Resonance form factors from finite-volume correlation functions with the external field method

Jonathan Lozano de la Parra

University of Bonn

lozano@hiskp.uni-bonn.de

Talk based on: arXiv:2205.11316

In collaboration with: Meißner, Romero-López, Rusetsky and Schierholz

Lattice 2022. August 9, 2022

Motivation

- Most of the hadrons are resonances.

Motivation

- Most of the hadrons are resonances.
- Nowadays, the calculation of resonance form factors (RFF) encounters difficulties.

Motivation

- Most of the hadrons are resonances.
- Nowadays, the calculation of resonance form factors (RFF) encounters difficulties.
- The straightforward analytic continuation is hindered by the presence of the triangle diagram.

Motivation

- Most of the hadrons are resonances.
- Nowadays, the calculation of resonance form factors (RFF) encounters difficulties.
- The straightforward analytic continuation is hindered by the presence of the triangle diagram.
- Subtracting this contribution turns the extraction of the RFF into a challenging task. Hoja et al. 2010, M. T. Hansen et al. 2019.

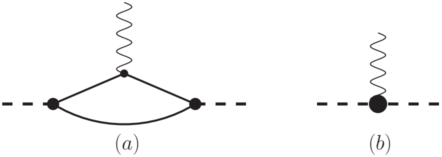
Motivation

- Most of the hadrons are resonances.
- Nowadays, the calculation of resonance form factors (RFF) encounters difficulties.
- The straightforward analytic continuation is hindered by the presence of the triangle diagram.
- Subtracting this contribution turns the extraction of the RFF into a challenging task. Hoja et al. 2010, M. T. Hansen et al. 2019.
- Recently, the Feynman-Hellmann theorem has been used to extract form factors of elementary particles in the Breit frame.
  A.J. Chambers et al. 2017

Motivation

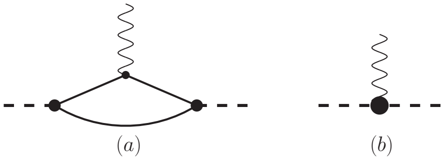
- Most of the hadrons are resonances.
- Nowadays, the calculation of resonance form factors (RFF) encounters difficulties.
- The straightforward analytic continuation is hindered by the presence of the triangle diagram.
- Subtracting this contribution turns the extraction of the RFF into a challenging task. Hoja et al. 2010, M. T. Hansen et al. 2019.
- Recently, the Feynman-Hellmann theorem has been used to extract form factors of elementary particles in the Breit frame. A.J. Chambers et al. 2017
- A natural question is: Can we extend this to composite particles?

Resonance form factor on the lattice



a) The triangle diagram, and b) contact term

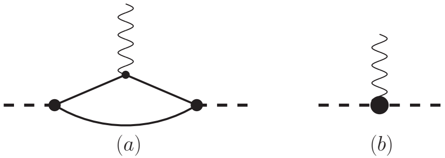
Resonance form factor on the lattice



a) The triangle diagram, and b) contact term

- Aim: Calculate the form factor of a resonance using lattice QCD.

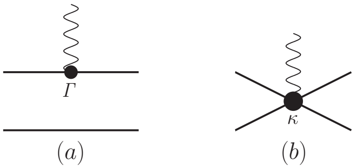
Resonance form factor on the lattice



a) The triangle diagram, and b) contact term

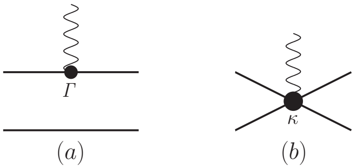
- Aim: Calculate the form factor of a resonance using lattice QCD.
- Standard method: Measure the three-point function on the lattice.

Our proposal



A method for the extraction of RFF by using a generalization of the Lüscher's method in the presence of an external source is presented.

Our proposal

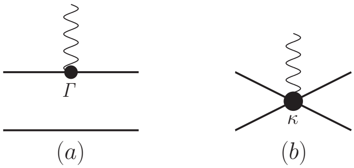


A method for the extraction of RFF by using a generalization of the Lüscher's method in the presence of an external source is presented.

Advantages:

- It suffices to determine the local contribution, proportional to κ.

Our proposal

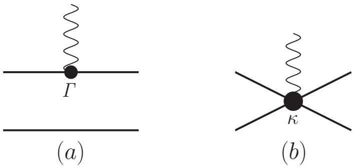


A method for the extraction of RFF by using a generalization of the Lüscher's method in the presence of an external source is presented.

Advantages:

- It suffices to determine the local contribution, proportional to κ.
- Finite-volume corrections in κ are suppressed.

Our proposal



A method for the extraction of RFF by using a generalization of the Lüscher's method in the presence of an external source is presented.

Advantages:

- It suffices to determine the local contribution, proportional to κ.
- Finite-volume corrections in κ are suppressed.
- The subtraction of the triangle diagram is not required.

Framework

What are the tools required for this derivation?

Framework

What are the tools required for this derivation?

- The system is placed in a spatially periodic external field.

Framework

What are the tools required for this derivation?

- The system is placed in a spatially periodic external field.
- We make use of a <u>non-relativistic</u> effective field theory.

Framework

What are the tools required for this derivation?

- The system is placed in a spatially periodic external field.
- We make use of a non-relativistic effective field theory.
- Include only linear terms in the coupling constant e of the external field:

Framework

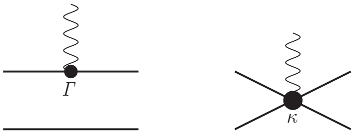
What are the tools required for this derivation?

- The system is placed in a spatially periodic external field.
- We make use of a non-relativistic effective field theory.
- Include only linear terms in the coupling constant e of the external field:

$$\mathcal{L} = \phi^\dagger \left(i\partial_t - m + eA^0 + \frac{eC_R}{6m^2} \triangle A^0 + \frac{\nabla^2}{2m} \right) \phi + C_0 \phi^\dagger \phi^\dagger \phi\phi$$
$$+ C_2 \left(\phi^\dagger \phi^\dagger (\phi \overset{\leftrightarrow}{\nabla}^2 \phi) + \text{h.c.} \right) + \frac{e\kappa}{4} \phi^\dagger \phi^\dagger \phi\phi \triangle A^0$$

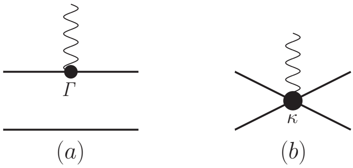
- The Breit frame is considered → Energy shift linear here.

Methodology I



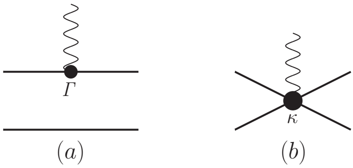
(a) (b)

Methodology I



To find the RFF, we must first calculate κ.

Methodology I



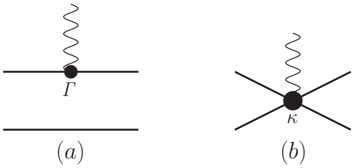
To find the RFF, we must first calculate κ.

- To achieve this, the Lüscher equation is derived in the presence of an external source,

$$\boxed{\det\left(X^{-1} - \frac{1}{2}\Pi\right) = 0}, \tag{1}$$

where X is a counterpart of the inverse K-matrix and Π is the Lüscher zeta-function.

Methodology I



To find the RFF, we must first calculate κ.

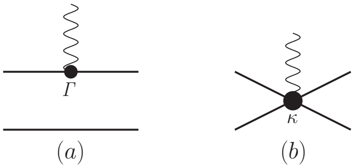
- To achieve this, the Lüscher equation is derived in the presence of an external source,

$$\boxed{\det\left(X^{-1} - \frac{1}{2}\Pi\right) = 0}, \tag{1}$$

where X is a counterpart of the inverse K-matrix and Π is the Lüscher zeta-function.

- By fitting to the energy levels in the external field, κ is determined.

Methodology I



To find the RFF, we must first calculate κ.

- To achieve this, the Lüscher equation is derived in the presence of an external source,

$$\boxed{\det\left(X^{-1} - \frac{1}{2}\Pi\right) = 0}, \qquad (1)$$

where X is a counterpart of the inverse K-matrix and Π is the Lüscher zeta-function.

- By fitting to the energy levels in the external field, κ is determined.
- With this, we can now evaluate the RFF at the complex pole. ✓

Methodology II

Methodology II

• To extract the complex pole position, P_R^0, we first solve the Lüscher equation in the infinite volume:

$$\det\left(X^{-1} - \frac{1}{2}\Pi_{\infty}\right) = 0, \tag{2}$$

Methodology II

• To extract the complex pole position, P_R^0, we first solve the Lüscher equation in the infinite volume:

$$\det\left(X^{-1} - \frac{1}{2}\Pi_{\infty}\right) = 0, \tag{2}$$

With the extracted value of κ, we determine the position of the pole on the second sheet.

Methodology II

• To extract the complex pole position, P_R^0, we first solve the Lüscher equation in the infinite volume:

$$\det\left(X^{-1} - \frac{1}{2}\Pi_{\infty}\right) = 0, \tag{2}$$

With the extracted value of κ, we determine the position of the pole on the second sheet.

• In the Breit frame, the RFF is given by the derivative of the resonance pole position with respect to e.

$$\boxed{\left.\frac{dP_R^0(e)}{de}\right|_{e=0} \propto F}. \tag{3}$$

Methodology II

• To extract the complex pole position, P_R^0, we first solve the Lüscher equation in the infinite volume:

$$\det\left(X^{-1} - \frac{1}{2}\Pi_{\infty}\right) = 0, \tag{2}$$

With the extracted value of κ, we determine the position of the pole on the second sheet.

• In the Breit frame, the RFF is given by the derivative of the resonance pole position with respect to e.

$$\boxed{\left.\frac{dP_R^0(e)}{de}\right|_{e=0} \propto F}. \tag{3}$$

• Thus, the Feynman-Hellmann theorem is generalized to the case of resonances. ✓

Methodology III

Methodology III

- The explicit form of the RFF (in dimensional regularization) is given by:

$$F(\omega) = \frac{\sqrt{-q_R^2}}{4\pi\left(1 + r\sqrt{-q_R^2}\right)}\left\{-\kappa\omega^2 q_R^2 \right.$$
$$\left. + 8\pi\Gamma\left(r + \frac{4}{\omega}\arcsin\frac{\omega}{\sqrt{\omega^2 - 16q_R^2}}\right)\right\}$$

we can see that the only parameter that has to extracted is κ.

Methodology III

- The explicit form of the RFF (in dimensional regularization) is given by:

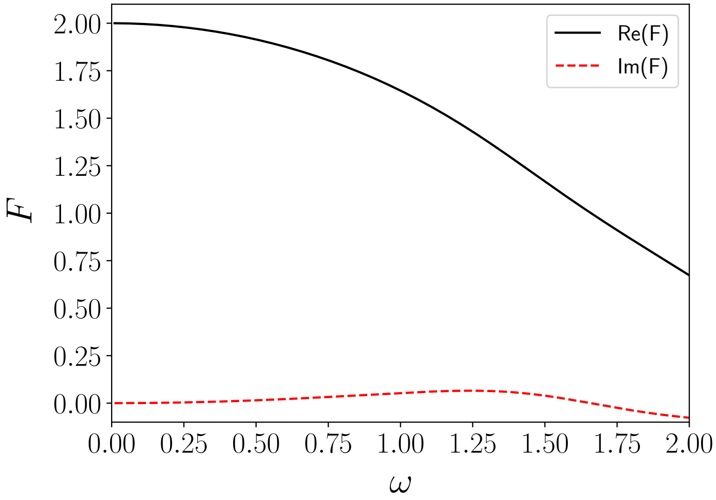
$$F(\omega) = \frac{\sqrt{-q_R^2}}{4\pi\left(1 + r\sqrt{-q_R^2}\right)}\Bigg\{-\kappa\omega^2 q_R^2 + 8\pi\Gamma\left(r + \frac{4}{\omega}\arcsin\frac{\omega}{\sqrt{\omega^2 - 16q_R^2}}\right)\Bigg\}$$

we can see that the only parameter that has to extracted is κ.

- For instance, with $\omega = 1$, we have:

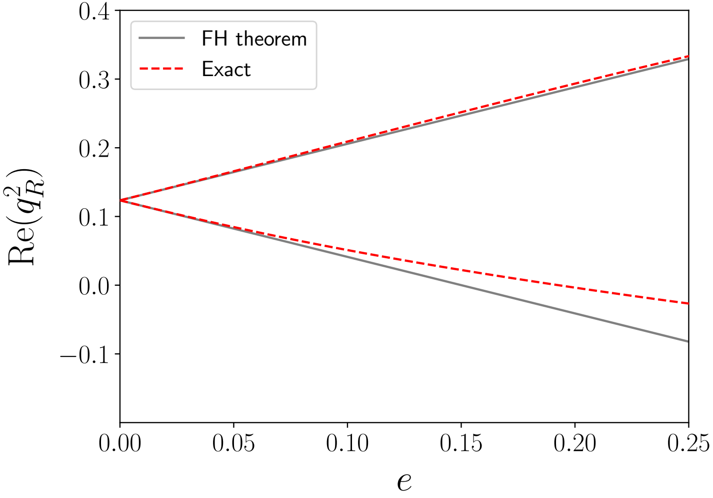
$$F(\omega) = 1.6454 + i0.0535\,, \qquad \frac{dP_R^0}{de}\bigg|_{e=0} = 1.6455 + i0.0534 \qquad \checkmark$$

Results



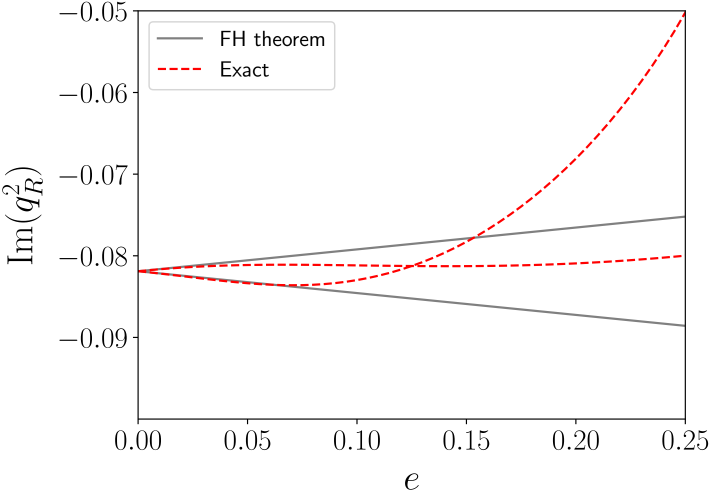
$$a = -1.5,\ \ r = -9,\ \ \kappa = 10,\ \ C_R = 0.9$$

Results



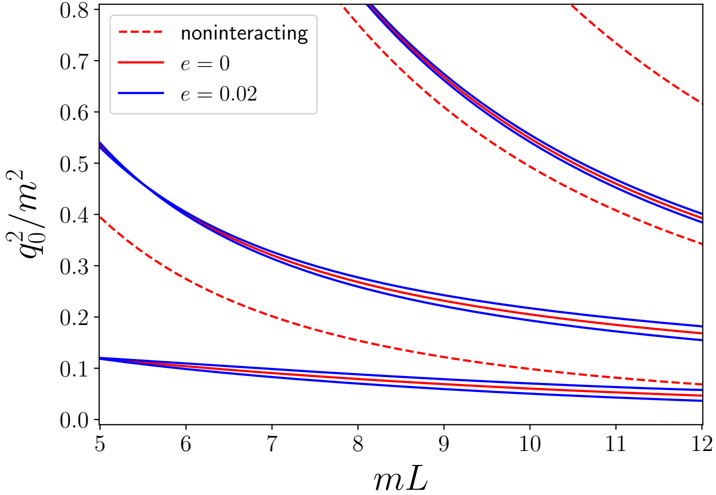
$$a = -1.5,\ r = -9,\ \kappa = 10,\ C_R = 0.9,\ \omega = 1$$

Results



$$a = -1.5, \ r = -9, \ \kappa = 10, \ C_R = 0.9, \ \omega = 1$$

Results



$a = -1.5, \ r = -9, \ \kappa = 10, \ C_R = 0.9$

Summary

Summary

- A novel method for the measurement of the resonance form factors on the lattice has been proposed.

Summary

- A novel method for the measurement of the resonance form factors on the lattice has been proposed.
- A generalization of the Lüscher equation in the presence of an external periodic field is obtained.

Summary

- A novel method for the measurement of the resonance form factors on the lattice has been proposed.
- A generalization of the Lüscher equation in the presence of an external periodic field is obtained.
- Additionally, the Feynman-Hellmann theorem is extended to the case of resonances:

$$\boxed{\left.\frac{dP_R^0(e)}{de}\right|_{e=0} \propto F.} \tag{4}$$

Summary

- A novel method for the measurement of the resonance form factors on the lattice has been proposed.
- A generalization of the Lüscher equation in the presence of an external periodic field is obtained.
- Additionally, the Feynman-Hellmann theorem is extended to the case of resonances:

$$\boxed{\left.\frac{dP_R^0(e)}{de}\right|_{e=0} \propto F.} \tag{4}$$

(More details in my talk at the Bethe Forum next Monday 15:50!)