Isovector nucleon form factors from 2+1-flavor dynamical domain-wall lattice QCD at the physical mass

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Domain-wall fermions (DWF) lattice Quantum Chromodynamics (QCD):

- preserve both chiral and flavor symmetries,
- started by RIKEN-BNL-Columbia Collaboration 23 years ago, using purpose-built parallel supercomputers.

Joint RBC+UKQCD Collaborations have been generating 2+1-flavor dynamical DWF ensembles:

- for more than a decade, and at physical mass for several years,
- with a range of momentum cut off, 1-3 GeV, and volumes $m_\pi L \sim 4$.

We have been calculating pion, kaon, $(g-2)_\mu$, and nucleon electroweak matrix elements.

An update.

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Nucleon form factors, measured in elastic scatterings or β decay or muon capture:

$$\langle p|V_\mu^+(x)|n\rangle = \bar{u}_p\left[\gamma_\mu F_1(q^2) - i\sigma_{\mu\lambda}q_\lambda\frac{F_2(q^2)}{2m_N}\right]u_n e^{iq\cdot x},$$

$$\langle p|A_\mu^+(x)|n\rangle = \bar{u}_p\left[\gamma_\mu\gamma_5 F_A(q^2) + \gamma_5 q_\mu\frac{F_P(q^2)}{2m_N}\right]u_n e^{iq\cdot x},$$

$$F_V = F_1, F_T = F_2; G_E = F_1 - \frac{q^2}{4m_N^2}F_2, G_M = F_1 + F_2.$$

Related to

- mean-squared charge radii, $F_1 = F_1(0) - \frac{1}{6}\langle r_E^2\rangle Q^2 + ...$
- anomalous magnetic moment, $F_2(0)$,
- $g_A = F_A(0) = 1.2752(13)g_V$ ($g_V = F_1(0) = G_{\text{Fermi}}\cos\theta_{\text{Cabibbo}}$).

$\langle r_E^2\rangle$ and g_A, in particular, are being revised:

- $\sqrt{\langle r_E^2\rangle} = 0.875(6)$ fm from electron scattering, 0.8409(4) and 0.833(10) from μ and e Lamb shift;
- $g_A/g_V = 1.264(2)$ pre 2002 (“cold neutron,”) 1.2755(11) post, (“ultra cold neutron.”)

The Goldberger-Treiman relation, $m_N g_A \propto f_\pi g_{\pi NN}$, determines much of nuclear physics, such as primordial and neutron-star nucleosyntheses.

The ratio of two- and three-point correlators, $\dfrac{C^{\Gamma,O}_{3\text{pt}}(t_{\text{src}}, t, t_{\text{snk}})}{C_{2\text{pt}}(t_{\text{src}}, t_{\text{snk}})}$ with

$$C^{(2)}(t_{\text{src}}, t_{\text{snk}}) = \sum_{\alpha,\beta} \left(\frac{1+\gamma_t}{2}\right)_{\alpha\beta} \langle N_\beta(t_{\text{snk}})\bar{N}_\alpha(t_{\text{src}})\rangle,$$

$$C^{(3)\Gamma,O}(t_{\text{src}}, t, t_{\text{snk}}) = \sum_{\alpha,\beta} \Gamma_{\alpha\beta}\langle N_\beta(t_{\text{sink}})O(t)\bar{N}_\alpha(0)\rangle,$$

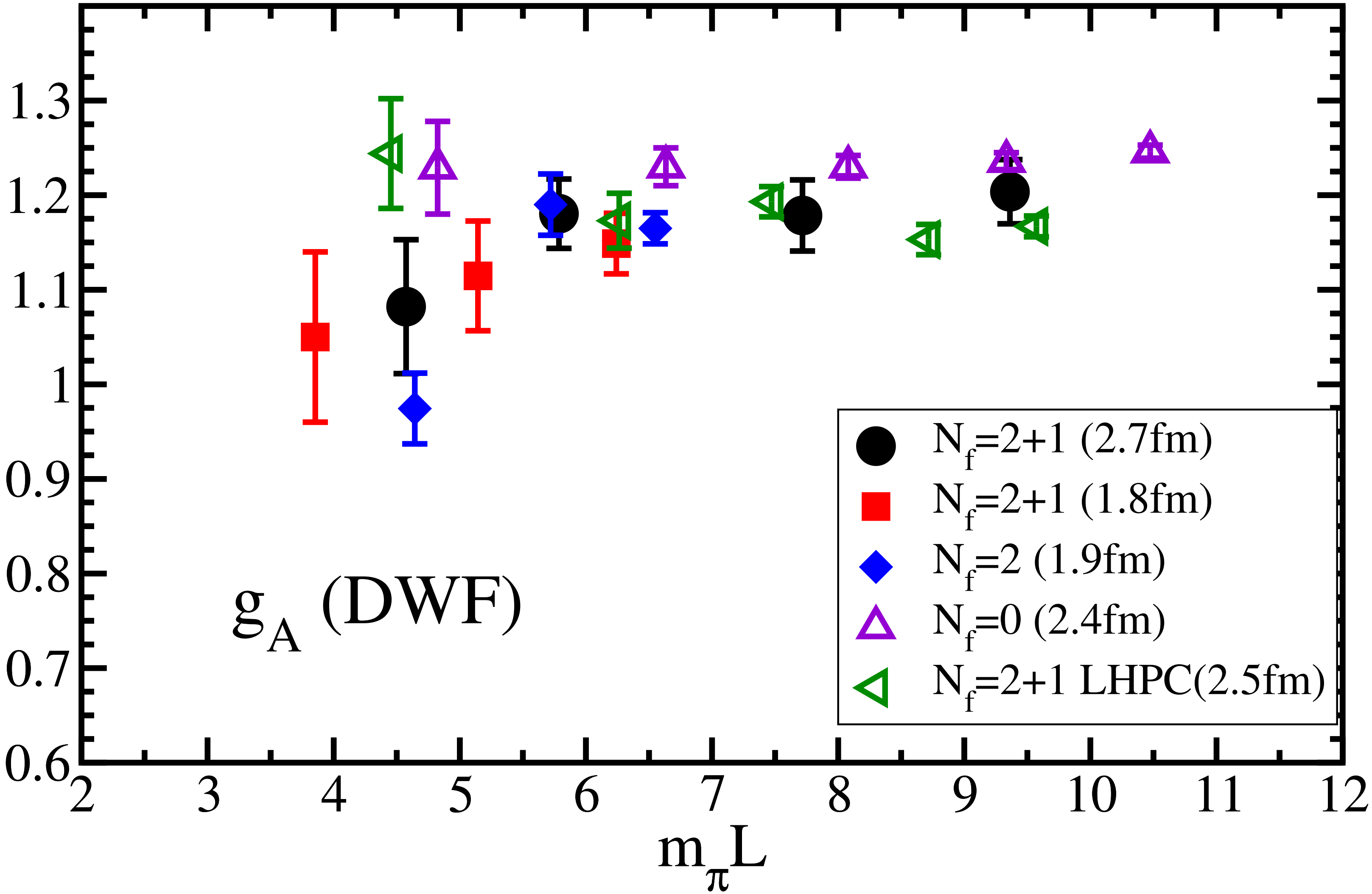
with appropriate nucleon operator, eg, $N = \epsilon_{abc}(u_a^T C\gamma_5 d_b)u_c$, gives a plateau in t for a lattice bare value $\langle O\rangle$ for the relevant observable, with appropriate spin ($\Gamma = (1+\gamma_t)/2$ or $(1+\gamma_t)i\gamma_5\gamma_k/2$) or momentum-transfer (if any) projections.

More specifically, for the form factors, ratios such as

$$\frac{C^{(3)\Gamma,O}_{\text{GG}}(t_{\text{src}}, t, t_{\text{snk}}, \vec{p}_{\text{src}}, \vec{p}_{\text{snk}})}{C^{(2)}_{\text{GG}}(t_{\text{src}}, t_{\text{snk}}, \vec{p}_{\text{src}}, \vec{p}_{\text{snk}})} \times \sqrt{\frac{C^{(2)}_{\text{LG}}(t, t_{\text{snk}}, \vec{p}_{\text{src}}))C^{(2)}_{\text{GG}}(t_{\text{src}}, t, \vec{p}_{\text{snk}}))C^{(2)}_{\text{LG}}(t_{\text{src}}, t_{\text{snk}}, \vec{p}_{\text{snk}}))}{C^{(2)}_{\text{LG}}(t, t_{\text{snk}}, \vec{p}_{\text{snk}}))C^{(2)}_{\text{GG}}(t_{\text{src}}, t, \vec{p}_{\text{src}}))C^{(2)}_{\text{LG}}(t_{\text{src}}, t_{\text{snk}}, \vec{p}_{\text{src}}))}}$$

with point (L) or Gaussian (G) smearings, give plateaux dependent only on momentum transfer.

Some time ago (2007) Takeshi Yamazaki reported unexpectedly large deficit in lattice calculation [1]:



[1] T. Yamazaki *et al.* [RBC+UKQCD Collaboration], Phys. Rev. Lett. **100**, 171602 (2008).

Long story short, by 2017: deficit in nucleon g_A/g_V calculated in lattice QCD with small volumes and heavy mass.

Yet a validation of lattice QCD: As of Lattice 2017, with similar quark mass and lattice cuts off,

- Calculations with overlap-fermion valence quarks on RBC+UKQCD DWF ensembles: ~ 1.2[2],
- Wilson-fermion unitary calculations now agree too once $O(a)$ systematics is removed:
  - PACS, 1.16(8)[3],
  - QCDSF$\sim$ 1.1[4],
- and even a Wilson valence on HISQ, PNDME[5], ~ 1.2,
- except for the then latest DWF valence[6] on HISQ staggered ensembles after an extrapolation.

g_A from different actions "blindly" agree with deficits once $O(a)$ systematics is removed,

The mass dependence is quite different from what the NR quark model or MIT bag model "predicted."

[2] J. Liang, Y. B. Yang, K. F. Liu, A. Alexandru, T. Draper and R. S. Sufian, arXiv:1612.04388 [hep-lat].

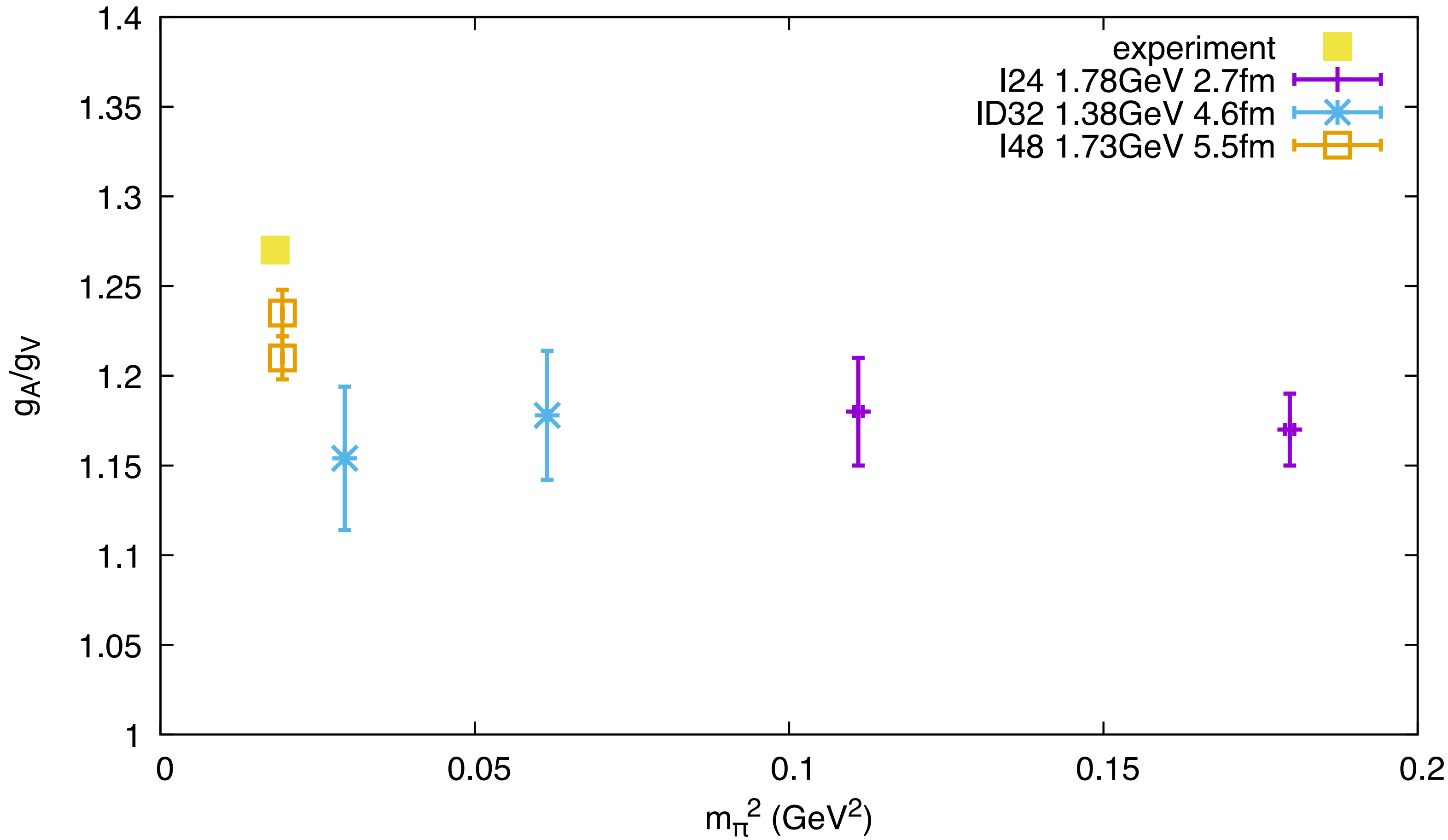
[3] A parallel talk by Tsukamoto at Lattice 2017, Granada; K. I. Ishikawa *et al.* [PACS Collaboration], Phys. Rev. D **98**, no. 7, 074510 (2018) doi:10.1103/PhysRevD.98.074510 [arXiv:1807.03974 [hep-lat]].

[4] J. Dragos *et al.*, Phys. Rev. D **94**, no. 7, 074505 (2016) doi:10.1103/PhysRevD.94.074505 [arXiv:1606.03195 [hep-lat]].

[5] T. Bhattacharya, V. Cirigliano, S. Cohen, R. Gupta, H. W. Lin and B. Yoon, Phys. Rev. D **94**, 054508 (2016) [arXiv:1606.07049].

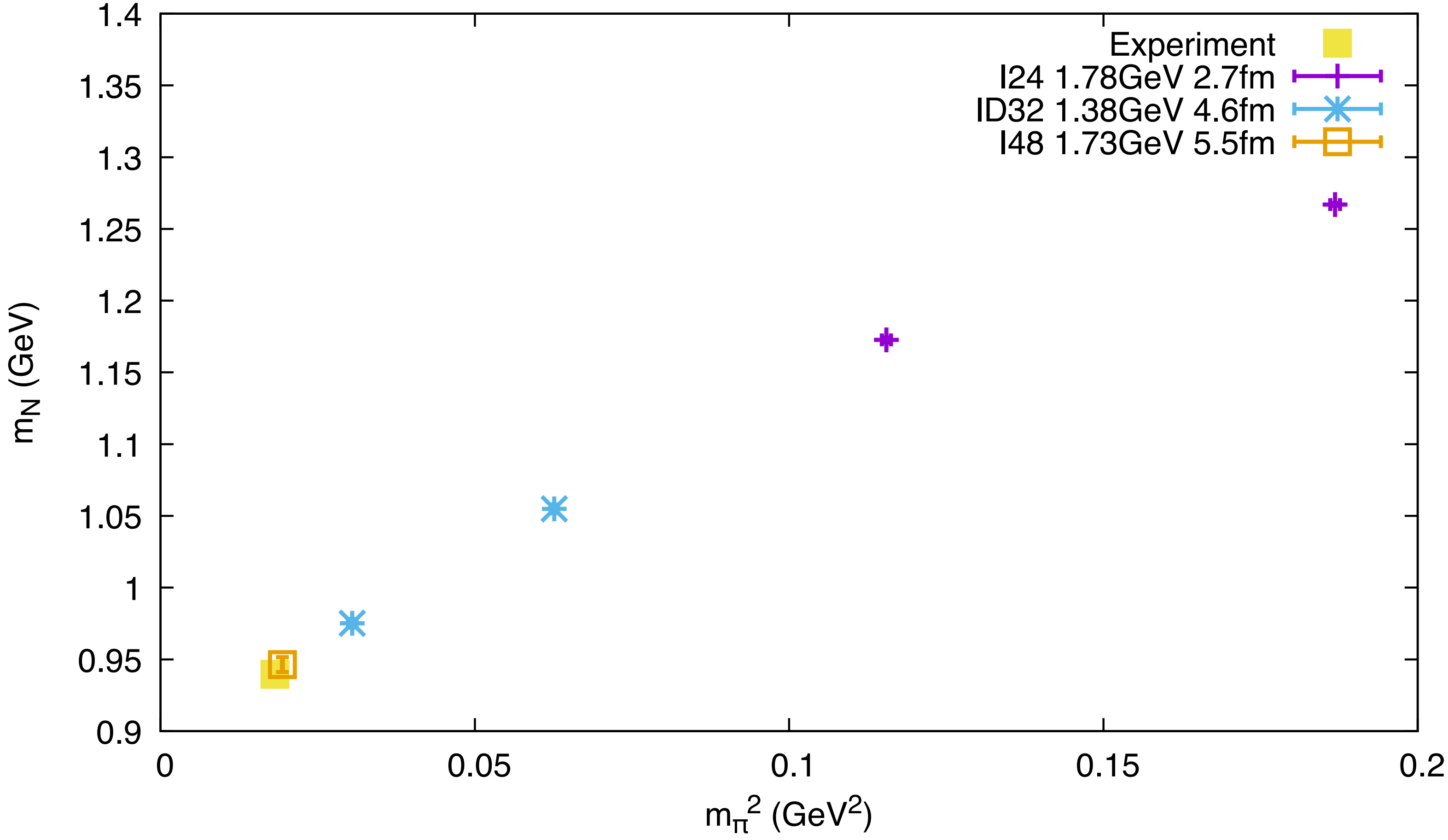
[6] E. Berkowitz *et al.*, arXiv:1704.01114 [hep-lat]; C. C. Chang *et al.*, Nature **558**, no. 7708, 91 (2018) [arXiv:1805.12130 [hep-lat]].

More recently, on RBC+UKQCD 2+1-flavor DWF, Iwasaki gauge, at 1.730(4) GeV and 5.5fm:

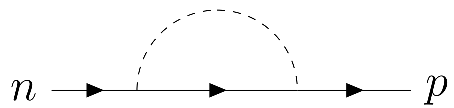


g_A/g_V calculated by RBC+LHP trends to the experiment.
Almost all other groups saw similar trends.

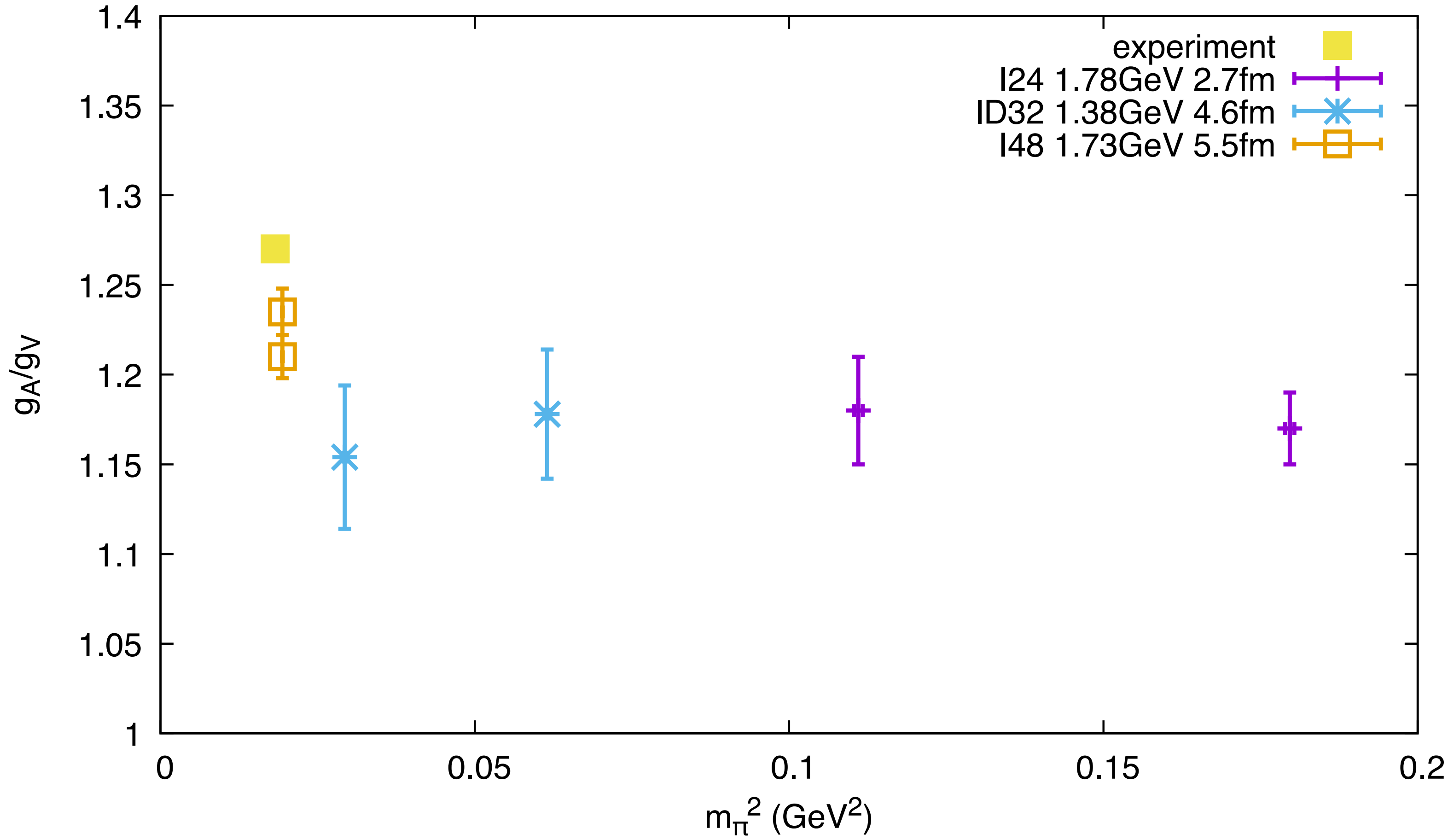
A nucleon-mass estimate of $m_N = 0.549(4)a^{-1} = 0.950(8)\mathrm{GeV}$:



along with a non-linear dependence on quark mass.
Chiral log, $m_\pi^2 \log m_\pi^2 \sim m_q \log m_q$?



And we still see a deficit: perhaps smaller but certainly more statistically significant than before,



So we like to check the systematic errors.

Previous RBC and RBC+UKQCD calculations addressed two important sources of systematics:

- Time separation between nucleon source and sink,
- Spatial volume.

And though not explicitly addressed yet, a better understanding of quark mass dependence is necessary.

No source or sink is purely ground state:

$$e^{-E_0 t}|0\rangle + A_1 e^{-E_1 t}|1\rangle + ...,$$

resulting in dependence on source-sink separation, $t_{\rm sep} = t_{\rm sink} - t_{\rm source}$,

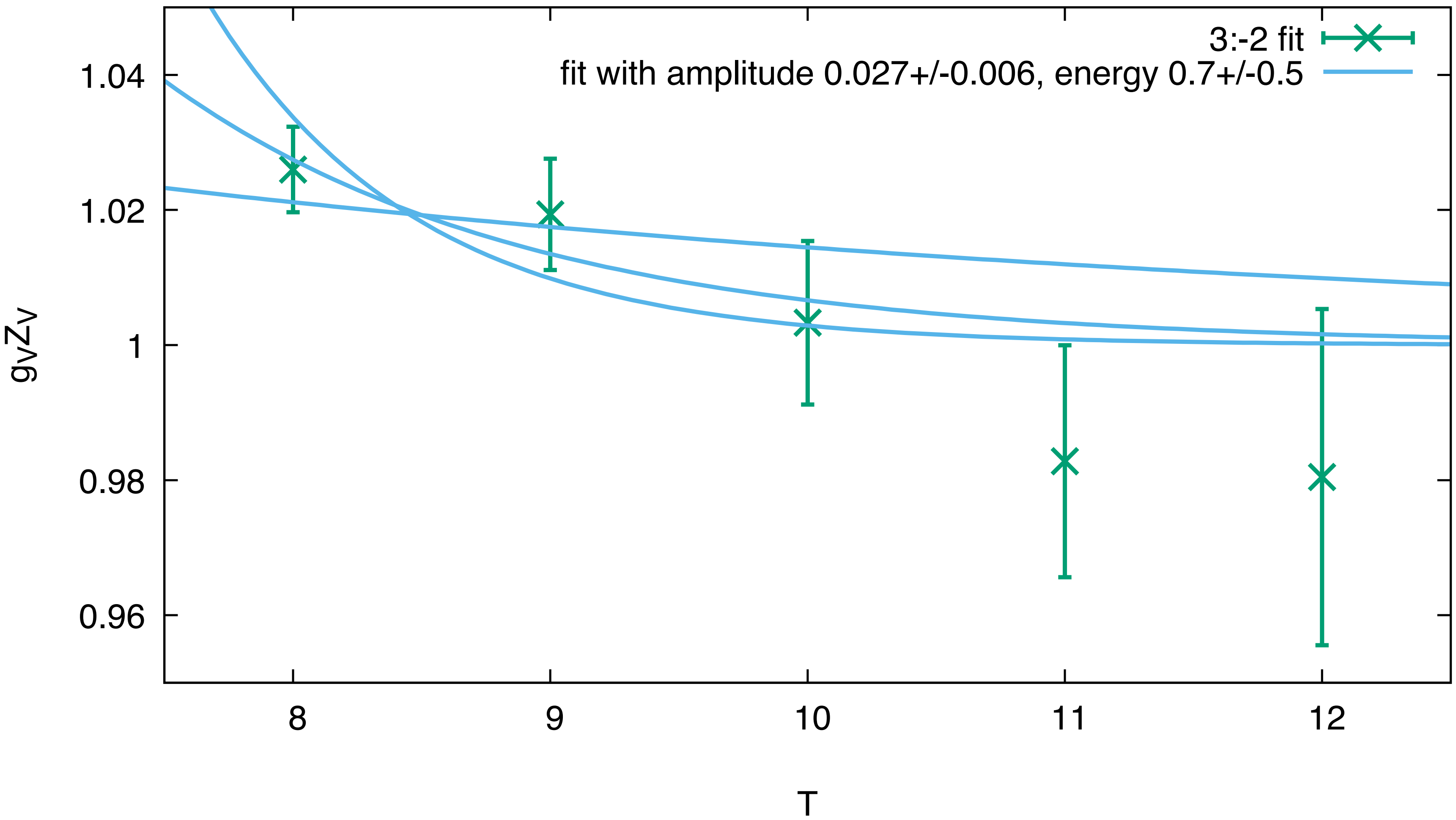
$$\langle 0|O|0\rangle + A_1 e^{-(E_1-E_0)t_{\rm sep}}\langle 1|O|0\rangle + ...$$

Any conserved charge, $O = Q$, $[H, Q] = 0$, is insensitive because $\langle 1|Q|0\rangle = 0$.

- g_V is clean,
- g_A does not suffer so much, indeed we never detected this systematics,
- structure function moments are not protected, so we saw the problem.

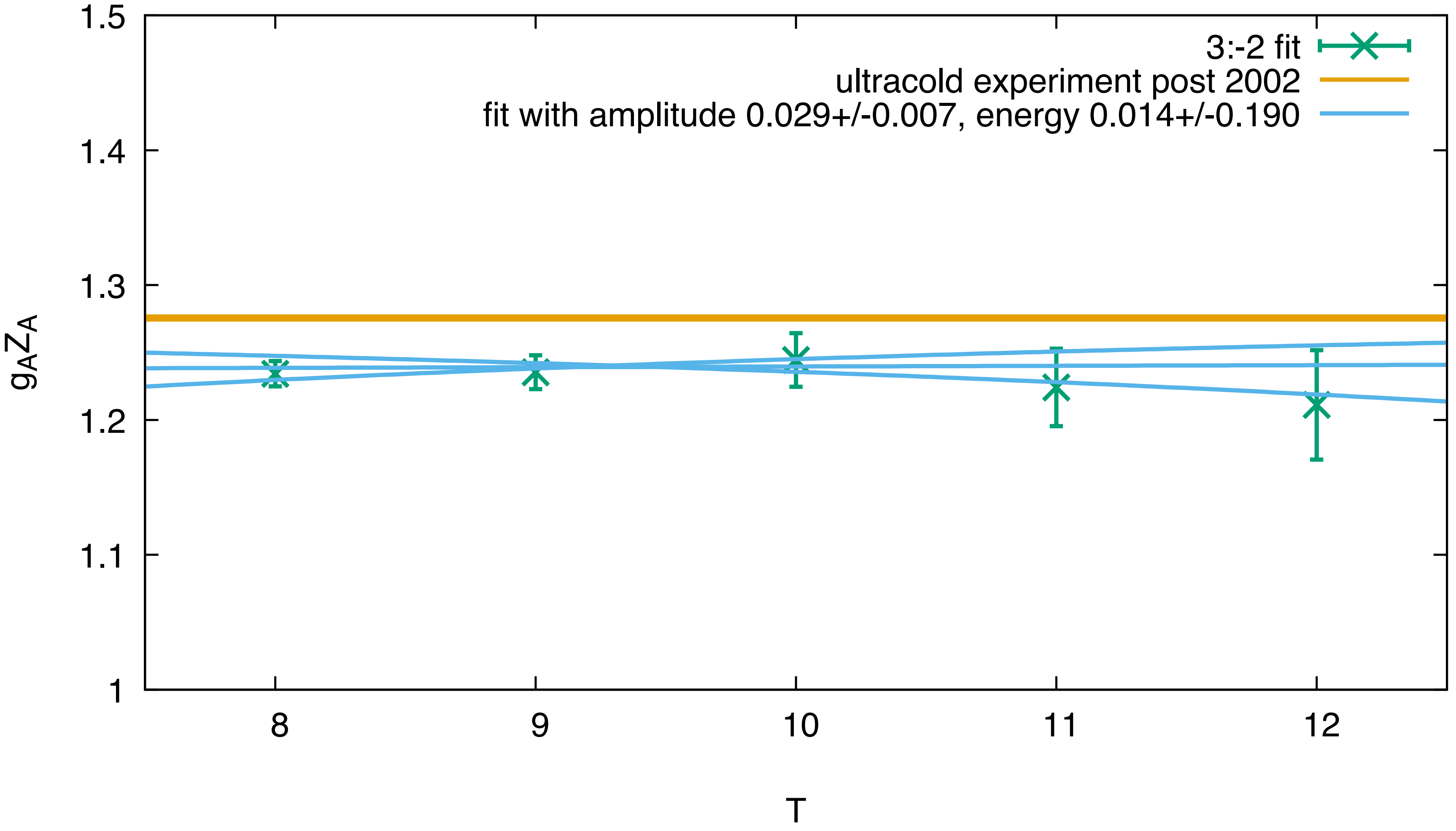
We can optimize the source so that A_1 is small, and we take sufficiently large $t_{\rm sep}$: Indeed with AMA we established there is no excited-state contamination present in any of our 170-MeV calculations.

Isovector vector charge, g_V, at $T = 8$ and 9, deviates from unity: possibly $O(a^2)$ mixing with excited states,



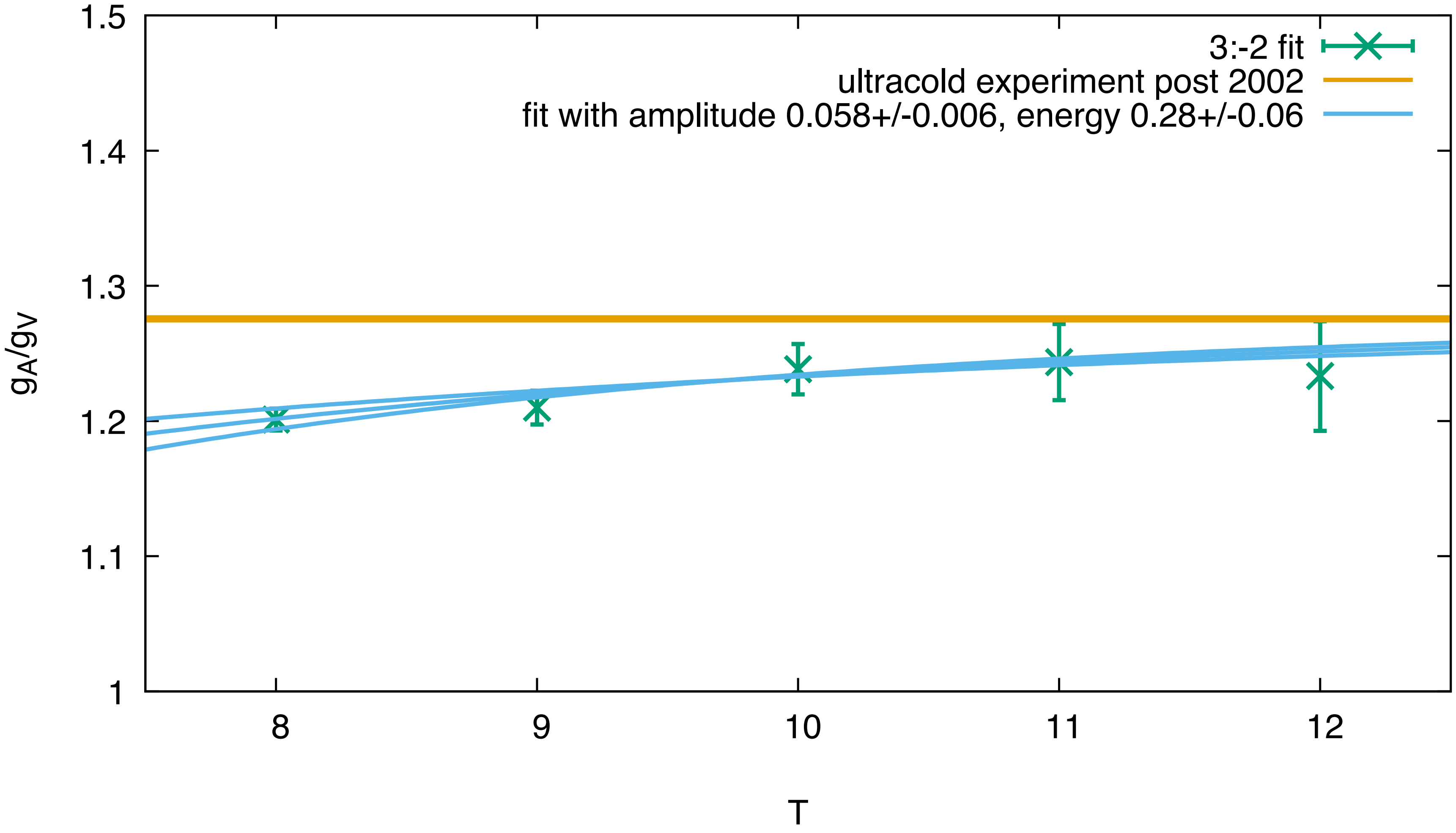
single-excitation fit is not so precise: we need shorter $T = 7$ and 6 calculations for further investigation.

Isovector axialvector charge, g_A, renormalized with Z_A^{meson}, undershoots the experiment by a few percent.



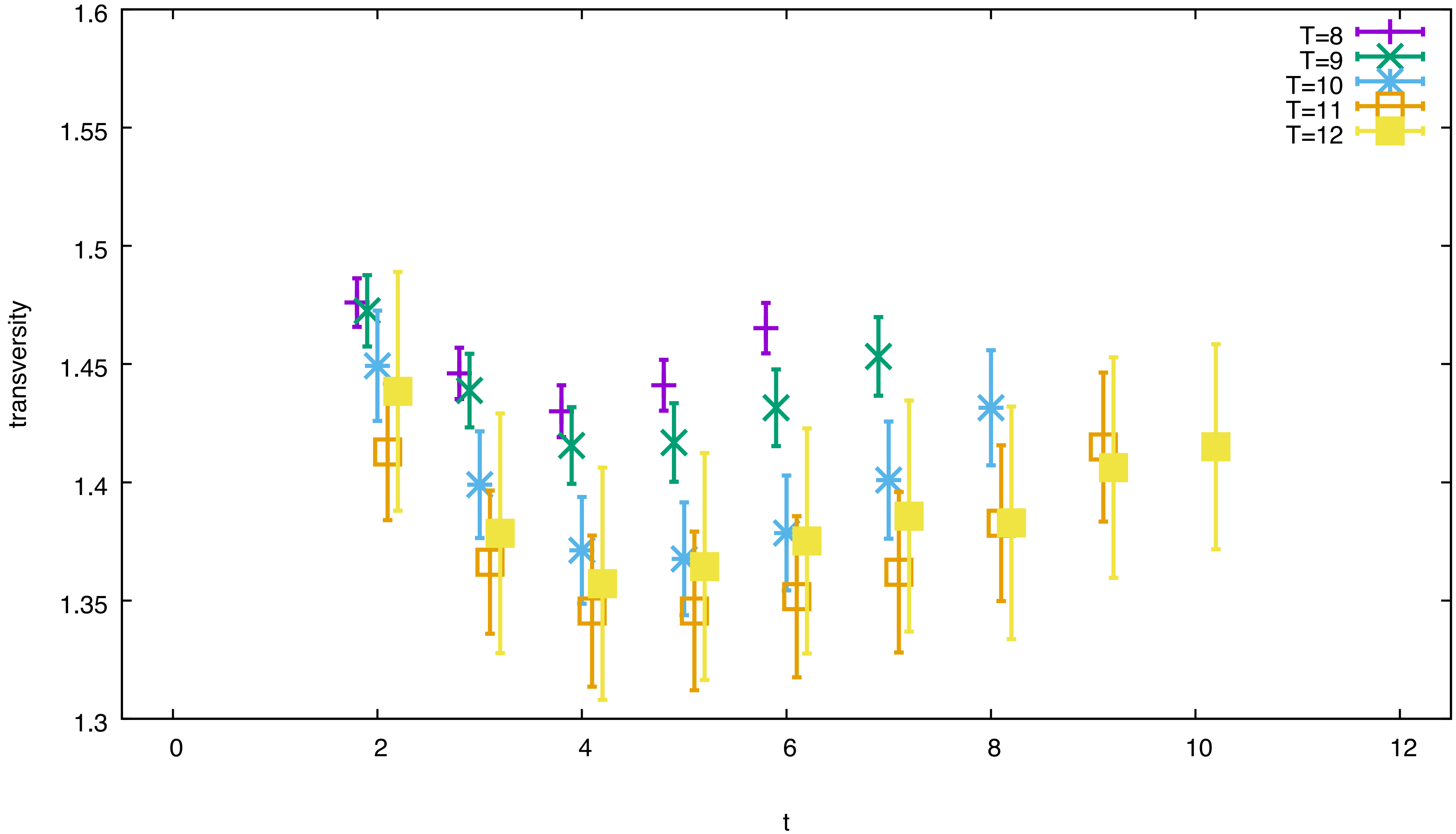
Excitation consistent with 0: this deficit appears independent of excited state contamination.

Isovector axialvector to vector charge ratio, g_A/g_V, undershoots the experiment by several percent.

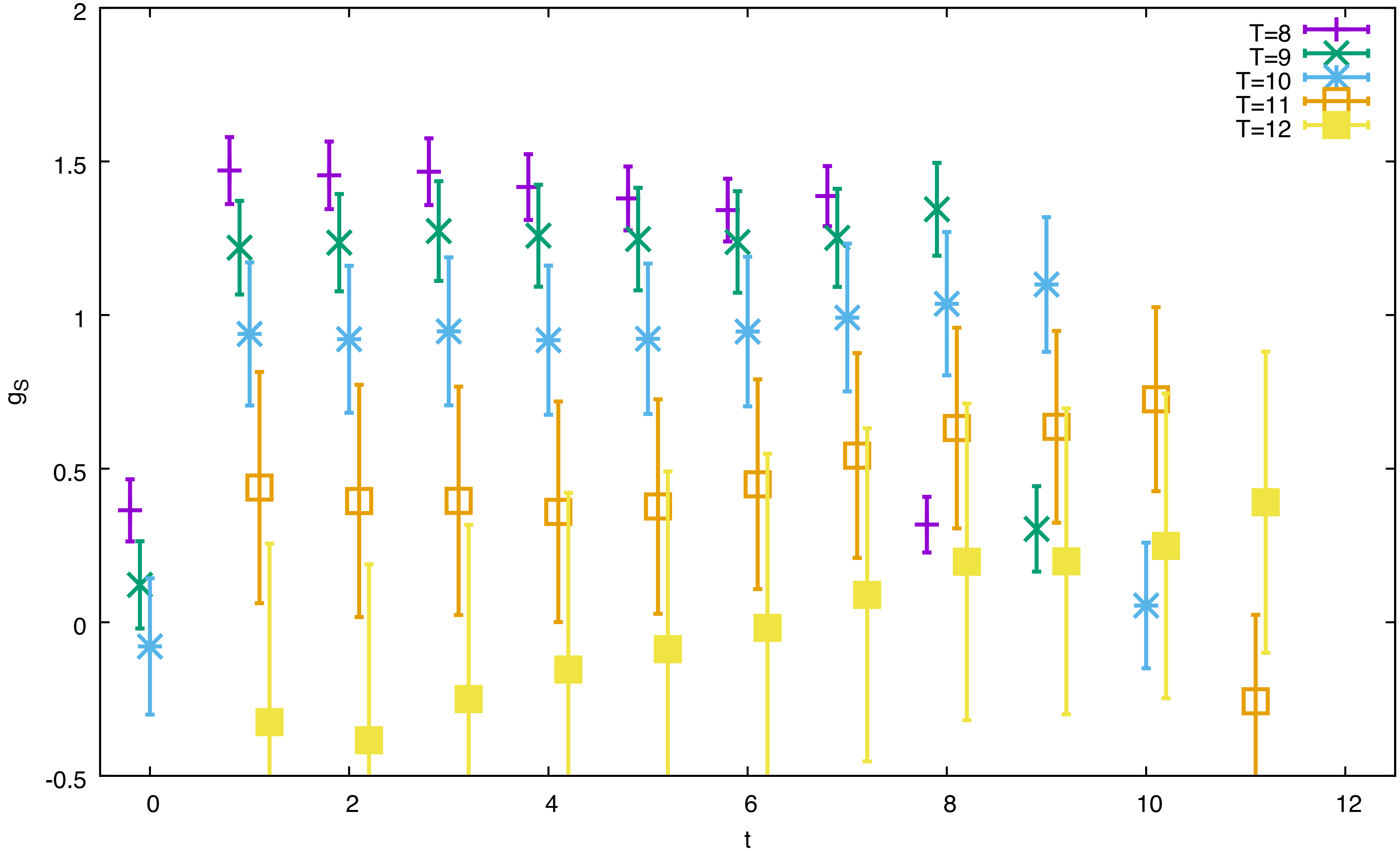


Perhaps affected by the g_V $T-$dependence?

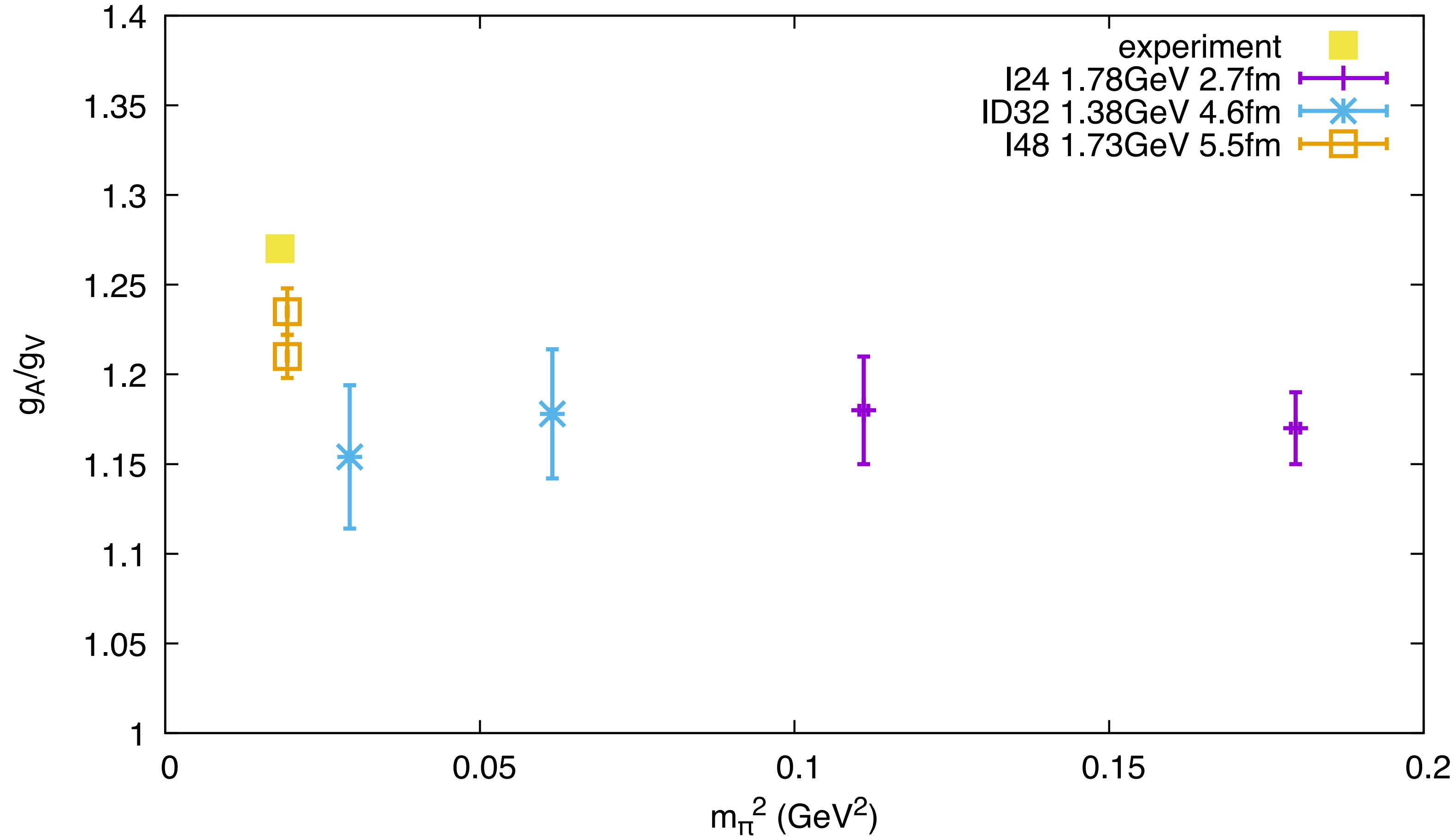
Isovector tensor coupling, g_T, gives decent signals.



Isovector scalar coupling, g_S, is very noisy.

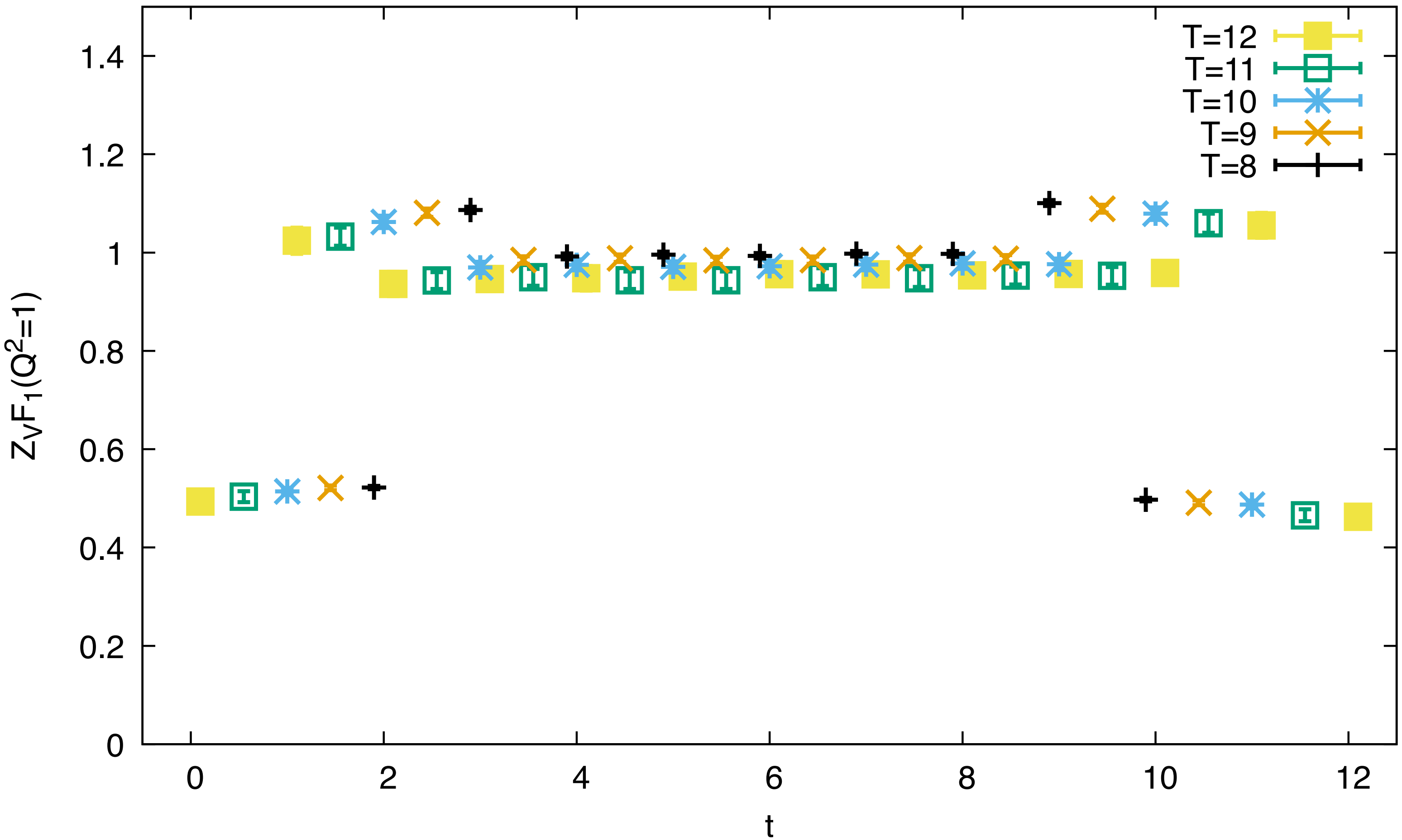


Thus we still see a deficit: perhaps smaller but certainly more statistically significant than before,



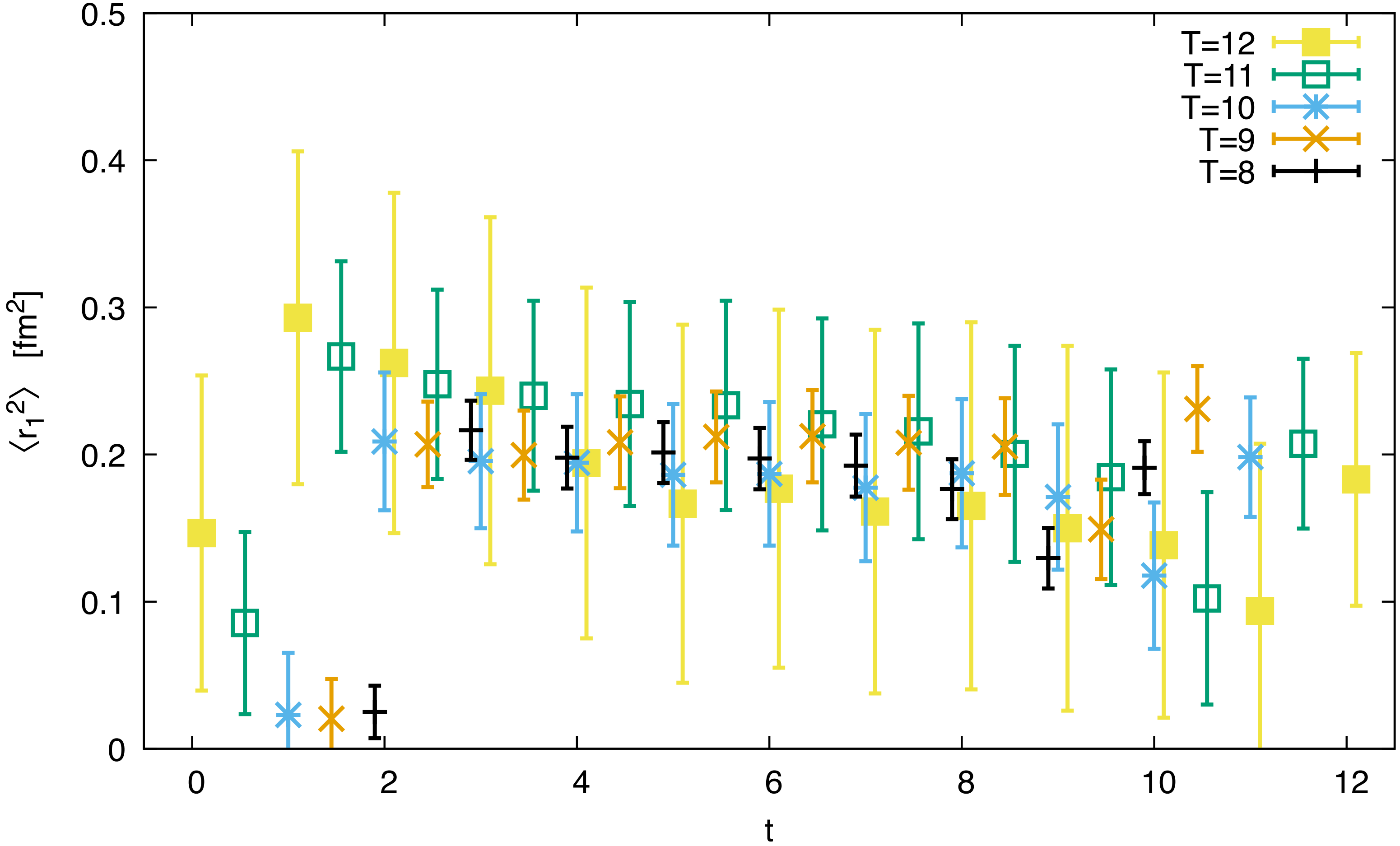
How do we clarify systematics from the nearby excited state(s)?

Can we quantify such possible excited states in momentum-dependent form factors?



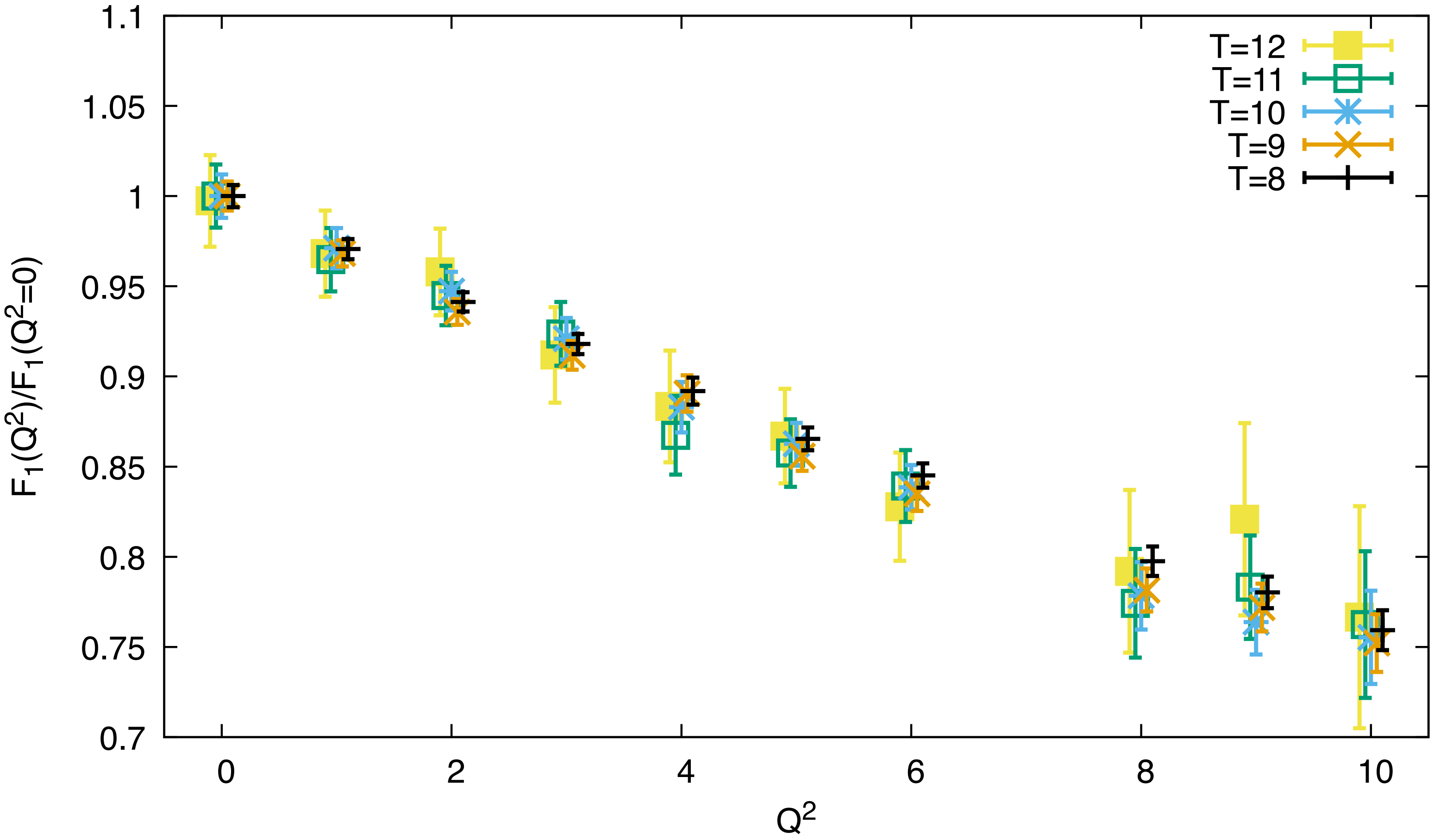
T-dependence perhaps is monotonic ...: would like more statistics.

Can "charge-squared radius," from $\langle r_1^2 \rangle = \frac{6[F_1(Q^2=0)-F_1(Q^2=1)]}{Q^2=1}$, tell these apart?



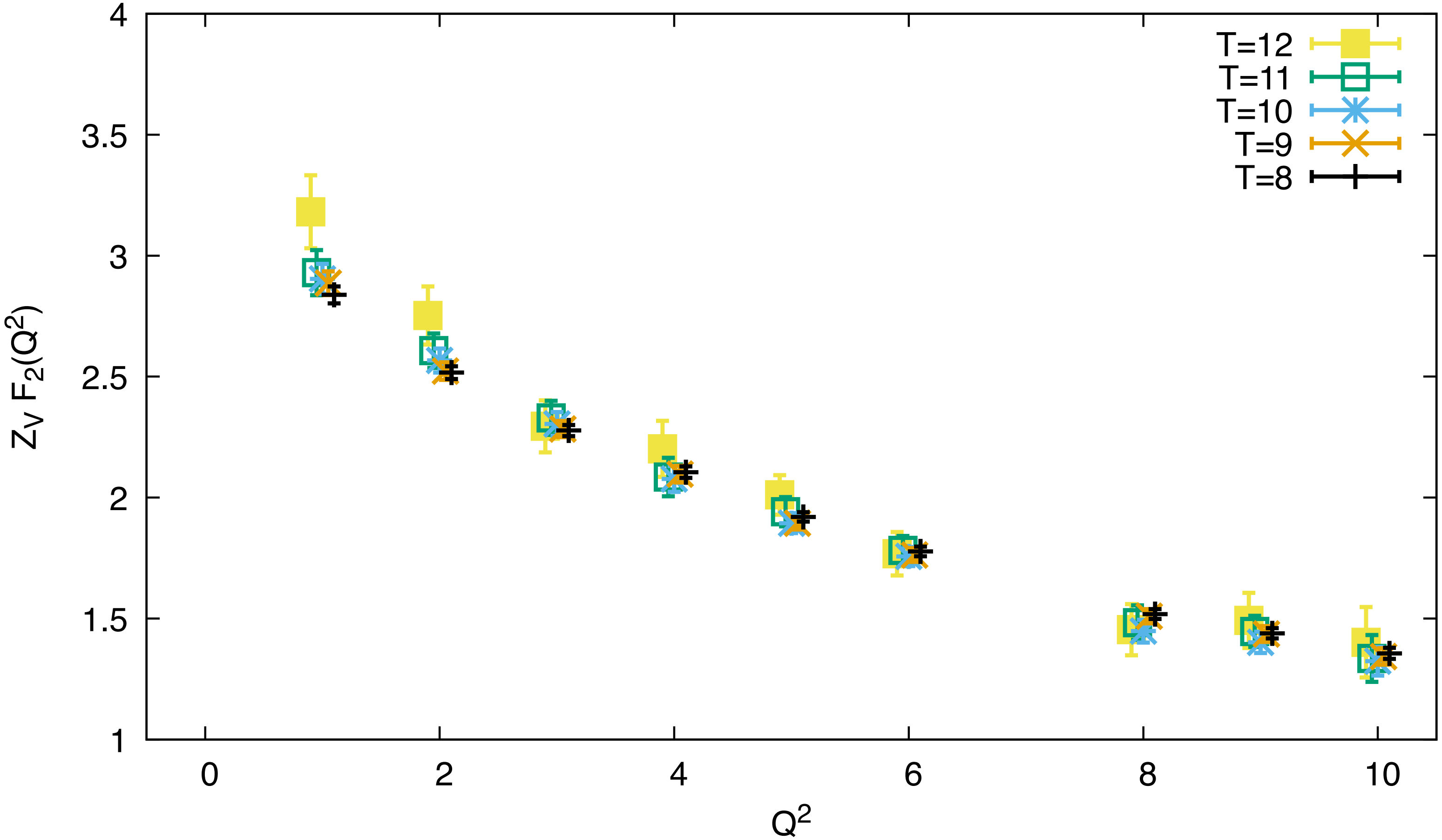
Not quite: yet good news as the shape may not be disturbed too much?
Average $\sim 0.20(2)\text{fm}^2$ as compared to experiment: $[(0.8409(4))^2 + 0.1161(22) = 0.8682(29)](\text{fm})^2$.

F_1 shape does not seem to depend on source-sink separation, T:

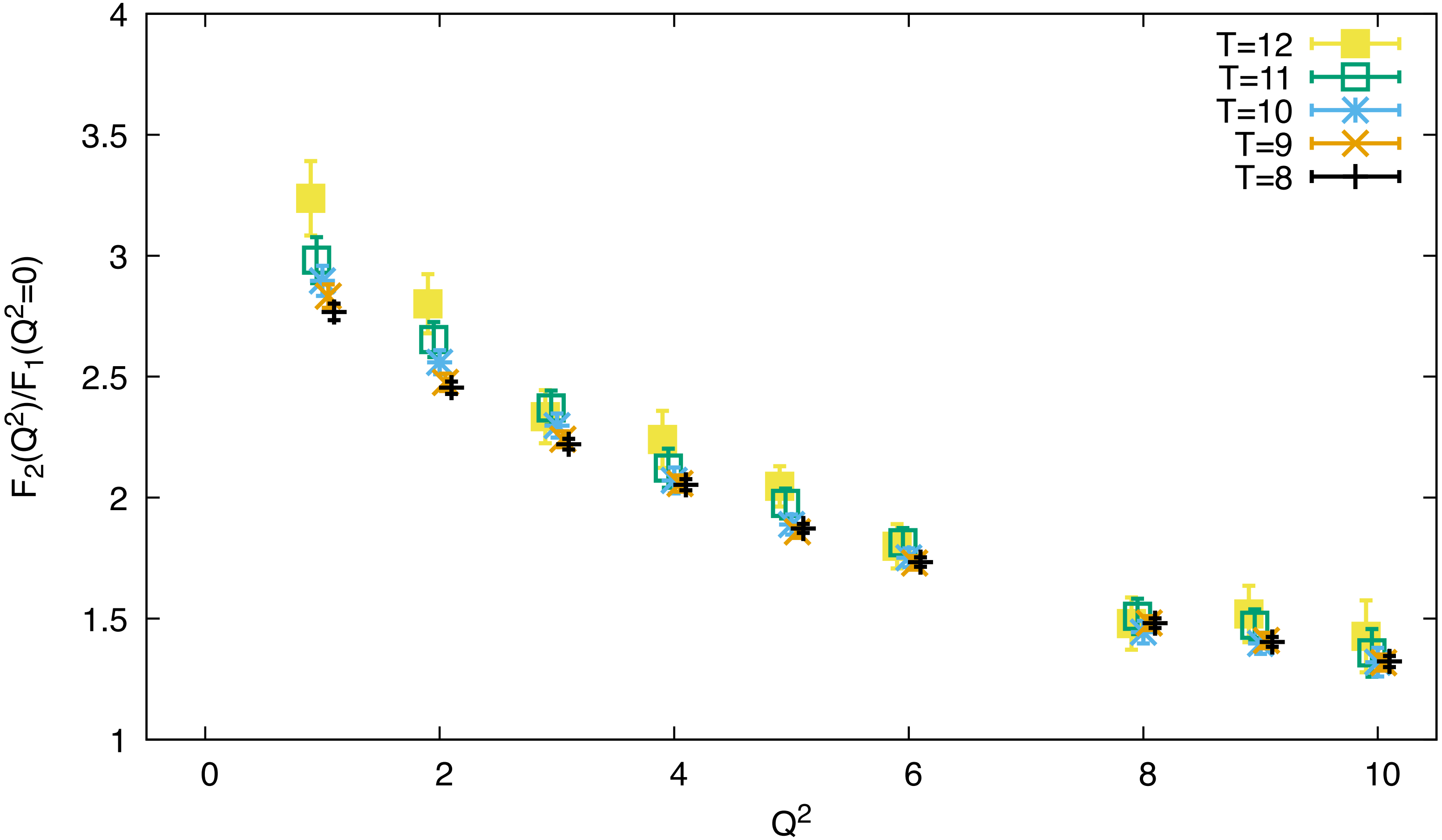


Form factors from $T = 8$, 9, and 10 are informative: no need for more statistics?

F_2 may not be affected by excited states either.

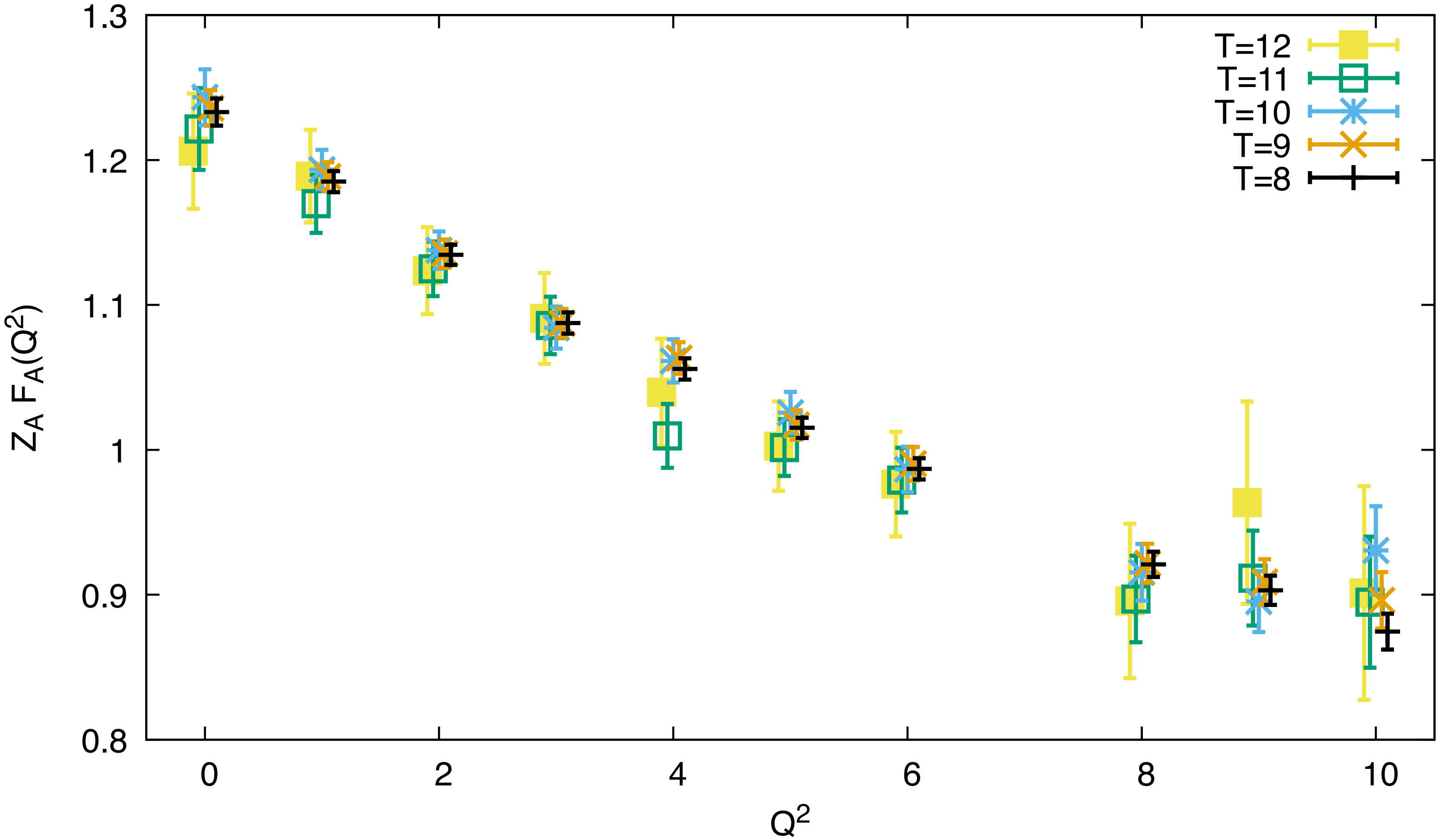


Or maybe?

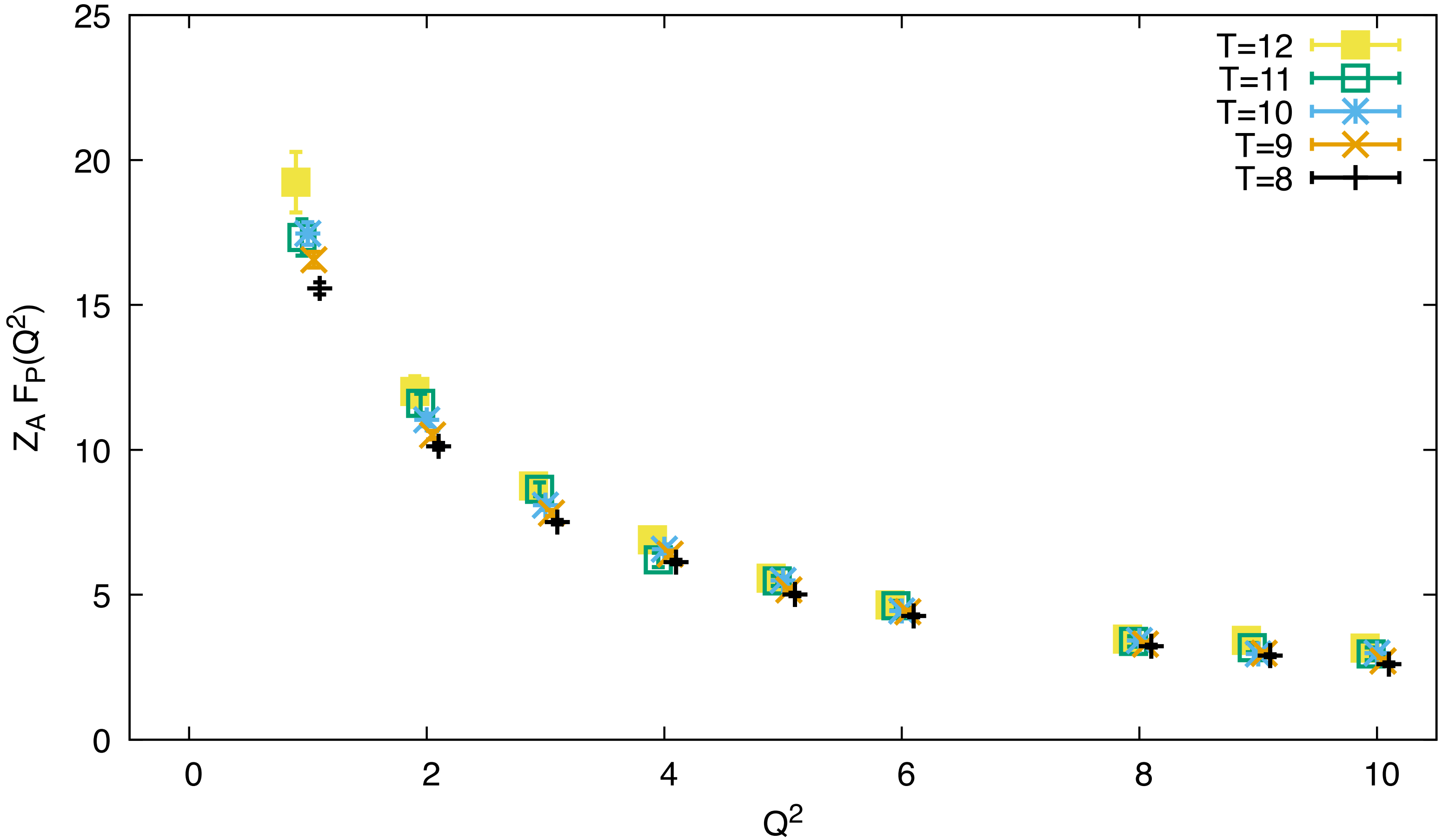


Extrapolate to $\sim 3.4(2)\mu_N$. Experiment: 2.79284734462(8) + 1.9130427(5) - 1 = 3.705874(5).
More statistics desired at larger T.

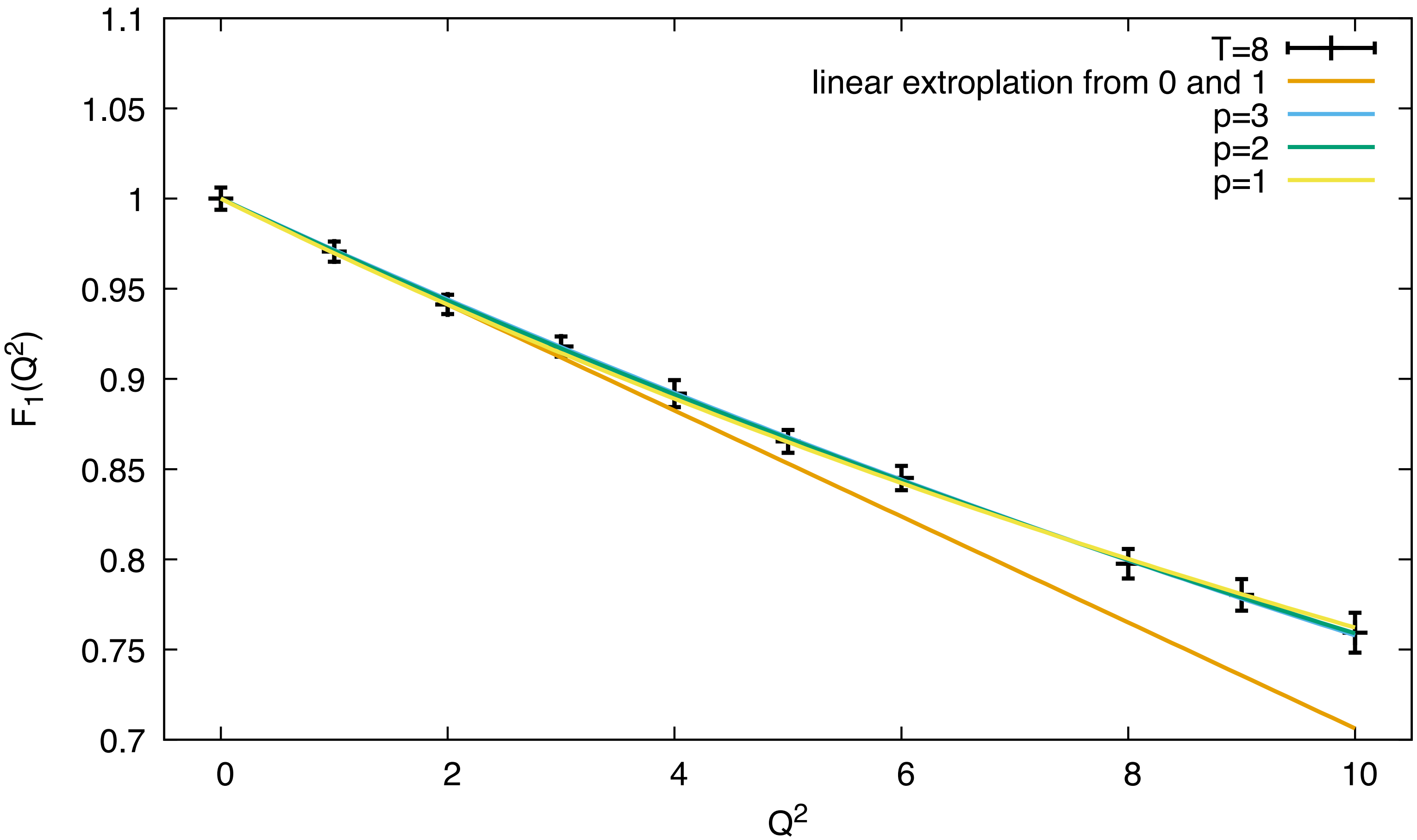
F_A:



F_P:



Extrapolations: "multipole" fits, to $F(Q^2) \sim F(0)\left(1+\frac{Q^2}{M_p^2}\right)^{-p}$, with $p = 1$, 2, and 3, appear to work well:



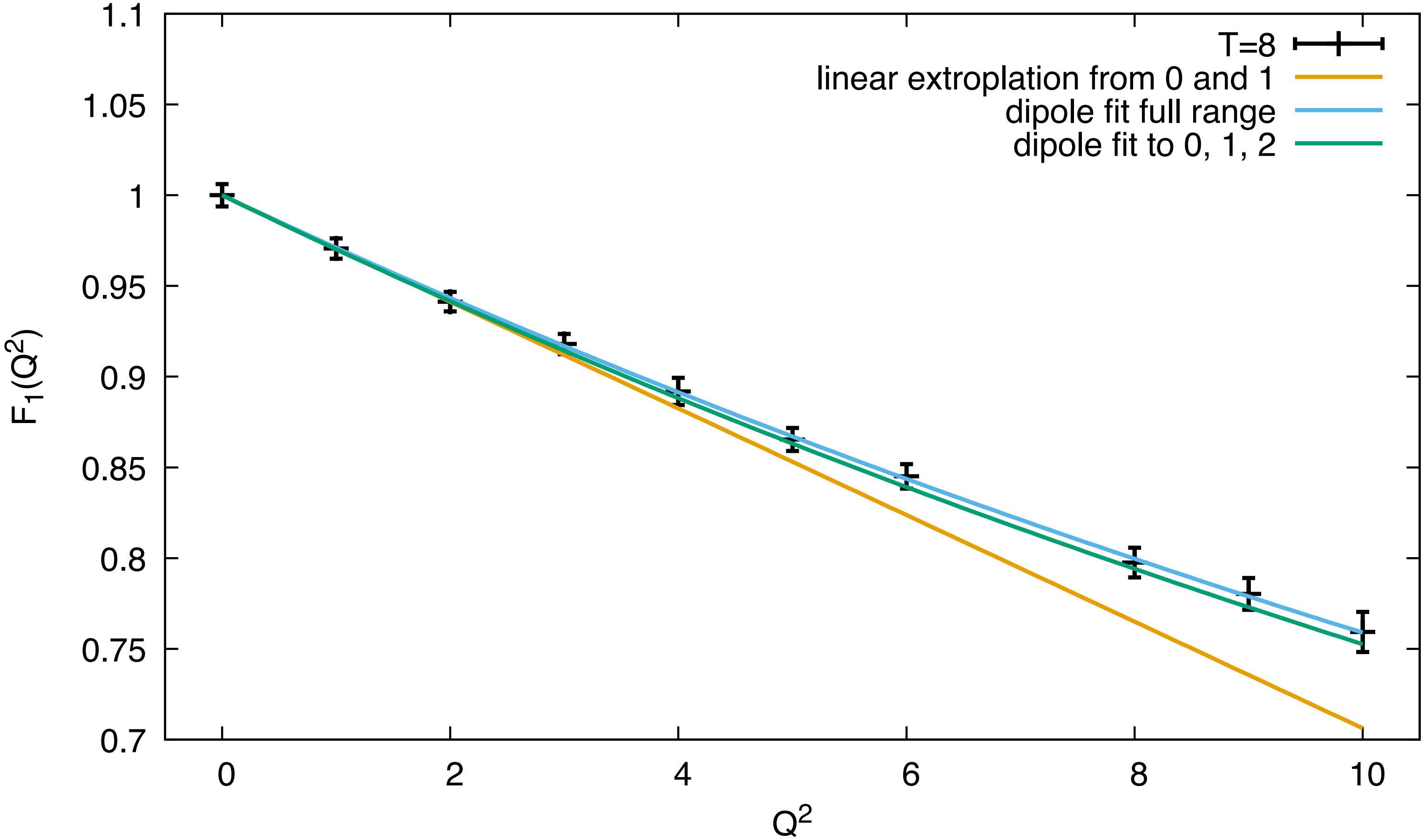
with similar $\langle r_1^2 \rangle = 6p/M_p^2 \sim 0.14\ \mathrm{fm}^2$ estimates.

Extrapolations by "linear" using the smallest two Q^2, and "dipole," $F(Q^2) \sim F(0)\left(1+\frac{Q^2}{M^2_{\text{dipole}}}\right)^{-2}$:

		$T=8$	9	10	11	12	experiment
$\langle r_1^2\rangle$	linear	0.134(14)	0.14(2)	0.13(3)	0.16(5)	0.13(8)	0.868(3)fm^2
	dipole	0.135(6)	0.143(8)	0.142(13)	0.14(2)	0.13(3)	
$F_2(0)$	linear	3.159(4)	3.250(6)	3.242(8)	3.252(13)	3.61(2)	3.705874(5)μ_N
	dipole	3.10(5)	3.15(6)	3.22(8)	3.24(11)	3.5(2)	
$\langle r_A^2\rangle$	linear	0.177(2)	0.174(2)	0.182(4)	0.192(5)	0.066(8)	–
	dipole	0.177(7)	0.174(10)	0.176(14)	0.18(2)	0.15(3)	
$F_P(0)$	linear	21.01(3)	22.61(5)	23.90(7)	23.04(11)	26.5(2)	–
	dipole	23(2)	25(2)	26(2)	26(2)	30(2)	

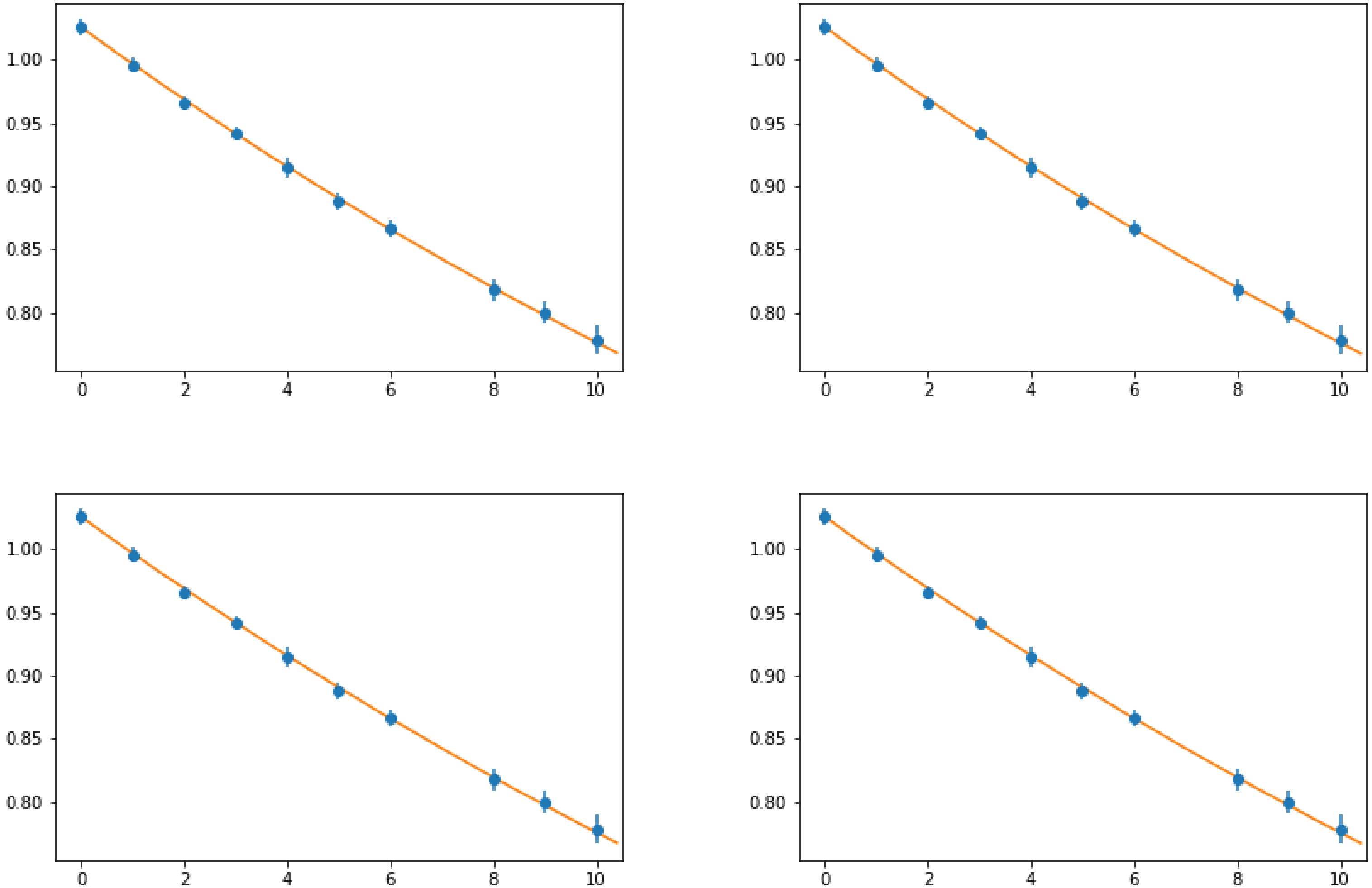
"Linear" and "dipole" fits agree with each other but do not agree with experiments.
What are we missing?

Multipole fits do not change much with the fitting range:



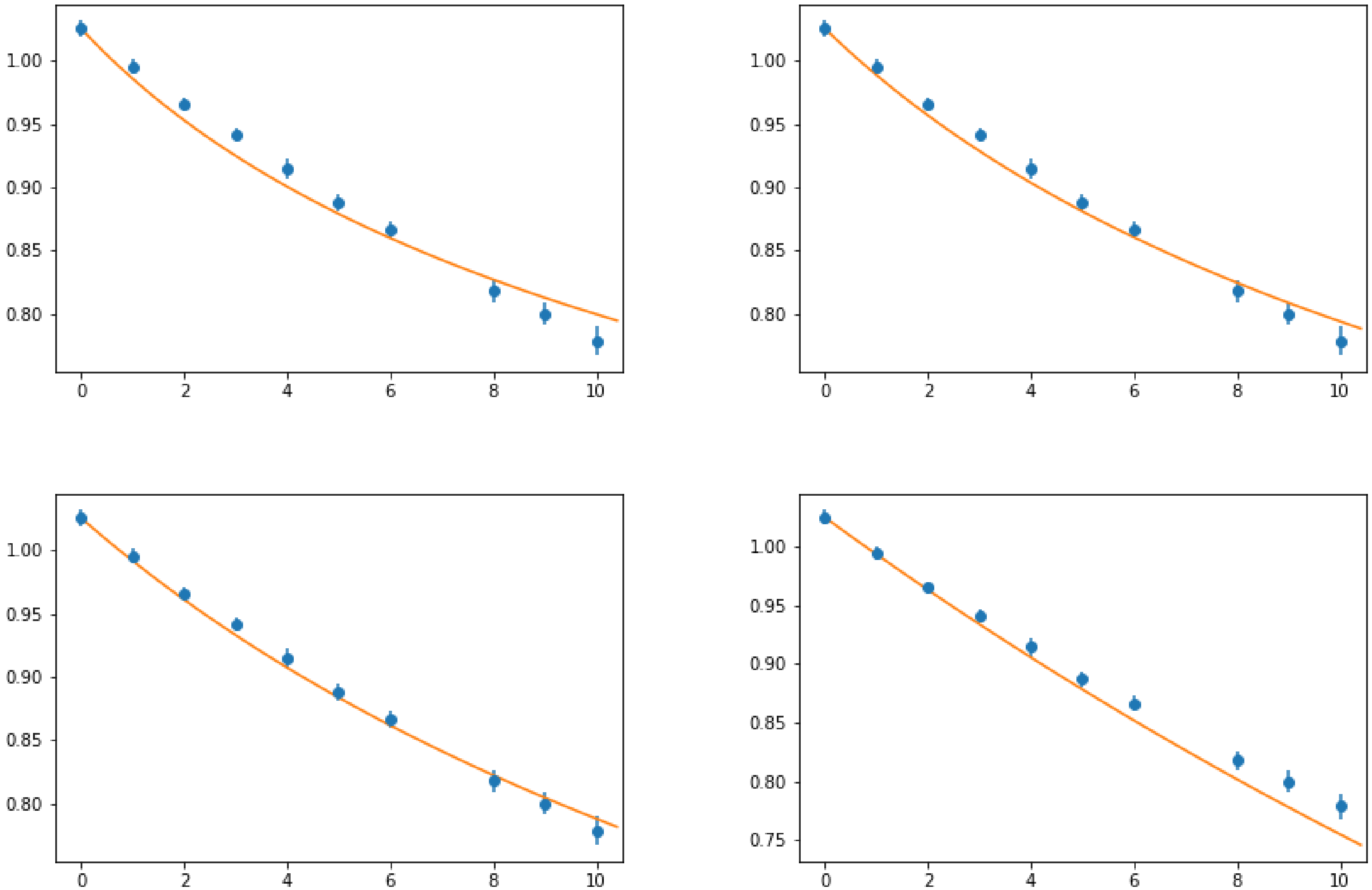
And they agree with the linear estimate for the smallest Q^2 pair.
Dubious if much more can be squeezed for $\langle r_{1,A}^2 \rangle$ or $F_{2,P}(0)$.

$p = 4, 5, 6, 7$ do not seem so different:



with $\chi^2/\text{dof} \sim 0.1$ or less.

$p = 1/4$, $1/3$, $1/2$, and 8 do not fit well:



$p = 1/4$ (upper left) gives $\langle r_1^2 \rangle = 0.77(5)\mathrm{fm}^2$ but with $\chi^2/\mathrm{dof} \sim 4$.

Nucleon on RBC+UKQCD DWF ensembles at physical mass and cutoff 1.730(4) GeV:

- Excited-state contamination is likely seen in isovector vector charge, g_V.
- Deficit in isovector vector charge, g_A, shrank but gained in statistical significance.
- Isovector form factor shapes may not be too sensitive to the excited-state contamination,
- yet $\langle r_1^2 \rangle$ and $F_2(0)$ are small.

Immediate/short-term: we are finishing up on I48 form factors:

- shorter T such as 7, and even 6, would help,
- as well as additional statistics, especially for $T = 8$, 9, and 10, perhaps even 11,
- above all, smaller momentum transfer most likely via twisted boundary condition.

Mid-term: isospin breaking,

- both u-d mass difference,
- and EM.

Longer term: finer lattice spacing,

- $a^{-1} = 2.359(7)$ GeV, and
- then ≥ 3 GeV to unquench charm, ...