Update on Flavor Diagonal Nucleon Charges

Sungwoo Park*,

T. Bhattacharya, R. Gupta, S. Mondal, B. Yoon, H-W Lin

[PNDME Collaboration]

* Jefferson Lab, VA, USA

Los Alamos National Laboratory, NM, USA

Michigan State University, MI, USA

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Physics from flavor diagonal nucleon charges

- $\boldsymbol{g_A^q = \Delta q}$: Quark contributions to the nucleon spin

$$\frac{1}{2} = \sum_{u,d,s,\cdots} \left(\frac{1}{2} \boldsymbol{\Delta q} + L_q \right) + J_g$$

X. Ji (1997), PNDME (2018)
L_q: orbital angular momentum of the quark
J_g: total angular momentum of the gluons

- $\boldsymbol{g_T^q}$: Quark EDM contributions to the neutron EDM d_n

$$|d_n| = |d_u^\gamma \boldsymbol{g_T^u} + d_d^\gamma \boldsymbol{g_T^d} + d_s^\gamma \boldsymbol{g_T^s} + \cdots| \leq 2.9 \times 10^{-26}\ e\ \mathrm{cm}$$

C. Baker et al. (2006)
PNDME (2018)

- $\boldsymbol{g_S^q = \frac{\partial M_N}{\partial m_q}}$: Slope of the nucleon mass with respect to the quark mass

$\sigma_{\pi N} = m_l \boldsymbol{g_S^{u+d}}$: Quark contributions to the nucleon mass
$\sigma_s = m_s \boldsymbol{g_S^s}$

PNDME (2021)

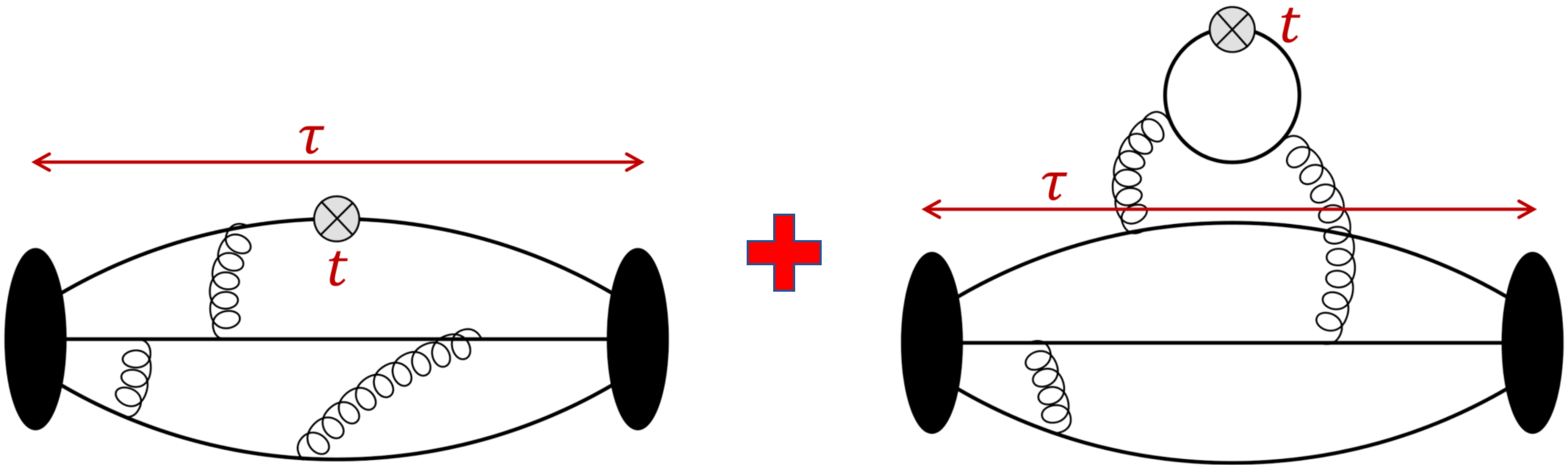
Clover fermions on MILC HISQ lattices

Ensemble ID	a [fm]	M_π [MeV]	$M_\pi L$	N_{cfg}^{conn}	$N_{cfg}^{disc,l}$	$N_{cfg}^{disc,s}$
a15m310	~0.15	320	3.93	1917	1917	1917
a12m310	~0.12	310	4.55	1013	1013	1013
a12m220	~0.12	228	4.38	744	958	870
a09m310	~0.09	313	4.51	2263	1017	1024
a09m220	~0.09	226	4.79	964	712	847
a09m130	~0.09	138	3.90	1290	1270	994
a06m310	~0.06	320	4.52	500	808	976
a06m220	~0.06	235	4.41	649	1001	1002

- 8 ensembles including one physical M_π^{phys} ensemble
- HYP smeared $N_f = 2 + 1 + 1$ MILC HISQ lattices
- Clover fermion with a tree-level tadpole improved c_{SW}

Connected and disconnected diagrams

- Flavor diagonal nucleon charges are obtained from the nucleon ME $\langle N|\bar{q}\,\Gamma\,q|N\rangle$
- Require high precision measurements of quark bilinear operators within the nucleon state for both "connected" and "disconnected" 3-point correlation functions,



Calculated with covariant Gaussian source smearing, multiple source-sink separation $0.9 \lesssim \tau \lesssim 1.4$, accelerated with coherent sequential inversions and the truncated solver method with bias correction. PNDME (2018)

All-to-all quark propagator estimated by stochastic method using Z_4 random sources, accelerated with the truncated solver method with bias correction and hoping parameter expansion. PNDME (2015)

Removing excited state contaminations (ESC)

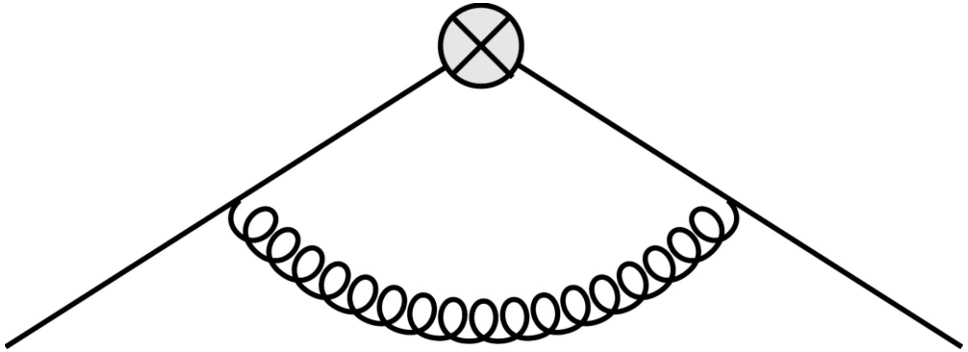
- Simultaneous fits to 2- and 3-point (connected + disconnected) functions using empirical Bayesian prior on the excited mass spectrum M_i and A_i

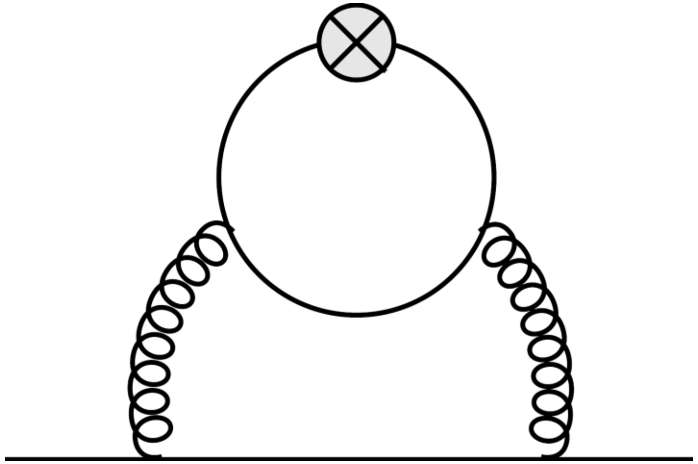
$$C^{2\text{pt}}(\tau) = \sum_{i=0} |\mathcal{A}_i|^2 e^{-M_i \tau}. \quad C_\Gamma^{3\text{pt}}(\tau; t) = \sum_{i,j=0} \mathcal{A}_i \mathcal{A}_j^* \langle i | O_\Gamma | j \rangle e^{-M_i t - M_j (t-\tau)},$$

- Repeat the analysis to quantify the model variation of the results by choosing different sets of (τ, t_{skip}) and number of states in the excited state fits (2 or 3^*-state fits)
  - t_{skip}: number of data points next to the source and the sink for each τ, skipped in the excited state fits
  - τ : source-sink separation
- The Final results are taken from the average over the model values, weighting each by its Akaike information criteria weights.

Nonperturbative renormalization

- We explicitly evaluated the 3×3 flavor mixing matrices in RI-sMOM scheme and convert into $\overline{\mathrm{MS}}$ scheme value 2 GeV.
- Results on the corrections from the flavor mixing
  - Small and negligible for $g_{A,T}^{u,d,s}$ and $g_S^{u,d}$
  - g_S^s gets a correction about ~20% at $a \approx 0.15$fm, and ~6% at $a \approx 0.06$fm from the off-diagonal $Z_S^{s,u+d}$.





Examples on ESC fits

with or without $N\pi$, $N\pi\pi$-state prior,

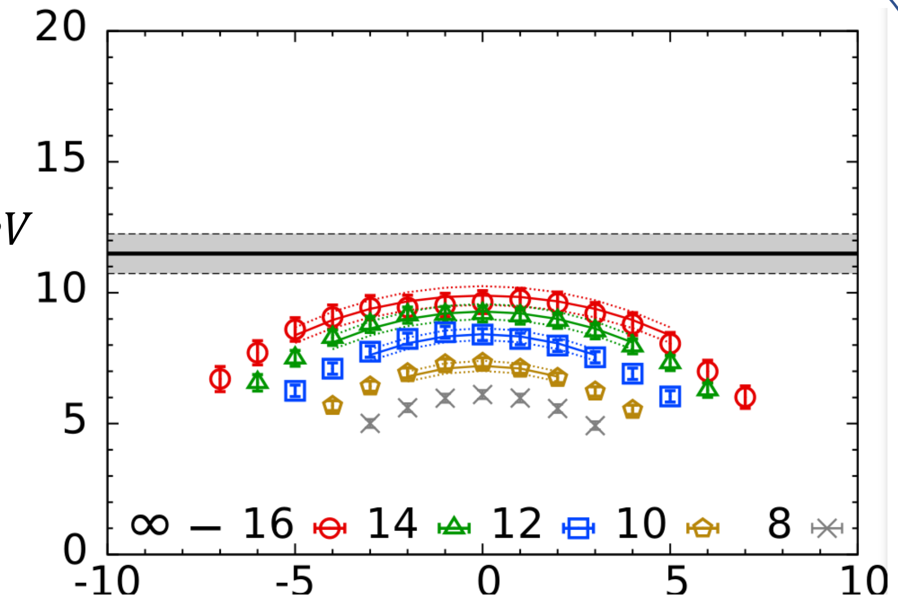
at physical M_π

Excited state fits at M_π^{phys}: g_S^{u+d}

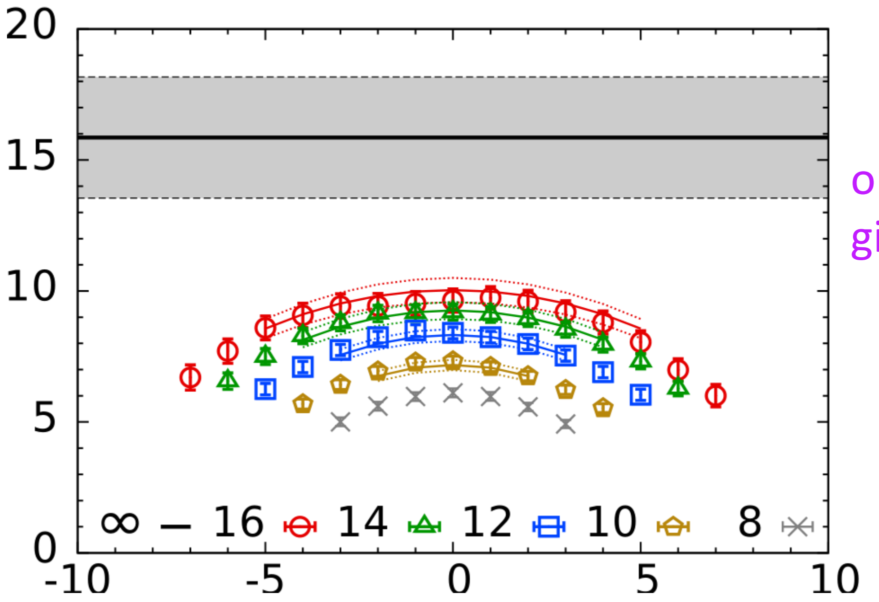
PNDME (2021)
R. Gupta, talk at 3:50pm Fri

$a \approx 0.09 fm$
$M_\pi \approx 135 MeV$

$M_1 \approx 1.6\ GeV$
without $N\pi$



$\sigma_{\pi N} = m_l g_S^{u+d}$ ~ 40 MeV, $\frac{\chi^2}{dof} = 1.1$



$N(1)\pi(-1)$
or, $N(0)\pi(0)\pi(0)$
gives $M_1 \approx 1.2\ GeV$

with $N\pi$

$\sigma_{\pi N} = m_l g_S^{u+d}$ ~ 60 MeV, $\frac{\chi^2}{dof} = 1.2$

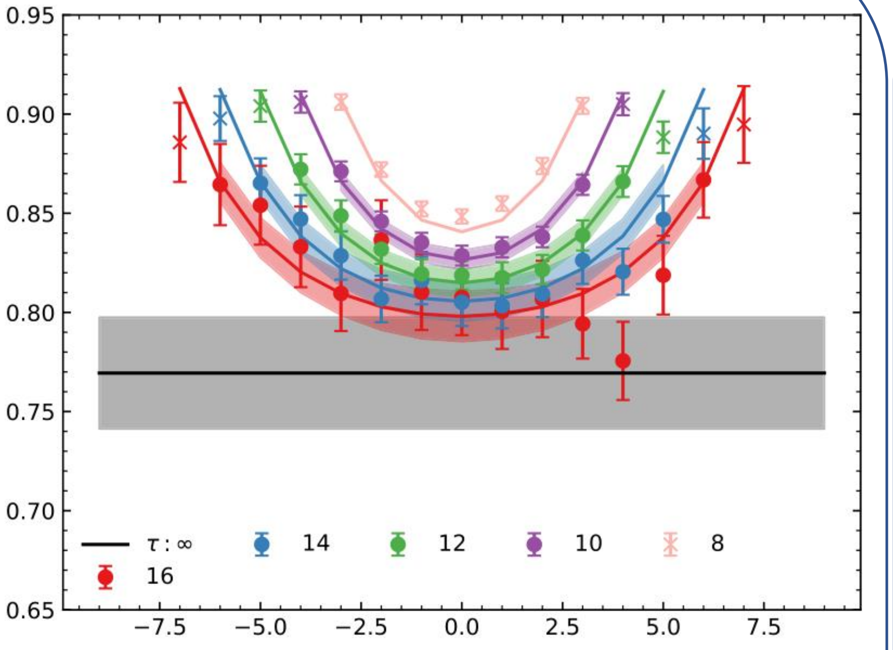
- Scalar is sensitive to $N\pi$ state

Excited state fits at M_π^{phys}: g_T^u

$a \approx 0.09 fm$
$M_\pi \approx 135 MeV$

$M_1 \approx 1.6\ GeV$

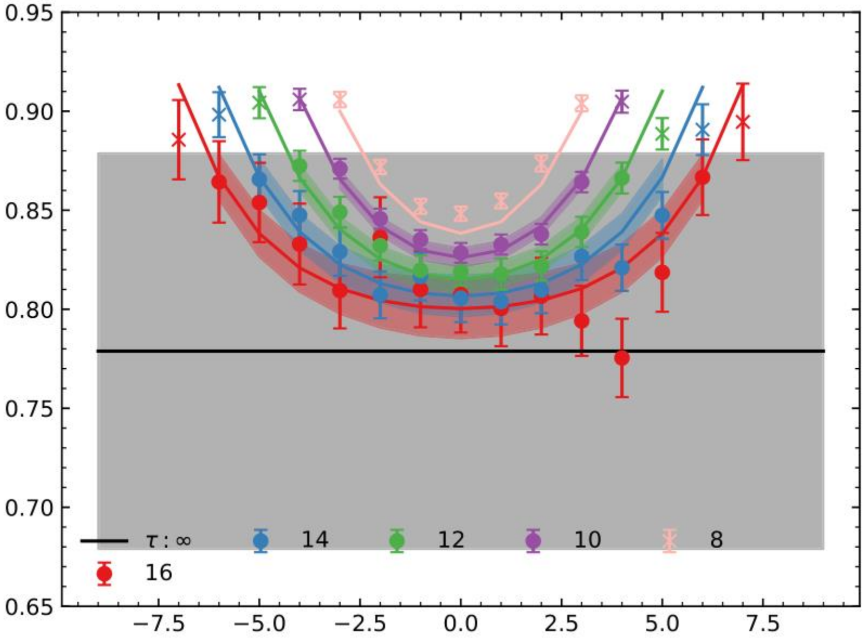
without $N\pi$



$0.769(28), \frac{\chi^2}{dof} = 1.1$

$N(1)\pi(-1)$
or, $N(0)\pi(0)\pi(0)$
gives $M_1 \approx 1.2\ GeV$

with $N\pi$



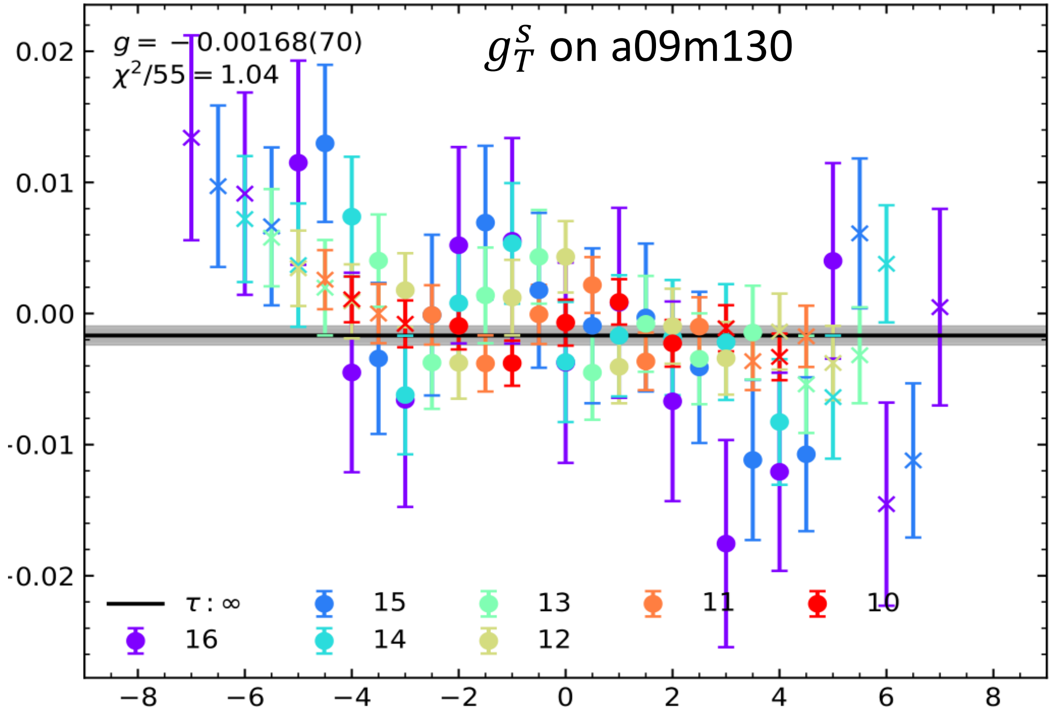
$0.78(10), \frac{\chi^2}{dof} = 1.1$

- Tensor is not sensitive to $N\pi$ state

$a \approx 0.09fm$
$M_\pi \approx 135MeV$

ESC in g_T^S is not resolved

- For g_T^S, 3pt function doesn't show excited state effect
- Constant fit to 3pt/2pt ratio.



Chiral-continuum extrapolated results

with or without $N\pi$, $N\pi\pi$-state prior

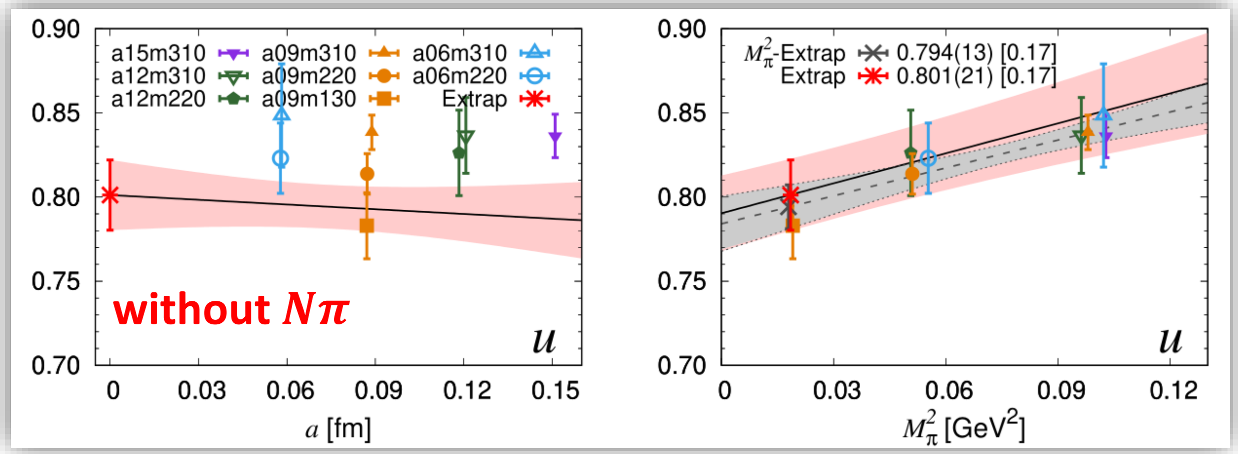
(All preliminary)

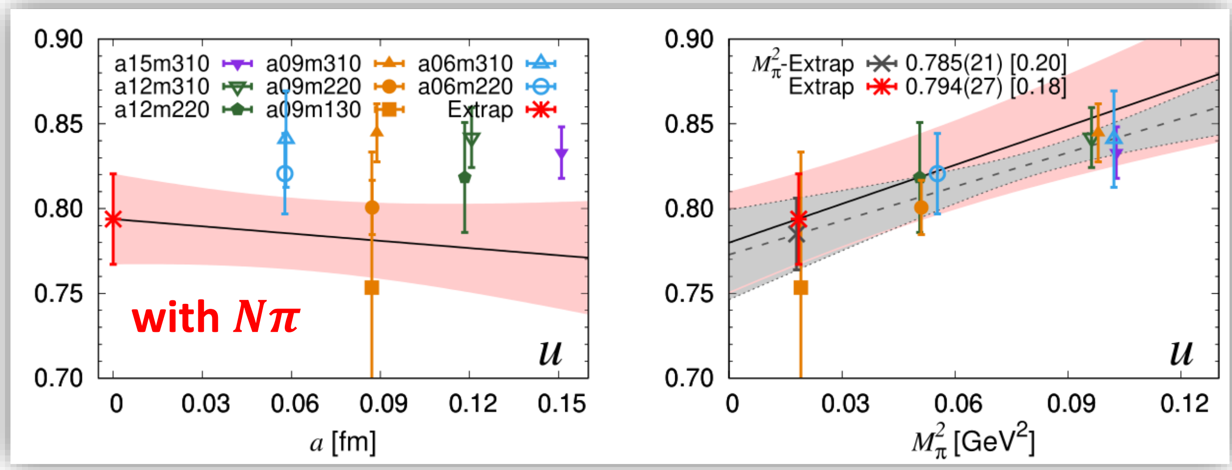
g_A^u: Chiral-continuum extrapolation

$$g + c_0 a + c_1 M_\pi^2$$

- $N\pi$ fit data points has larger errors
- Dominant dependence on M_π
- extrapolated values are consistent

$g_A^u = 0.794(27)_{stat}(07)_{sys}$, preliminary



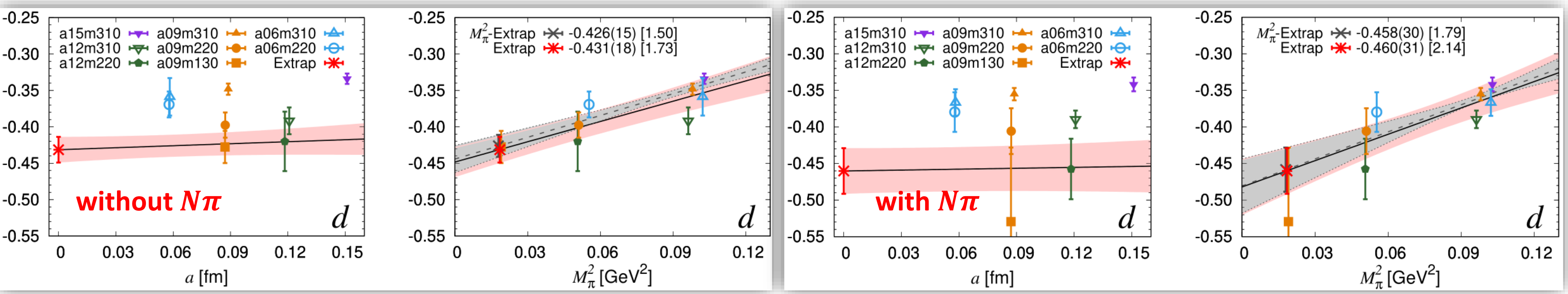


g_A^d: Chiral-continuum extrapolation

$$g + c_0 a + c_1 M_\pi^2$$

- $N\pi$ fit data points has larger errors
- Dominant dependence on M_π
- extrapolated values have $\sim 1\sigma$ difference,
- $\frac{\chi^2}{dof}$ is relatively poor

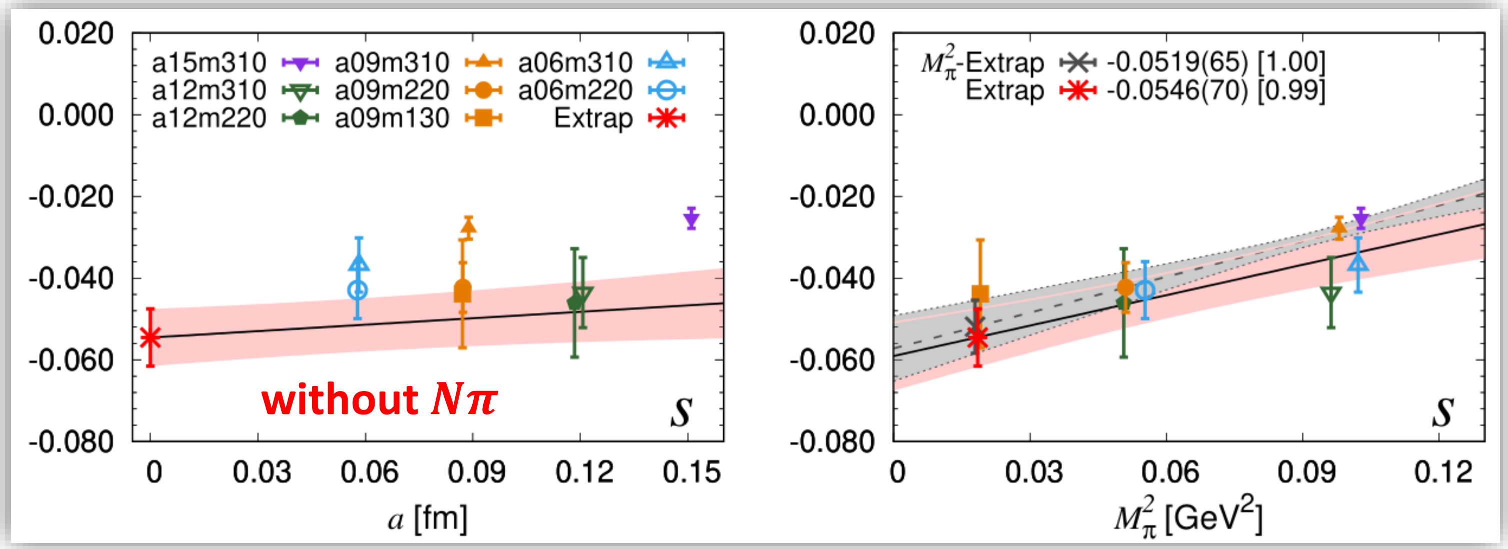
$g_A^d = -0.460(31)_{stat}(29)_{sys}$, **preliminary**



g_A^s: Chiral-continuum extrapolation

$$g + c_0 a + c_1 M_\pi^2$$

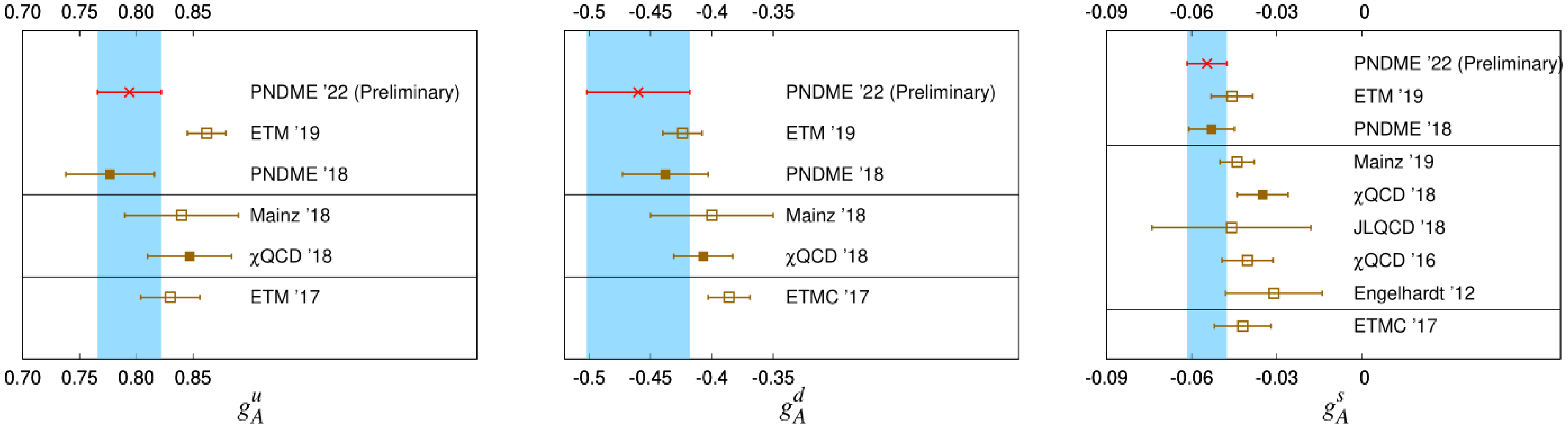
- Dominant dependence on M_π
- mild a dependence



$g_A^s = -0.0547(70)_{stat}$,
preliminary

The leading multihadron excited state is expected to be ΣK which has a larger mass gap than $N\pi$

g_A: Comparison with FLAG 2021



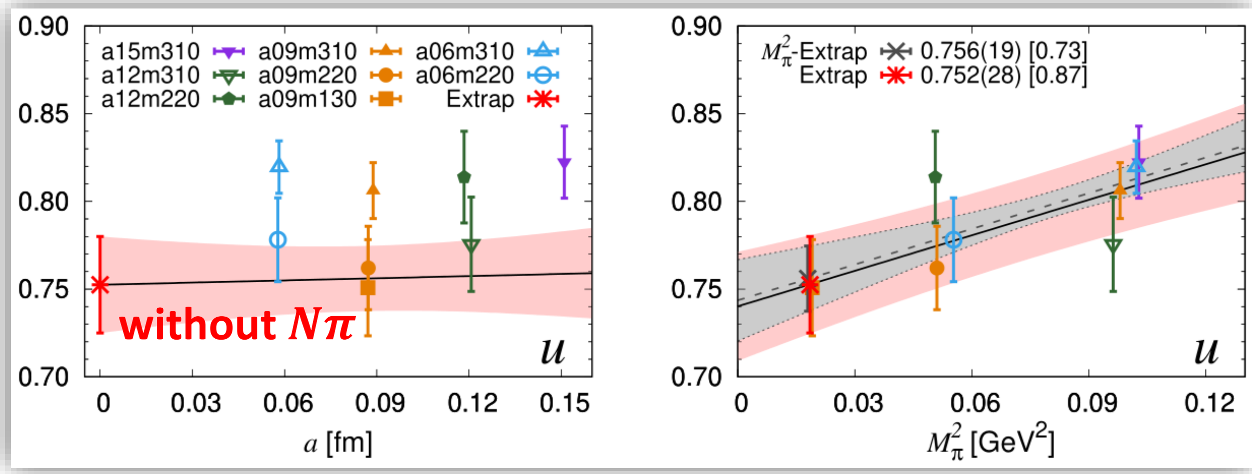
$g_A^u = 0.794(27)_{stat}(07)_{sys}$, $g_A^d = -0.460(31)_{stat}(29)_{sys}$, $g_A^s = -0.0547(70)_{stat}$

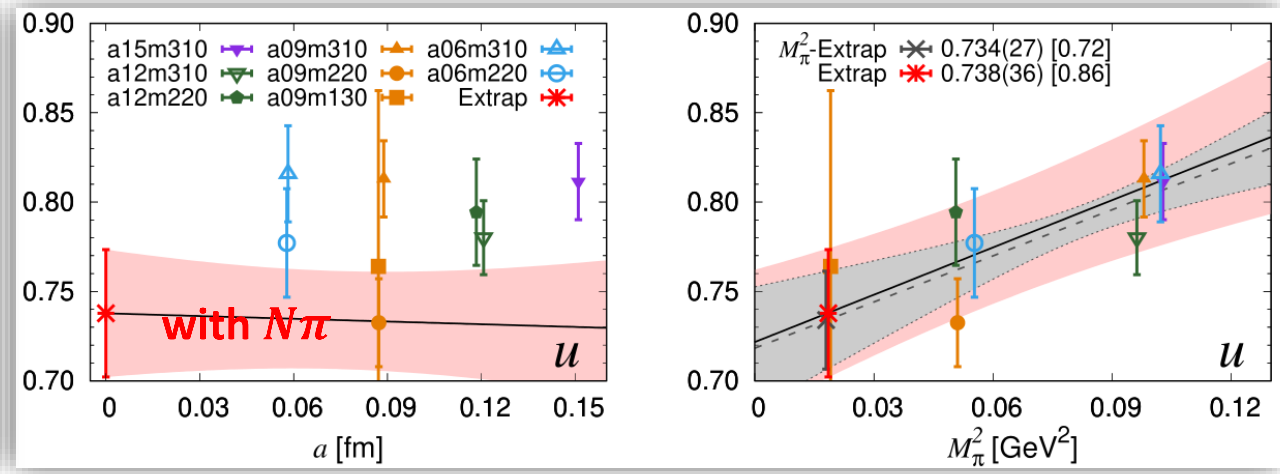
g_T^u: Chiral-continuum extrapolation

$$g + c_0 a + c_1 M_\pi^2$$

- $N\pi$ fit data points has larger errors
- Dominant dependence on M_π
- extrapolated values are consistent

$g_T^u = 0.752(28)_{stat}(14)_{sys}$, preliminary



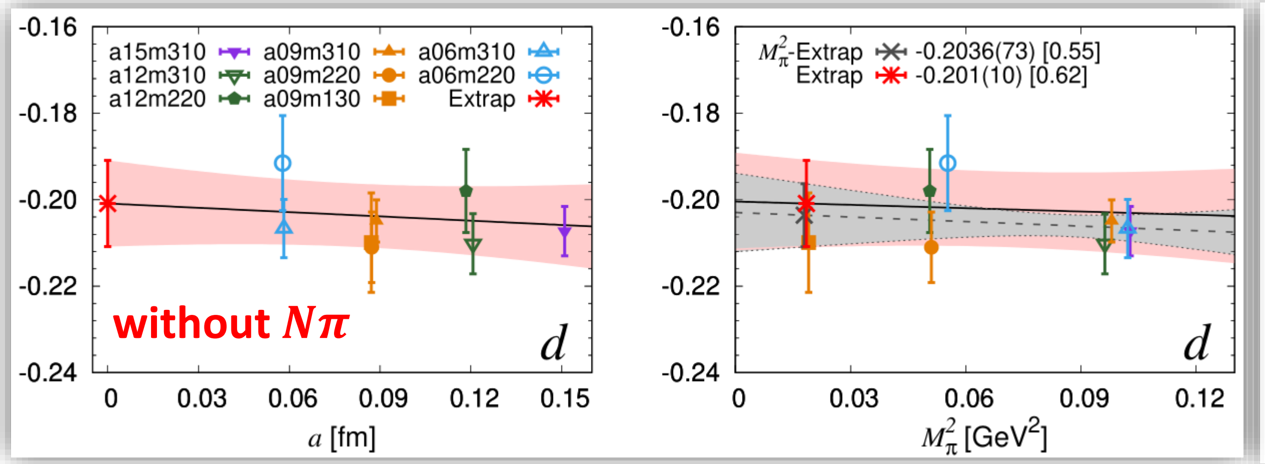


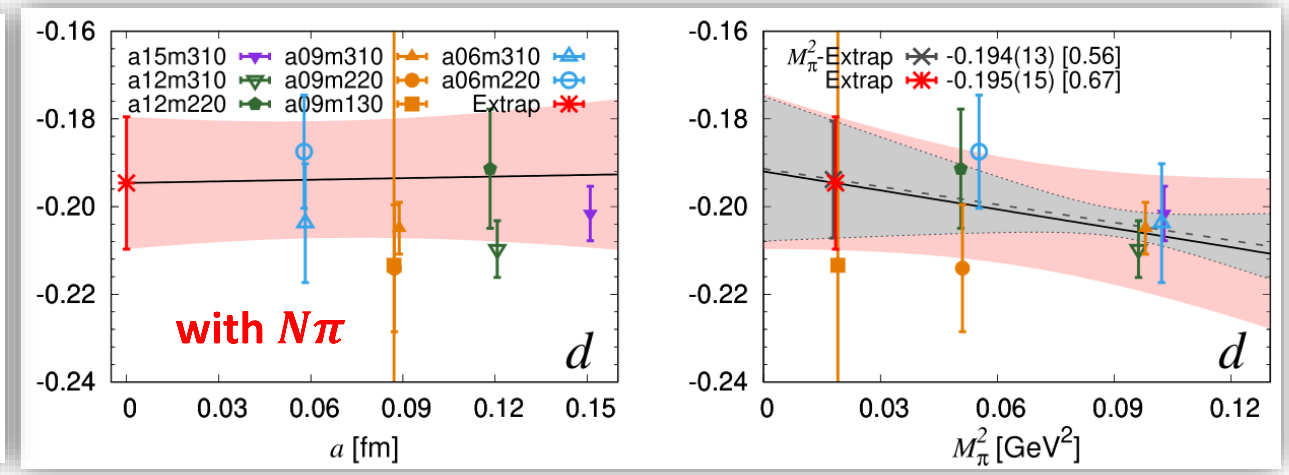
g_T^d: Chiral-continuum extrapolation

$$g + c_0 a + c_1 M_\pi^2$$

- $N\pi$ fit data points has larger errors
- mild M_π dependence with $N\pi$ fit data
- extrapolated values are consistent

$g_T^d = -0.201(10)_{stat}(06)_{sys}$, preliminary

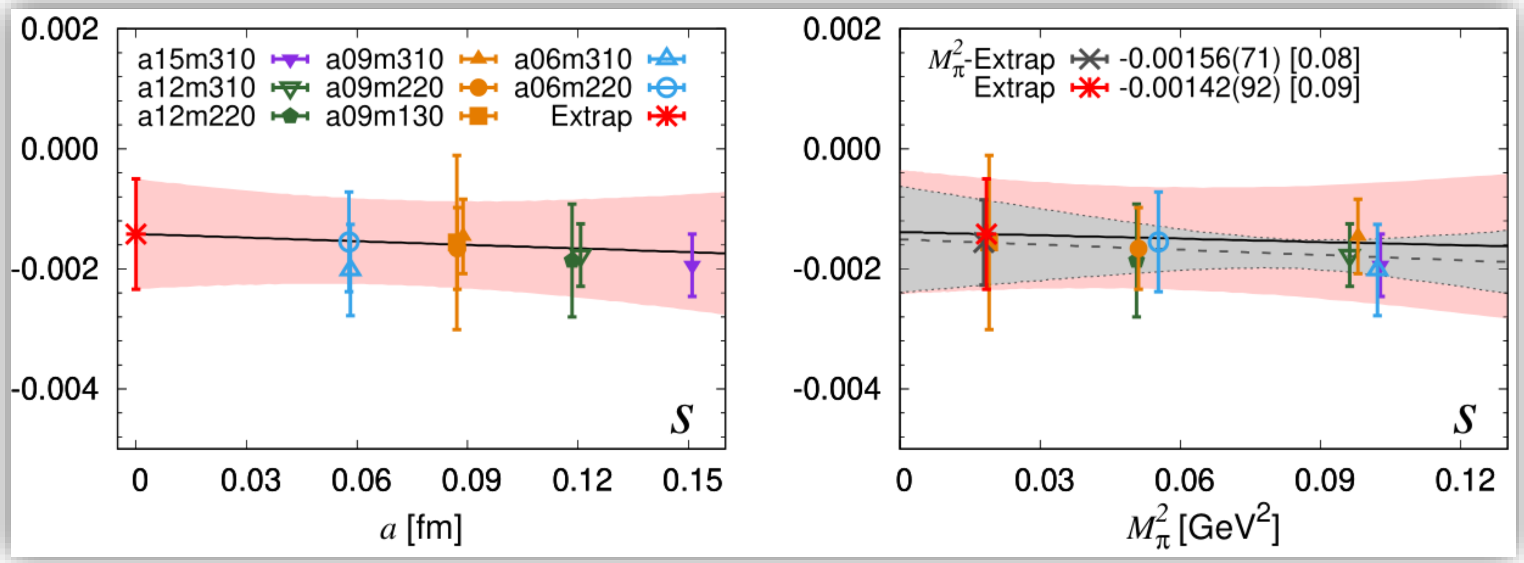




g_T^s: Chiral-continuum extrapolation

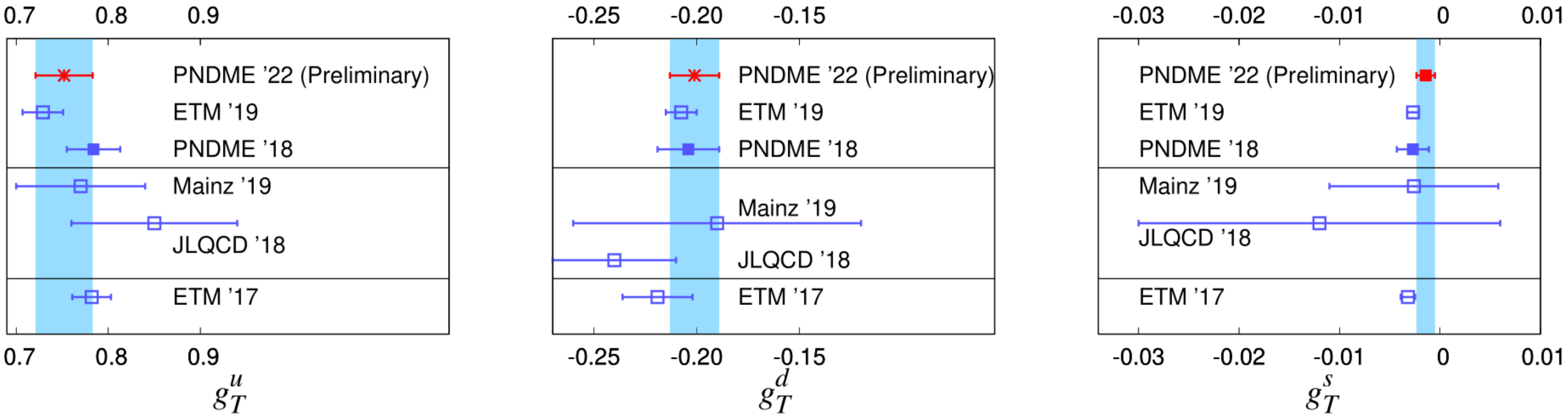
$g + c_0 a + c_1 M_\pi^2$

- constant fit to 3pt/2pt ratio, ignoring ESC
- no a or M_π dependence



$g_T^s = -0.00142(92)_{stat}$,
preliminary

g_T: Comparison with FLAG 2021



$g_T^u = 0.752(28)_{stat}(14)_{sys},$ $g_T^d = -0.201(10)_{stat}(06)_{sys},$ $g_T^s = -0.00142(92)_{stat}$

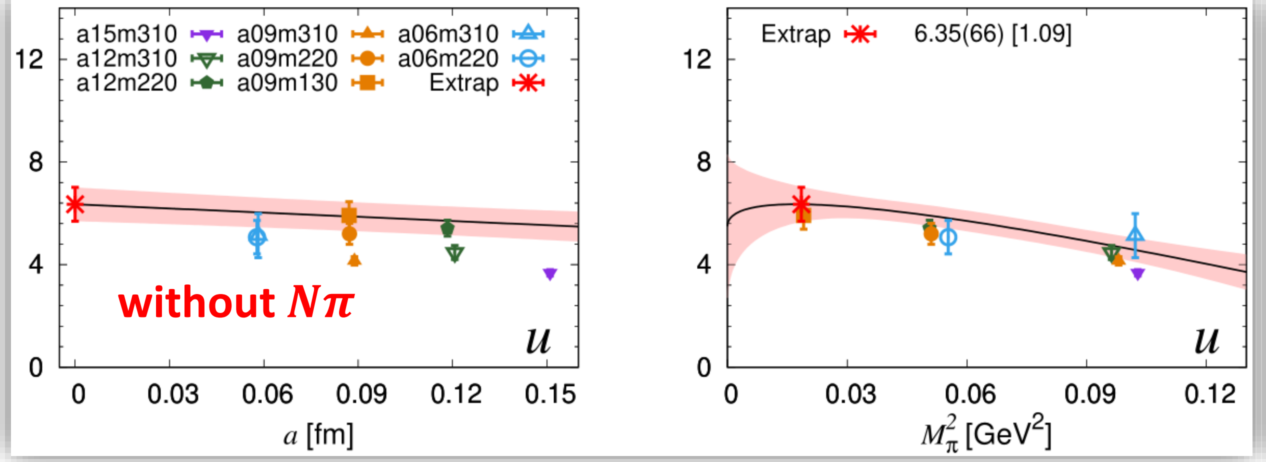
g_S^u: Chiral-continuum extrapolation

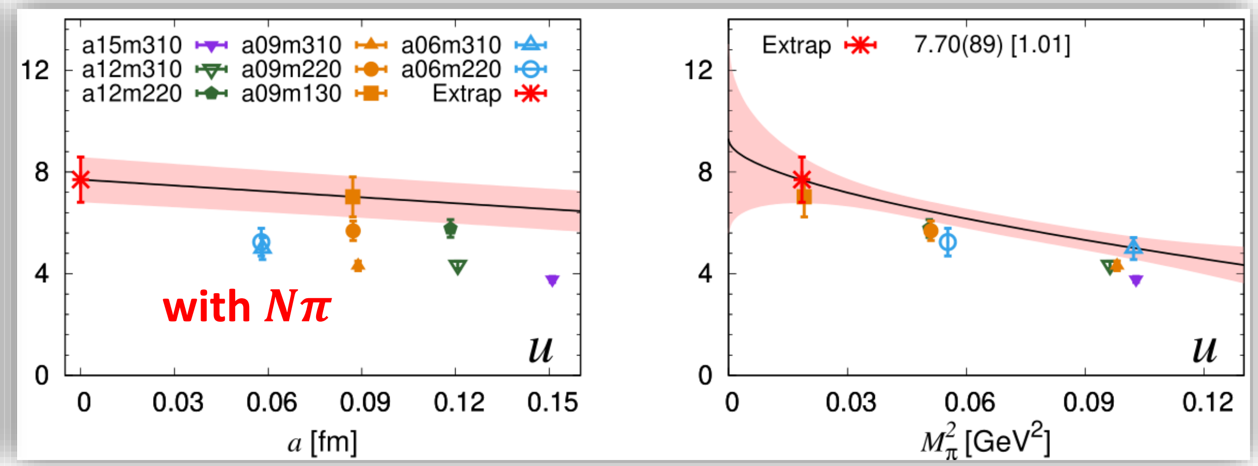
$$g + c_0 a + c_1 M_\pi + c_2 M_\pi^2$$

- chiral fit motivated by $g_S^q = \frac{\partial M_N}{\partial m_q}$
- mild a-dependence
- Very sensitive to including $N\pi$ state

$g_S^u = 6.35(66)_{stat}$,

$g_S^u = 7.70(89)_{stat}$, preliminary





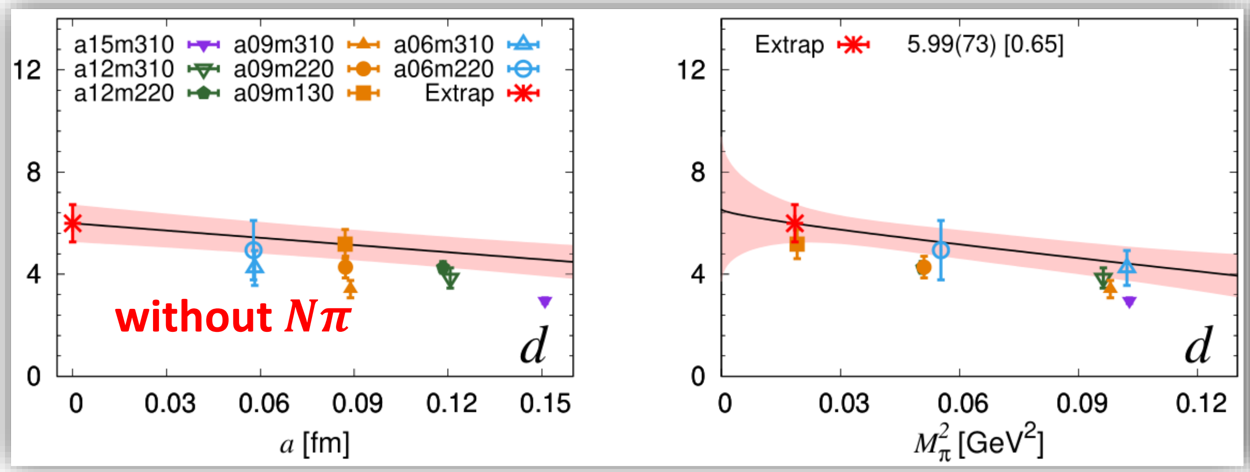
g_S^d: Chiral-continuum extrapolation

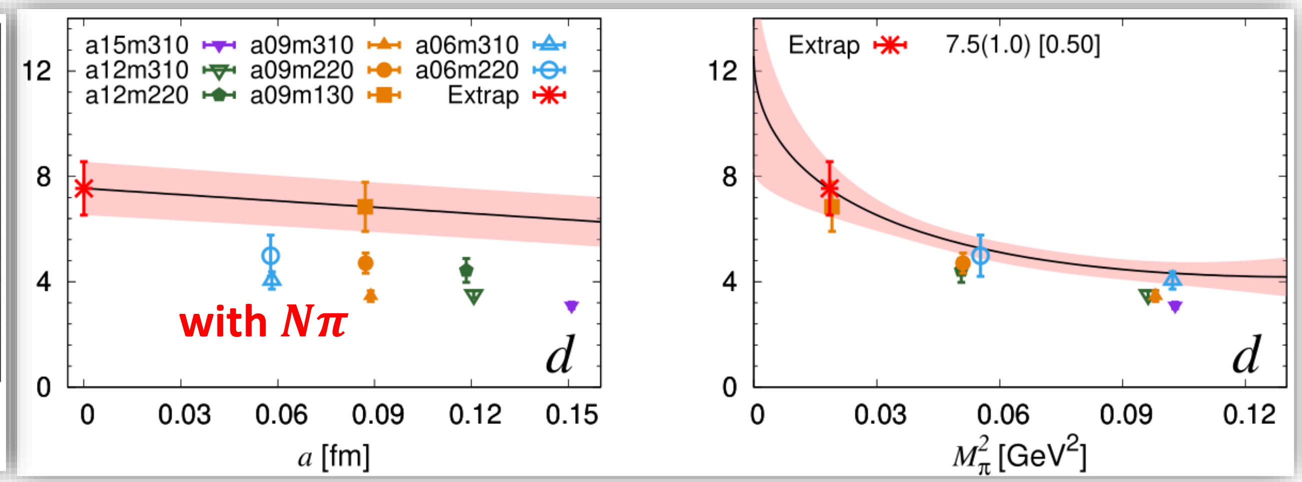
$$g + c_0 a + c_1 M_\pi + c_2 M_\pi^2$$

- chiral fit motivated by $g_S^q = \frac{\partial M_N}{\partial m_q}$
- mild a-dependence
- Very sensitive to including $N\pi$ state

$g_S^d = 5.99(73)_{stat}$,

$g_S^d = 7.5(1.0)_{stat}$, preliminary

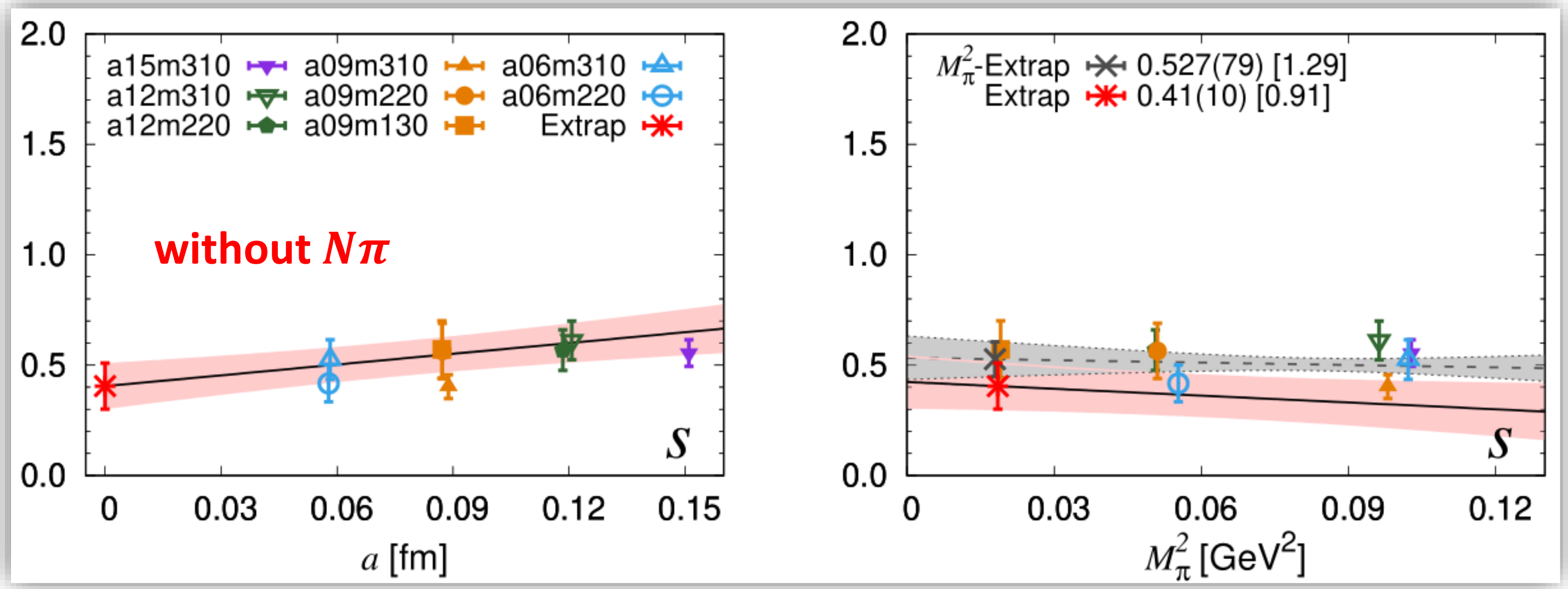




g_S^s: Chiral-continuum extrapolation

$$g + c_0 a + c_1 M_\pi^2$$

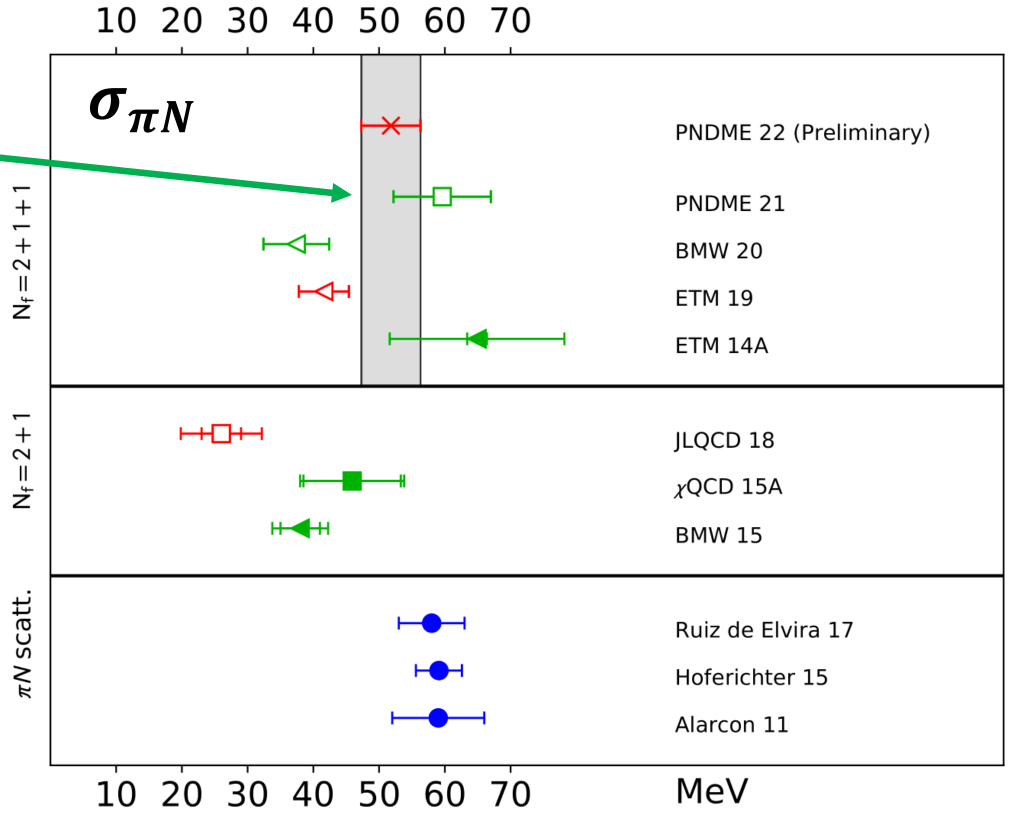
- chiral fit motivated by $g_S^s = \frac{\partial M_N}{\partial m_s}$
- mild a- and M_π-dependence

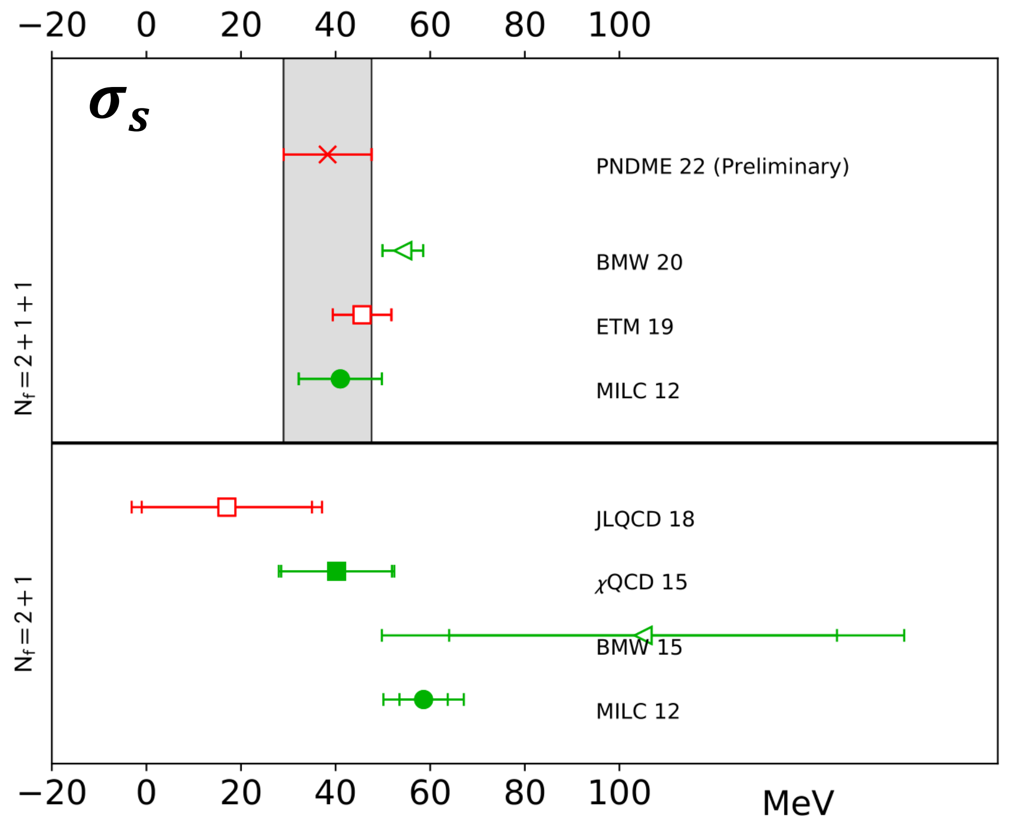


$g_S^s = 0.41(10)_{stat}$, preliminary

g_S: Comparison with FLAG 2021 + PNDME 21

PNDME (2021) $\sigma_{\pi N}$ which does not require renormalization





$g_S^u = 7.70(89)_{stat}$, $g_S^d = 7.5(1.0)_{stat}$, $g_S^s = 0.41(10)_{stat}$

$\sigma_{\pi N} = m_{ud}\left(g_S^u + g_S^d\right) = 51.8(4.5)$, $\sigma_s = m_s g_S^s = 38.3(9.3)$

$(m_{ud}, m_s$ from FLAG 19)

Summary

- We analyzed flavor diagonal nucleon charges using clover fermion on 8 MILC HISQ lattices
- Excited state fits
  - $g_{A,T}^{u,d,s}$: not sensitive to the $N\pi/N\pi\pi$ state mass prior
  - $g_S^{u,d,s}$ ($\sigma^{\pi N}$ and σ^s): sensitive to the $N\pi/N\pi\pi$ state mass prior
- Renormalization: no significant flavor mixing for $g_{A,T}^{u,d,s}, g_S^{u,d}$
- Finite volume correction is small for $M_\pi L > 4$
- Leading chiral logarithm $M_\pi^2 \log M_\pi^2$: cannot resolve
- Comparison with clover-on-clover calculation in progress

Acknowledgement

- We thank the MILC collaboration for providing the 2+1+1-flavor HISQ lattices.
- The calculations used the CHROMA software suite.
- We thank DOE for allocations at NERSC and OLCF.
- We thank the USQCD collaboration and Institutional Computing at LANL for allocations.