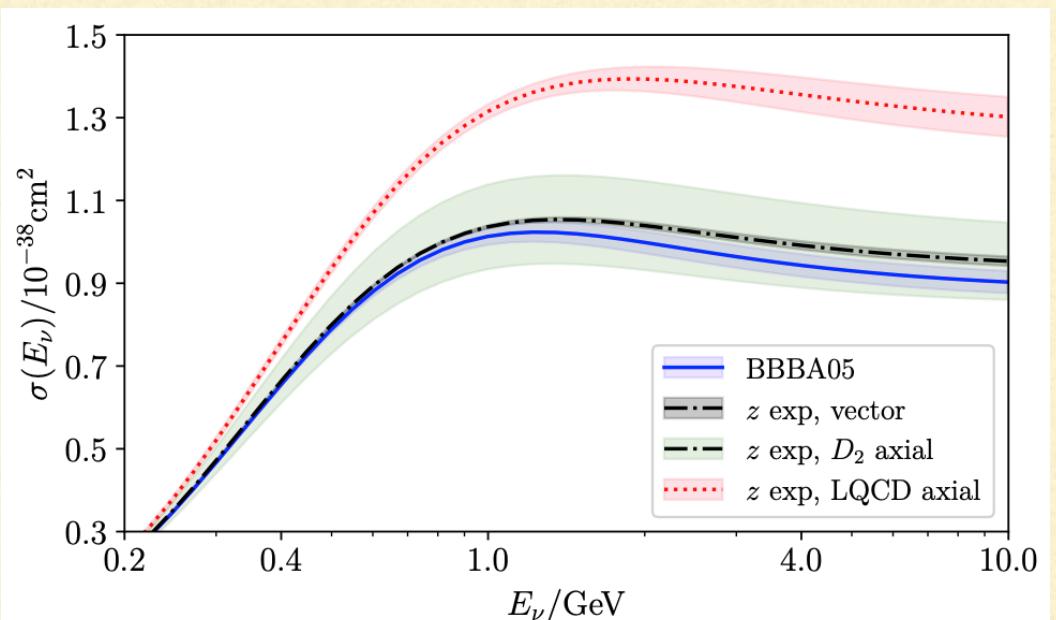
# Magnetic and Axial FF



- **Magnetic FF (exp. vs exp.)**

Different parameterizations exhibits clear discrepancy over all  $Q^2 > 0$

LQCD: NOT enough precise  
→ A percent level precision is needed



- **Axial FF (exp. vs lat.)**

Less known  $F_A(q^2)$  behavior causes large uncertainties on  $\nu N$  cross section

**LQCD: enough precise**  
→ **Theoretical prediction**

\* dipole ansatz with a dipole mass of 0.84 GeV

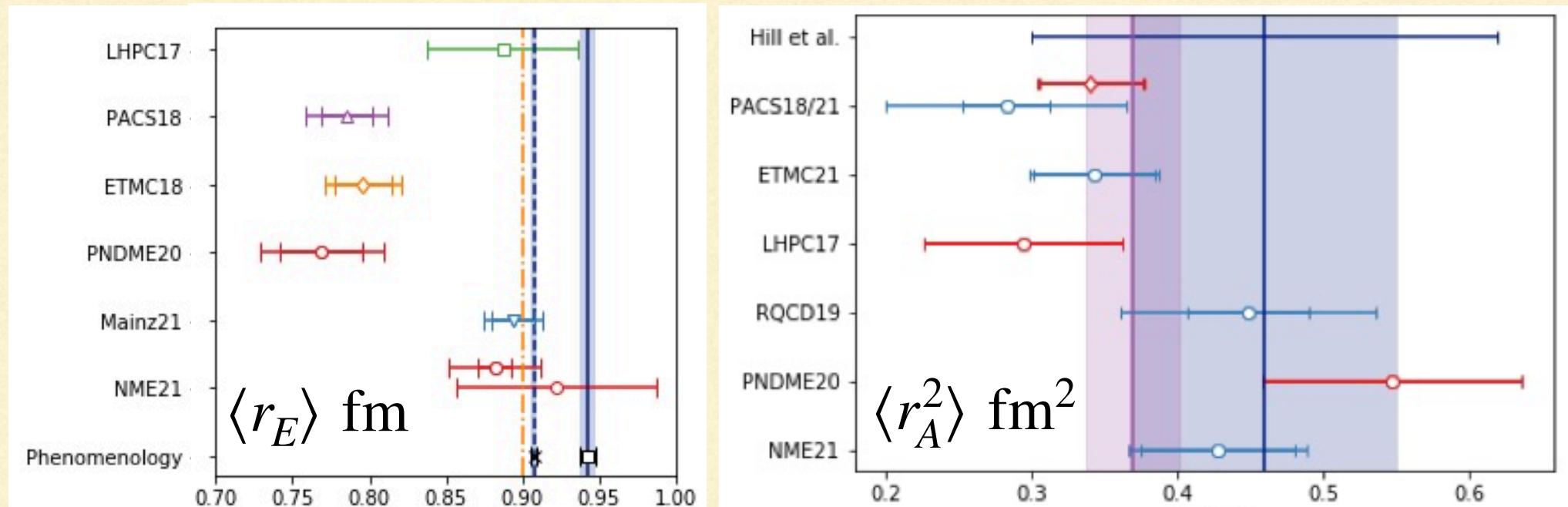
[I] Taken from Aaron A. S. Mayer et al., arXiv:2201.01839 (2022). 4

# From lattice QCD community

Sources of uncertainties:

- Statistical accuracy
- Excited states contamination
- Model dependences
- Chiral Continuum Finite-size

- PACS('18) :
- ✓ All-mode-averaging
  - ✓ More than two choice of  $t_{\text{sep}}$
  - ✓ Conventional + Direct
  - Large vol. + Phys.  $m_\pi$



High-precision(PACS'18) + Continuum limit = Next PACS(this work)

[1] K.-I. Ishikawa et al. for PACS Collaboration, Phys.Rev.D **104**, 074514(2021)

[2] Taken from Dalibor Djukanovic ,The 38th International Symposium on lattice field theory

---

# Methodology

- Lattice QCD
- Major systematic uncertainties
- Methodology for measuring  $\sqrt{\langle r_l^2 \rangle}$  using LQCD

# Calculation strategy using lattice QCD

**Targets** : RMS radius of  $G_l(q^2)$  :  $\sqrt{\langle r_{l2} \rangle} = -\frac{6}{G_l(0)} \left. \frac{dG_l(q^2)}{dq^2} \right|_{q^2 \rightarrow 0}$

Lattice accessible

$$\langle N(p') | \bar{q} \gamma_\mu q | N(p) \rangle = \bar{u}(p') \left[ \frac{(p' + p)^\mu}{2M} \frac{G_E(q^2) - \frac{q^2}{4M^2} G_M(q^2)}{1 - \frac{q^2}{4M^2}} + i\sigma^{\mu\nu} \frac{q_\nu}{2M} G_M(q^2) \right] u(p)$$

$$\rightarrow \langle r_E^2 \rangle, \mu, \langle r_M^2 \rangle$$

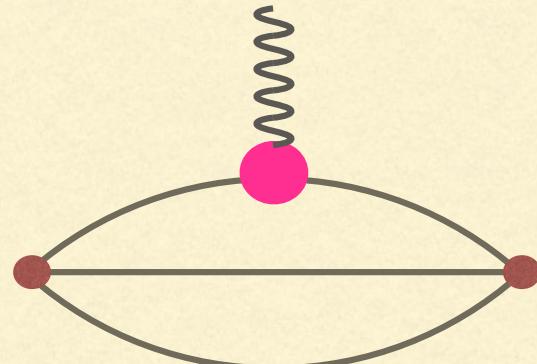
$$\langle N(p') | \bar{q} \gamma_\mu \gamma_5 q | N(p) \rangle = \bar{u}(p') \left[ \gamma_\mu F_A(q^2) + i \frac{q^\mu}{2M} F_P(q^2) \right] u(p)$$

$$\rightarrow \langle r_A^2 \rangle, g_A$$

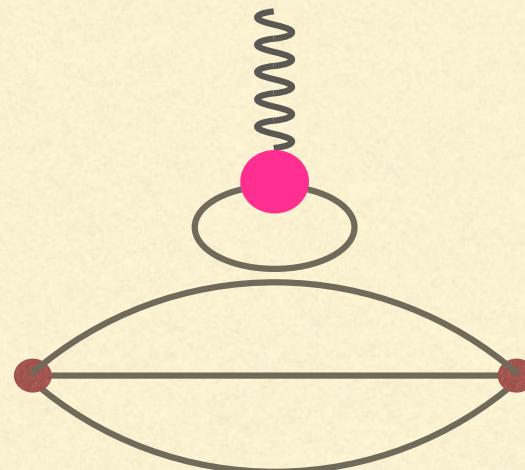
**Determination of a slope at  $q^2 = 0$  : z-expansion today**

# Isovector quantities with 2+1 LQCD

**CAUTION:** Only connected contributions are considered



Connected contribution



Disconnected contribution  
= High computational costs

Isovector with 2+1 LQCD dose NOT suffer from disconnected  
For a comparison,

$$G_E^v(q^2) = G_E^p(q^2) - G_E^n(q^2)$$

$$\rightarrow \sqrt{\langle (r_E^v)^2 \rangle} = \sqrt{\langle (r_E^p)^2 \rangle - \langle (r_E^n)^2 \rangle} = \begin{cases} 0.939(6) \text{ fm : scatt.} \\ 0.907(1) \text{ fm : } \mu\text{H spec.} \end{cases}$$

---

# Numerical results

- Nucleon renormalized axial charge
- Dispersion relation
- Nucleon form factors and RMS radii

# Simulation details -PACS10 configuration[1][2]

Eliminate major uncertainties

Finite size effect  
Chiral extrapolation

$\otimes$  Low  $q^2$  data are accessible  
 $q^2 = (2\pi/L)^2 \times |\vec{n}^2|$  = PACS10

Lattice size

$128^4$  [1]

$160^4$  [2]

Spacial vol. $\gg$ nucleon	$\sim (10.9 \text{ fm})^3$	$\sim (10.1 \text{ fm})^3$
----------------------------	----------------------------	----------------------------

Pion mass $\sim m_\pi^{\text{exp.}}$	135 MeV	135 MeV
--------------------------------------	---------	---------

Nucleon mass	$\sim 0.935 \text{ GeV}$	$\sim 0.946 \text{ GeV}$
--------------	--------------------------	--------------------------

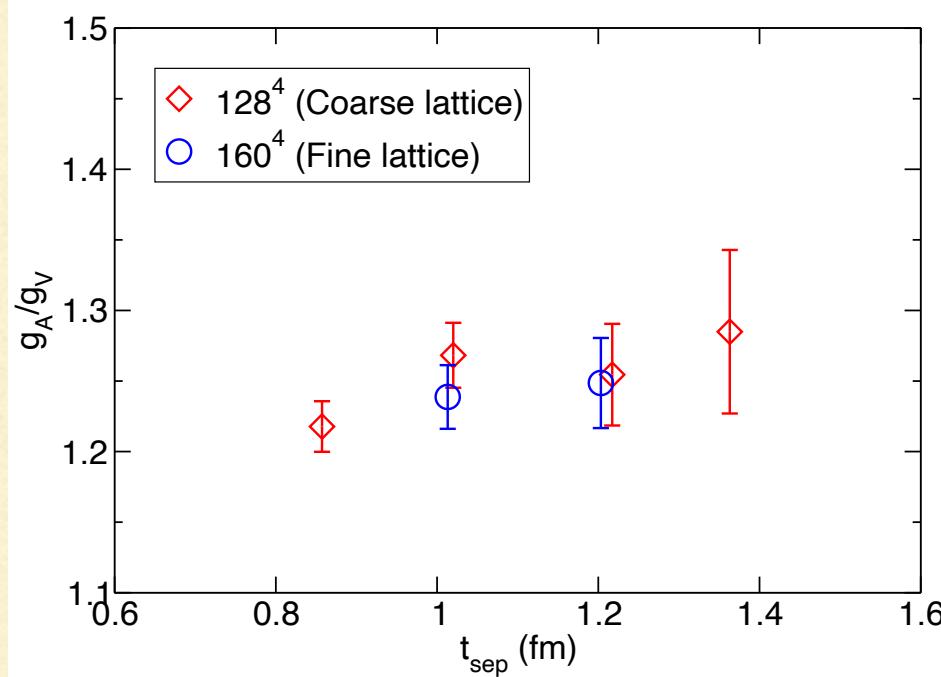
$ t_{\text{sink}} - t_{\text{src}} /a$	10, 12, 14, 16	16, 19
--	----------------	--------

Lattice spacing	coarse $\sim 0.086 \text{ fm}$	fine $\sim 0.063 \text{ fm}$
-----------------	-----------------------------------	---------------------------------

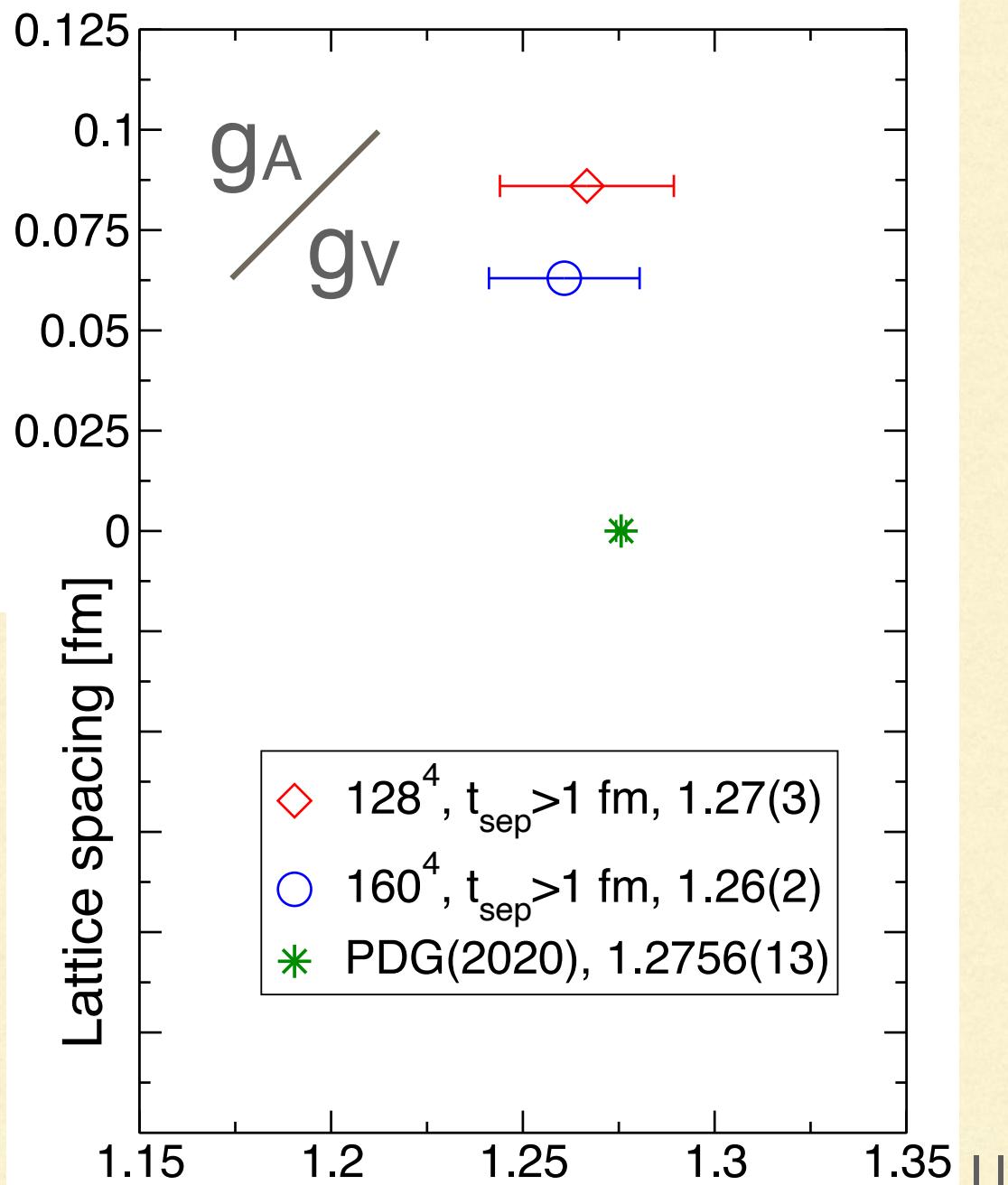
[1] E. Shintani et al., Phys. Rev. D 99, 014510(2019), (Erratum; Phys. Rev. D 102 (2020) 019902.)

[2] E. Shintani and Y.Kuramashi, Phys.Rev. D 100, 034517(2019)

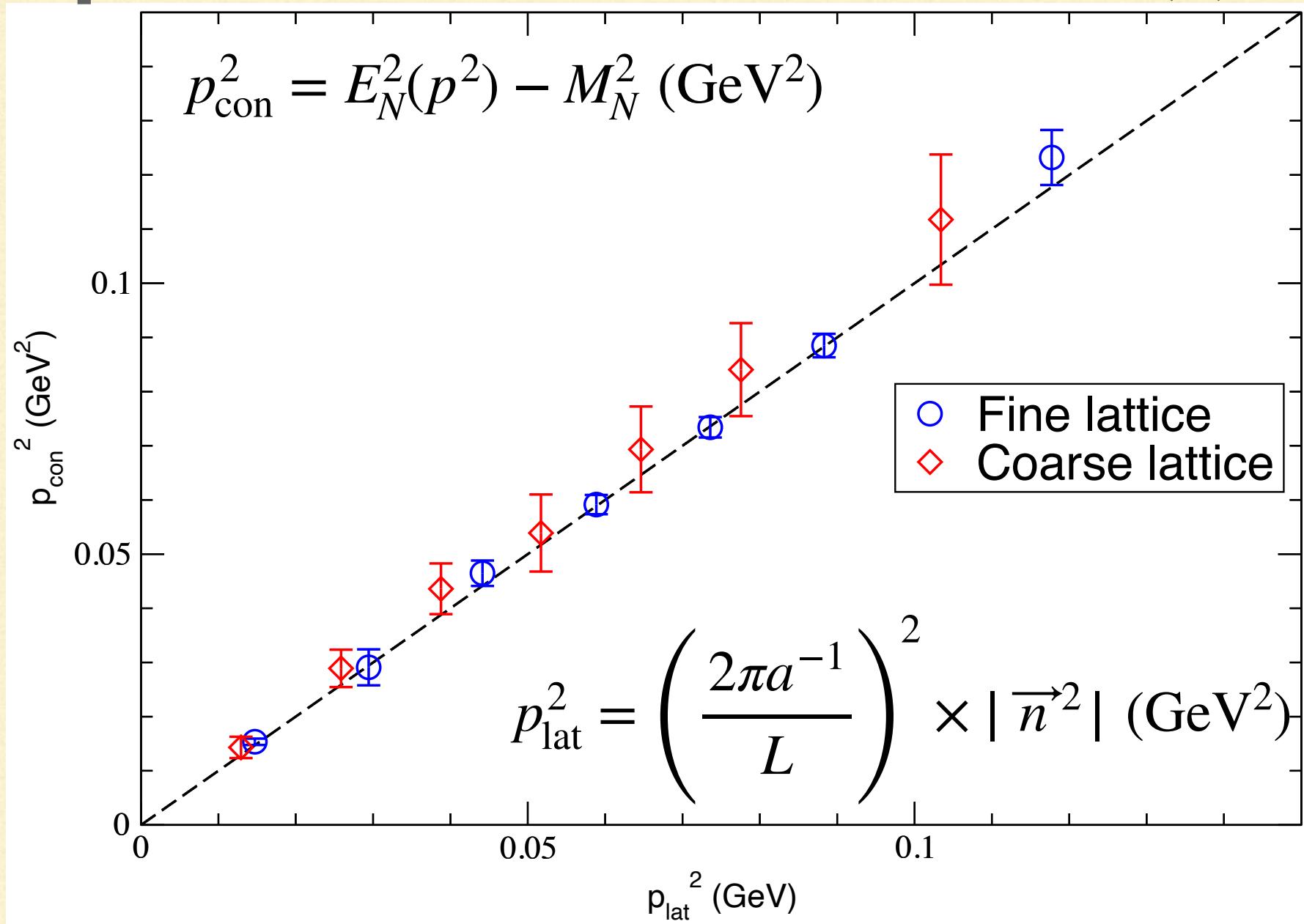
# Nucleon axial charge



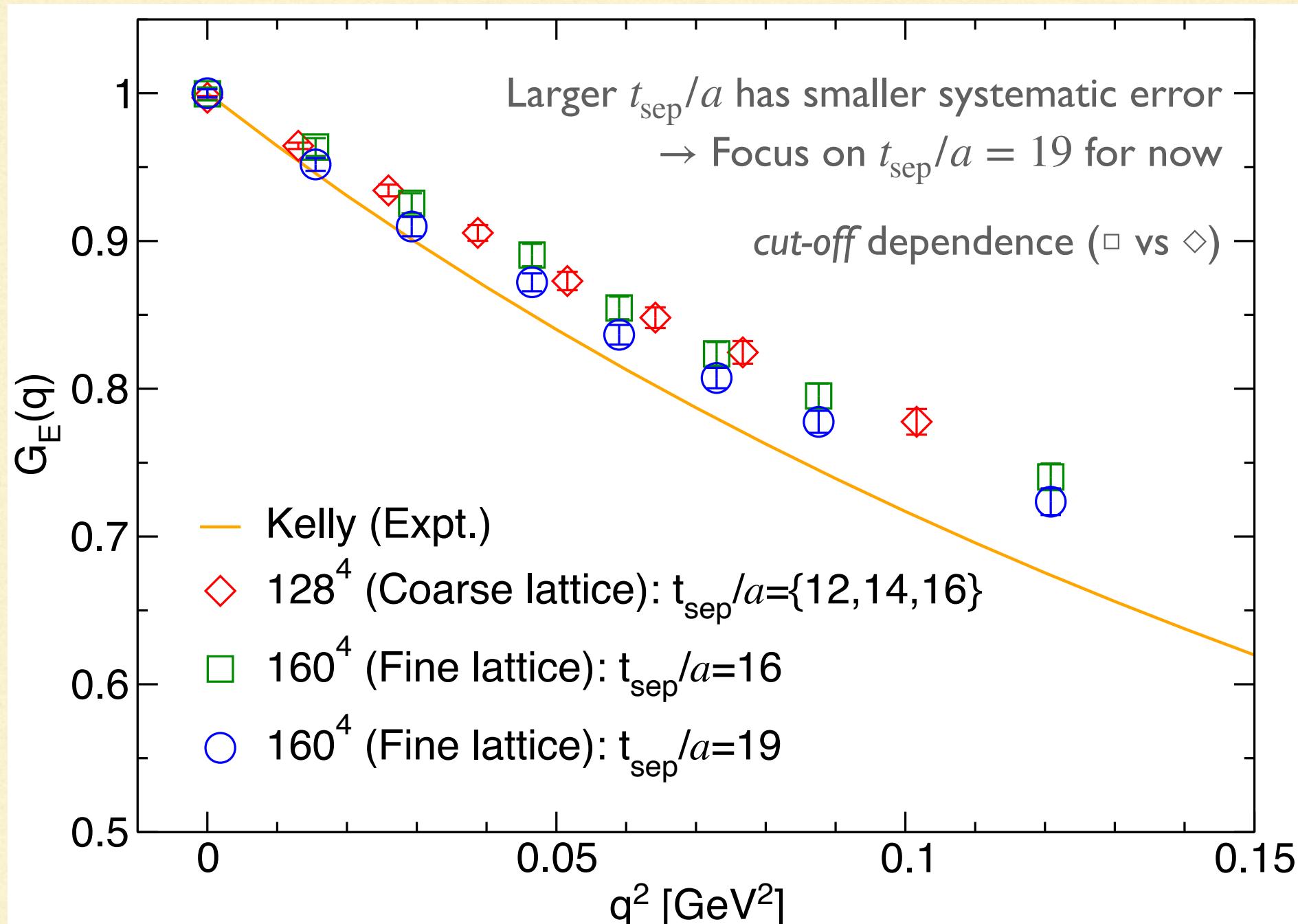
$t_{sep} > 1$  (fm) seems large enough for g.s. saturation  
Both lattice (**coarse** & **fine**) reproduce the **PDG(2020)** within statistical errors (2%)  
= discretization err.  $\leq 2\%$



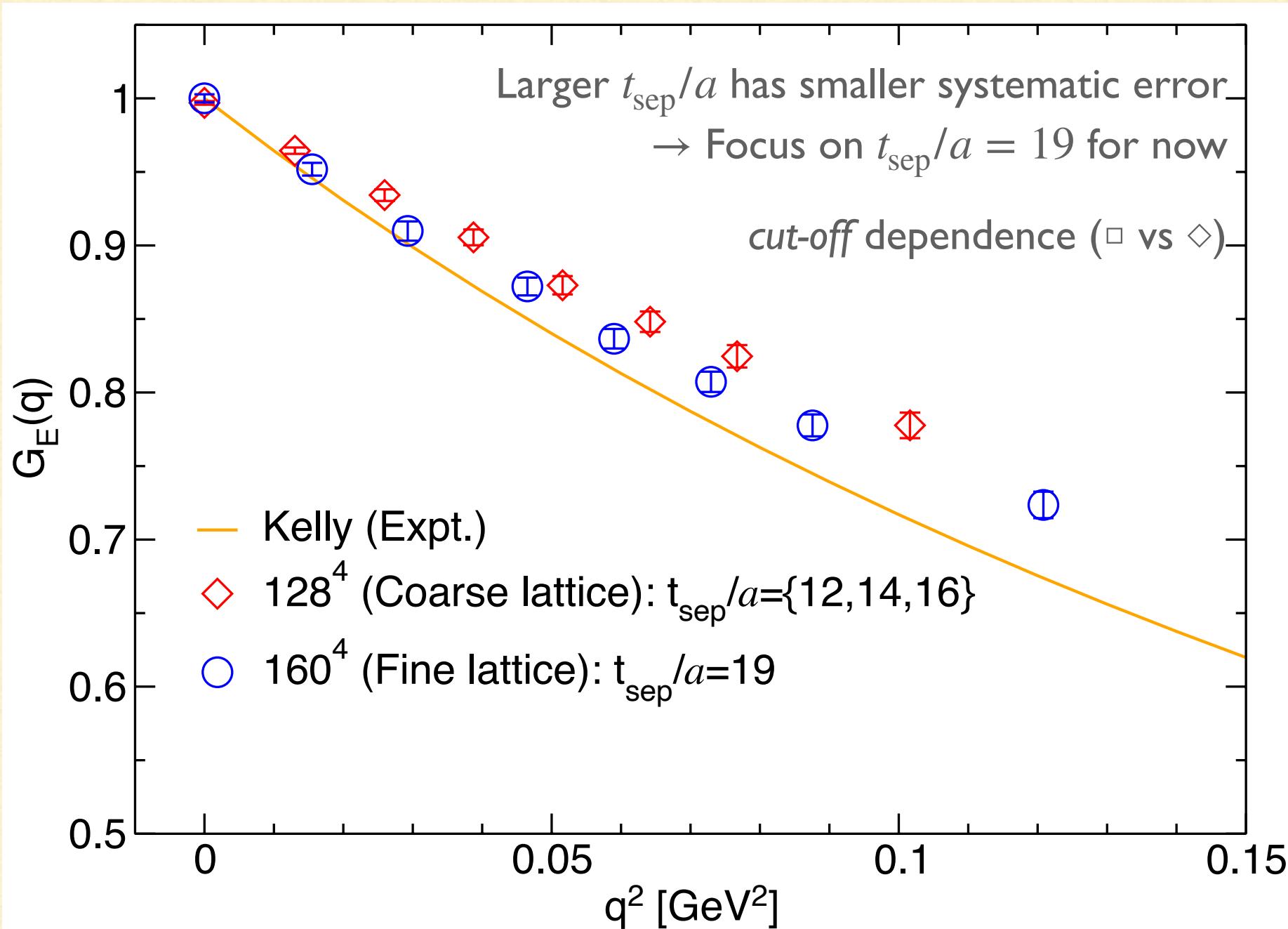
# Dispersion relation -On-shell $O(a)$ imp.



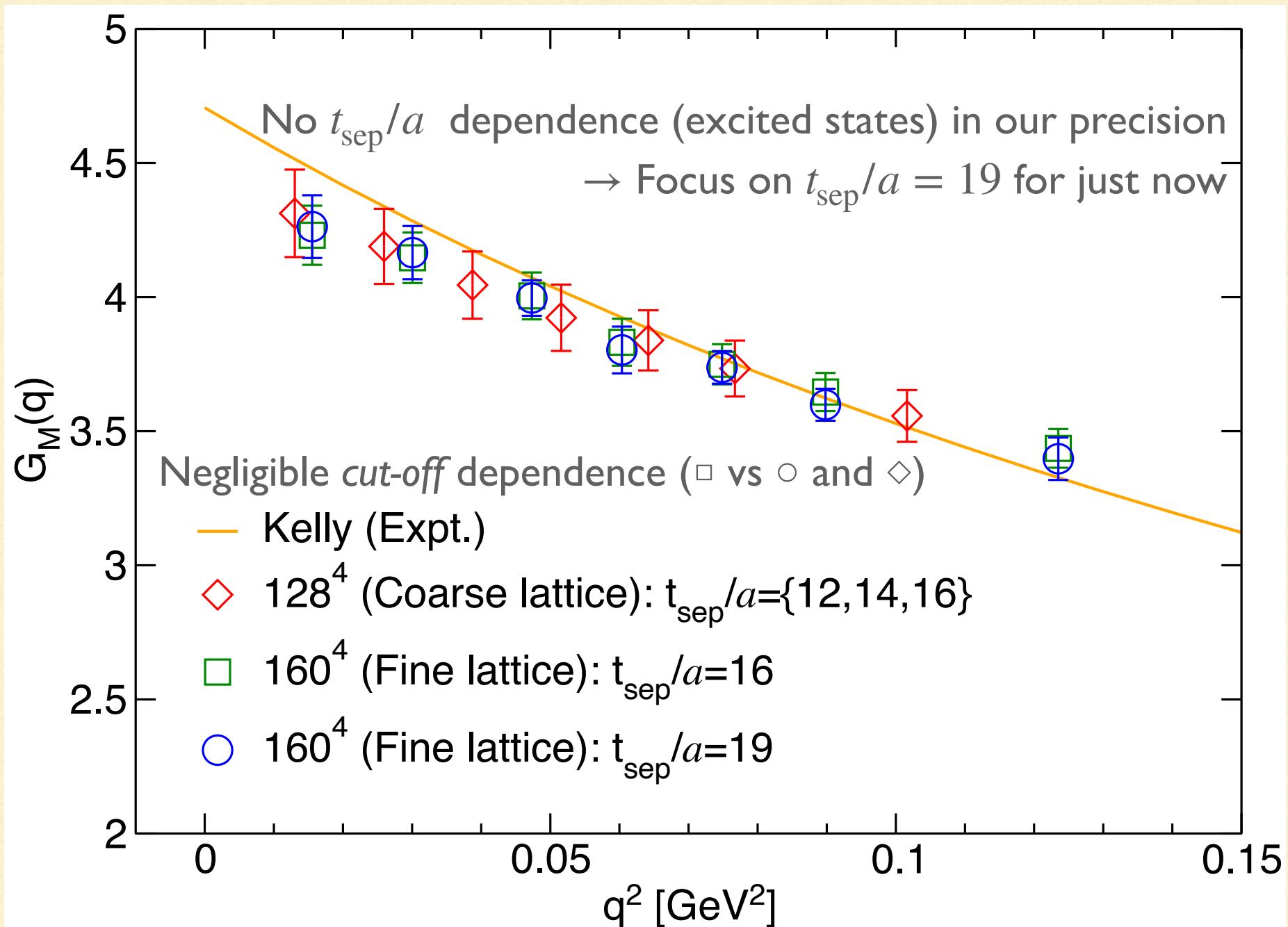
# Electric form factor



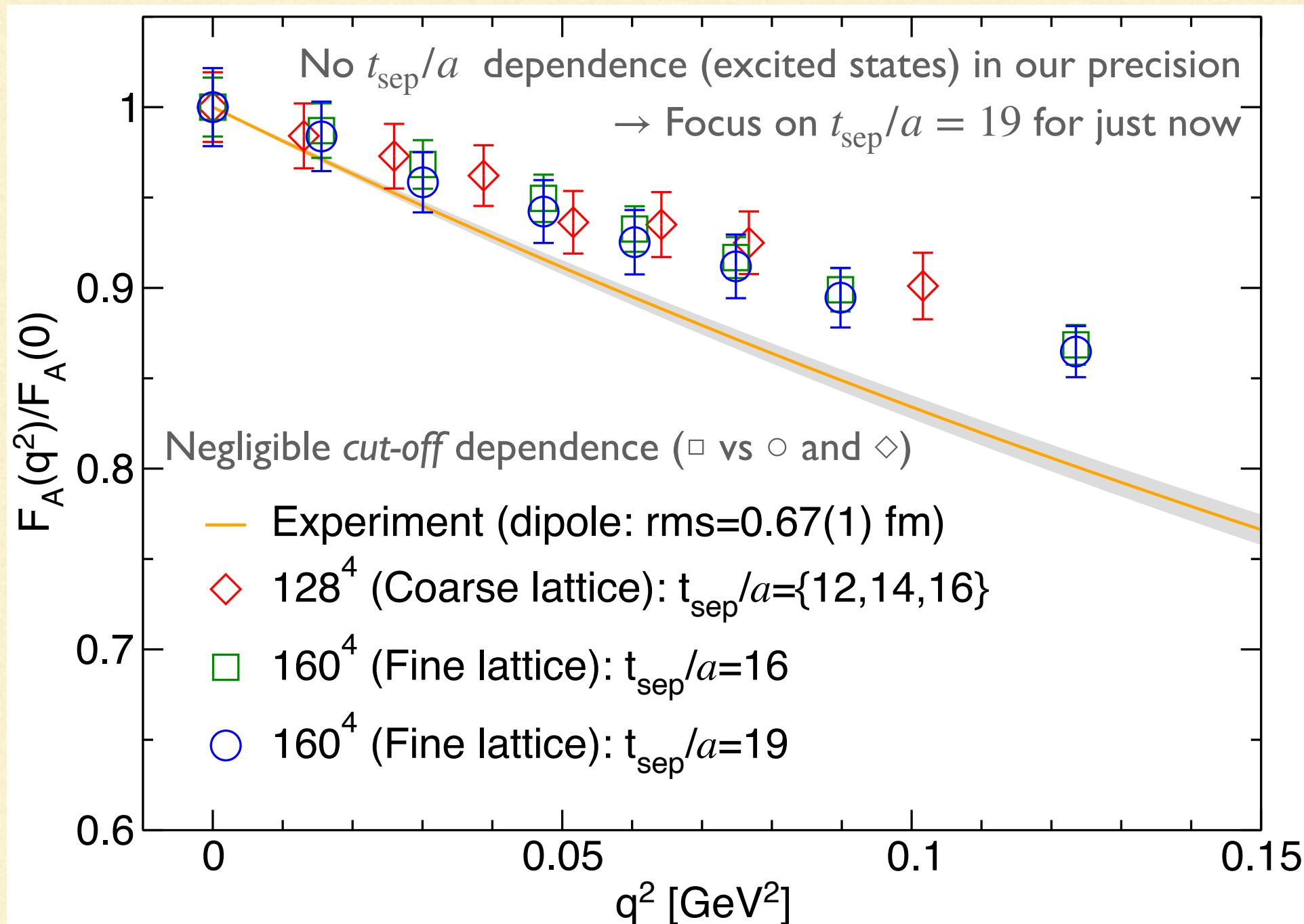
# Electric form factor



# Magnetic form factor

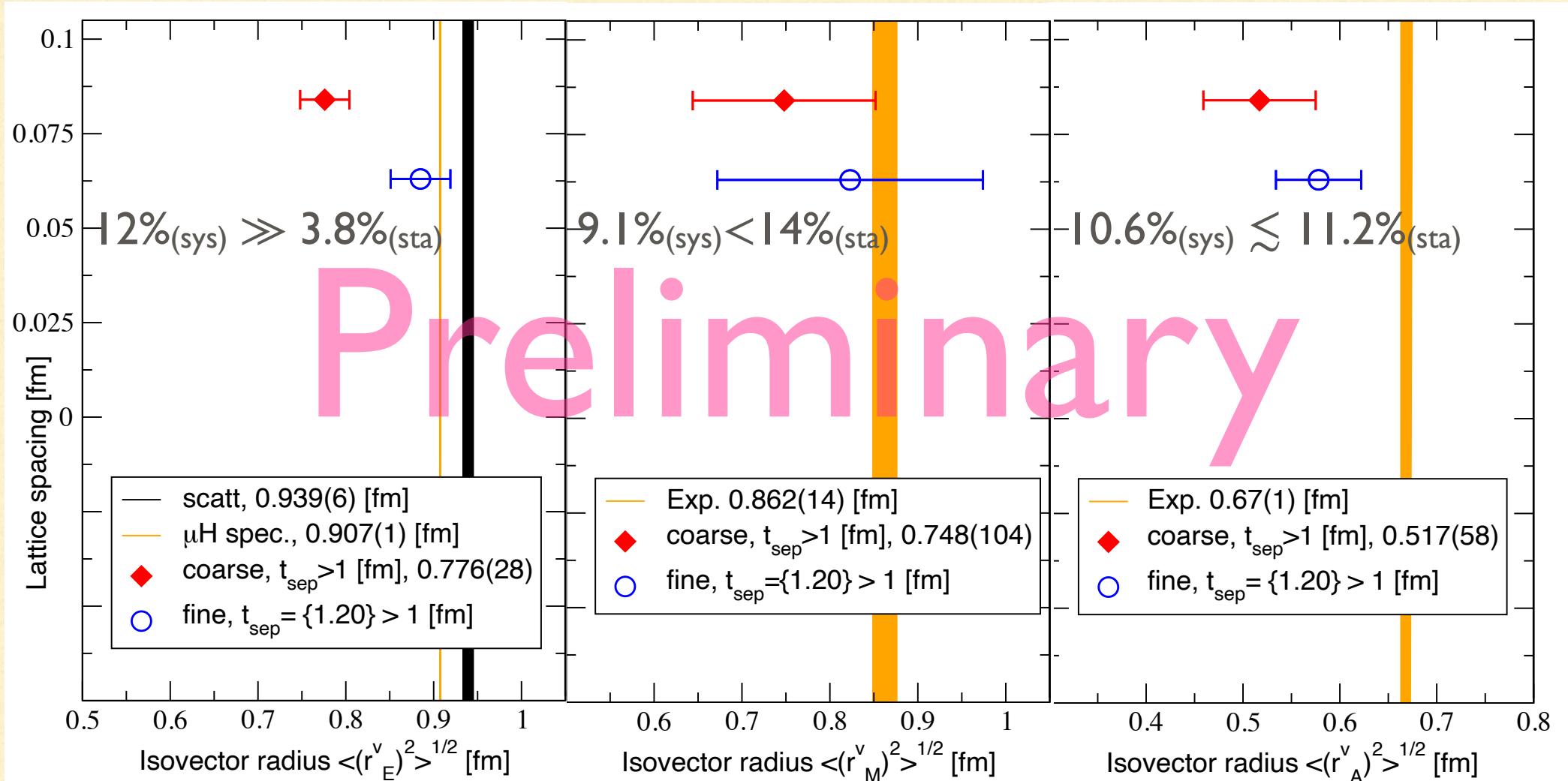


# Axial form factor



# Isovector radii

Discretization err.  $\gg$  Statistical err. in Electric FF



Discretization err.  $\lesssim$  Statistical err. in Magnetic and Axial FF

---

# Summary and Discussion

- Conclusion of this talk
- Future works

# Summary of our Form Factor projects

PACS Collaboration for Nucleon projects:

- AMA technique → High statistical precision
- Physical point ( $m_\pi = 135$  MeV) → No chiral extrapolations
- Large physical volume ( $\sim 10^4$  fm $^4$ ) → Low  $Q^2$  information
- *Fully dynamical lattice QCD simulations* towards **continuum limit**

We attempted to eliminate the sources of uncertainties.

Our **preliminary** results:

- For  $g_A$ , both **coarse** & **fine** reproduce the **PDG** within stat. err. (2%)
- $\sqrt{\langle (r_E^v)^2 \rangle}$  with high statistical precision (better than 5%) exposes the presence of large disc. err. ( $\sim 12\%$ ) on our course lattice.
- Both  $\sqrt{\langle (r_M^v)^2 \rangle}$  and  $\sqrt{\langle (r_A^v)^2 \rangle}$  are show the smiler size of disc. err. ( $\sim 10\%$ ), but their stat. precision are still comparable ( $\sim 10\%$ ).