

# Gravitational structure of the proton and pion

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# Gravitational form factors (GFFs)

Gluons  $T_g^{\{\mu\nu\}} = 2 \operatorname{Tr}[G^{\alpha\{\mu} G^{\nu\}\alpha}]$

Quarks  $T_q^{\{\mu\nu\}} = \bar{q} \gamma^{\{\mu} i \overleftrightarrow{D}^{\nu\}} q$

$$\begin{aligned} a^{\{\mu} b^{\nu\}} &\equiv \frac{1}{2}(a^\mu b^\nu + a^\nu b^\mu) \\ \vec{D} &= (\vec{D} - \vec{D})/2 \\ u, \bar{u} &= \text{Dirac spinors} \\ P &= (p' + p)/2 \\ \Delta &= p' - p \\ t &= \Delta^2 \end{aligned}$$

Momentum fraction

$$A_{q,g}(0) = \langle x \rangle_{q,g}$$

$$A_g(0) + \sum_q A_q(0) = 1$$

Spin fraction

$$J = (A + B)/2$$

$$J_g(0) + \sum_q J_q(0) = \frac{1}{2}$$

$$\begin{aligned} \left\langle N(p') \left| T_{g,q}^{\{\mu\nu\}} \right| N(p) \right\rangle &= \bar{u}(p') \left[ A_{g,q}(t) \gamma^{\{\mu} P^{\nu\}} + B_{g,q}(t) \frac{i P^{\{\mu} \sigma^{\nu\}\rho} \Delta_\rho}{2M} \right. \\ &\quad \left. + D_{g,q}(t) \frac{\Delta^{\{\mu} \Delta^{\nu\}} - g^{\mu\nu} \Delta^2}{4M} + \bar{c}_{g,q}(t) M g^{\mu\nu} \right] u(p) \end{aligned}$$

Internal forces

$$D(0) = D_g(0) + \sum_q D_q(0)$$

“the last global unknown”

~ trace anomaly

Power-divergent mixing

Not conserved  $\sum_q c_q + c_g = 0$

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$$\begin{aligned} \langle N(p') | T_{g,q}^{\{\mu\nu\}} | N(p) \rangle &= \bar{u}(p') \left[ A_{g,q}(t) \gamma^{\{\mu} P^{\nu\}} + B_{g,q}(t) \frac{i P^{\{\mu} \sigma^{\nu\}\rho} \Delta_\rho}{2M} \right. \\ &\quad \left. + D_{g,q}(t) \frac{\Delta^{\{\mu} \Delta^{\nu\}} - g^{\mu\nu} \Delta^2}{4M} + \bar{c}_{g,q}(t) M g^{\mu\nu} \right] u(p) \end{aligned}$$

$$\langle \pi(p') | T_{g,q}^{\{\mu\nu\}} | \pi(p) \rangle = A_{g,q}(t) 2P^\mu P^\nu + D_{g,q}(t) \frac{1}{2} (\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2) + \bar{c}_{g,q}(t) 2M^2 g^{\mu\nu}$$

# Lattice calculation

Ensemble [“a091m170” (JLab/W&M/MIT/LANL)]

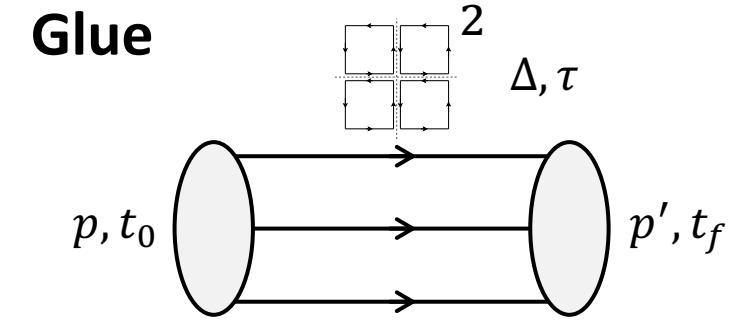
Tree-level tadpole-improved Symanzik gauge

2+1 stout-smeared Wilson clover

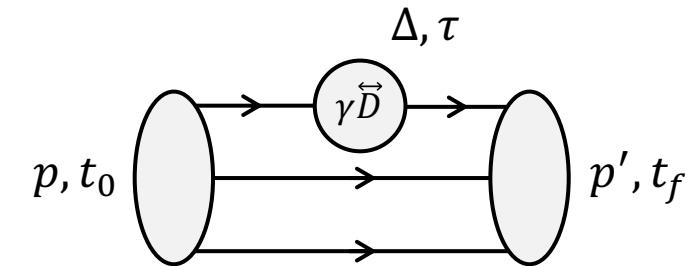
$M_\pi = 170$  MeV

$a = 0.091$  fm

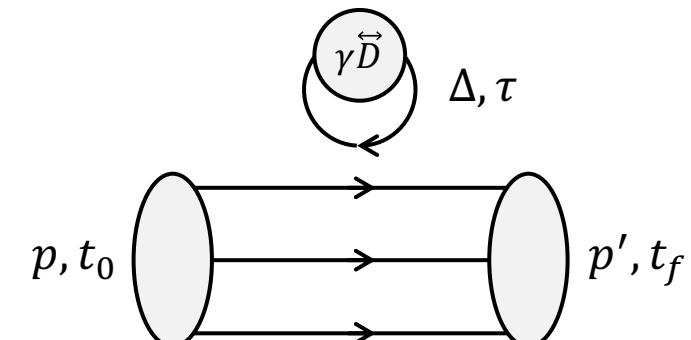
$48^3 \times 96$



**Connected quark ( $u, d$ )**



**Disconnected quark ( $u + d, s$ )**



Sketch of calculation (same for  $N, \pi$ ):

1. Compute three-point functions
2. Fit to extract bare matrix elements
3. Analyze to extract bare GFFs
4. Renormalize

# Lattice EMT operators

Quark:  $T_q^{\{\mu\nu\}} = \bar{q}\gamma^{\{\mu}i\overleftrightarrow{D}^{\nu\}}q$

Discretized covariant derivative

$$\overleftrightarrow{D} = (\vec{D} - \overleftarrow{D})/2$$

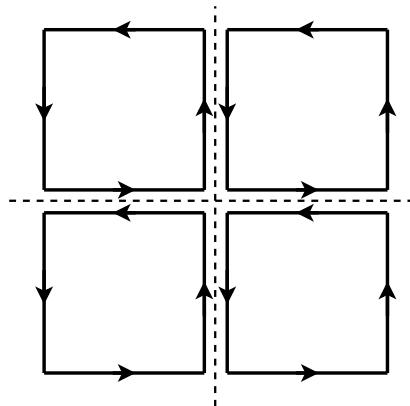
$$(\vec{D}_\mu \psi)(x) = \frac{1}{2} [U_\mu(x)\psi(x + \mu) - U_\mu^\dagger(x - \mu)\psi(x - \mu)]$$

$$(\bar{\psi} \overleftrightarrow{D}_\mu)(x) = \frac{1}{2} [\bar{\psi}(x + \mu)U_\mu^\dagger(x) - \bar{\psi}(x - \mu)U_\mu(x - \mu)]$$

Glue:  $T_g^{\{\mu\nu\}} = 2 \operatorname{Tr}[G^{\alpha\{\mu} G^{\nu\}\alpha}]$

Clovers flowed to  $t/a^2 = 2$

$$G_{\mu\nu} \propto (Q_{\mu\nu} - Q_{\mu\nu}^\dagger)$$



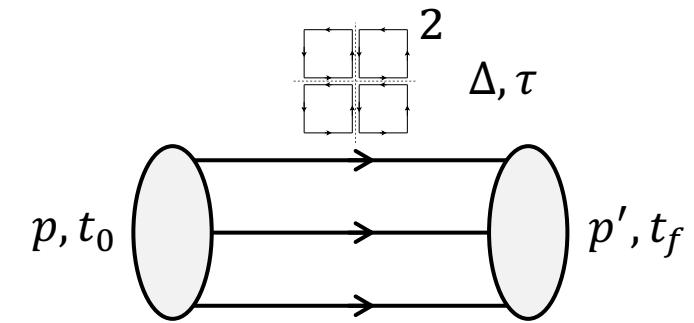
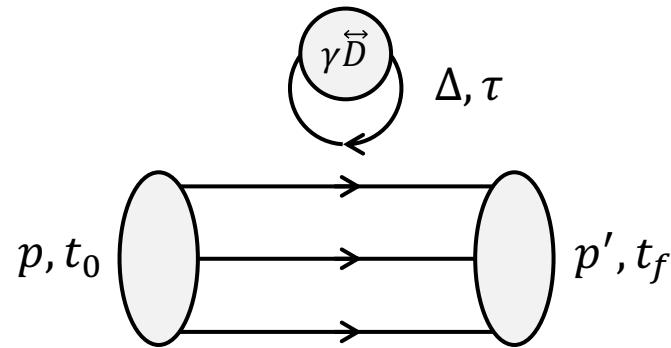
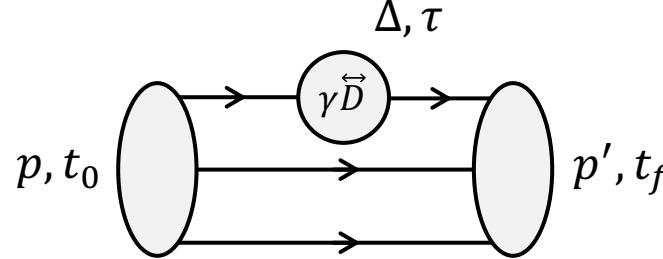
## Operator Bases

$$\tau_1^{(3)}: \quad \frac{1}{2}(O^{11} + O^{22} - O^{33} + O^{00}), \quad \frac{1}{\sqrt{2}}(O_{33} + O_{00}), \quad \frac{1}{\sqrt{2}}(O_{11} - O_{22})$$

$$\tau_3^{(6)}: \quad \left\{ \frac{i\delta_{\mu 0}}{\sqrt{2}}(O^{\mu\nu} + O^{\nu\mu}), \quad 0 \leq \mu \leq \nu \leq 3 \right\}$$

WIP

# 1. Compute three-point functions



## Connected Quark ( $u, d$ )

Sequential source (thru sink)

- 16 sources / config
- 3 sink momenta
- 4 sink times

Stats:

- **Pion:** 684 configs
- **Nucleon:** 999 config

## Disconnected Quark ( $u + d, s$ )

Hierarchical probing

- 512 Hadamard vectors
- 512 sources / config

Stats (both  $\pi$  and  $N$ ):

- **$u + d$ :** 760 configs
- **$s$ :** 793 configs

## Glue (disconnected)

Clover EMT

- Flowed to  $t/a^2 = 2$
- 512 sources / config

Stats:

- **Pion:** 999 configs
- **Nucleon:** 1411 configs

**Note:** all  $p^2 \leq 10$  &  $\Delta^2 \leq 25$ , all operators, all spin channels, etc

## 2. Fit to extract bare matrix elements

Strategies:

1. Fit three-point functions directly

$$C^{3pt}(p, p'; t_f, \tau) \propto \langle p' | T | p \rangle e^{-E_0 \tau - E'_0(t_f - \tau)} + (\text{excited states})$$

2. Construct and analyze ratios

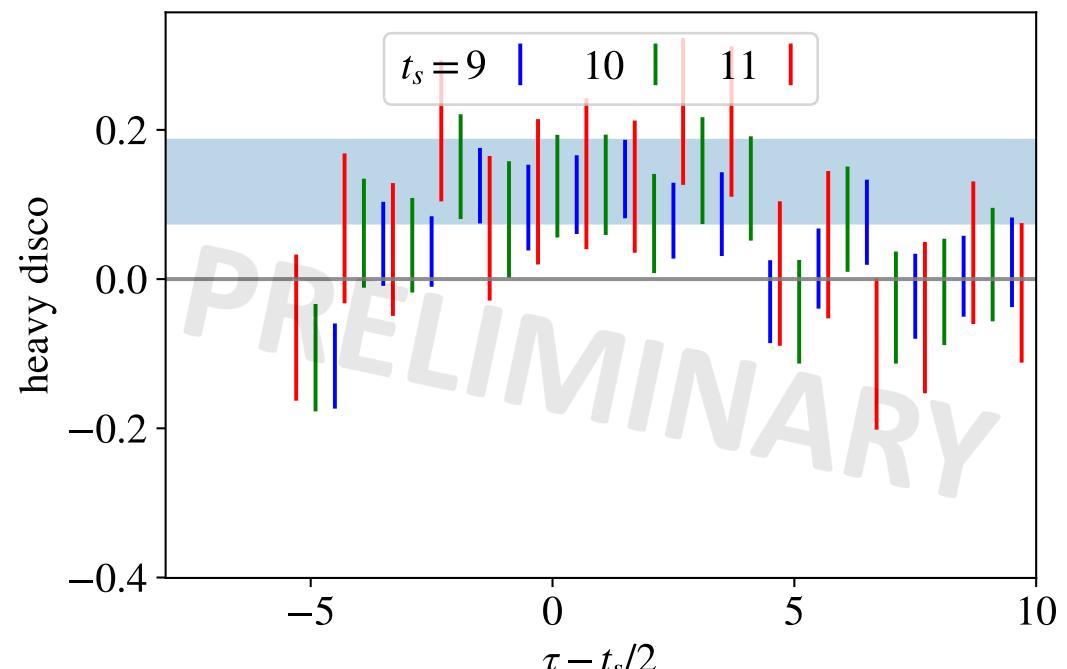
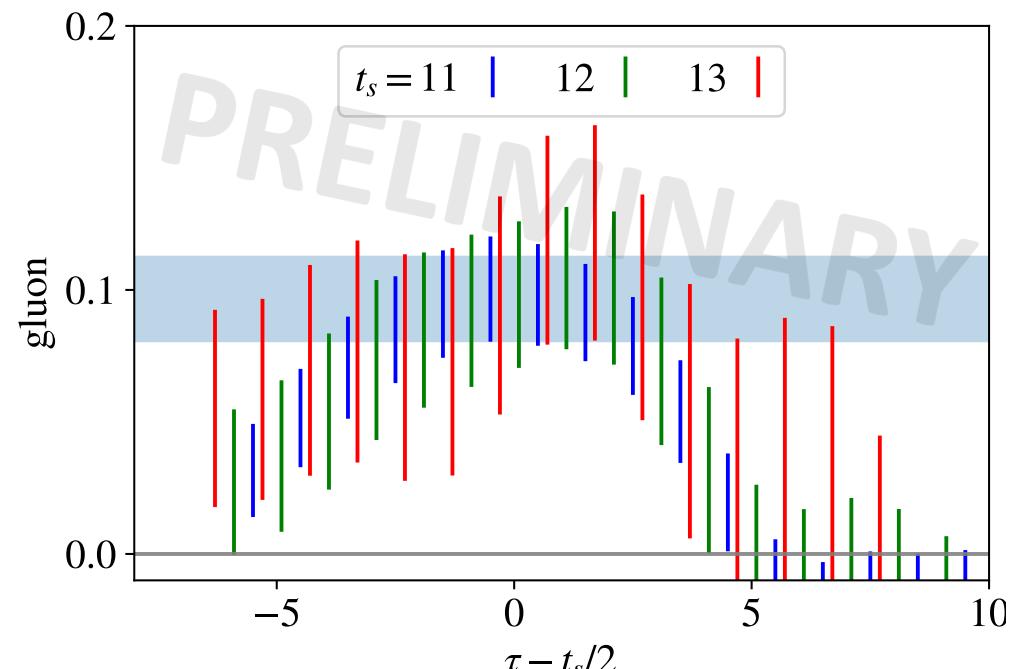
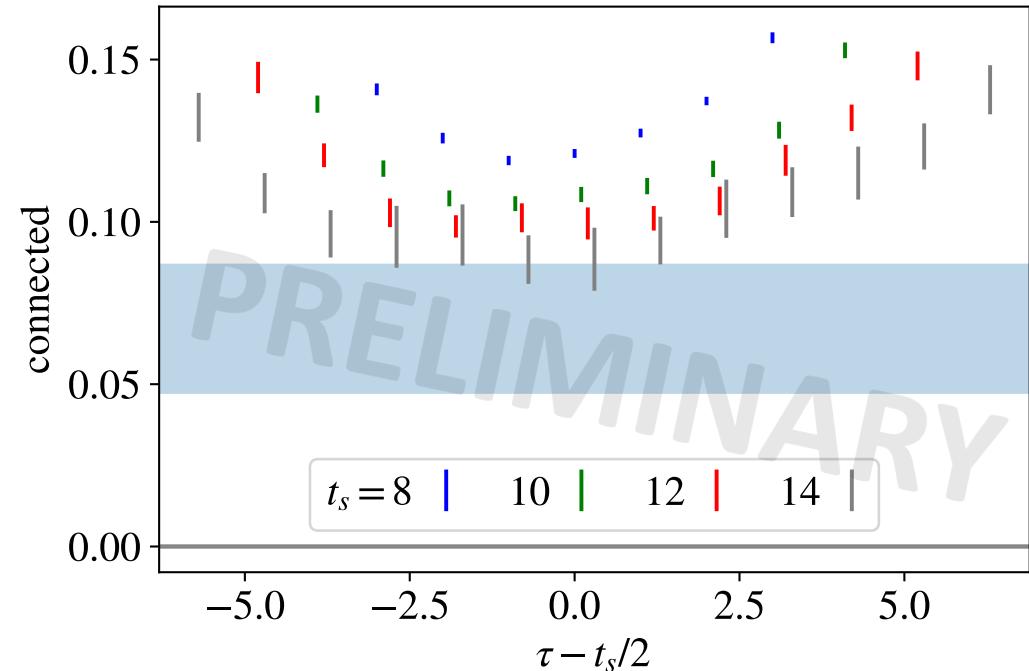
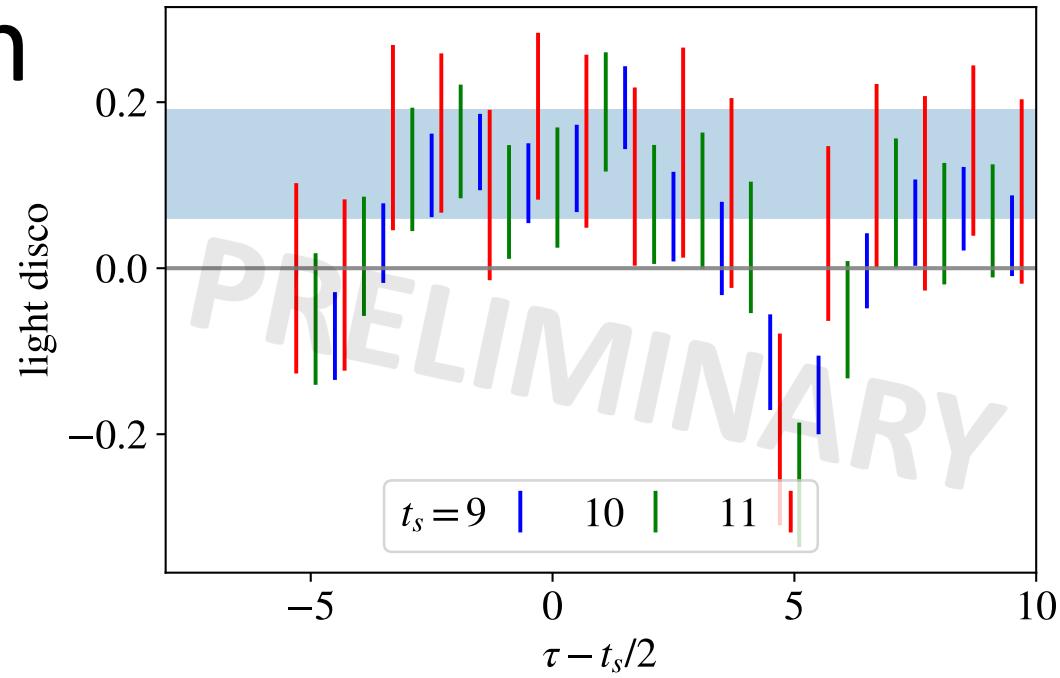
$$R(p, p'; \tau, t_f) = \frac{C^{3pt}(p, p'; t_f, \tau)}{C^{2pt}(p'; t_f)} \sqrt{\frac{C^{2pt}(p; t_f - \tau)}{C^{2pt}(p'; t_f - \tau)} \frac{C^{2pt}(p'; t_f)}{C^{2pt}(p; t_f)} \frac{C^{2pt}(p'; \tau)}{C^{2pt}(p; \tau)}} \\ \propto \langle p' | T | p \rangle + O\left(e^{-\Delta E \tau - \Delta E'(t_f - \tau)}\right)$$

Options: Plateau fits

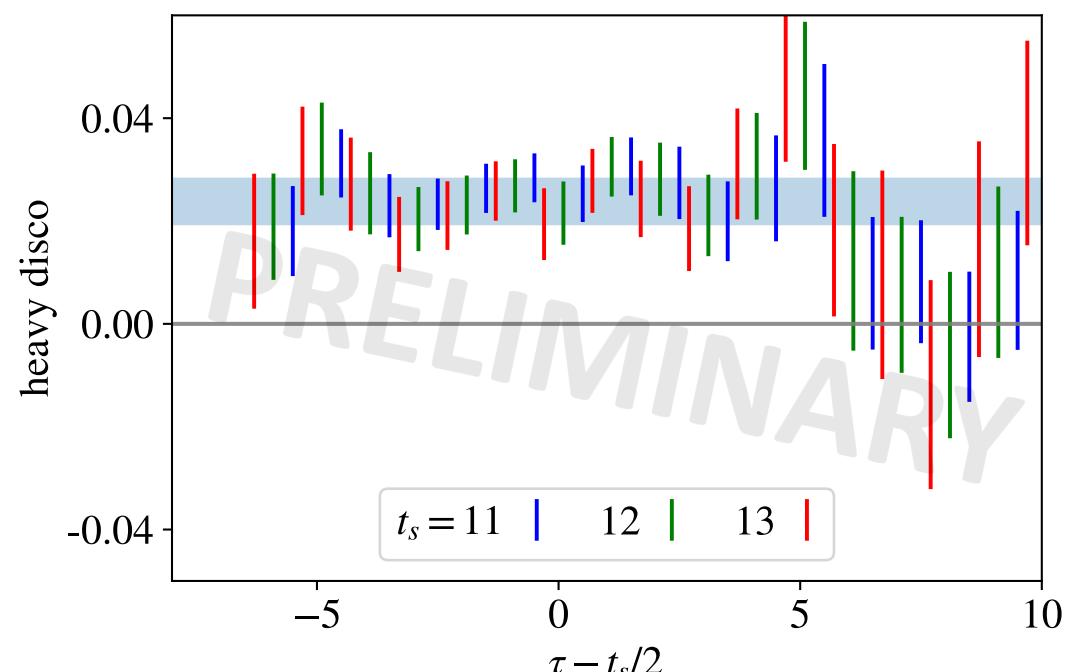
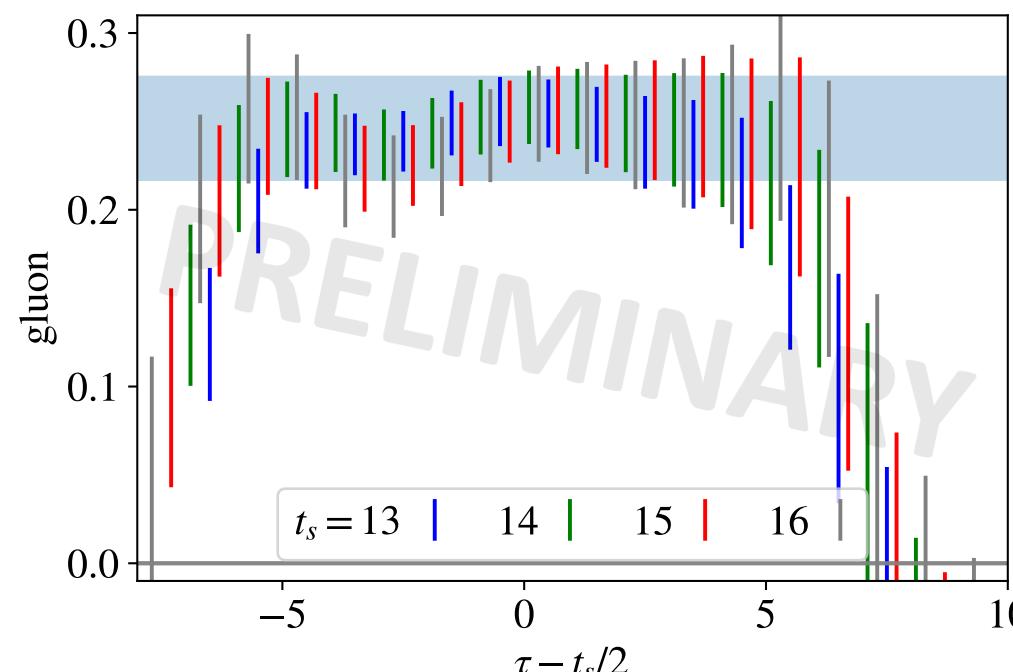
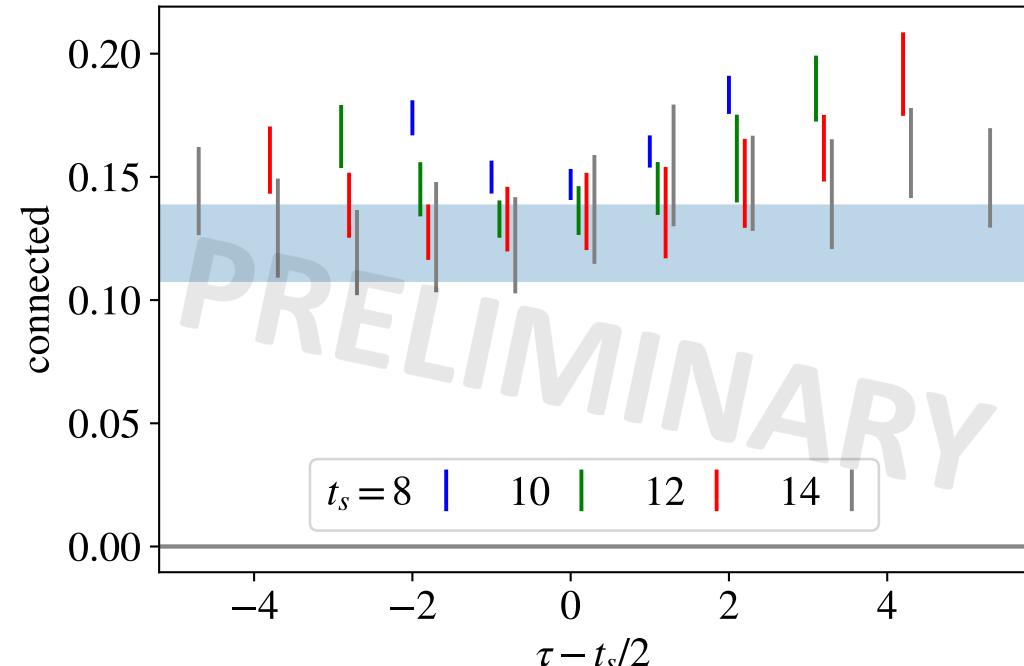
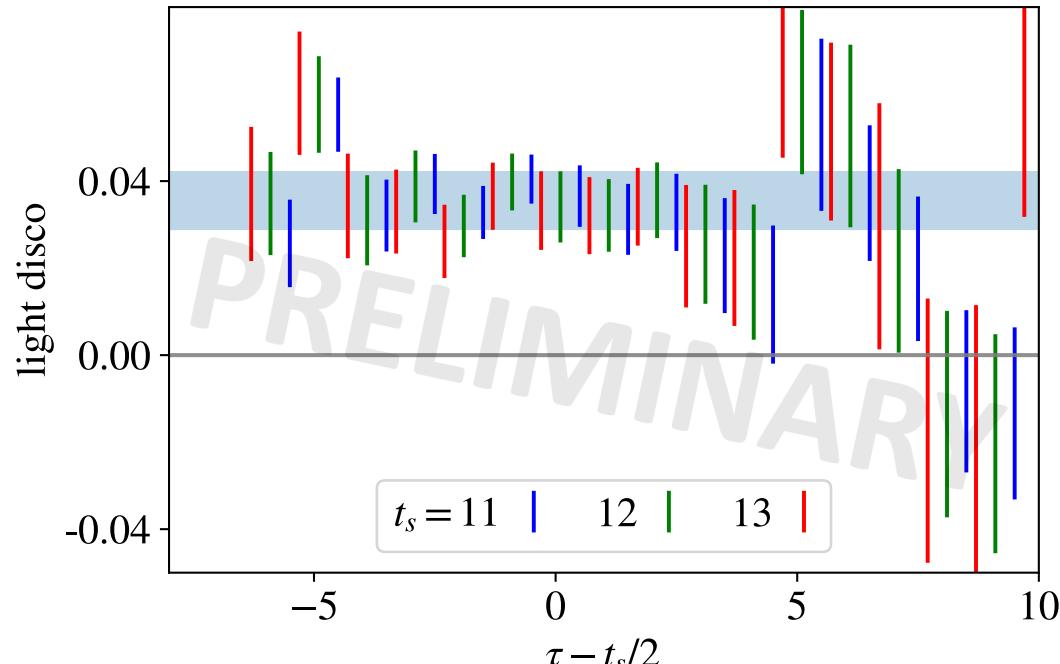
$n$ -state fits

“Summation method”

# Nucleon



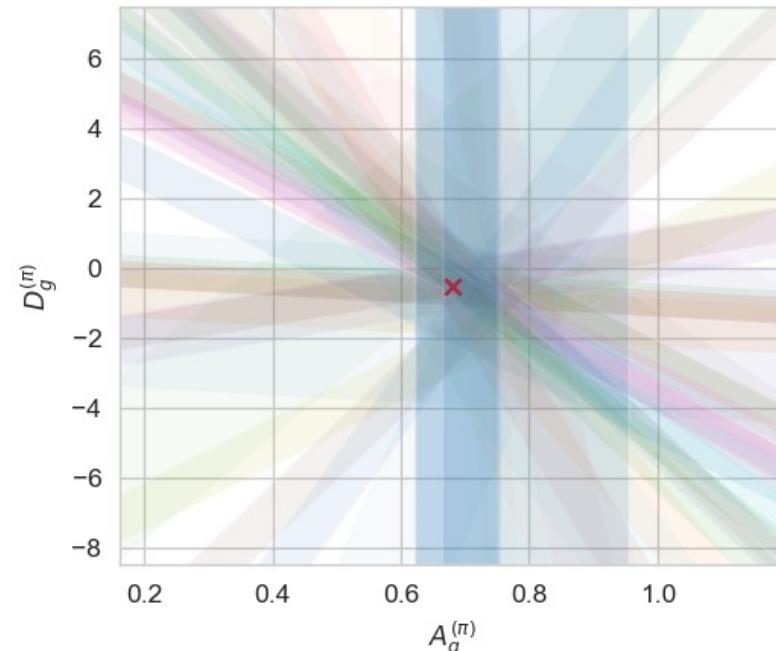
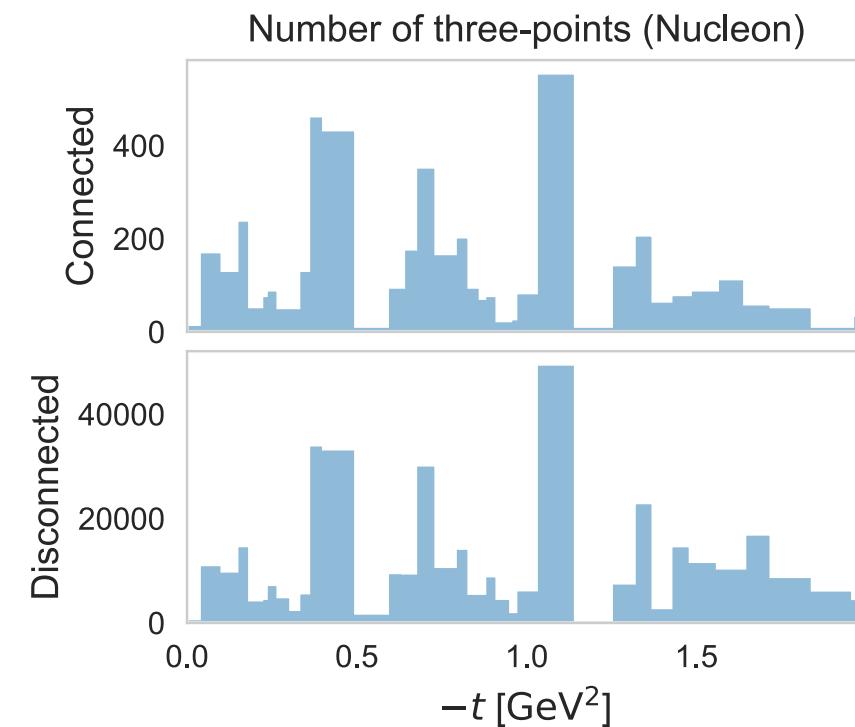
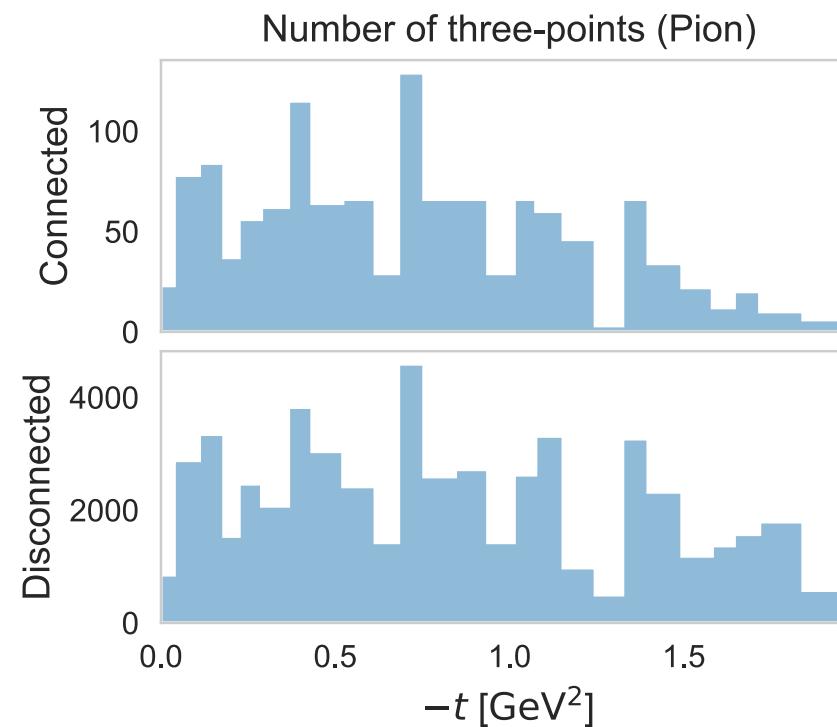
# Pion



### 3. Analyze to extract bare GFFs

$$\langle p' | T(\Delta) | p \rangle = (\text{kinematic coeffs}) \cdot (\text{GFFs})(t)$$

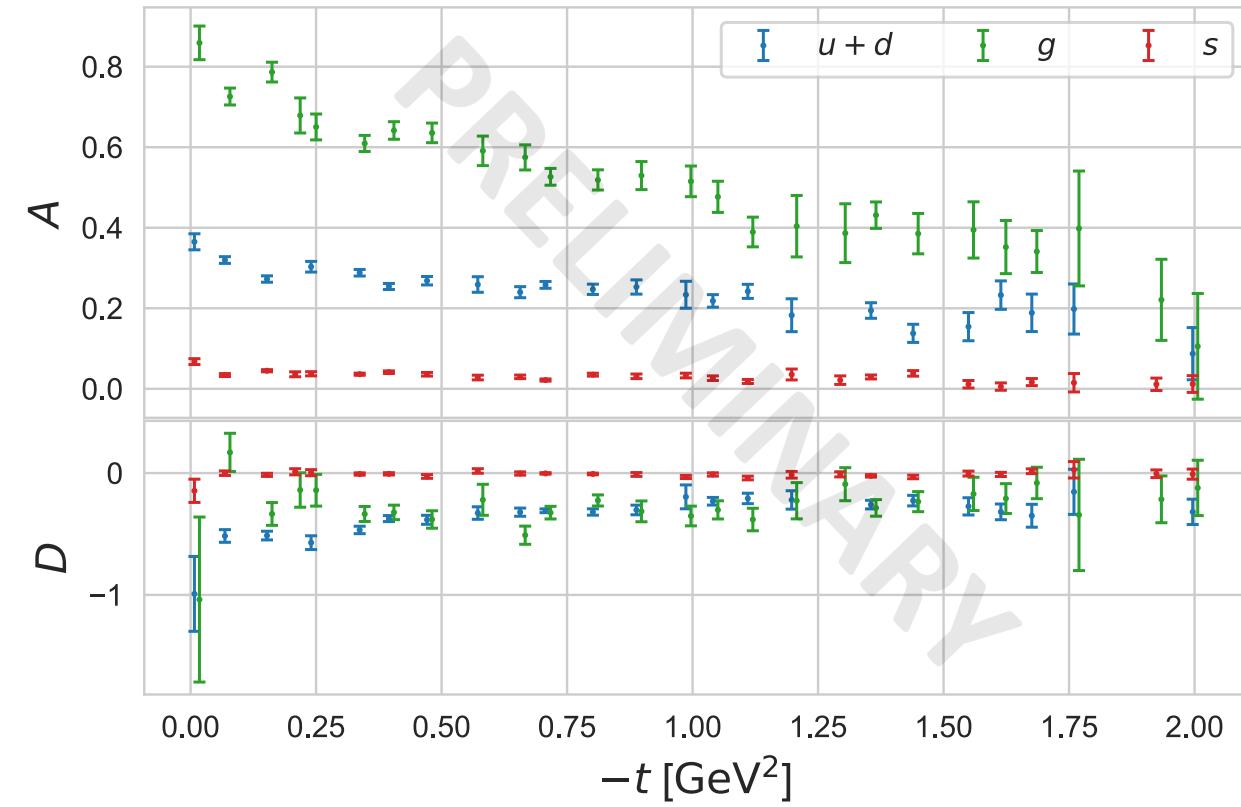
⇒ Bin constraints by  $t = \Delta^2$  and fit



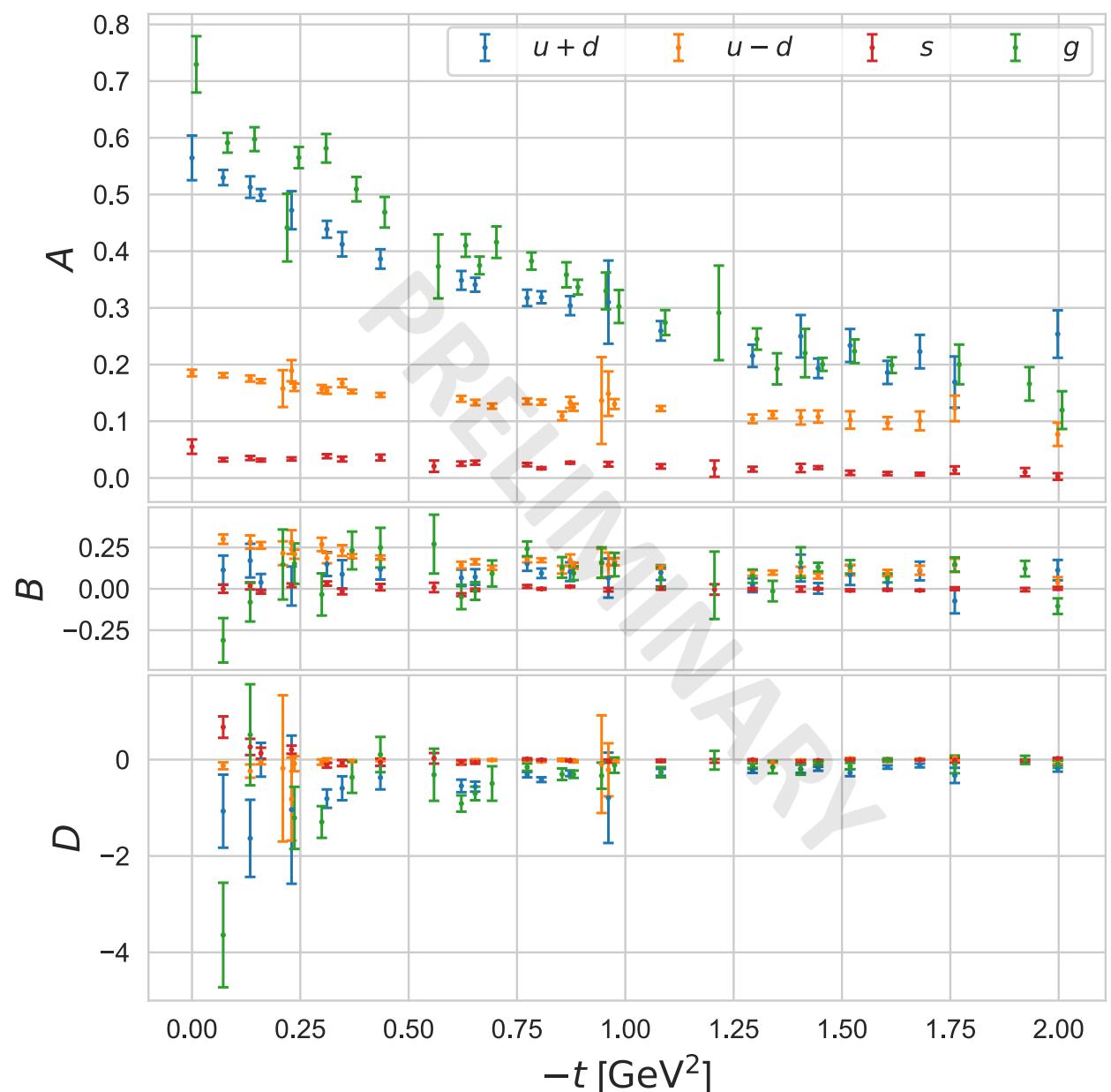
# Bare results

For irrep  $\tau_1^{(3)}$

Pion



Nucleon



## 4. Renormalize

For bare GFFs  $X_f = \{X_u, X_d, X_s\}$  and  $X_g$

$$X_u^R = Z_{qq}^{\overline{\text{MS}}} X_u + \delta Z_{qq}^{\overline{\text{MS}}} \sum_f X_f + Z_{qg}^{\overline{\text{MS}}} X_g$$
$$X_g^R = Z_{gg}^{\overline{\text{MS}}} X_g + Z_{gq}^{\overline{\text{MS}}} \sum_f X_f$$

WIP

RI-MOM w/ perturbative matching to  $\overline{\text{MS}}$

Strategy:

- Compute on smaller volumes w/ different params
- Compare w/ direct results & CDER on target ensemble to quantify systematics

**PRELIMINARY (for  $\tau_1^{(3)}$ )**

$$Z_{qq}^{\overline{\text{MS}}} = 1.02(5)$$

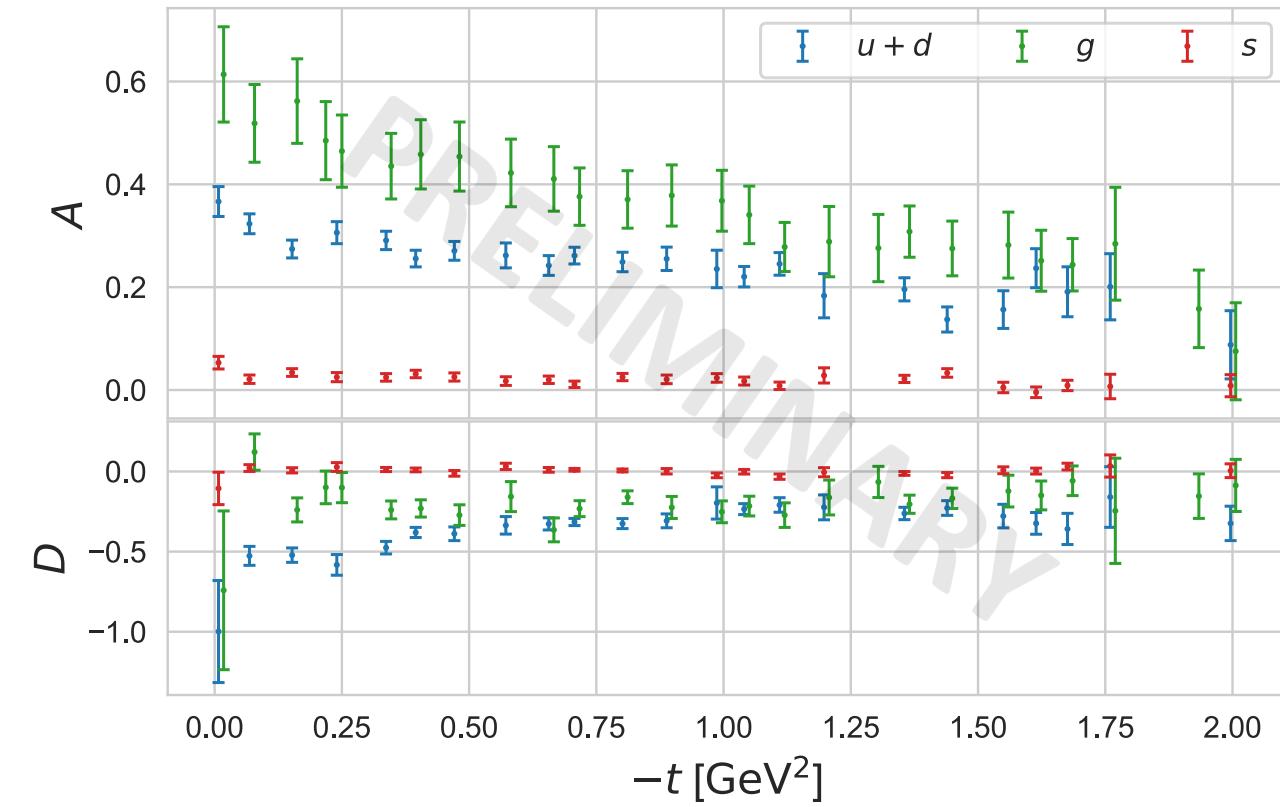
$$\delta Z_{qq}^{\overline{\text{MS}}} = -0.04(2)$$

$$Z_{gg}^{\overline{\text{MS}}} = 0.7(2)$$

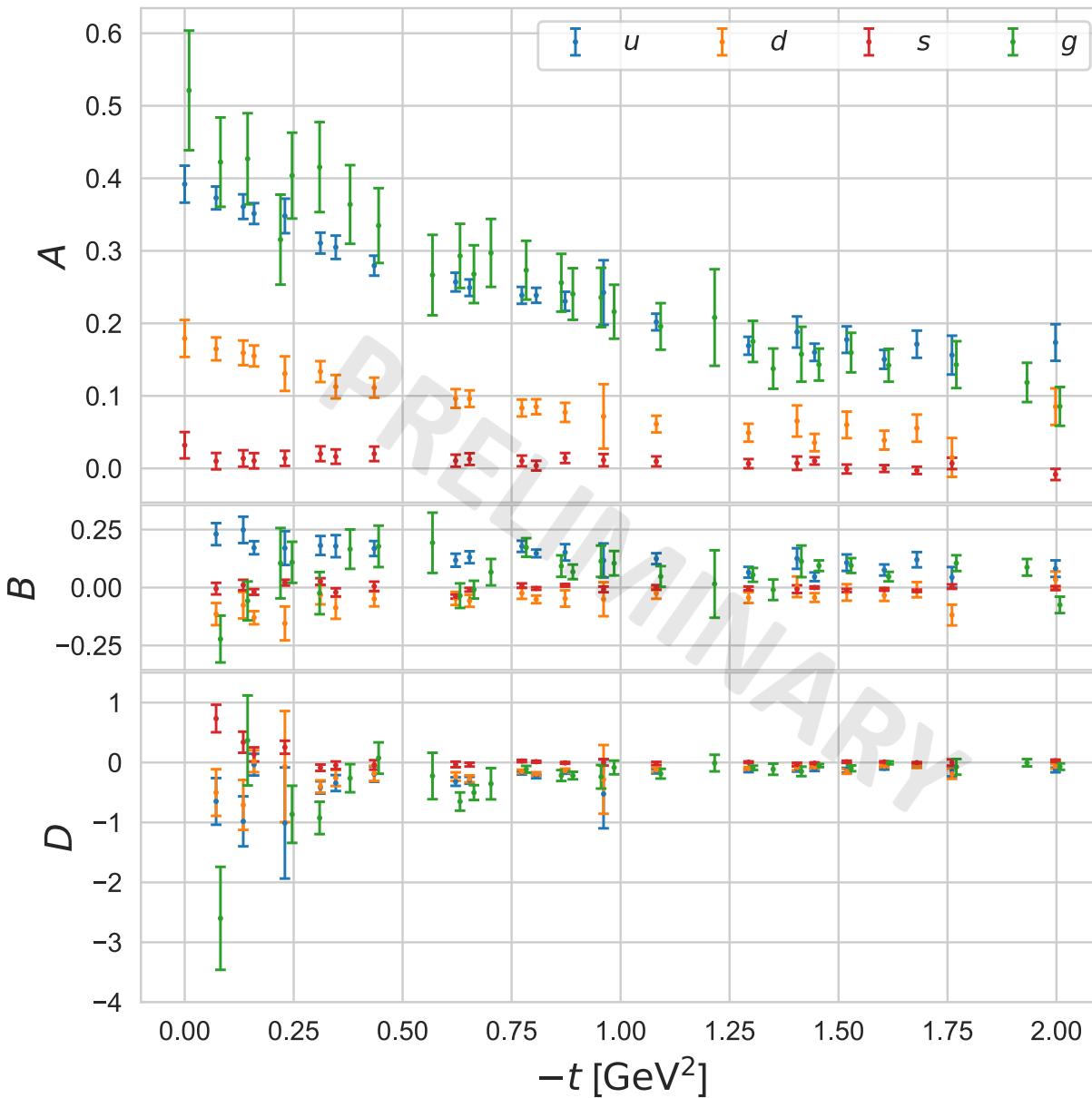
# Renormalized results

Note: quark/glue mixing not accounted for!

Pion



Nucleon



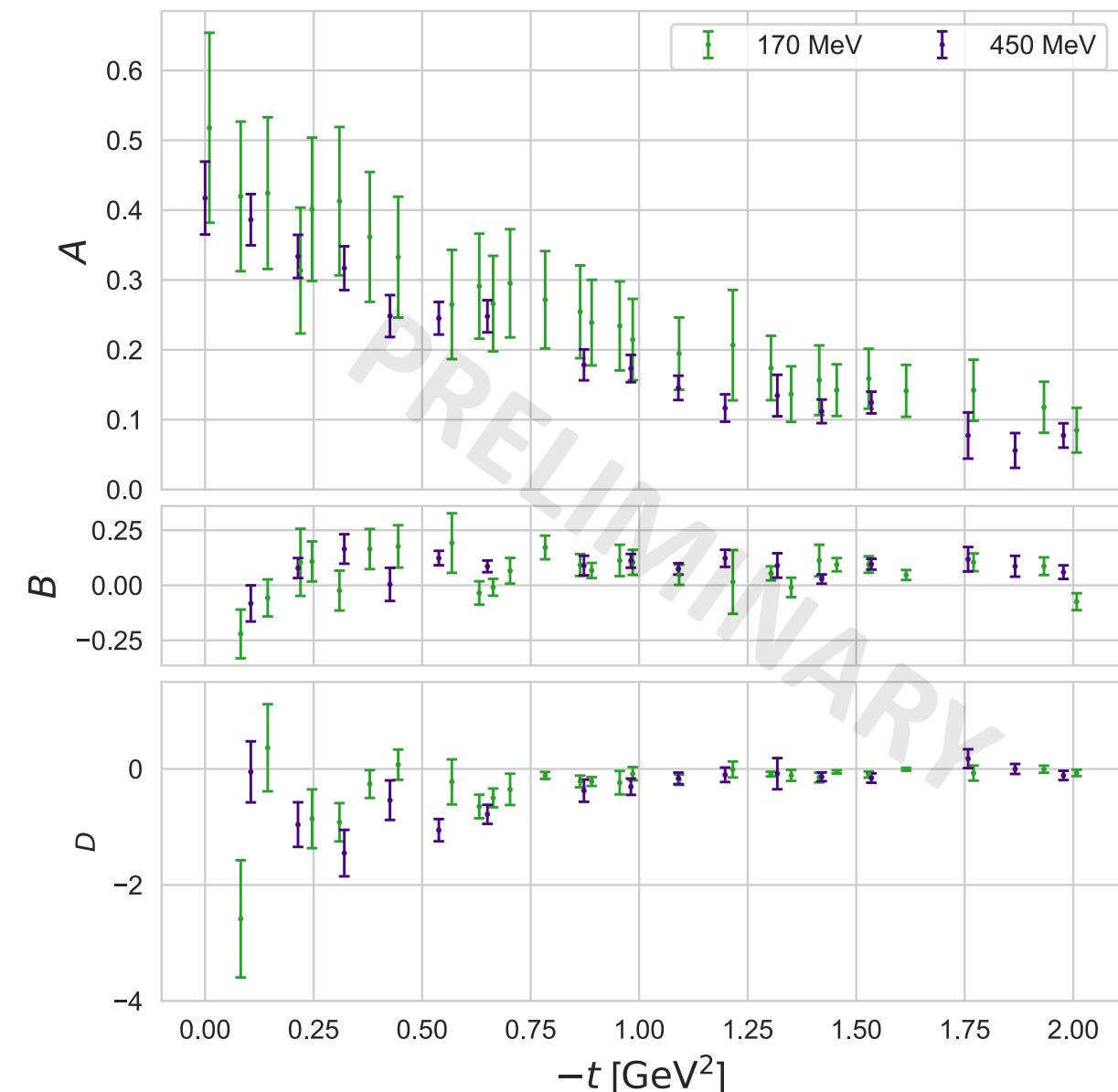
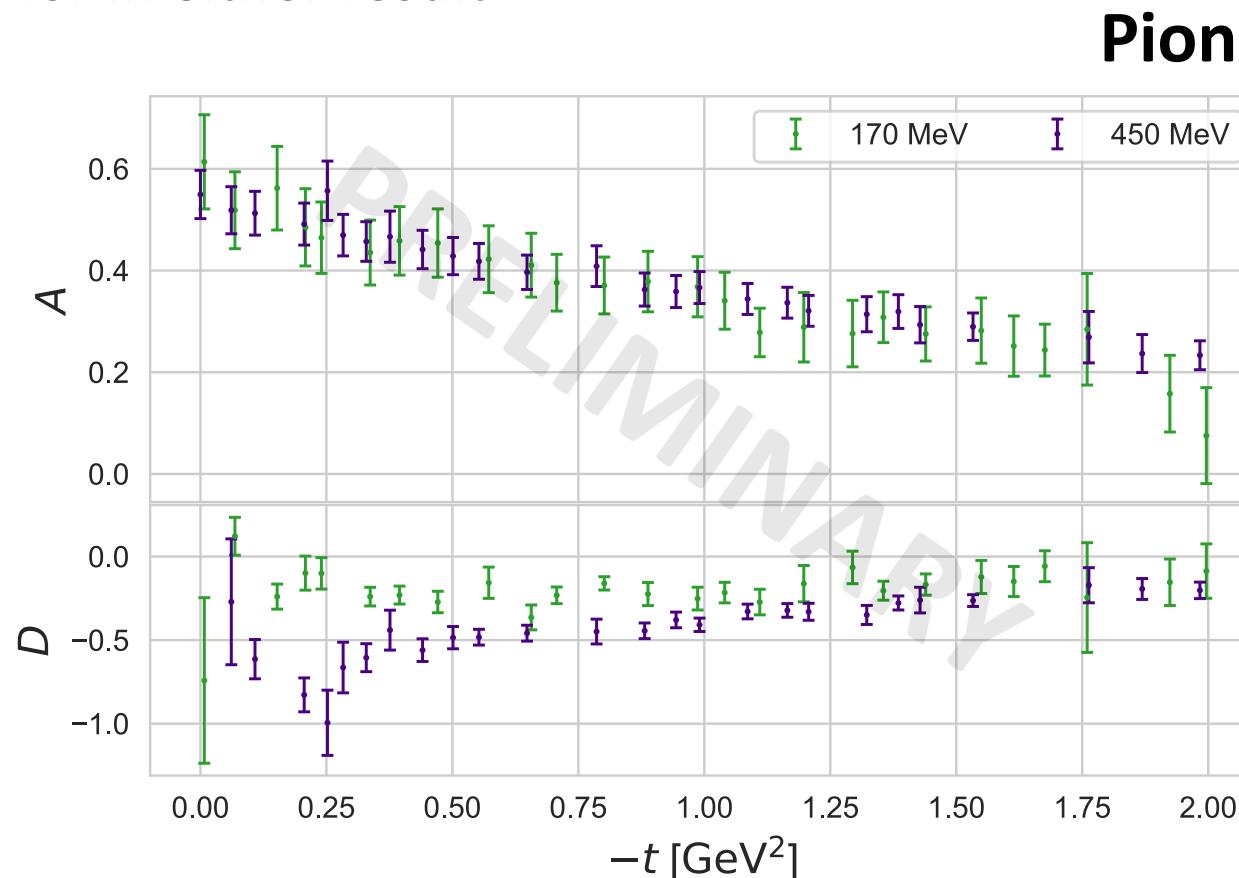
# Mass dependence in gluon GFFs(?)

Nucleon

Compare: 170 MeV vs 450 MeV (similar  $a$ )

[Pefkou DH Shanahan 2107.10368]

Note: quark/glue mixing not accounted for in either result



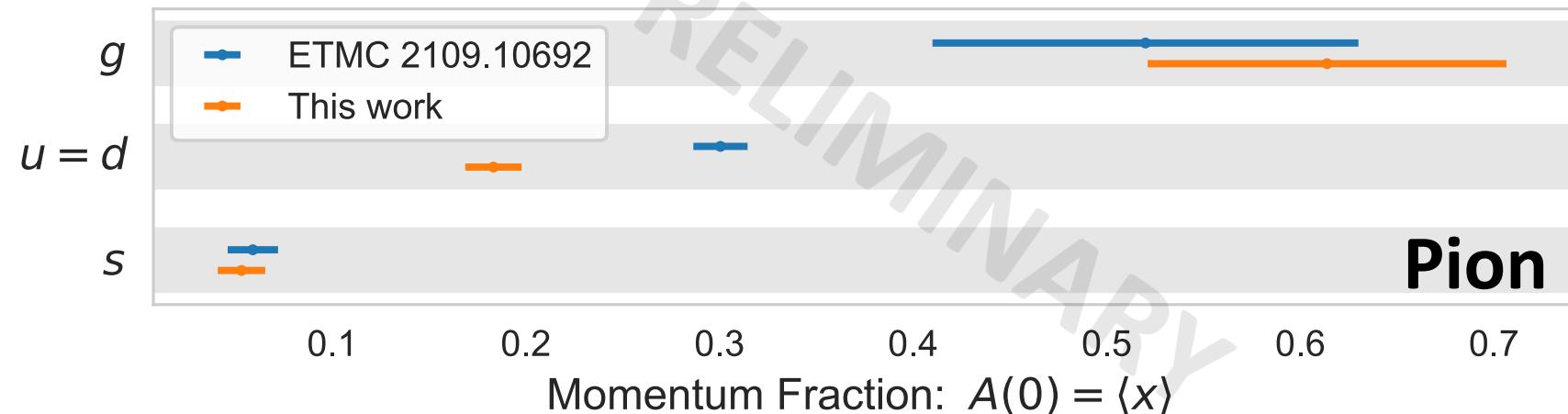
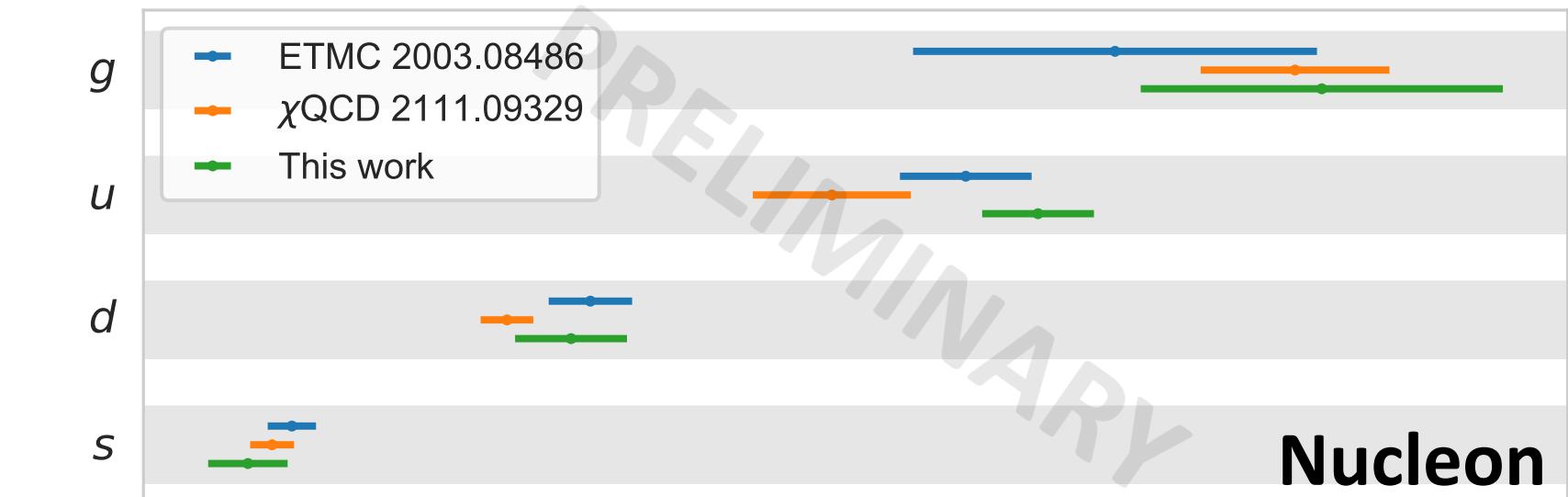
# Momentum fractions vs. other lattice results

**Note:** quark/glue mixing  
not accounted for!

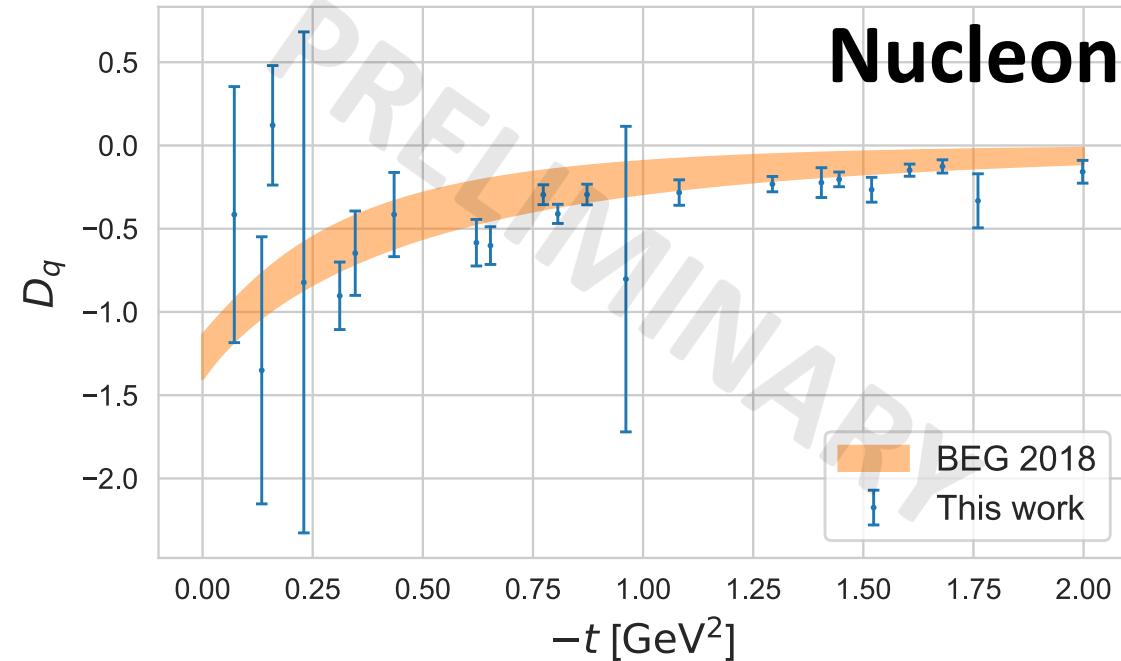
**Note:** other studies also  
treated charm quark; not  
shown here

**Pion:**  
 $\sum_{u,d,s,g} \langle x \rangle = 1.09(23)$

**Nucleon:**  
 $\sum_{u,d,s,g} \langle x \rangle = 1.12(10)$



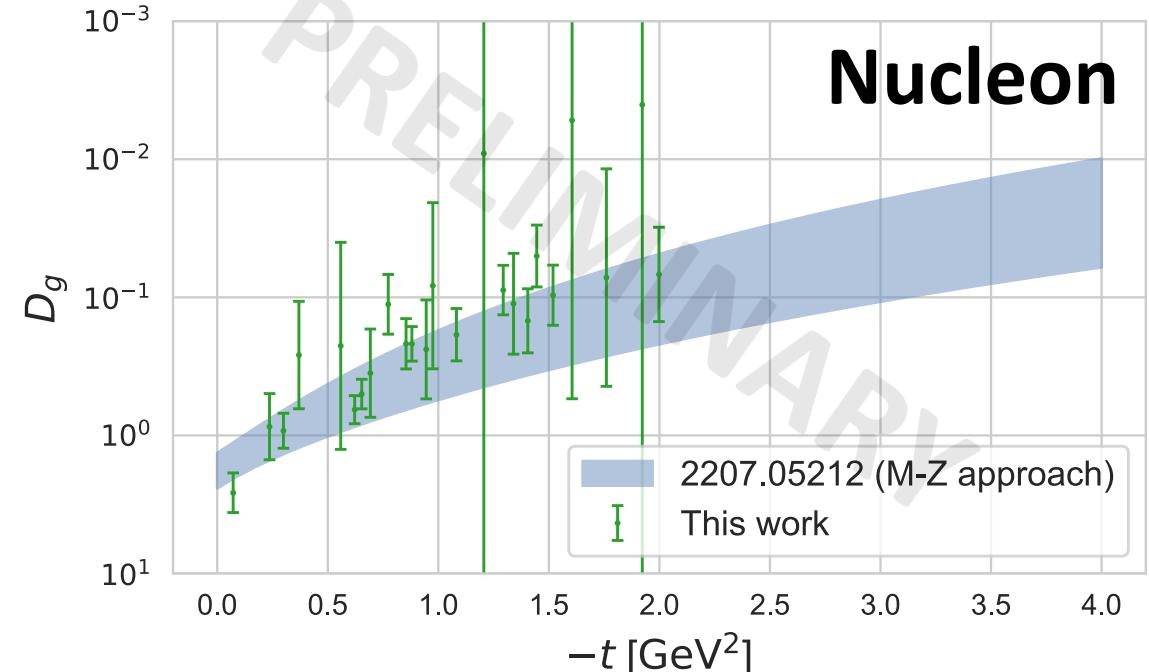
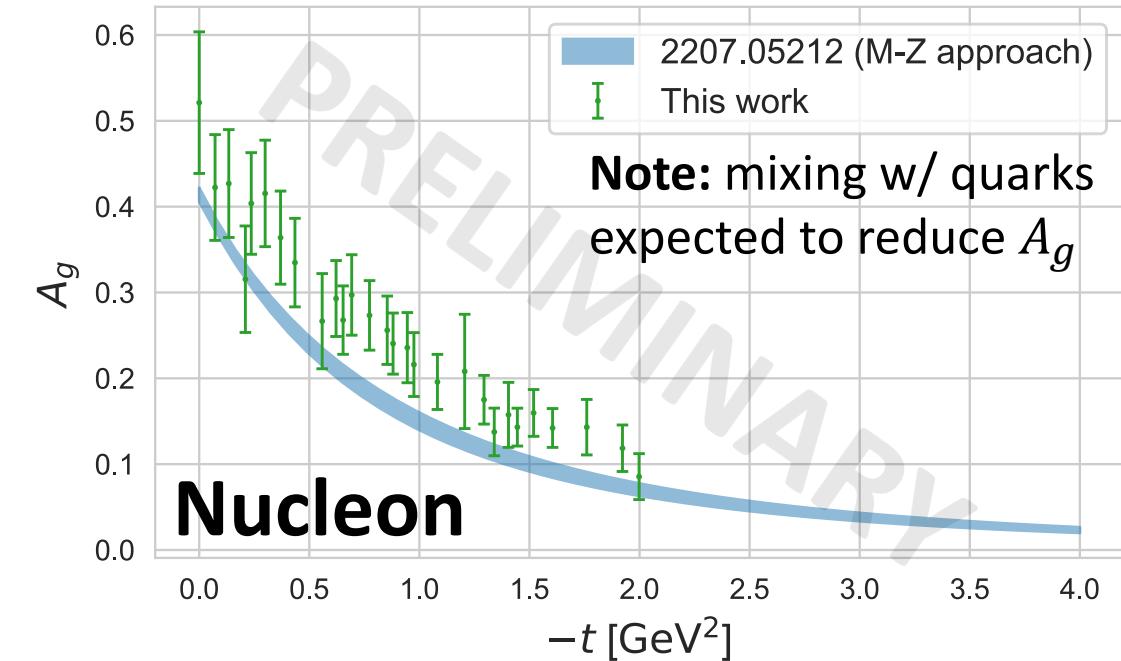
# Versus experimental results



- Experimental quark extraction from DVCS  
[\[Burkert Elouadrhiri Girod 2018\]](#)
- Experimental glue extraction from  $J/\Psi$  photoproduction  
[\[JLab HallC 2207.05212\]](#)

**Note:** unpublished at present

**Note':** strong method dependence



# Outlook

TODO:

Finalize bare GFF extraction

Compute mixings

Finish quantifying systematics, especially excited states and NPR

Results already compare well w/ other lattice, experiment

First calculation of complete set of  $N, \pi$  GFFs on a single ensemble  
(Away from forward limit!)

→ Compute *physical* densities of energy, pressure, shear forces