Leading power accuracy in Large-Momentum effective theory

39th Lattice Symposium

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Parton Physics

Feynman's parton model describes the contents of a hadron in the limit of infinite momentum (i.e. on the light-cone). In this limit, the hadron is composed of non-interacting particles.

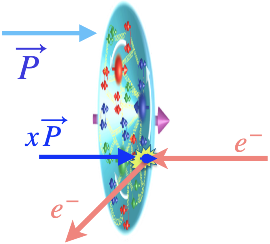

Figure 1: Feynman's parton model[1].

[1]Image taken from Yong Zhou's presentation at CSSM/CDMPP seminar at University of Adelaide (13/4/22).

Parton Physics

The dependence of the parton model on real time makes it inaccessible on the lattice. In addition, we cannot impose the light-cone gauge $A^3 + iA^4 = 0$ on the lattice.

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The advent of large-momentum effective theory (LaMET) in 2013 by Xiangdong Ji showed that parton physics can be approximated on the lattice[2]. Several topics have been studied with this method such as Parton Distribution Functions (PDF), Generalised PDFs (GPD) and Transverse Momentum distributions (TMD).

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LaMET

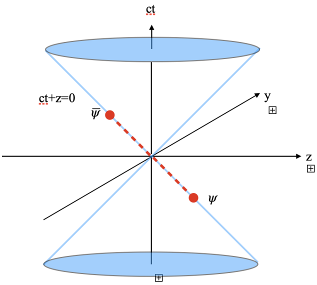

Figure 2: Operator used for parton physics.

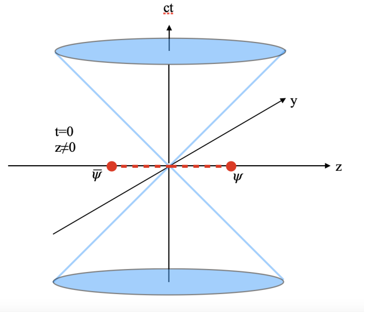

Figure 3: Operator used in LaMET.

We replace the operator on the left with the non-local "quasi-"operator on the right. The two are related by a Lorentz boost.

LaMET

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A Distribution Amplitude, $\phi(x)$, is the probability of finding a $q\bar{q}$ pair with momentum-fractions x and $1-x$, respectively. (x is the fraction of the hadron momentum carried by the constituent parton.)

Renormalon ambiguity

Perturbative expansions are not convergent to all orders.

The coefficients of the terms in the peturbative series

$$f(\alpha_s) = \sum_n c_n \alpha_s^{n+1} \quad (1)$$

grow factorially as more and more diagrams have to be computed. The series begins to diverge when $n \sim \alpha_s^{-1}$ and we must truncate it.

This introduces a renormalon ambiguity $\mathcal{O}(a\Lambda_{\mathrm{QCD}})$.

Renormalon ambiguity

Another way of understanding the series $f(\alpha_s)$ is to consider its Borel transformation:

$$B[f](t) \equiv \sum_n c_n \frac{t^n}{n!} \qquad (2)$$

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Assuming $c_n = N s^n (n+b)!$,

$$B[f](t) = \frac{N b!}{(1-st)^{b+1}}. \tag{4}$$

Singularity at $s = t^{-1}$; "renormalon".

Renormalon ambiguity

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In a paper recently submitted to PRL, we describe a new method known as Leading Renormalon Resummation (LRR).

Renormalon ambiguity

The renormalisation group equation (RGE) relates an observable, M, at energy scale μ and anomalous dimension, γ, via

$$\begin{aligned}
&\frac{\partial M^{\overline{\mathrm{MS}}}(z,\mu)}{\partial \ln \mu^2} = \gamma(\mu) M^{\overline{\mathrm{MS}}}(z,\mu) \\
\implies &M^{\overline{\mathrm{MS}}}(z,\mu) = M^{\overline{\mathrm{MS}}}(z,z^{-1}) \exp\left(\int_{\alpha(z^{-1})}^{\alpha(\mu)} \mathrm{d}\alpha' \, \frac{\gamma(\alpha')}{\beta(\alpha')}\right) \\
\implies &M^{\overline{\mathrm{MS}}}(z,\mu) \equiv M^{\overline{\mathrm{MS}}}(z,z^{-1}) e^{\mathcal{I}(\mu)} e^{-\mathcal{I}(z^{-1})}.
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\end{aligned}$$

On the lattice

$$M^{\mathrm{lat}}(z,a^{-1}) = e^{-\delta m z} f^{\mathrm{lat}}(z,z^{-1}) e^{-\mathcal{I}^{\mathrm{lat}}(z^{-1})} e^{\mathcal{I}^{\mathrm{lat}}(a^{-1})} \qquad (6)$$

where f^{lat} is the lattice perturbation series.

Renormalon ambiguity

The dependence on z and the dependence on scales are completely factorised in eqs. (5) and (6). The physics is contained in the z-dependent terms and should be independent of the method used to calculate them. Thus we can interchange the z-dependence between the two formulae:

$$M^{\mathrm{lat}}(z, a^{-1}) = M^{\overline{\mathrm{MS}}}(z, z^{-1}) e^{-\mathcal{I}(z^{-1})} e^{\mathcal{I}^{\mathrm{lat}}(a^{-1})} e^{-\delta m z} \tag{7}$$

Renormalon ambiguity

We show in our PRL paper that the renormalon term obeys

$$\frac{1}{z}\left(\ln\left[M^{\text{lat}}(z,a^{-1})e^{-\mathcal{I}^{\text{lat}}(a^{-1})}e^{\frac{m_{-1}(a)}{a}z}\right] - \ln\left[M^{\overline{\text{MS}}}(z,z^{-1})e^{-\mathcal{I}(z^{-1})}\right]\right) = -m_0^{\text{eff}}(1+\mathcal{O}(z\Lambda_{\text{QCD}})). \tag{8}$$

At large z-values, the perturbative calculation fails. At small z-values the lattice data become unreliable due to discretisation effects. Thus, we expect a window of z-values for which m_0 is constant.

However, we found that – even within the window of valid z-values – m_0 was not a constant, even statistically.

Renormalon ambiguity

The way around the z-dependence would be to resum the series $M^{\overline{\text{MS}}}(z, z^{-1})$. However, the only terms that we can realistically resum are the bubble-chain diagrams which dominate in the large-n_f limit.

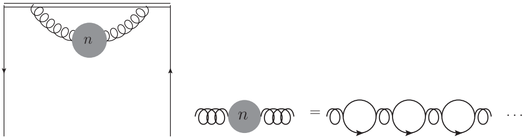

Figure 4: The tadpole diagram of the quasi-PDF dominating in the large-n_f limit. n indicates that the gluon propagator is dressed with n fermion loops.

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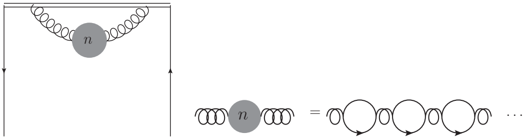

Figure 4: The tadpole diagram of the quasi-PDF dominating in the large-n_f limit. n indicates that the gluon propagator is dressed with n fermion loops.

The contribution to $M^{\overline{\text{MS}}}$ from the tadpole diagrams is[3]

$$M_{\text{tp}}(z,\mu)|_{\text{PV}} = \text{P.V.} \int_0^\infty dw e^{-4\pi w/\alpha(\mu)\beta_0} \frac{2C_F}{\beta_0} \times \left(\frac{\Gamma(1-w) e^{\frac{5}{3}w} (z^2\mu^2/4)^w}{(1-2w)\Gamma(1+w)} - 1 \right) / w. \tag{9}$$

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Renormalon ambiguity

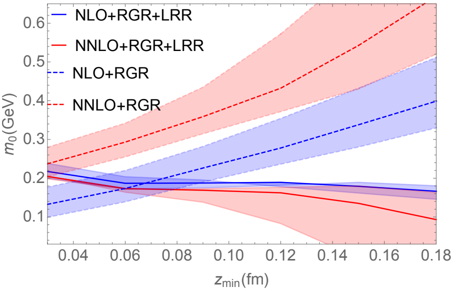


Figure 5: Value of m_0 parameter with original method and with LRR method. The renormalon is fitted in the range $[z_{\text{min}}, z_{\text{min}} + 0.06\text{ fm}]$

Method outline

Computing the DAs from the matrix elements is a lengthy task:

- Matrix elements (in coordinate space) are renormalized in the self-renormalization scheme[4].
- The effects of IR renormalons must be removed.
- The DA must be matched to the light-cone (boosted to infinite momentum).
- The DA must be Fourier transformed to momentum space.

This method is the same as that of "Pion and Kaon Distribution Amplitudes from Lattice QCD"[5] but with the LRR treatment.

[4] "Self-renormalization of quasi-light-front correlators on the lattice": 2103.02965
[5] 2201.09173

Self-Renormalisation

Self-renormalisation of the DA matrix elements requires studying the PDF matrix elements. The bare matrix element for the quasi-PDF is

$$\mathcal{M}(z,a)_B = \langle M(P_z=0)|\overline{\psi}(0)\Gamma U(0,z)\psi(z)|M(P_z=0)\rangle \quad (10)$$

where

- $|M(P_z=0)\rangle$ is the meson state at zero momentum;
- a is the lattice spacing;
- Γ is the Dirac structure;
- $U(0,z)$ is the Wilson line from the origin to $(0,0,0,z)$:

$$U(0,z) = \exp\left(-ig\int_0^z \mathrm{d}z'\, A_z(z')\right). \quad (11)$$

The Wilson line is necessary to ensure gauge invariance but it introduces a new problem…

Self-Renormalisation

Expanding the PDF operator to 1-loop gives

$$\langle O_\Gamma(z)\rangle = \Gamma\left(1 + g^2\gamma\ln\left(\frac{z^2}{a^2}\right) + \underbrace{m_{-1}\frac{z}{a}}_{\text{linear divergence}} + \dots\right) \tag{12}$$

The linear divergence becomes more severe at large-z and at small-a. We must remove it in order to compute a meaningful continuum limit.

Self-Renormalisation

The outcome of the self-renormalisation scheme is that the bare matrix element, $\mathcal{M}_B(z,a)$ is related to the renormalised one by

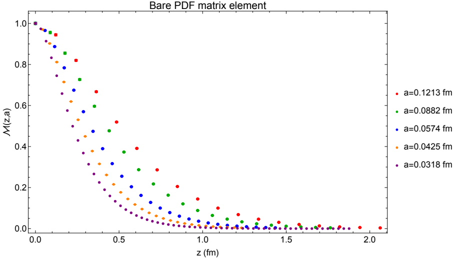
$$\mathcal{M}_R(z) = \frac{\mathcal{M}_B(z,a)}{Z_R(z,a)} \tag{13}$$

where

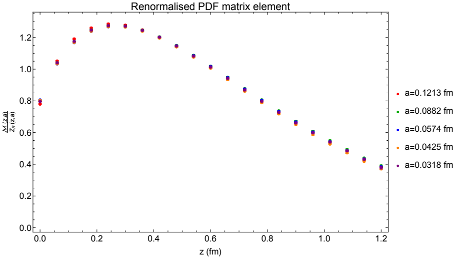
$$Z_R(z,a) = \exp\left[\frac{kz}{a\ln(a\Lambda_{\mathrm{QCD}})} + m_0 z + f_1(z)a + \frac{3C_F}{4\pi\beta_0}\ln\left(\frac{\ln(a\Lambda_{\mathrm{QCD}})}{\ln(\Lambda_{\mathrm{QCD}}/\mu)}\right) + \ln\left(1 + \frac{d}{\ln(a\Lambda_{\mathrm{QCD}})}\right)\right]. \tag{14}$$

The first term describes the linear divergence due to the Wilson line, m_0 describes the effects of renormalons, f_1 describes discretisation effects, the double-logs describe the logarithmic divergence and the d terms is used to match the matrix element to perturbation theory for $z \lesssim 0.2$ fm. C_F is the quadratic Casimir for $SU(3)$, β_0 is the coefficient of the one-loop beta function.

Comparison



(a) Bare



(b) Renormalised

Figure 6: PDF matrix element before and after renormalisation including treatment of the linear divergence. Not only is the divergence removed but the renormalised matrix elements are independent of a.

Lightcone matching

After removing the effects of renormalons, we match our quasi-DA, $\tilde{\phi}(x, \mu, P_z)$, to the lightcone DA, $\phi(x, \mu)$, with the matching kernel $\mathcal{C}$:

$$\tilde{\phi}(x, \mu, P_z) = \int_{-1}^{1} \frac{\mathrm{d}y}{|y|} \mathcal{C}(x/y, \mu, P_z)\phi(x, \mu) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{x^2 P_z^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(1-x)^2 P_z^2}\right) \tag{15}$$

With the renormalon terms subtracted, corrections are $\mathcal{O}(\Lambda_{\mathrm{QCD}}^2/P_z^2)$ as opposed to $\mathcal{O}(\Lambda_{\mathrm{QCD}}/P_z)$.

Provisional DA results

Data from the MILC collaboration was used to compute the DAs using LaMET.

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These data use $2 + 1$ quark flavours: u, d and s. We compute the DA for each of

1. π
2. K
3. η_s.

Note that there is exact isospin symmetry for the π and η_s.

Pion DA

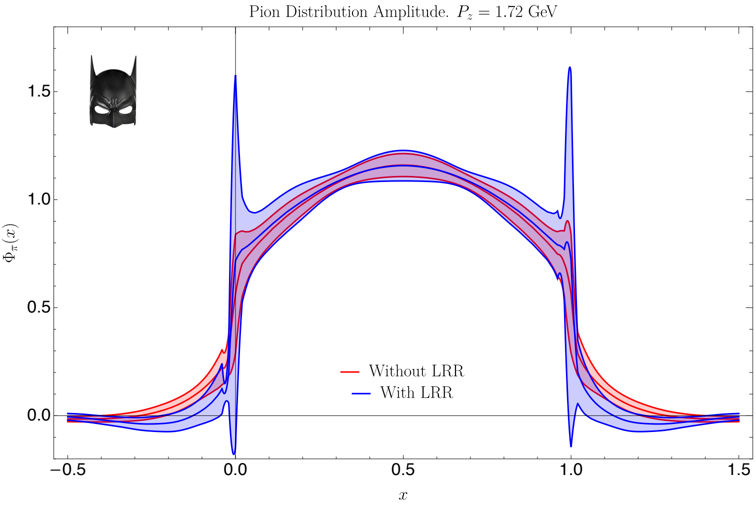

Figure 7: Pion DA (provisional).

Conclusions

- We have presented provisional results for the pion DA using LRR.
- My collaborators, Yushan Su and Rui Zhang, are working with me on finalising the DA calculations.
- Su and Zhang are applying LRR to the calculation of parton distribution functions.

Thank you