Nucleon form factors with sLapH
OR
Nucleon-pion sigma term from MDWF on HISQ

Lattice 2022

André Walker-Loud

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High Energy Physics – Lattice

[Submitted on 8 Aug 2022]

Elastic nucleon-pion scattering at $m_\pi \approx 200$ MeV from lattice QCD

John Bulava, Andrew D. Hanlon, Ben Hörz, Colin Morningstar, Amy Nicholson, Fernando Romero-López, Sarah Skinner, Pavlos Vranas, André Walker-Loud

Elastic nucleon-pion scattering at $M\pi \approx 200$ MeV from lattice QCD

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  arXiv:2208.03867

- Single CLS ensemble (D200)
  - $a \approx 0.063$ fm, $V=64^3\text{x}128$, $N_{cfg}=2000$
  - $M\pi \approx 200$ MeV, $M\pi L \approx 4.2$, $\mathrm{tr}(M_q) = \mathrm{tr}(M_q^{phys})$, $\rightarrow M_K \approx 480$ MeV

Elastic nucleon-pion scattering at $M\pi \approx 200$ MeV from lattice QCD

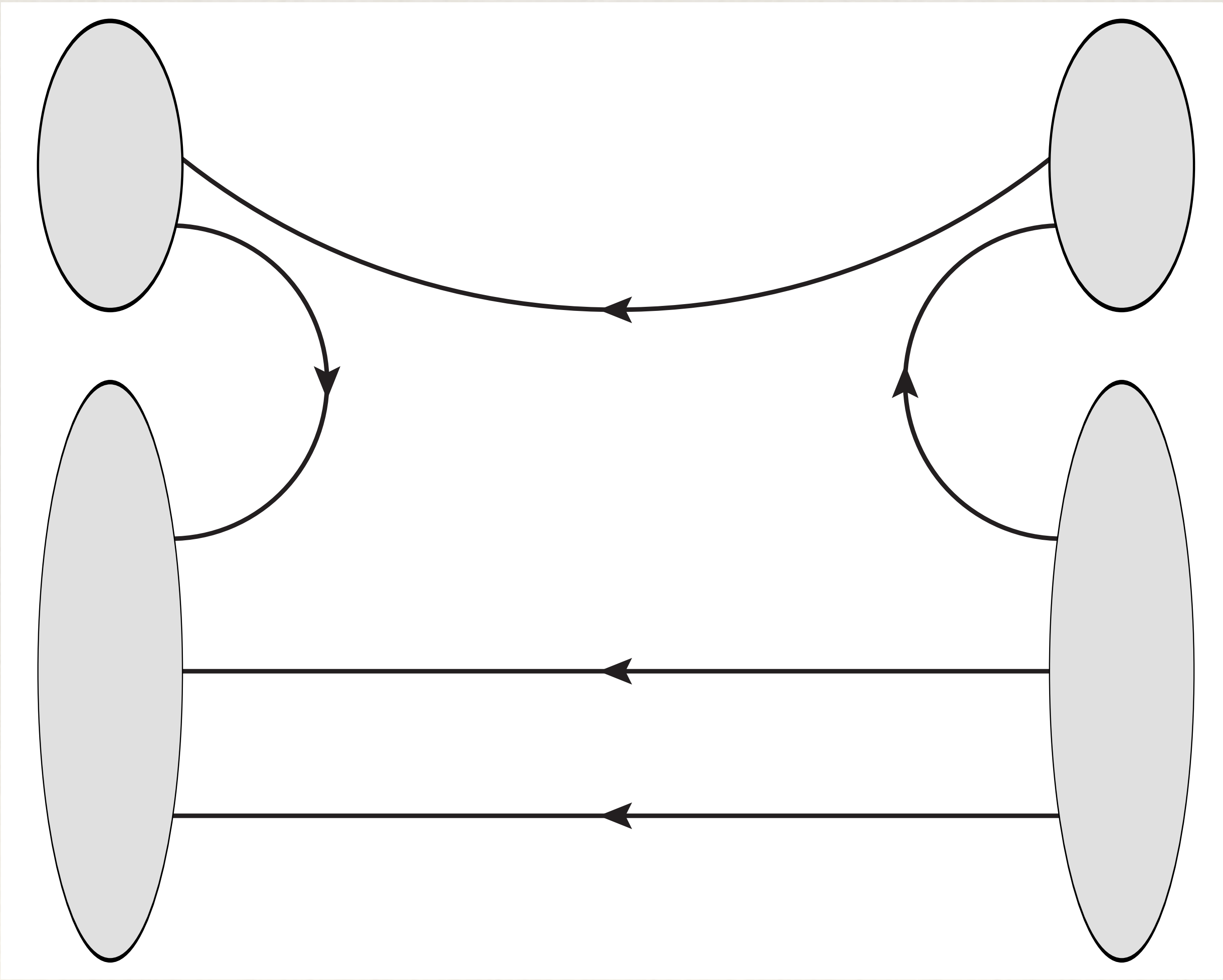
- What were the goals of this calculation?
  - This is part of a large effort to understand two (and more) hadron interactions from QCD
    - NN, Nπ, YN, YY, …, N$\pi\pi$, NNN?
  - Nπ scattering is important to include in single-nucleon structure/form-factor calculations to test our understanding of excited states
  - Nπ scattering is essential to study the N→ Nπ, Δ transition matrix elements
  - We are utilizing a variational basis of interpolating operators to have positive-definite two-hadron correlators
    - To mitigate the growth in contraction cost of the distillation method
      the number of eigenvectors of the 3D Laplacian grows with V_3 for fixed smearing profile
      we are using the stochastic Laplacian Heaviside (sLapH) method

      Peardon et al. 0907.1913, Morningstar et al. 1104.3870
      - use a noise basis between the Laplacian eigenvectors and solutions of D^{-1}

Elastic nucleon-pion scattering at $M\pi \approx 200$ MeV from lattice QCD

- What were the goals of this calculation?
  - Is the sLapH method capable — with reasonable statistics/resources — in achieving precise estimates of the interaction energies/scattering amplitudes at light pion masses?
  - It seems the answer is yes, fortunately
  - To carry out the study, we used

$N_{\rm D}$	(ρ, n_ρ)	$N_{\rm ev}$	$N_{\rm R}^{\rm fix}$	$N_{\rm R}^{\rm rel}$	Noise dilution	N_{t_0}
~~2560~~ 2176	(0.1,36)	448	6	2	$({\rm TF,SF,LI16})_{\rm fix}({\rm TI8,SF,LI16})_{\rm rel}$	4

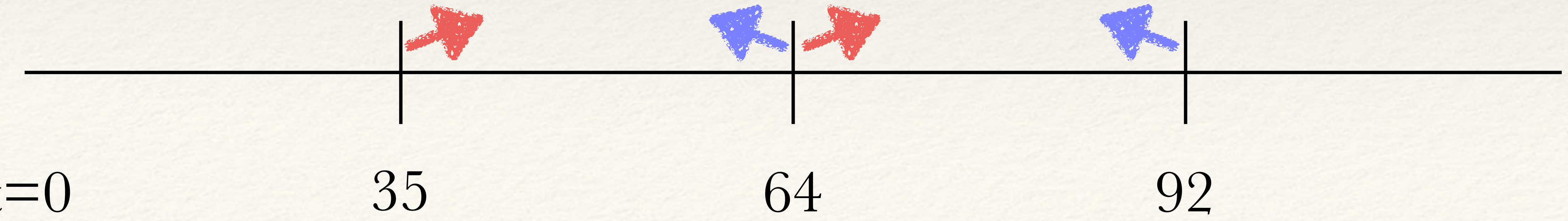


$N_{\rm D}$ Dirac inversions per config

$N_{\rm ev}$ No. of eigenvectors of 3D Laplacian

$N_{\rm R}^{\rm fix}$ No. of noise sources for fixed lines

$N_{\rm R}^{\rm rel}$ No. of noise sources for relative lines



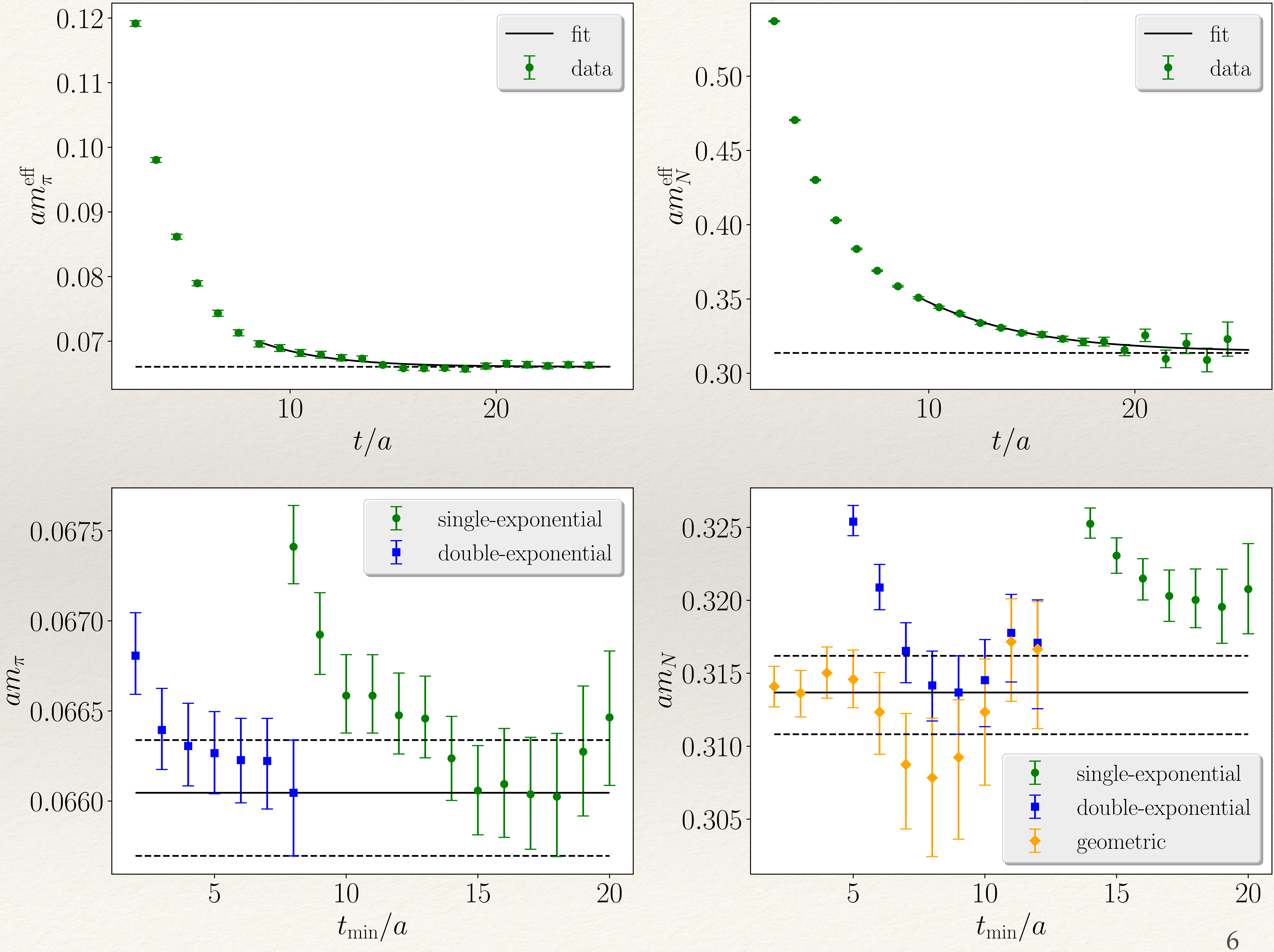
Elastic nucleon-pion scattering at $M\pi \approx 200$ MeV from lattice QCD

- **Results**
  - We explored multi-exponential fits as well as a "geometric series" fit

$$C_{\text{geom.}}(t) = \frac{Ae^{-E_0 t}}{1 - Be^{-\Delta E t}}$$

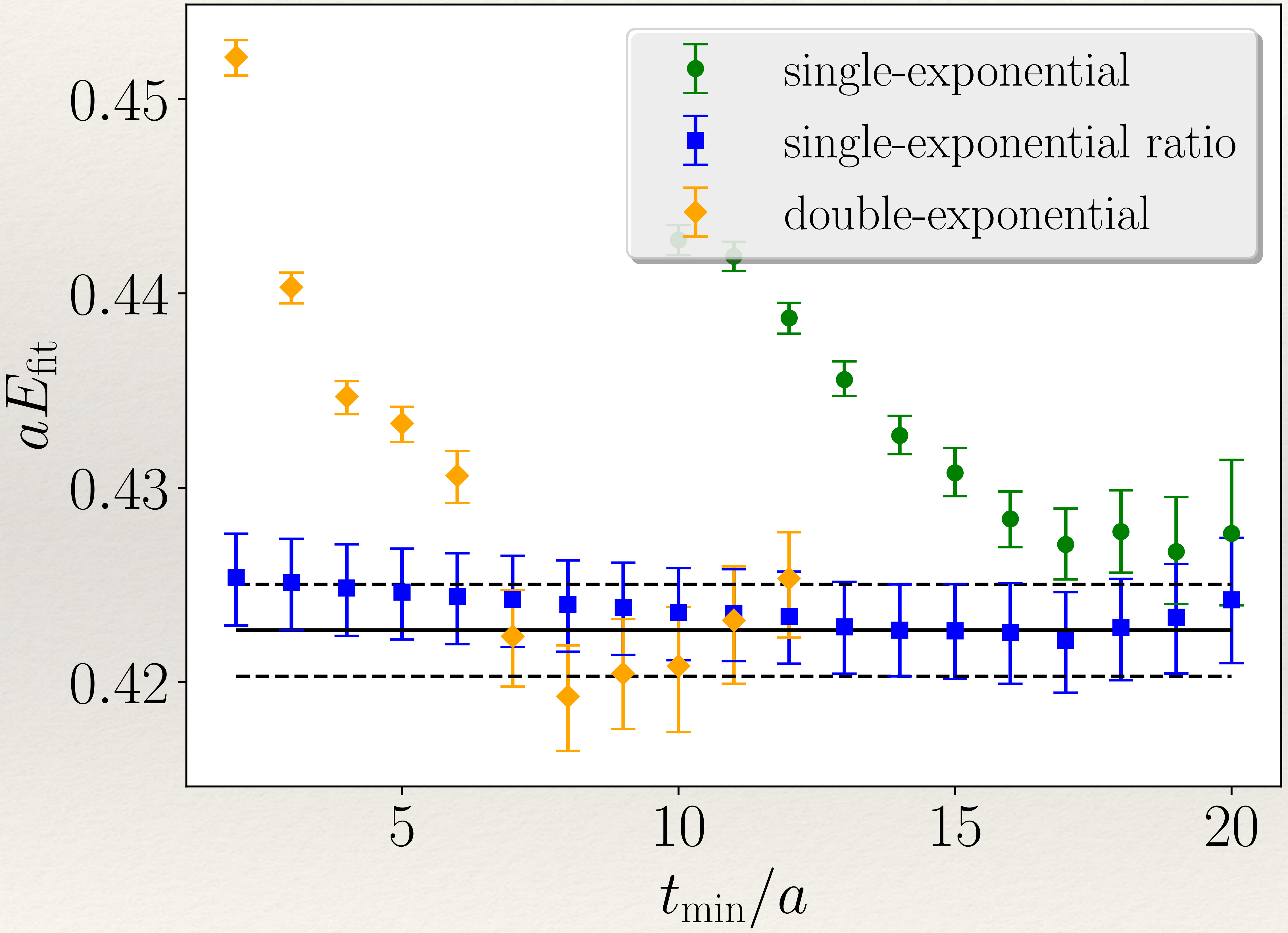
  - This GS fit does quite well
  - Our interest is quantifying uncertainty on ground state
  - We also tried a multi-state version of the GS ansatze

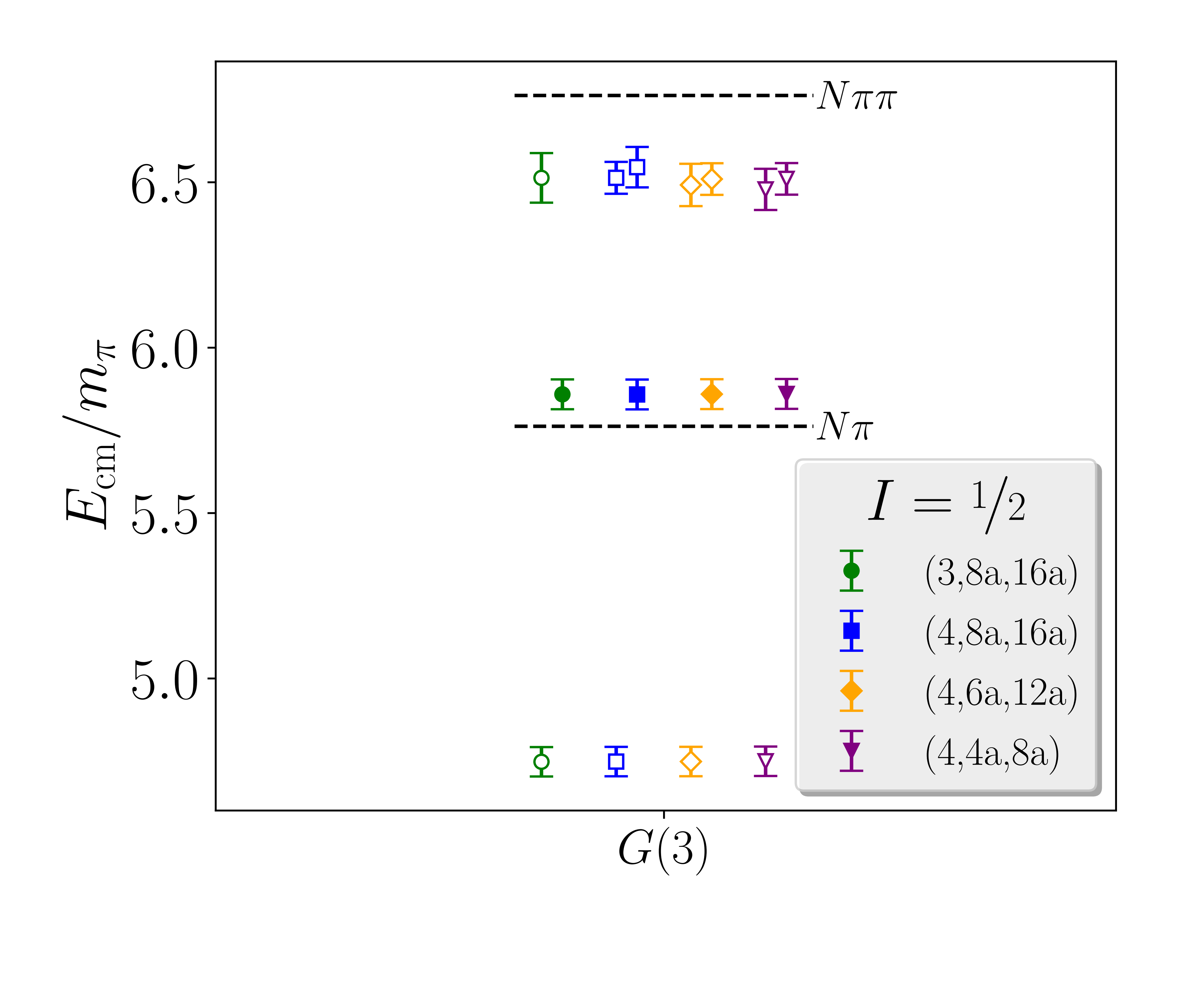
$$C^N_{\text{geom.}}(t) = \frac{Ae^{-E_0 t}}{1 - \sum_{n=1}^{N-1} B_n e^{-\Delta E_n t}}$$



Elastic nucleon-pion scattering at $M\pi \approx 200$ MeV from lattice QCD

Parity Odd Results - S-wave $N\pi$

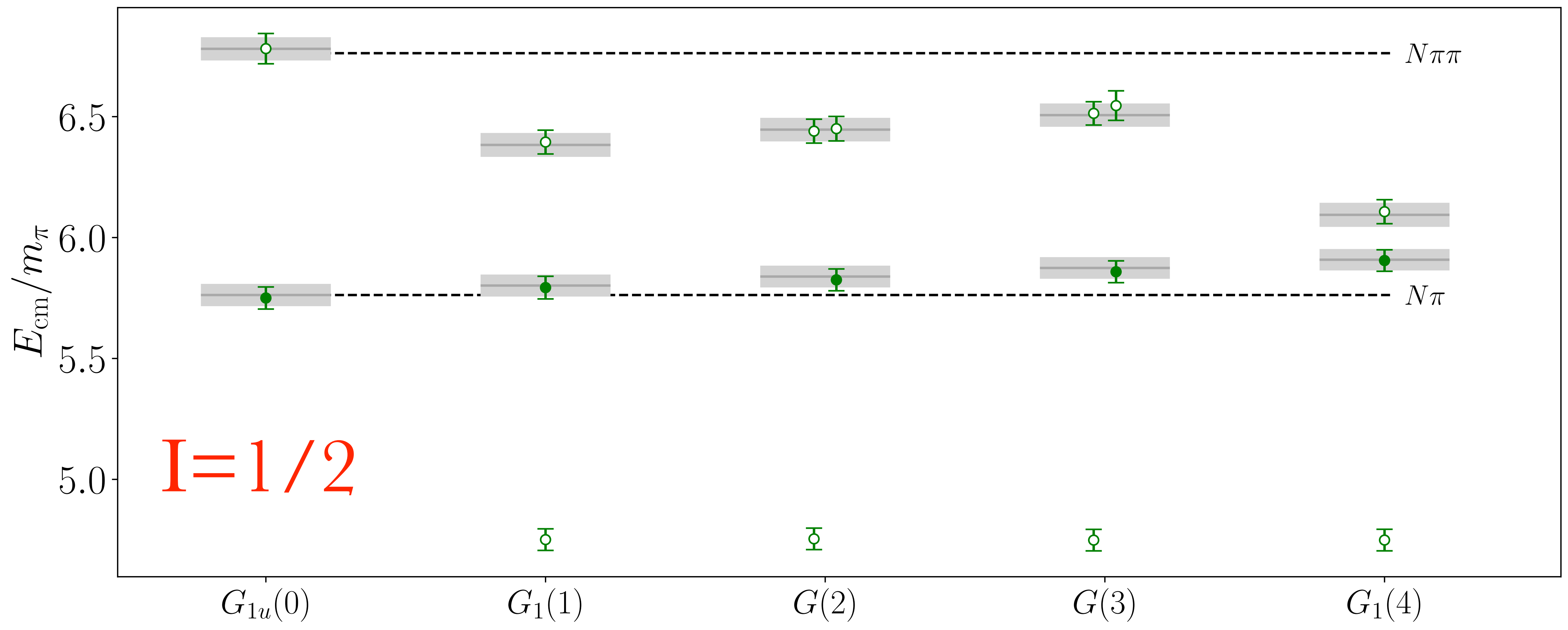


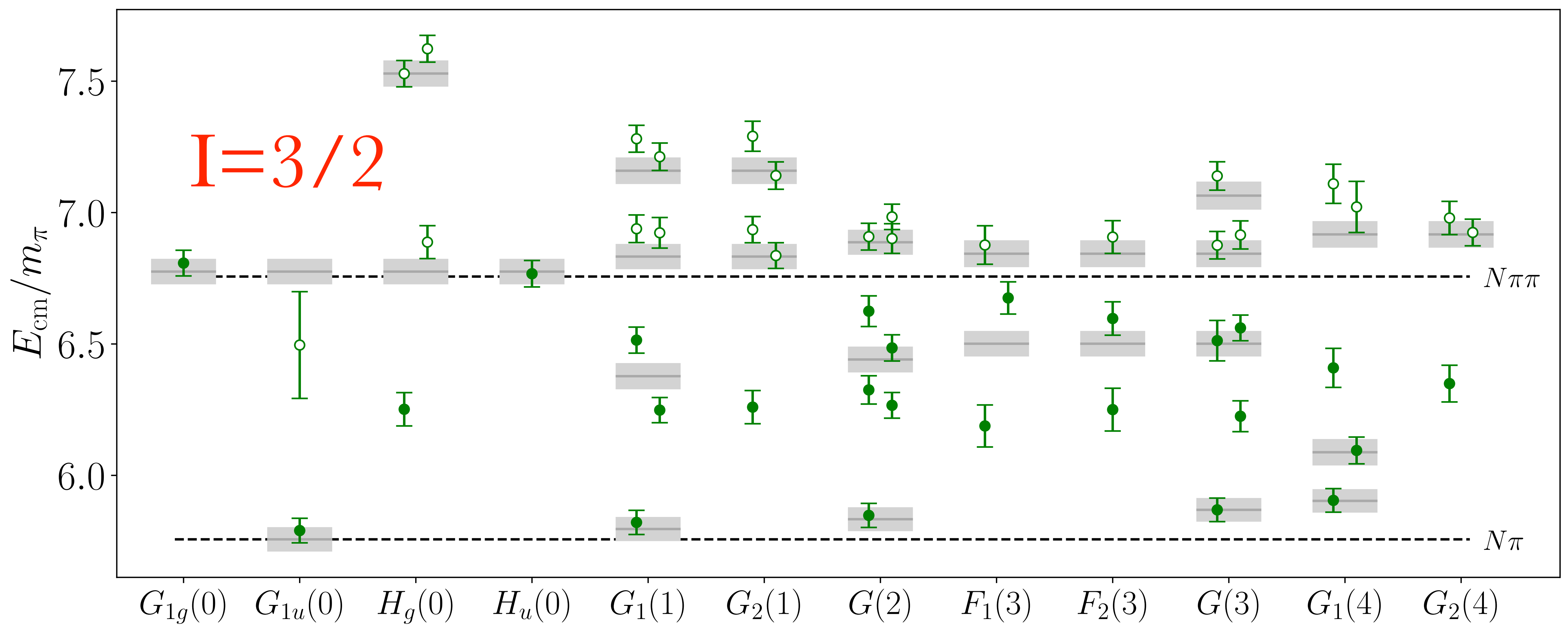


Elastic nucleon-pion scattering at Mπ ≈ 200 MeV from lattice QCD

Various irreps used to determine the spectrum

$\boldsymbol{d}$	Λ	dim.	contributing $(2J,\ell)^{n_{\rm occ}}$ for $\ell_{\rm max}=2$
$(0,0,0)$	$G_{1\rm u}$	2	$(1,0)$
	$G_{1\rm g}$	2	$(1,1)$
	$H_{\rm g}$	4	$(3,1)$, $(5,2)$
	$H_{\rm u}$	4	$(3,2)$, $5,2)$
	$G_{2\rm g}$	2	$(5,2)$
$(0,0,n)$	G_1	2	$(1,0)$, $(1,1)$, $(3,1)$, $(3,2)$, $(5,2)$
	G_2	2	$(3,1)$, $(3,2)$, $(5,2)^2$
$(0,n,n)$	G	2	$(1,0)$, $(1,1)$, $(3,1)^2$, $(3,2)^2$, $(5,2)^3$
(n,n,n)	G	2	$(1,0)$, $(1,1)$, $(3,1)$, $(3,2)$, $(5,2)^2$
	F_1	1	$(3,1)$, $(3,2)$, $(5,2)$
	F_2	1	$(3,1)$, $(3,2)$, $(5,2)$





Note: the gray bands and green energy levels are correlated, which is not reflected visually in the plots

Elastic nucleon-pion scattering at Mπ ≈ 200 MeV from lattice QCD

- FV Spectrum to Scattering Amplitudes [Lüscher, … many others]

$$\det[\tilde{K}^{-1}(E_{\mathrm{cm}}) - B^{\boldsymbol{P}}(E_{\mathrm{cm}})] + \mathrm{O}(\mathrm{e}^{-ML}) = 0$$

- $\tilde{K}$ proportional to the K-matrix
- $B^{\mathrm{P}}(E_{\mathrm{cm}})$ is the "Box Matrix" that encodes information about the finite-volume and BCs

Elastic nucleon-pion scattering at Mπ ≈ 200 MeV from lattice QCD

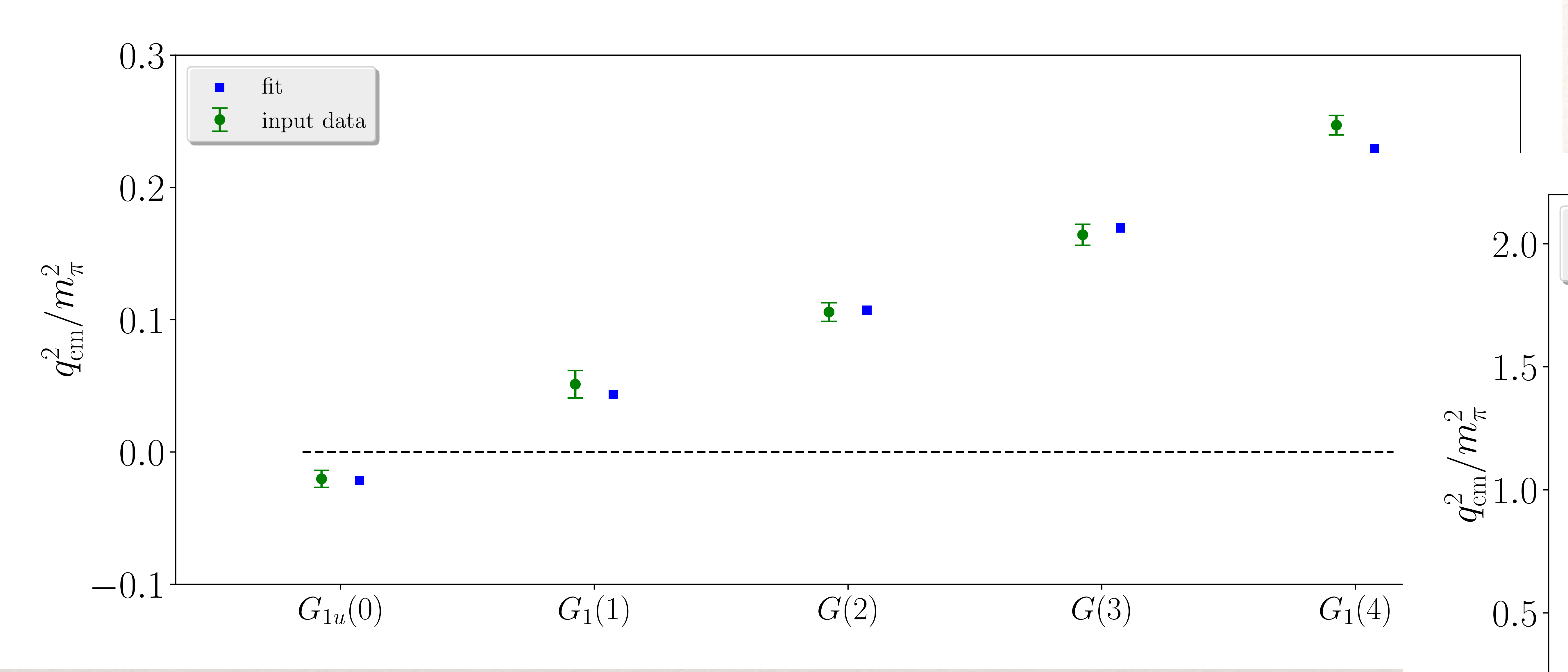
- FV Spectrum to Scattering Amplitudes [Lüscher, … many others]

$$\det[\tilde{K}^{-1}(E_{\rm cm}) - B^{\boldsymbol{P}}(E_{\rm cm})] + \mathrm{O}(\mathrm{e}^{-ML}) = 0$$

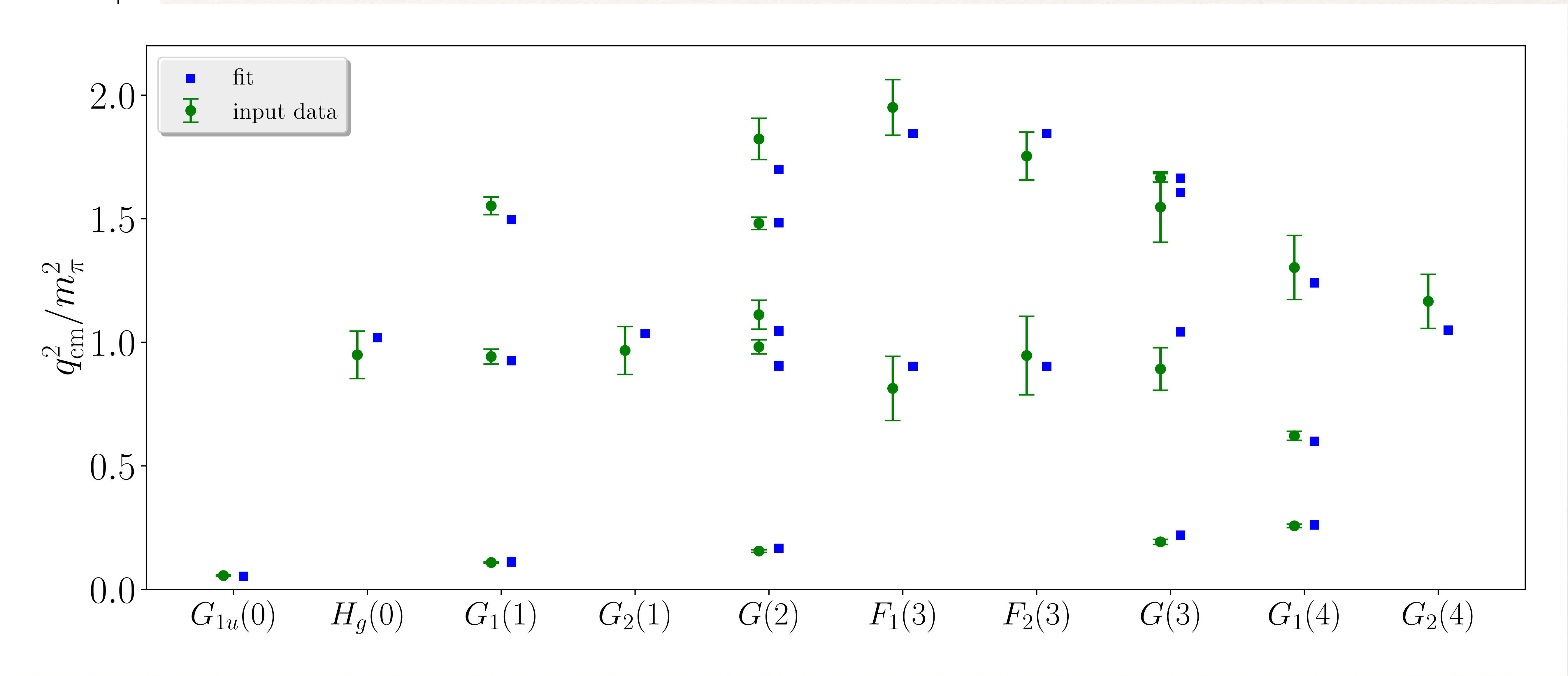
- We try 2 strategies to fit the results:
  - Spectrum Method
    - Use model parameters of interaction to predict energies (effective range expansion)
    - Vary parameters to minimize discrepancy with numerically determined spectrum
  - Determinant Residual Method
    - Use energies and model parameters to evaluate the determinant (above)
    - Vary parameters to minimize the determinant
- Both methods give consistent results
  - spectrum method is more precise, and more able to constrain higher partial waves
  - net-residual method is noisier and typically has smaller $\chi 2$

Elastic nucleon-pion scattering at Mπ ≈ 200 MeV from lattice QCD

- FV Spectrum to Scattering Amplitudes - spectrum method comparison



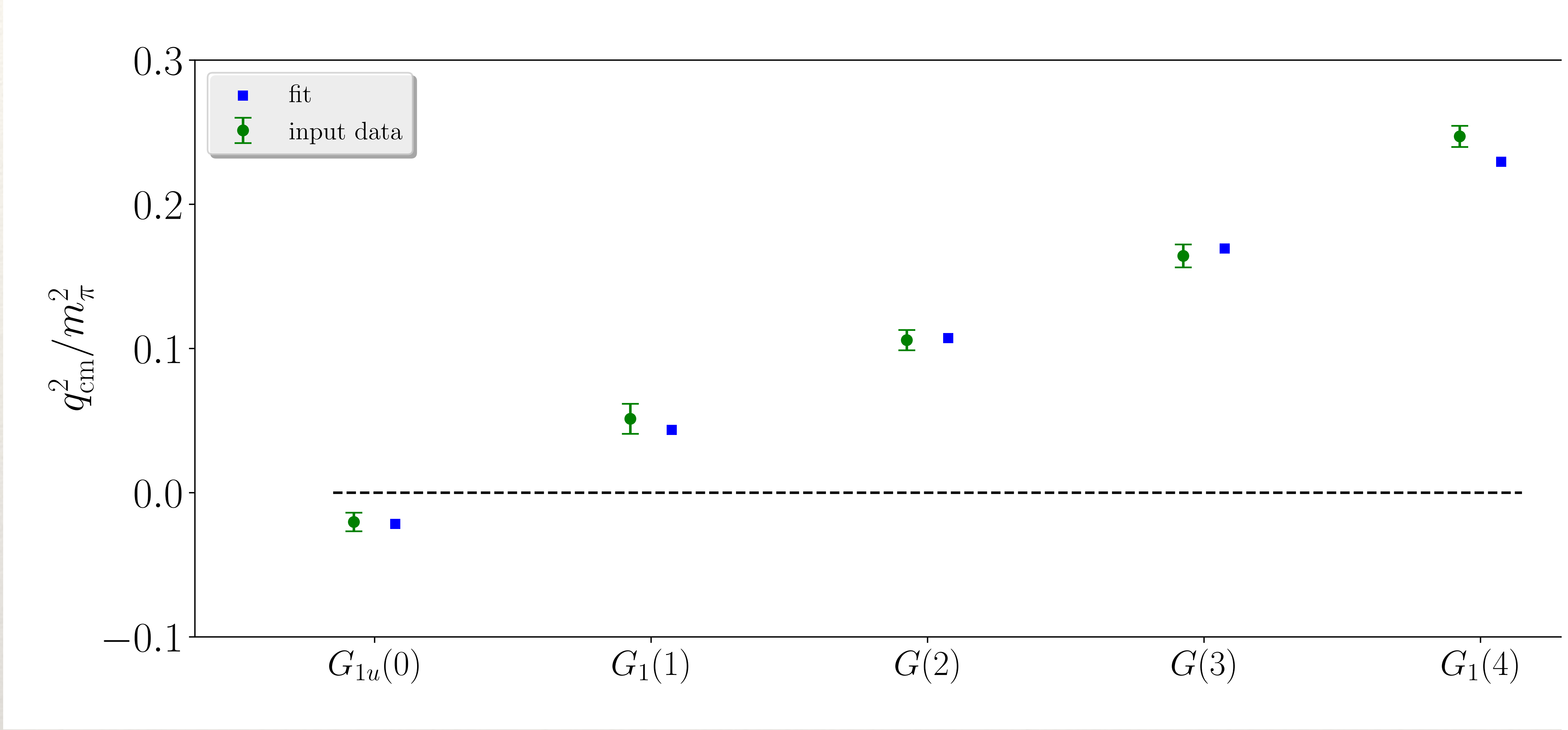
I=1/2 fit using s-wave only approximation

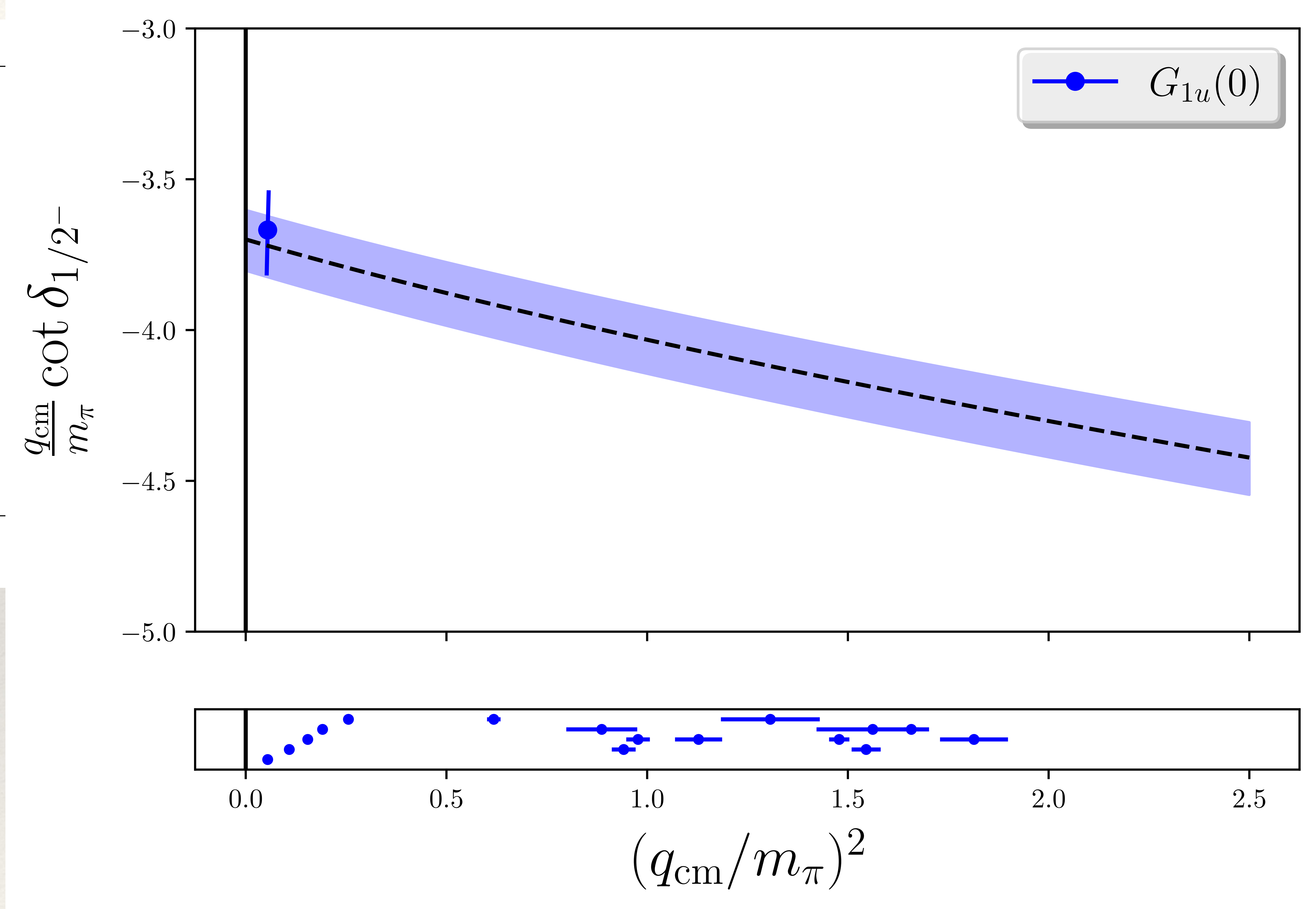


I=3/2 fit using s- and p-wave approximation

Elastic nucleon-pion scattering at $M\pi \approx 200$ MeV from lattice QCD

❑ FV Spectrum to Scattering Amplitudes - spectrum method comparison - resulting amplitude



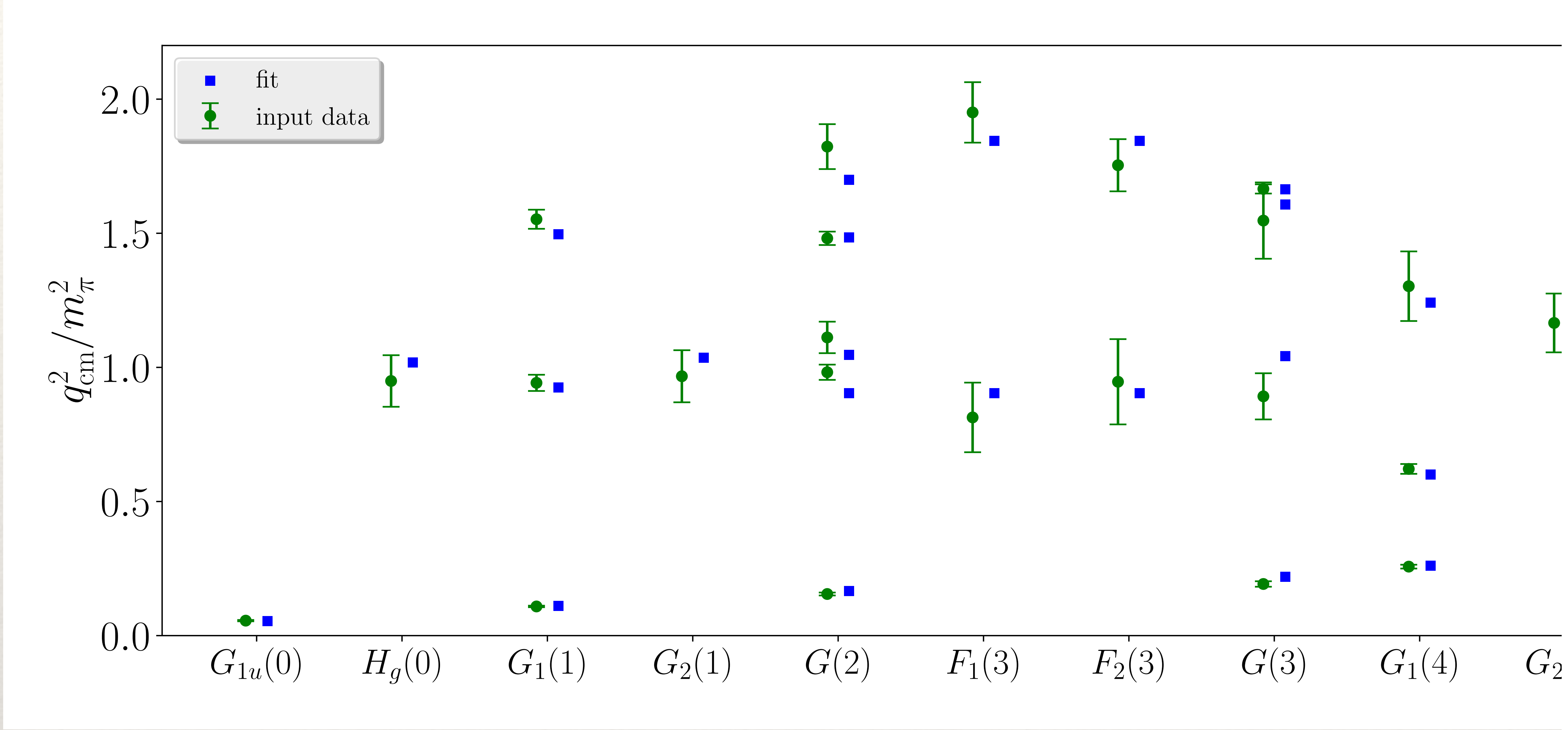


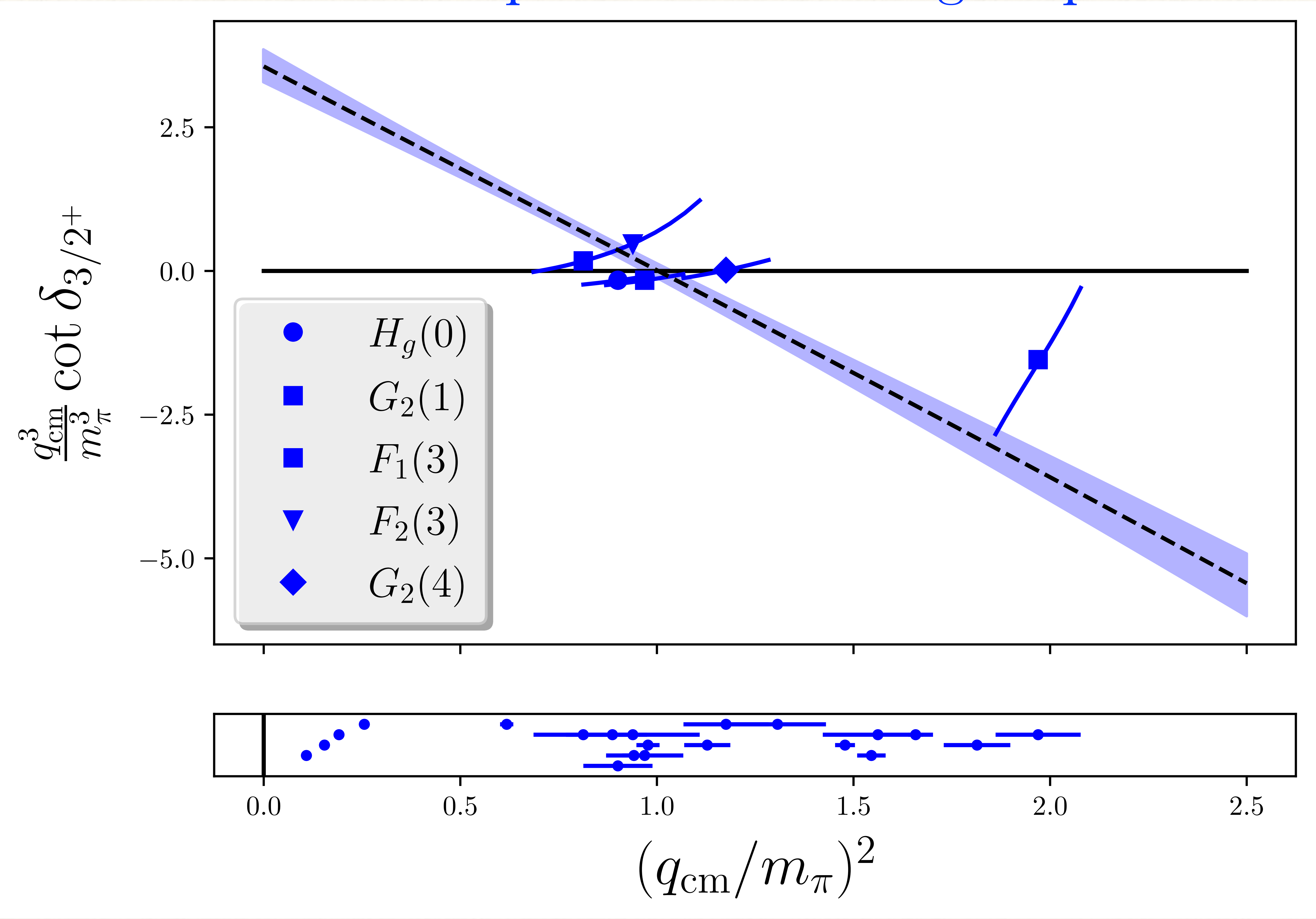
I=1/2 fit using s-wave only approximation

NOTE: global fit - only showing data from 1 irrep

Elastic nucleon-pion scattering at Mπ ≈ 200 MeV from lattice QCD

FV Spectrum to Scattering Amplitudes - spectrum method comparison - resulting amplitude





I=3/2 fit using s- and p-wave approximation

Elastic nucleon-pion scattering at Mπ ≈ 200 MeV from lattice QCD

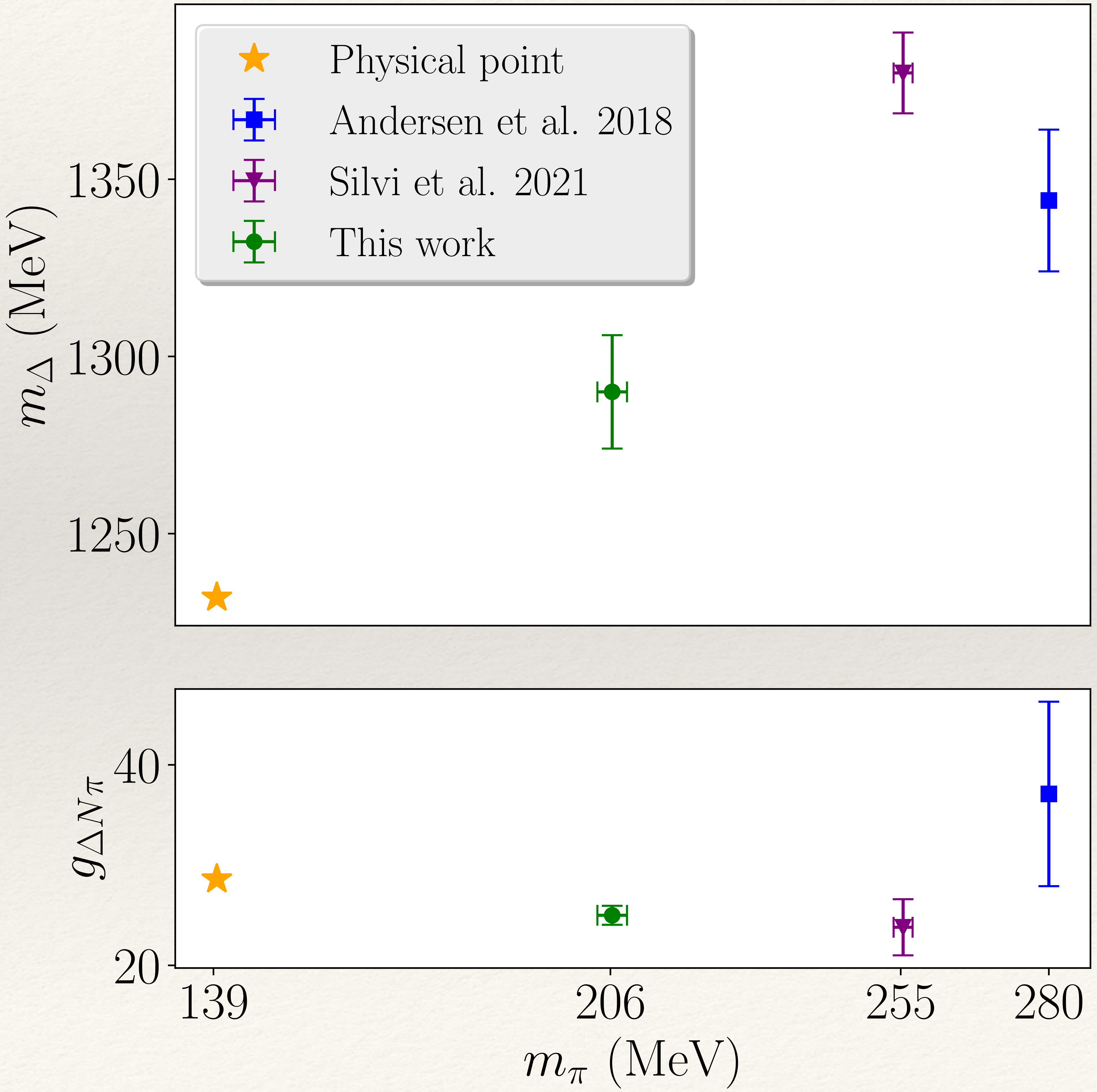
❑ Results for scattering lengths and effective Delta-resonance parameters

$$m_\pi a_0^{3/2} = -0.2735(81)\,, \qquad m_\pi a_0^{1/2} = 0.142(22),$$

$$\frac{m_\Delta}{m_\pi} = 6.257(35), \qquad g_{\Delta N\pi} = 14.41(53)$$

$$m_\pi = 202.7(2.5)\ \text{MeV}$$

$$m_\Delta = 1268(17)\ \text{MeV}$$



❑ Delta-resonance parameterized in amplitude as

$$\frac{q_{\text{cm}}^3}{m_\pi^3}\cot\delta_{3/2^+} = \frac{6\pi\sqrt{s}}{m_\pi^3 g_{\Delta N\pi}^2}(m_\Delta^2 - s),$$

Compare with χPT

- The formula for the scattering length are known at 4th order in the chiral expansion
  - They are expressed in terms of what is called scalar and vector scattering lengths

$$a_0^{3/2} = a_0^+ - a_0^- \,, \qquad a_0^{1/2} = a_0^+ + 2a_0^-$$

- At N[4]LO, these are given by

- Hoferichter et al, 1510.06039, Hoferichter et al, 1507.07552
- Fettes, Meissner [Steininger] [hep-ph/9803266] hep-ph/0002162

$$m_\pi a_0^- = \epsilon_\pi^2 \frac{2\pi}{1+\mu}\left\{1 + \epsilon_\pi^2\left[2 + \frac{\Lambda_\chi^2}{m_N^2}\left(\frac{g_A^2}{4} + 8D\right)\right]\right\},$$

$$m_\pi a_0^+ = -\epsilon_\pi^3 \frac{\pi}{1+\mu}\frac{\Lambda_\chi}{m_N}\left\{g_A^2 + 8C - \epsilon_\pi\, 3\pi g_A^2 \frac{m_N}{\Lambda_\chi}\right.$$
$$\left. + \epsilon_\pi^2\left[8 - 3g_A^2 + 2g_A^4 + 4C' + 4\frac{\Lambda_\chi^2}{m_N^2}\left(\frac{g_A^2}{16} + 16C'' - g_A D_{18} - 16E\right)\right]\right\}$$

$$\epsilon_\pi = \frac{m_\pi}{4\pi F_\pi}, \qquad \mu = \frac{m_\pi}{m_N}, \qquad \Lambda_\chi = 4\pi F_\pi,$$
$$C = m_N(2c_1 - c_2 - c_3), \qquad C' = m_N(2c_1 - c_3), \qquad C'' = m_N^2 c_1 c_2,$$
$$D = m_N^2(\bar{d}_1 + \bar{d}_2 + \bar{d}_3 + 2\bar{d}_5), \qquad D_{18} = m_N^2 \bar{d}_{18}, \qquad E = m_N^3(\bar{e}_{14} + \bar{e}_{15} + \bar{e}_{16}),$$

Compare with χPT

- Use LECs from N^2,3,4^LO order fit to Nπ scattering: predict scattering lengths at Mπ≈200 MeV

$$m_\pi a_0^- = \epsilon_\pi^2 \frac{2\pi}{1+\mu}\left\{1+\epsilon_\pi^2\left[2+\frac{\Lambda_\chi^2}{m_N^2}\left(\frac{g_A^2}{4}+8D\right)\right]\right\},$$

$$m_\pi a_0^+ = -\epsilon_\pi^3 \frac{\pi}{1+\mu}\frac{\Lambda_\chi}{m_N}\left\{g_A^2+8C-\epsilon_\pi\, 3\pi g_A^2\frac{m_N}{\Lambda_\chi} + \epsilon_\pi^2\left[8-3g_A^2+2g_A^4+4C'+4\frac{\Lambda_\chi^2}{m_N^2}\left(\frac{g_A^2}{16}+16C''-g_A D_{18}-16E\right)\right]\right\}$$

- Hoferichter et al, 1510.06039, Hoferichter et al, 1507.07552

$$\epsilon_\pi^{\mathrm{D200}} = 0.1759(12)\,, \qquad \mu^{\mathrm{D200}} = 0.2102(19)\,, \qquad \left(\frac{m_N}{\Lambda_\chi}\right)^{\mathrm{D200}} = 0.8368(72)$$

Quantity	χPT prediction at $m_\pi \approx 200$ MeV NLO	N^{2}LO	N^{3}LO	This work at $m_\pi \approx 200$ MeV	RS at m_π^{phys} [91]
$m_\pi a_0^{1/2}$	0.2526(45)	0.444(10)	0.1660(93)	0.142(22)	0.1699(23)
$m_\pi a_0^{3/2}$	-0.2291(46)	-0.2020(63)	-0.0756(98)	$-0.2735(81)$	-0.0863(17)

- LECs determined from experimental data do a BAD job predicting our results
- We have not yet tried fitting results to understand where the tension might be

Conclusions

- We have performed $N\pi$ scattering — for the first time — at a pion mass of $M\pi \approx 200$ MeV
- We used the stochastic Laplacian Heaviside (sLapH) method - a stochastic variant of distillation
- We found that the achievable precision with sLapH is good enough to get precise determinations of the scattering parameters
- We found that our results are in tension with predictions from SU(2) χPT and LECs determined to high-precision $N\pi$ scattering phase shift data
- We are therefore not yet in a position to usefully contribute to the nucleon-pion sigma term "puzzle" — the contrast between the pheno determination and most precise LQCD ones
- Lighter pion masses seem reachable (given our results) and are necessary to understand the apparent tension with SU(2) χPT predictions

Thank You