Nucleon Electromagnetic Form Factors Using Physical Point N_f=2+1+1 Twisted Mass Fermion Ensembles

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Extended Twisted Mass Collaboration

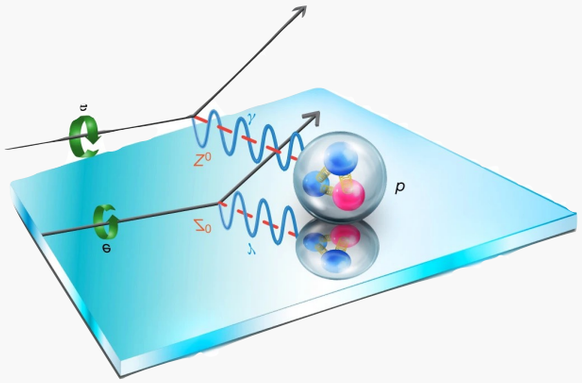
LATTICE 2022

The Cyprus Institute
RESEARCH•TECHNOLOGY•INNOVATION

Nucleon Electromagnetic Form Factors

Nucleon EM form factors from twisted mass fermions at physical point

- Lattice setup
  - Correlation functions (connected & disconnected)
  - Ensembles used and statistics
  - Excited state analysis
- Electromagnetic form factors
  - Isovector form-factors
    - Analysis of excited states
  - Disconnected isoscalar
  - Proton/neutron combining disconnected
  - Preliminary continuum limit



Nucleon Electromagnetic Form Factors

Matrix element:

$$\langle N(p',s')|j^\mu|N(p,s)\rangle = \sqrt{\frac{M_N^2}{E_N(\mathbf{p}')E_N(\mathbf{p})}}\bar{u}(p',s')\mathcal{O}^\mu u(p,s)$$

$$\mathcal{O}^\mu = \gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2M_N}F_2(q^2), \quad q = p' - p$$

Dirac and Pauli (F_1 and F_2) **/ Sachs Electric and Magnetic** (G_E and G_M) **form-factors:**

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{(2M_N)^2}F_2(q^2) \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

Isovector & Isoscalar currents:

$$j^v_\mu = \bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d,$$

$$j^s_\mu = \bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d$$

Assuming mass degenerate up and down quarks

$$F^p - F^n = F^u - F^d$$

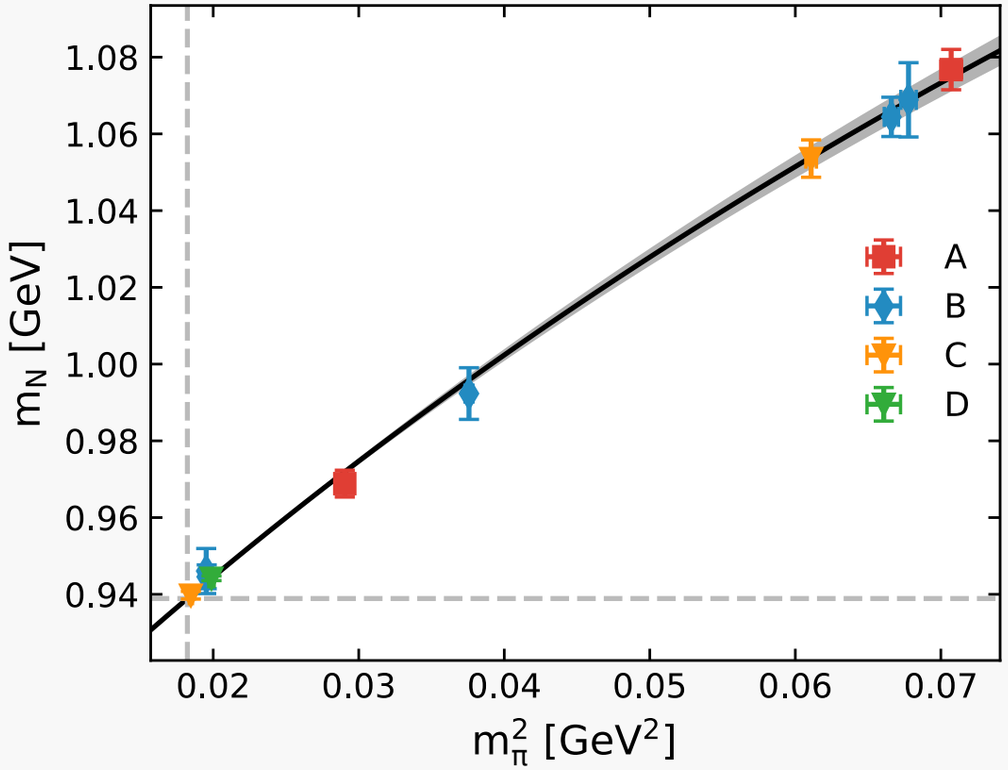
$$F^p + F^n = \frac{1}{3}(F^u + F^d)$$

Ensembles

Three N_f=2+1+1 ensembles at physical pion mass

Ens. ID (abbrv.)	Vol.	a [fm]
cB211.072.64 (cB64)	64×128	0.080
cC211.060.80 (cC80)	80×160	0.068
cD211.054.96 (cD96)	96×192	0.057

- Lattice spacings quoted as determined from nucleon mass
  - "*Quark masses*" paper for details – Phys.Rev.D 104 (2021) 074515 • arXiv:2104.13408



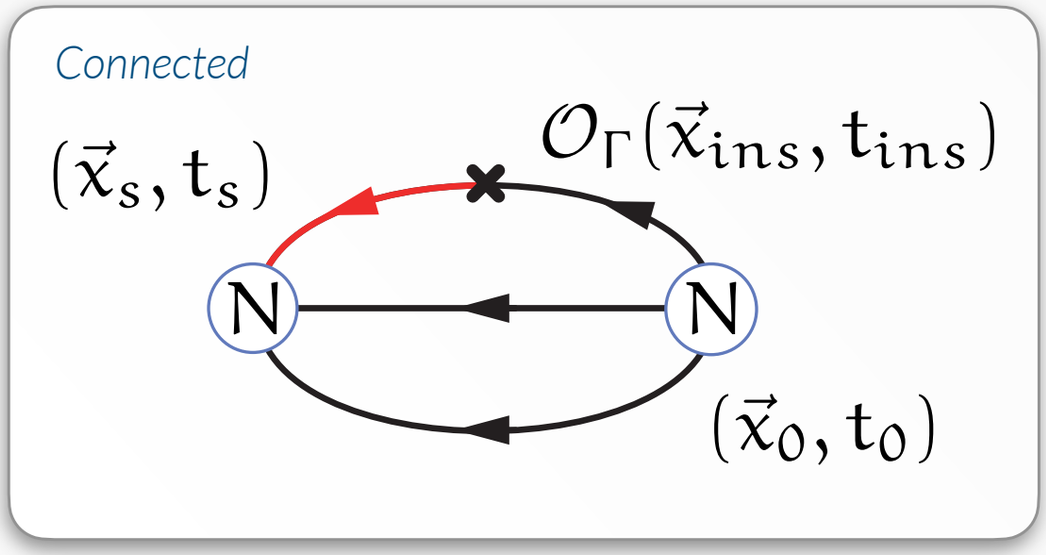
- EM form-factors for cB64 reported:
  - Proton/neutron – Phys.Rev.D100 (2019), 014509 • arXiv:1812.10311
  - Strange – Phys.Rev.D97 (2018) 094504 • arXiv:1801.09581

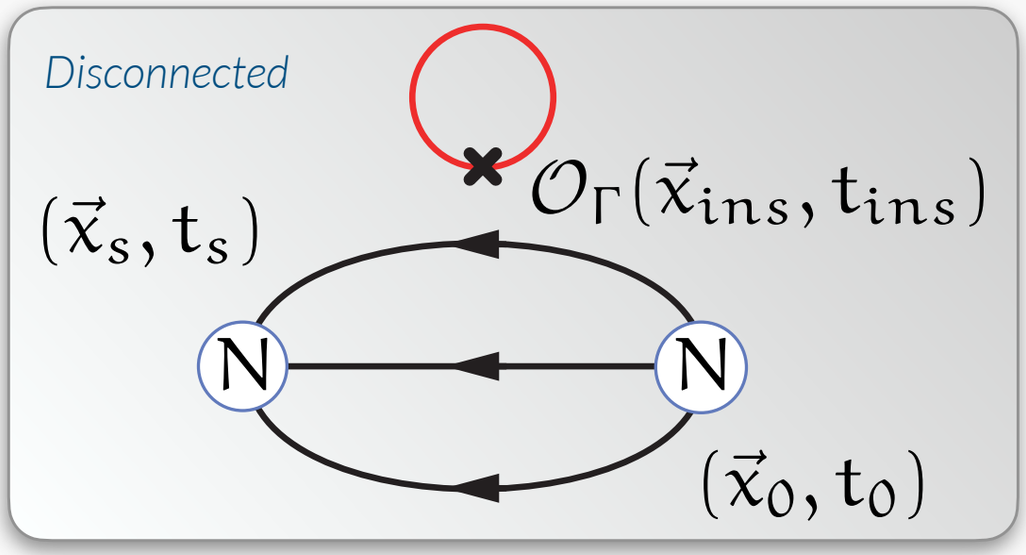
- **Here:** Preliminary analysis of two additional lattice spacings + reanalysis of excited state effects

More on ETMC simulations by **B. Kostrzewa**, Tuesday @ 15:50 CEST – parallel to this session :(

Evaluation of matrix elements

$$G_\Gamma(P;\vec{q};t_s,t_{ins}) = \sum_{\vec{x}_s\,\vec{x}_{ins}} e^{-i\vec{q}.\vec{x}_{ins}}\, P^{\alpha\beta}\, \langle \bar{\chi}_N^\beta(\vec{x}_s;t_s)|\mathcal{O}^\Gamma(\vec{x}_{ins};t_{ins})|\chi_N^\alpha(\vec{0};0)\rangle$$





Sequential source through sink

- Inversions for each of the 4 projectors

$$P : \begin{cases} P_0 = \frac{1+\gamma_0}{4} \\ P_k = i\gamma_5\gamma_k P_0,\ k = 1,\ 2,\ 3 \end{cases}$$

- Inversions for each choice of t_s

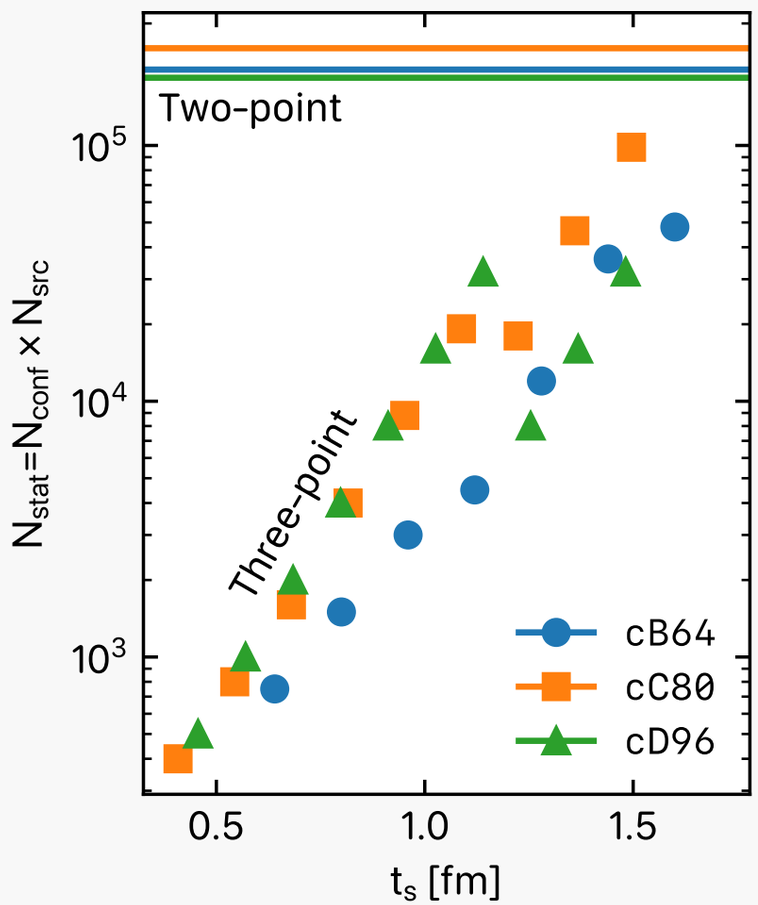
Disconnected

- Two-point: re-use statistics from connected + additional source positions
- Deflation, hierarchical probing, and dilution for fermion loop (details follow)

Statistics

$$R_\Gamma(P;\vec{q};t_s;t_{ins}) = \frac{G_\Gamma(P;\vec{q};t_s;t_{ins})}{G(\vec{0};t_s)}\sqrt{\frac{G(\vec{p};t_s-t_{ins})G(\vec{0};t_{ins})G(\vec{0};t_s)}{G(\vec{0};t_s-t_{ins})G(\vec{p};t_{ins})G(\vec{p};t_s)}}$$

Connected: Increasing N_{src} with t_s



Ideally: Aim for *constant statistical errors* over all values of t_s of a given ensemble

- Robust analysis of excited states: summation method, two- or three-state fits, etc.

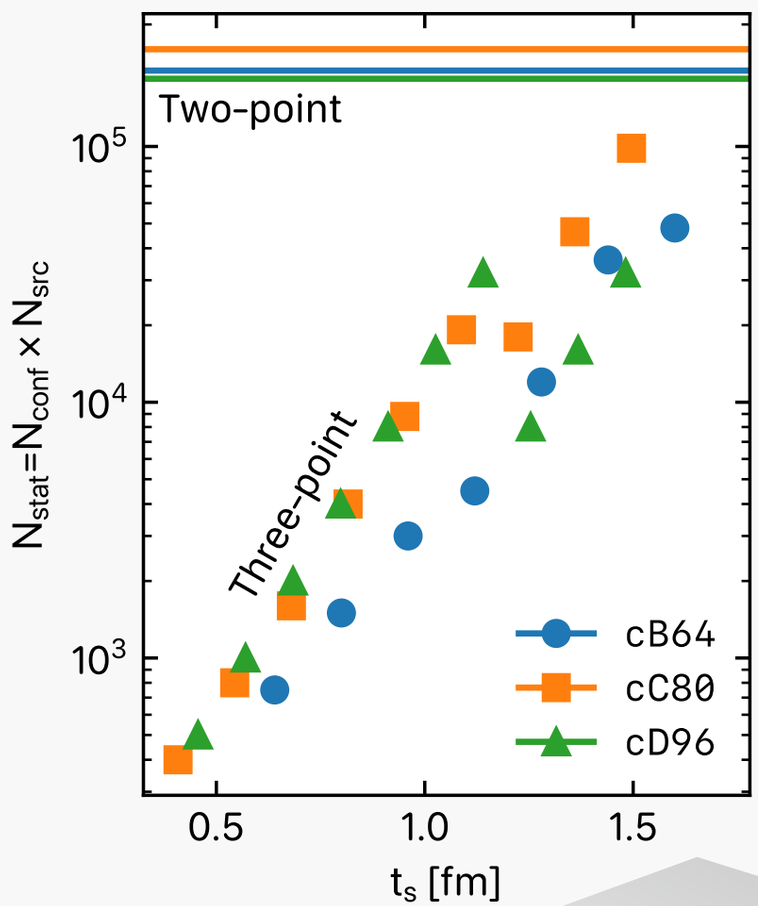
Two-point function: High number of sources per config

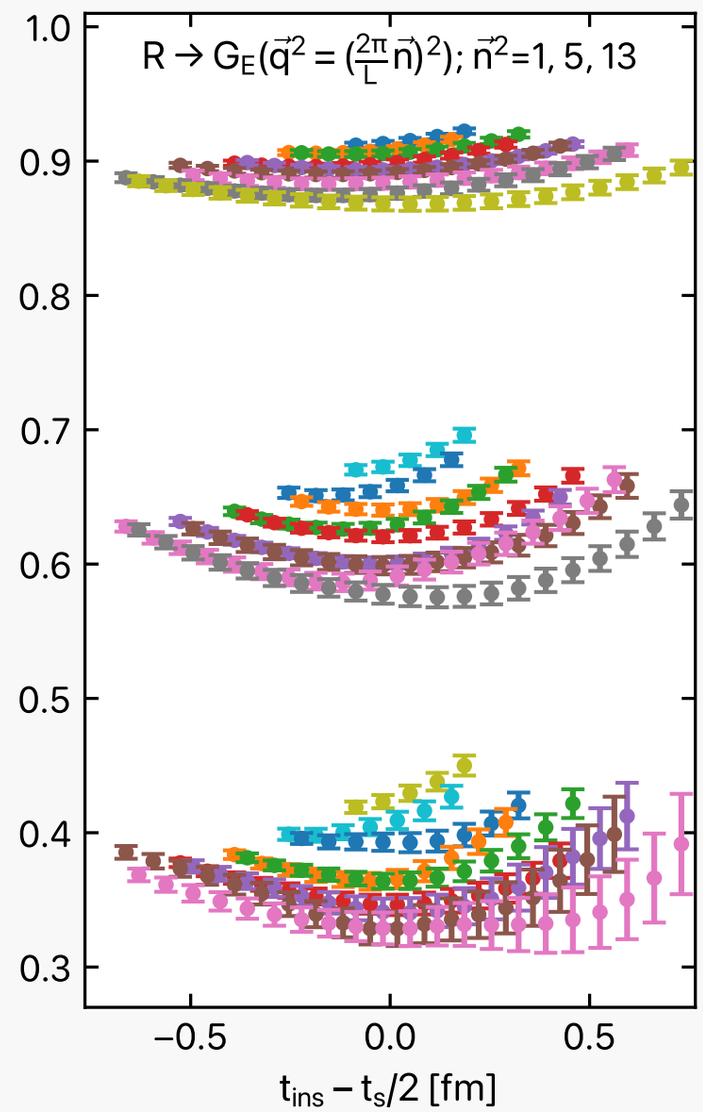
- Needed for disconnected contributions

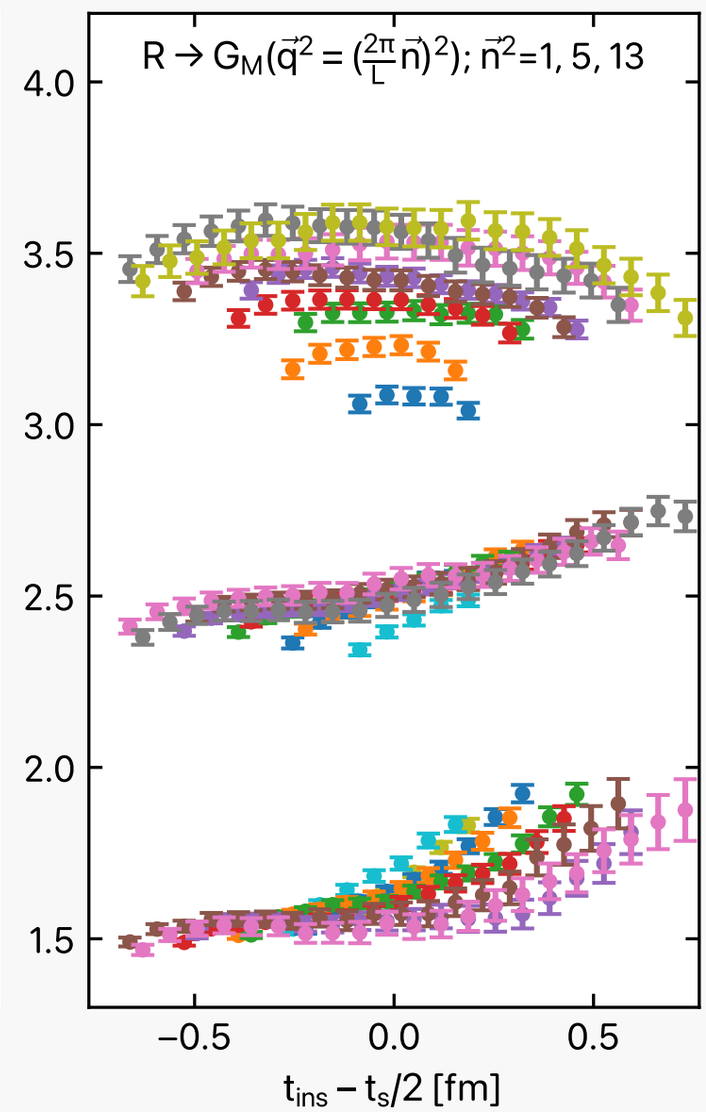
Statistics

$$R_\Gamma(P;\vec{q};t_s;t_{ins}) = \frac{G_\Gamma(P;\vec{q};t_s;t_{ins})}{G(\vec{0};t_s)}\sqrt{\frac{G(\vec{p};t_s-t_{ins})G(\vec{0};t_{ins})G(\vec{0};t_s)}{G(\vec{0};t_s-t_{ins})G(\vec{p};t_{ins})G(\vec{p};t_s)}}$$

Connected: Increasing N_{src} with t_s







Example for cC80

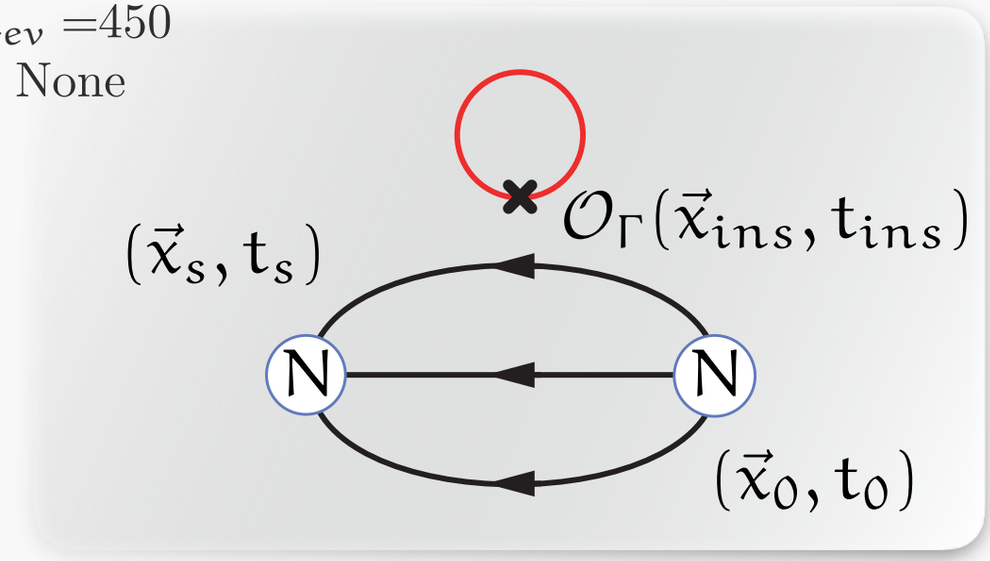
Statistics

$$R_\Gamma(P;\vec{q};t_s;t_{ins}) = \frac{G_\Gamma(P;\vec{q};t_s;t_{ins})}{G(\vec{0};t_s)}\sqrt{\frac{G(\vec{p};t_s-t_{ins})G(\vec{0};t_{ins})G(\vec{0};t_s)}{G(\vec{0};t_s-t_{ins})G(\vec{p};t_{ins})G(\vec{p};t_s)}}$$

Disconnected: High number of sources per config. + hierarchical probing, color/spin dilution and exact of low-mode estimation of loops

Ens.	Light Stochastic	Deflation
cB64:	$n_{vec} = 12_{col./spin} \times 512_{nhad.}$	n_{ev} =200
cC80:	$n_{vec} = 12_{col./spin} \times 512_{nhad.}$	n_{ev} =450
cD96:	$n_{vec} = 12_{col./spin} \times 512_{nhad.} \times 8_{stoch.}$	None

Ens.	Strange
cB64:	$n_{vec} = 12_{col/spin} \times 12_{stoch.} \times 32_{nhad.}$
cC80:	$n_{vec} = 12_{col/spin} \times 4_{stoch.} \times 512_{nhad.}$
cD96:	$n_{vec} = 12_{col/spin} \times 4_{stoch.} \times 512_{nhad.}$



Treatment of excited states

Summation method

$$S_\Gamma(\vec{q};t_s) = \sum_{t_{ins}=\tau}^{t_s-\tau} R_\Gamma(\vec{q};t_s;t_{ins}) \to \mathcal{M}t_s + C$$

Two-state fit

$$G_\Gamma(\vec{q};t_s,t_{ins}) = \sum_{i=0}^{1}\sum_{j=0}^{1} c_{ij} e^{-E_i(0)(t_s-t_{ins})} e^{-E_j(\vec{q})t_{ins}}$$

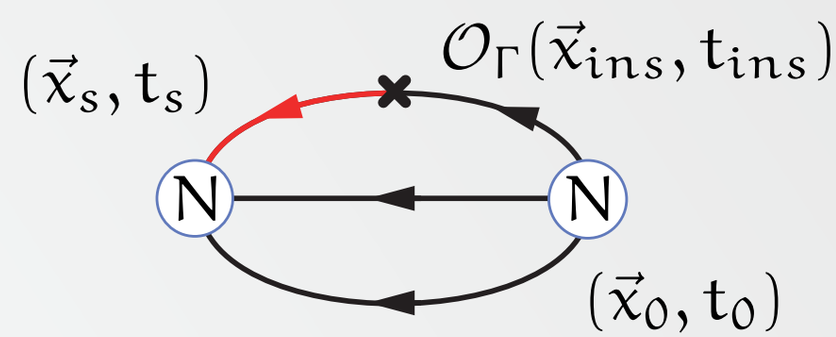
$$\mathcal{M} = \frac{c_{00}}{\sqrt{a_0(\vec{0})a_0(\vec{q})}}$$

$$G(\vec{q};t_s) = a_0(\vec{q})e^{-\varepsilon_0(\vec{q})t_s} + a_1(\vec{q})e^{-\varepsilon_1(\vec{q})t_s}$$

Ground state:

$$E_0(0) = \varepsilon_0(0)$$

$$E_0(\vec{q}) = \varepsilon_0(\vec{q}) = [\varepsilon_0^2(\vec{0}) + (\frac{2\pi}{L}\vec{q})^2]^{\frac{1}{2}}$$

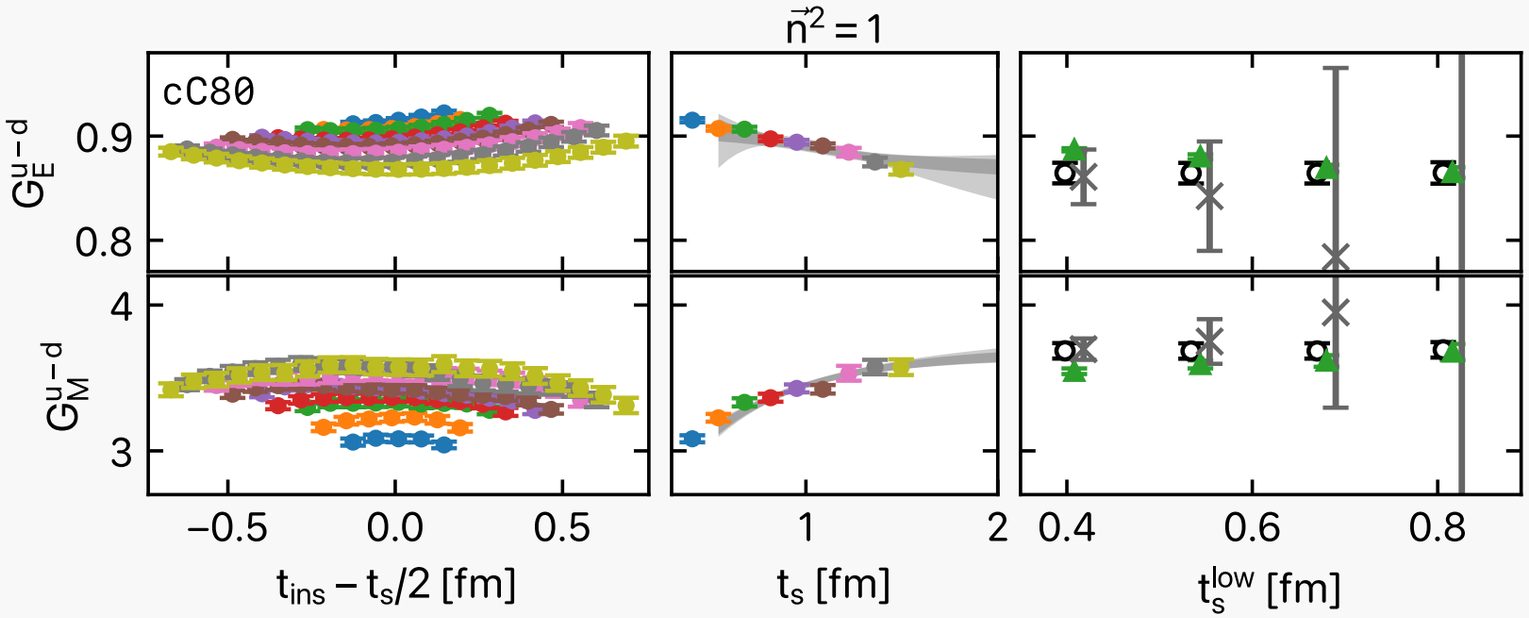


Excited states:

In general, $E_1(\vec{q}) \neq \varepsilon_1(\vec{q})$ and $E_1(0) \neq \varepsilon_1(0)$*

*E.g. arXiv:1905.06470

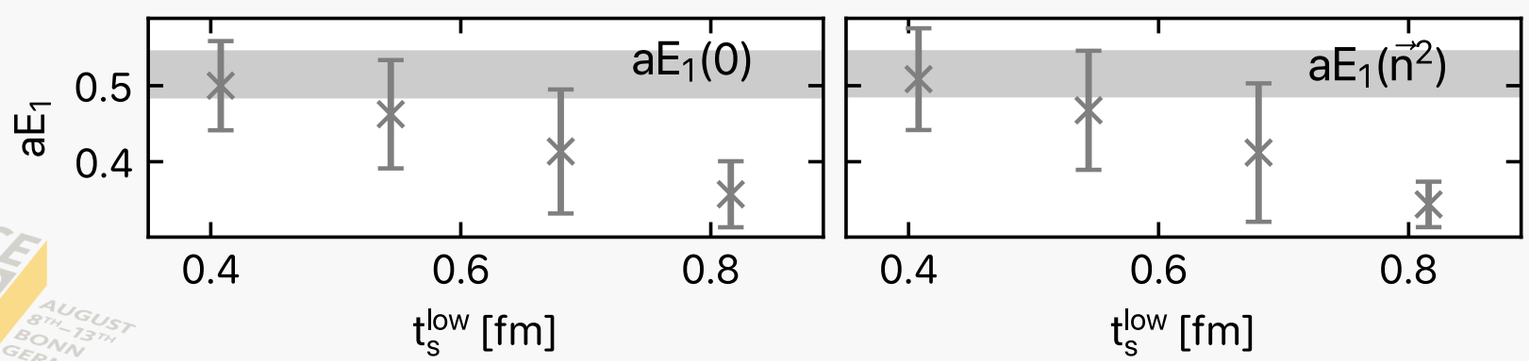
Treatment of excited states



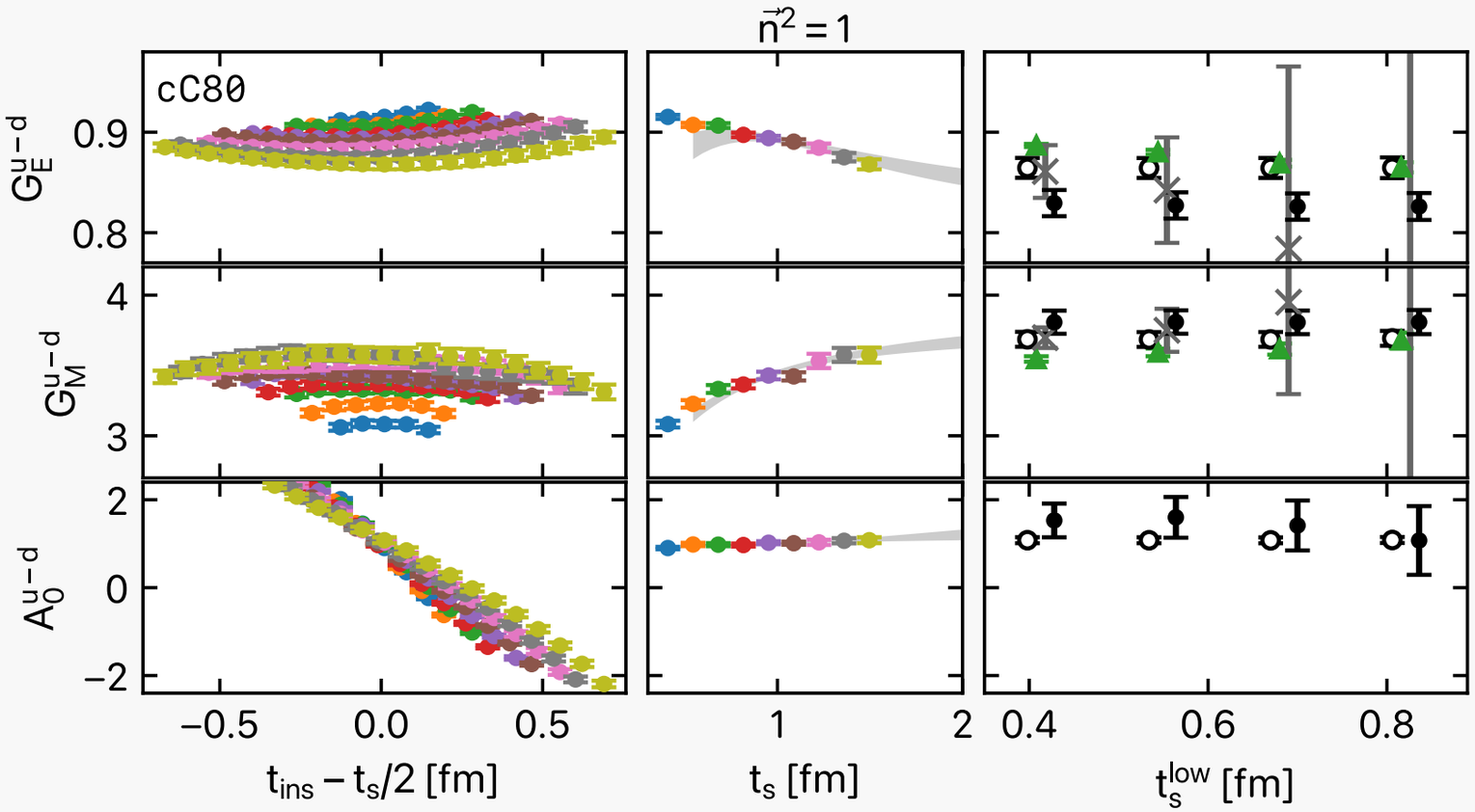
Summation

Two-state,
$E_1(0) = \varepsilon_1(0)$
$E_1(\vec{q}) = \varepsilon_1(\vec{q})$

Two-state,
$E_1(\vec{q}) \neq \varepsilon_1(\vec{q})$
$E_1(0) \neq \varepsilon_1(0)$



Treatment of excited states

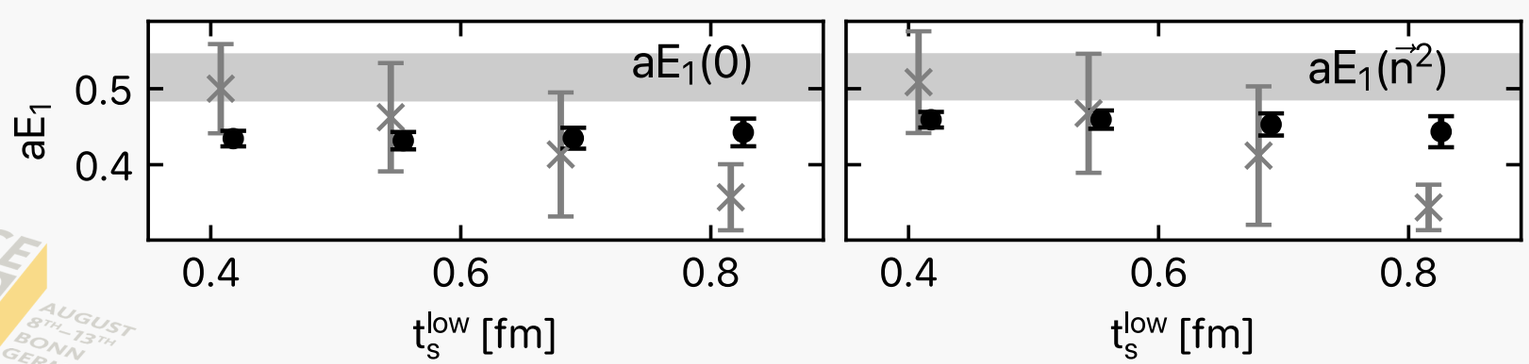


Summation

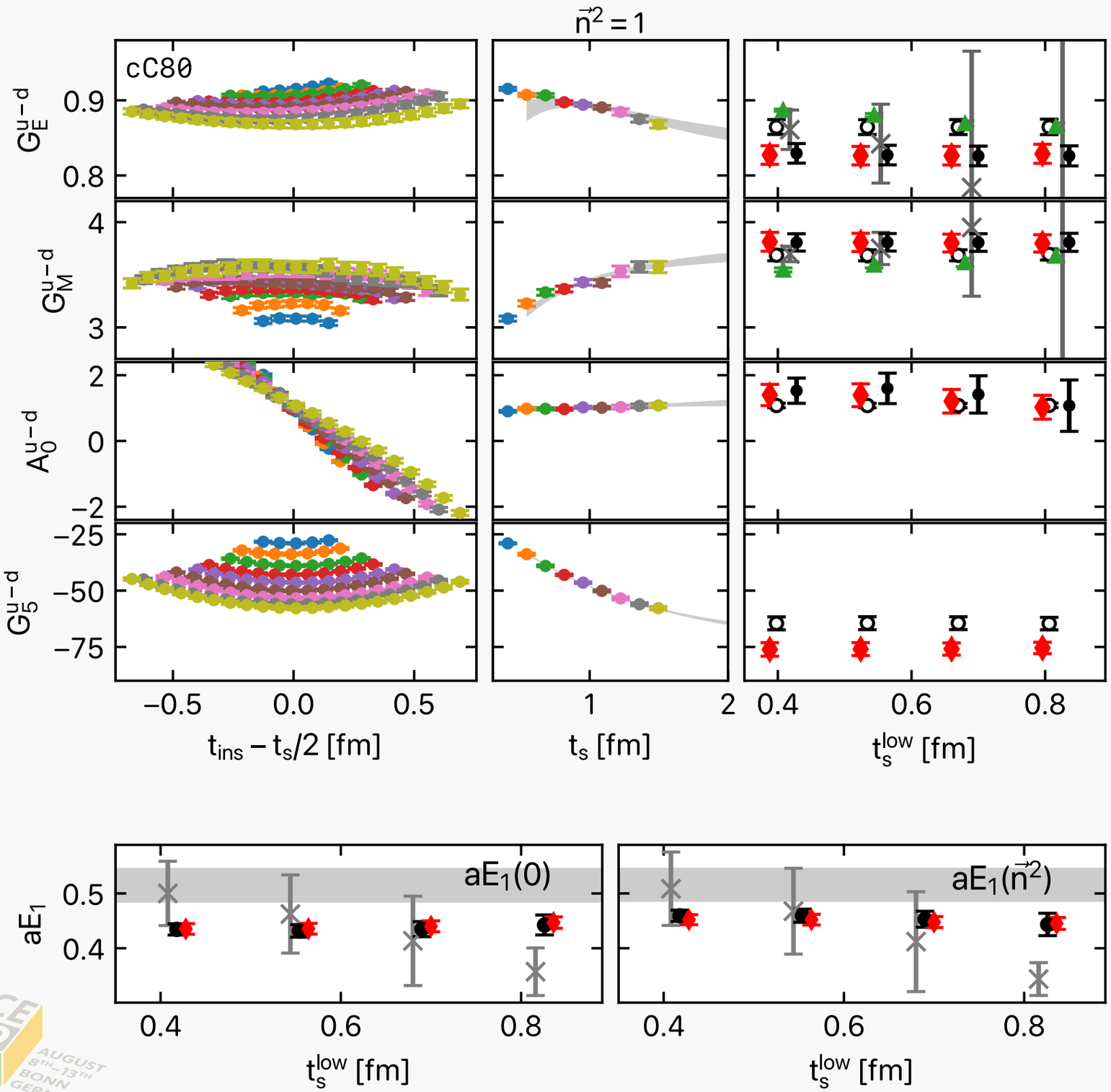
Two-state,
$E_1(0) = \varepsilon_1(0)$
$E_1(\vec{q}) = \varepsilon_1(\vec{q})$

Two-state,
$E_1(\vec{q}) \neq \varepsilon_1(\vec{q})$
$E_1(0) \neq \varepsilon_1(0)$

Two-state with A_0
$E_1(\vec{q}) \neq \varepsilon_1(\vec{q})$
$E_1(0) \neq \varepsilon_1(0)$



Treatment of excited states



Summation

Two-state,
$E_1(0) = \varepsilon_1(0)$
$E_1(\vec{q}) = \varepsilon_1(\vec{q})$

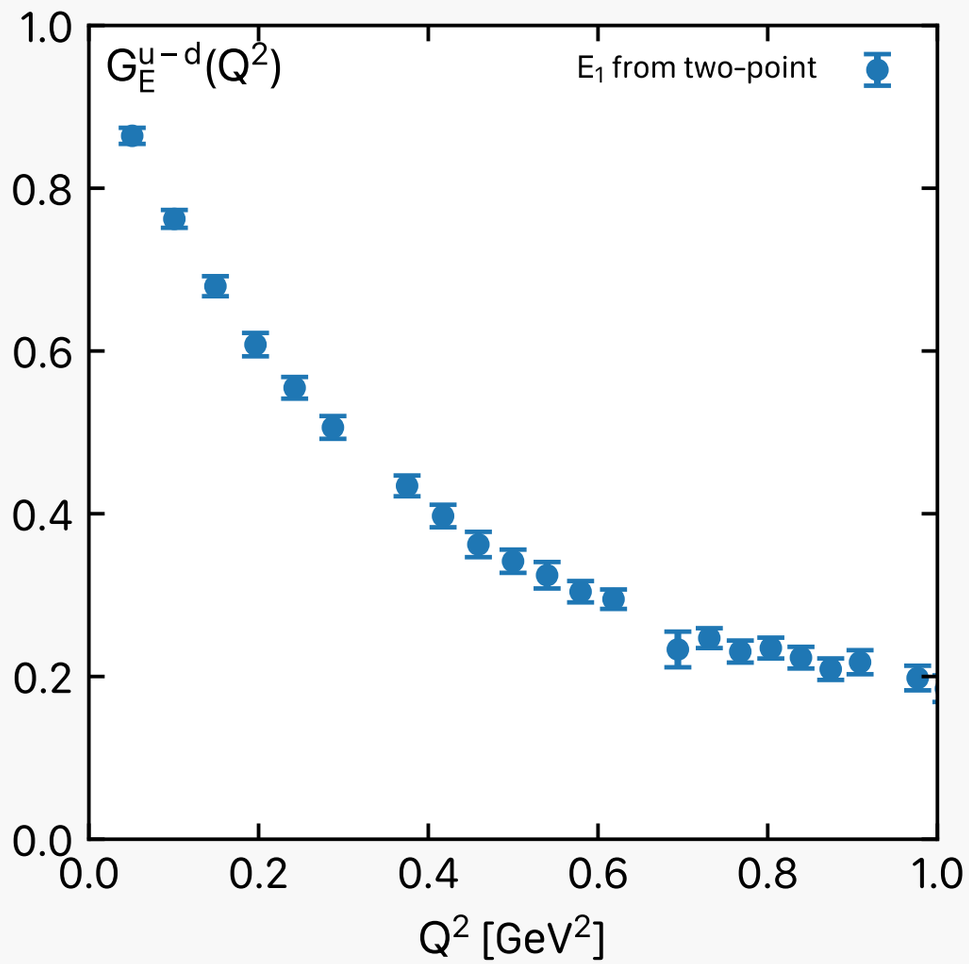
Two-state,
$E_1(\vec{q}) \neq \varepsilon_1(\vec{q})$
$E_1(0) \neq \varepsilon_1(0)$

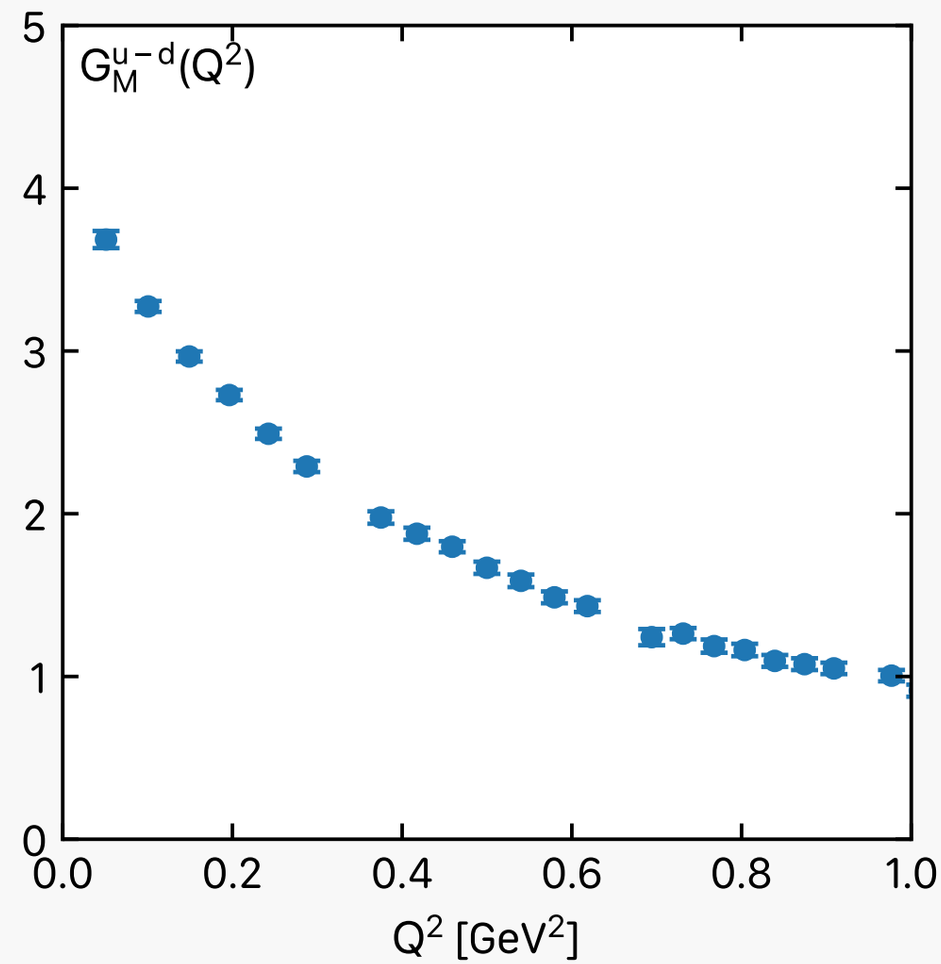
Two-state with A_0
$E_1(\vec{q}) \neq \varepsilon_1(\vec{q})$
$E_1(0) \neq \varepsilon_1(0)$

Two-state with A_0 & G_5
$E_1(\vec{q}) \neq \varepsilon_1(\vec{q})$
$E_1(0) \neq \varepsilon_1(0)$

Isovector Form Factors

Excited states: Effect of fit choice





Isovector Form Factors

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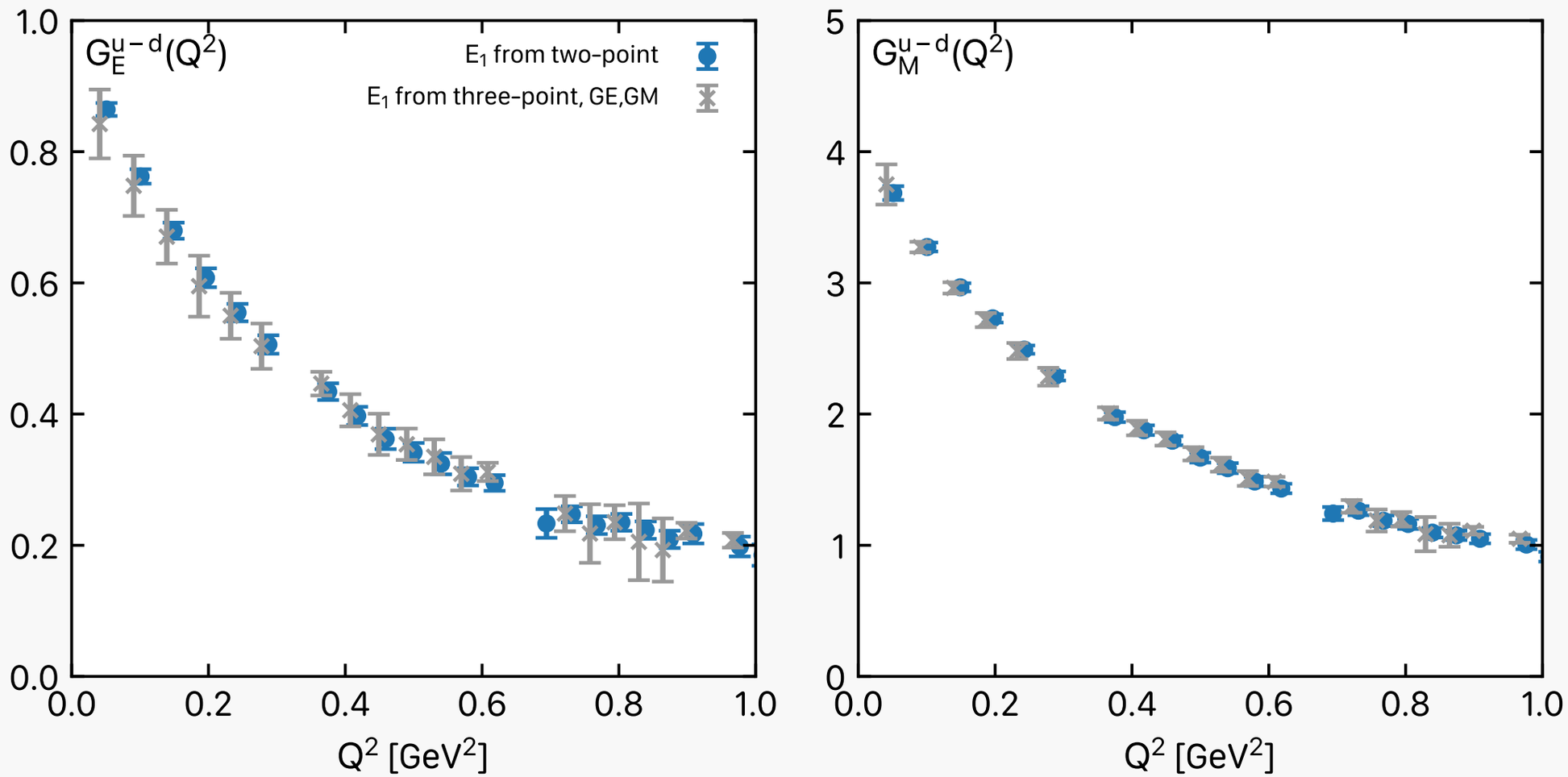

Isovector Form Factors

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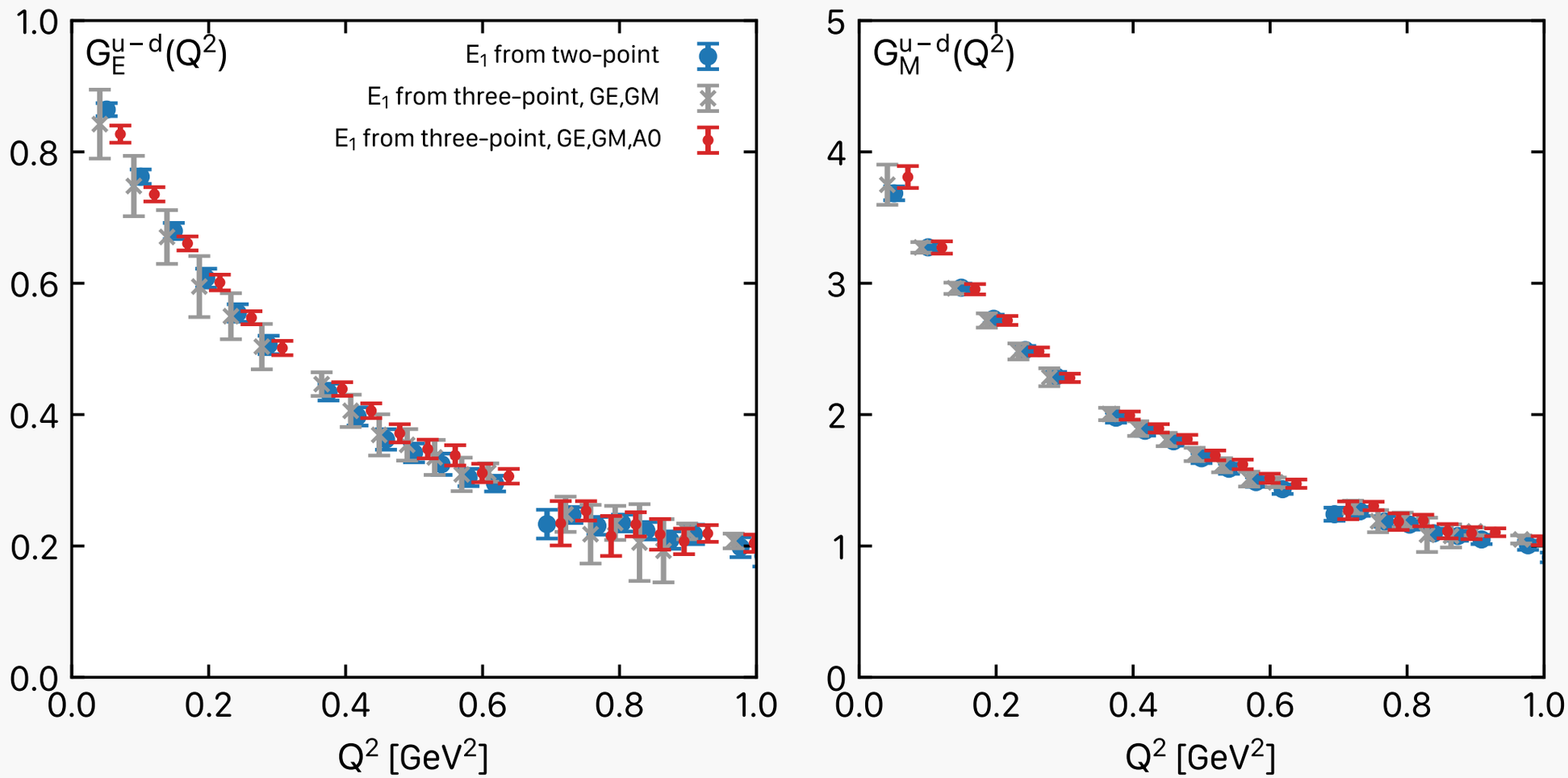
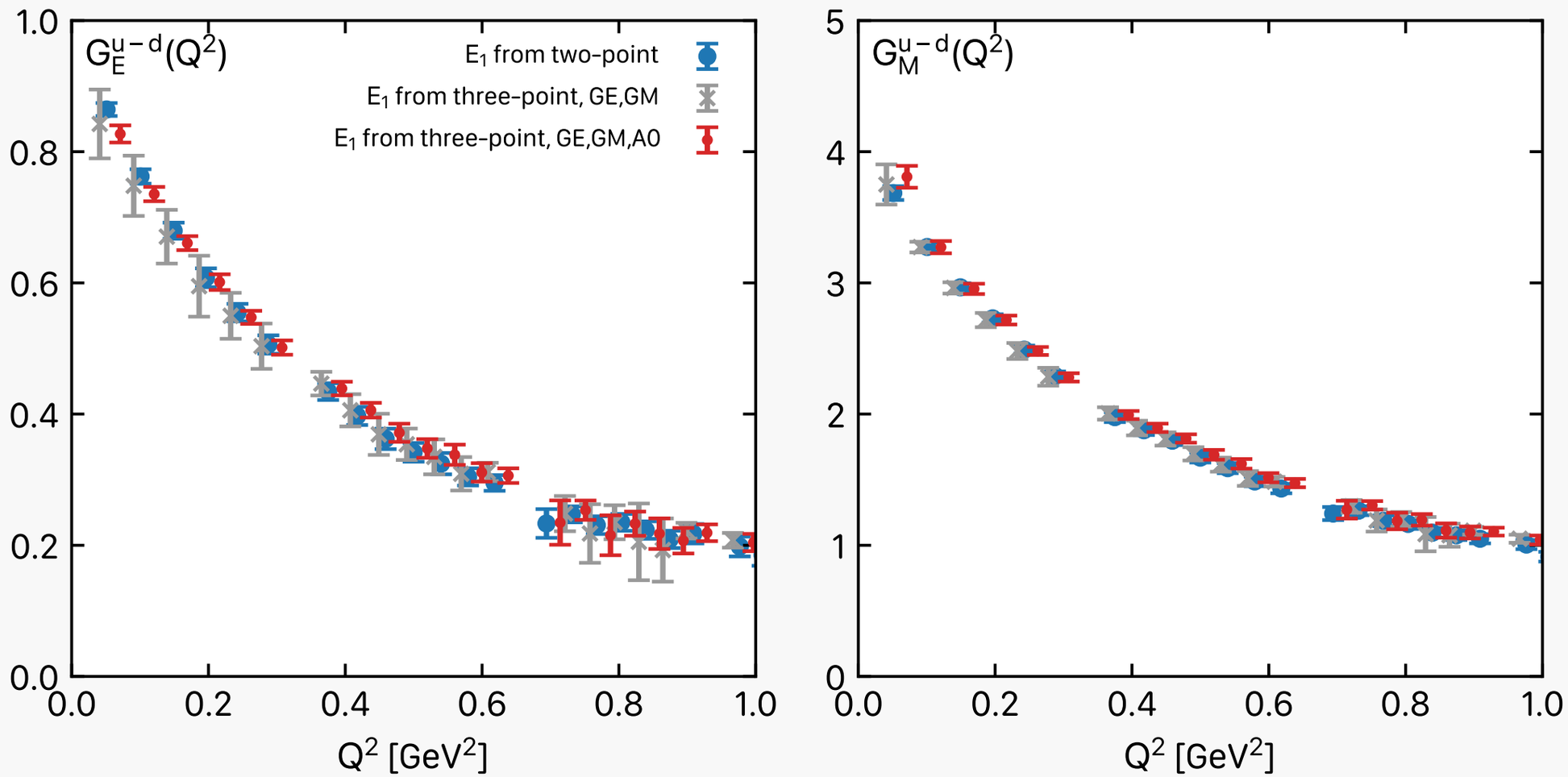


Example from **cC80** (intermediate a)

- Low-Q^2:
  - A_0 fit $\sim$1–2σ from fit with energies fixed from two-point function
  - As E_1 fitted to smaller values: G_E decreases; G_M increases

Isovector Form Factors

Excited states: Effect of fit choice

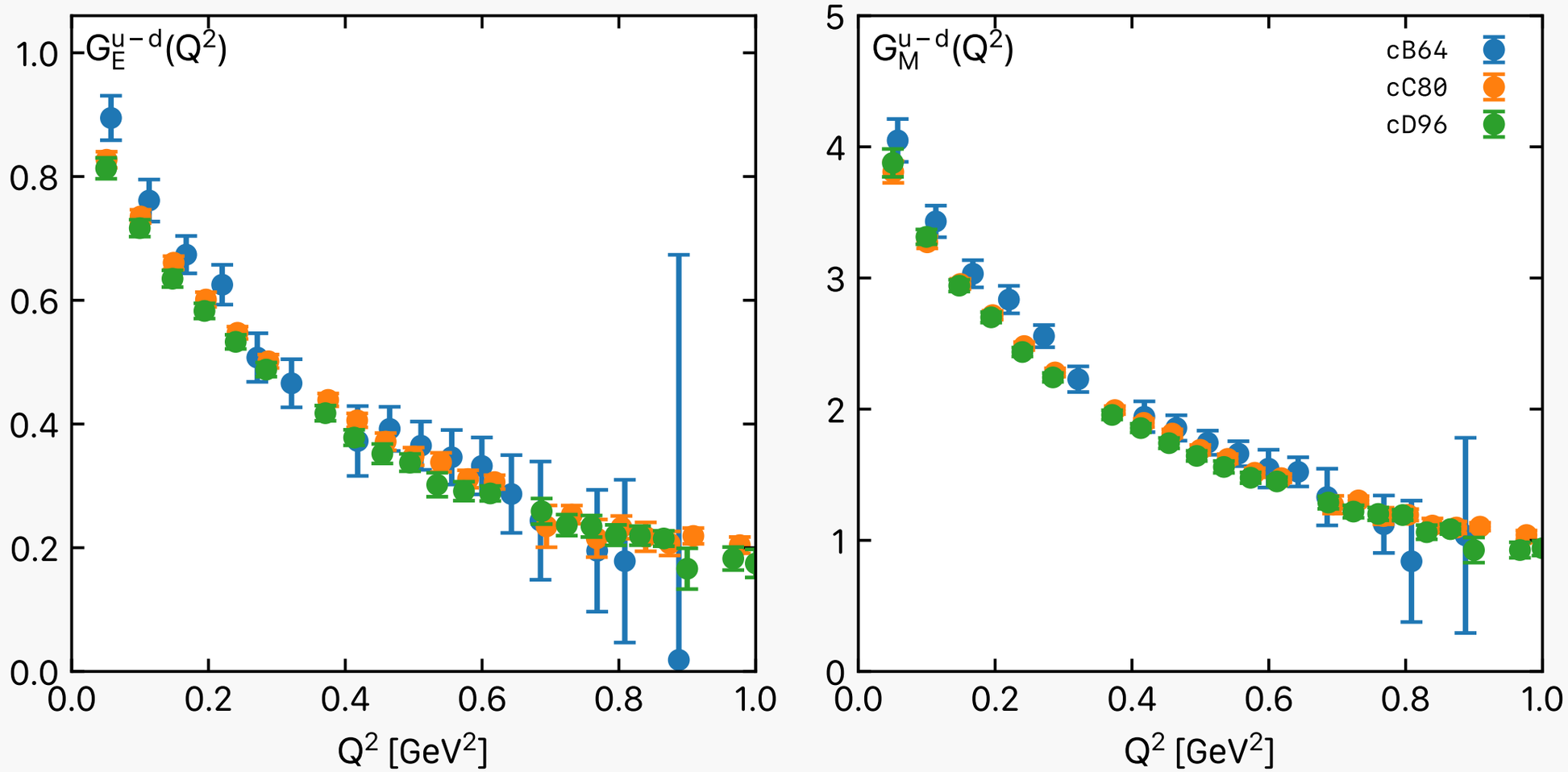


Example from **cC80** (intermediate a)

- High-Q^2:
  - Consistent for $Q^2 > 0.5\ GeV^2$

Isovector Form Factors

Excited states: Effect of fit choice

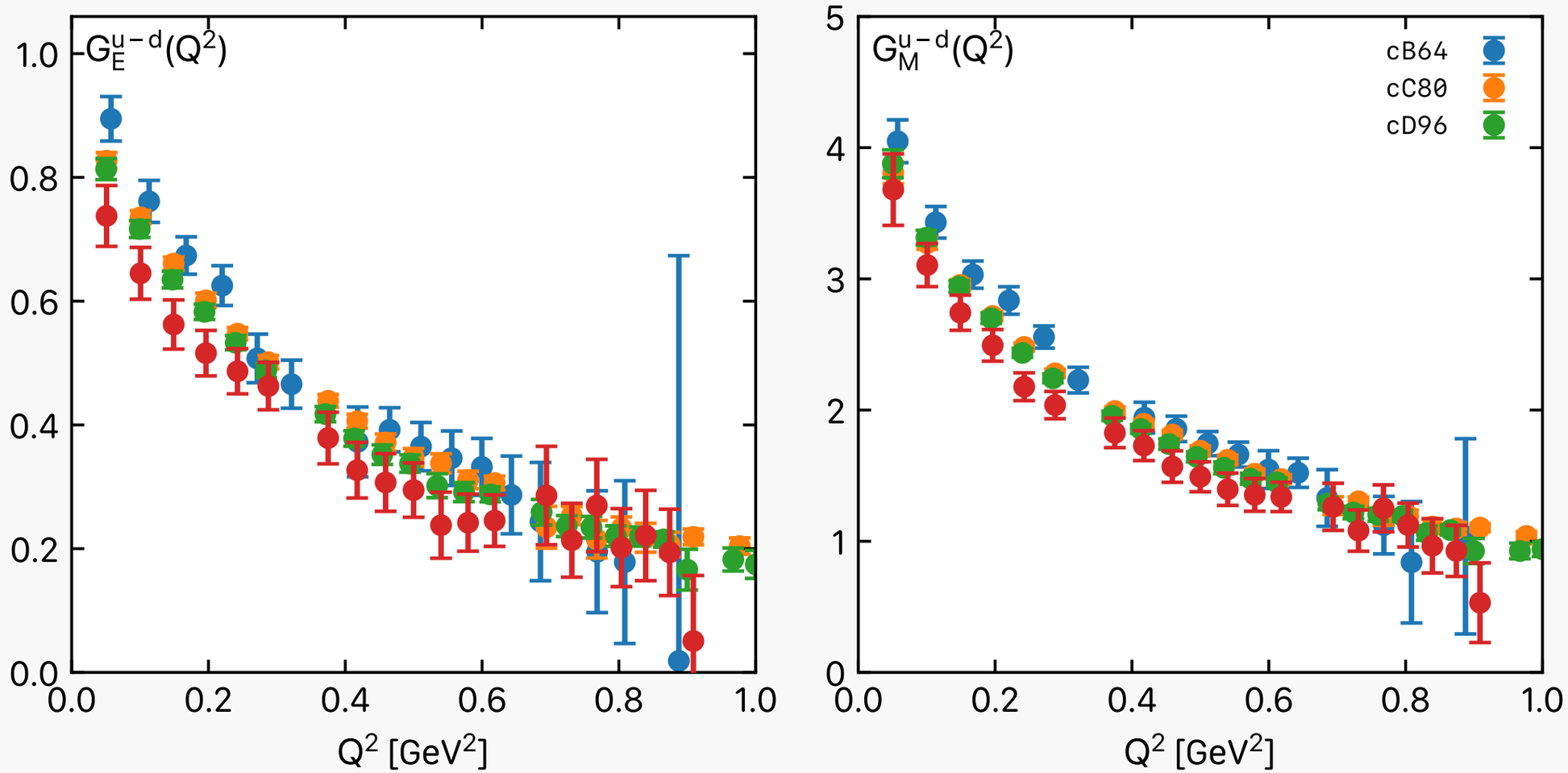


Three ensembles with: $a = 0.08,\ 0.068,\ 0.057\ \mathrm{fm}$

- Approach to continuum (at low-Q^2):
  - G_E towards smaller values
  - G_M towards larger values

Isovector Form Factors

Excited states: Effect of fit choice

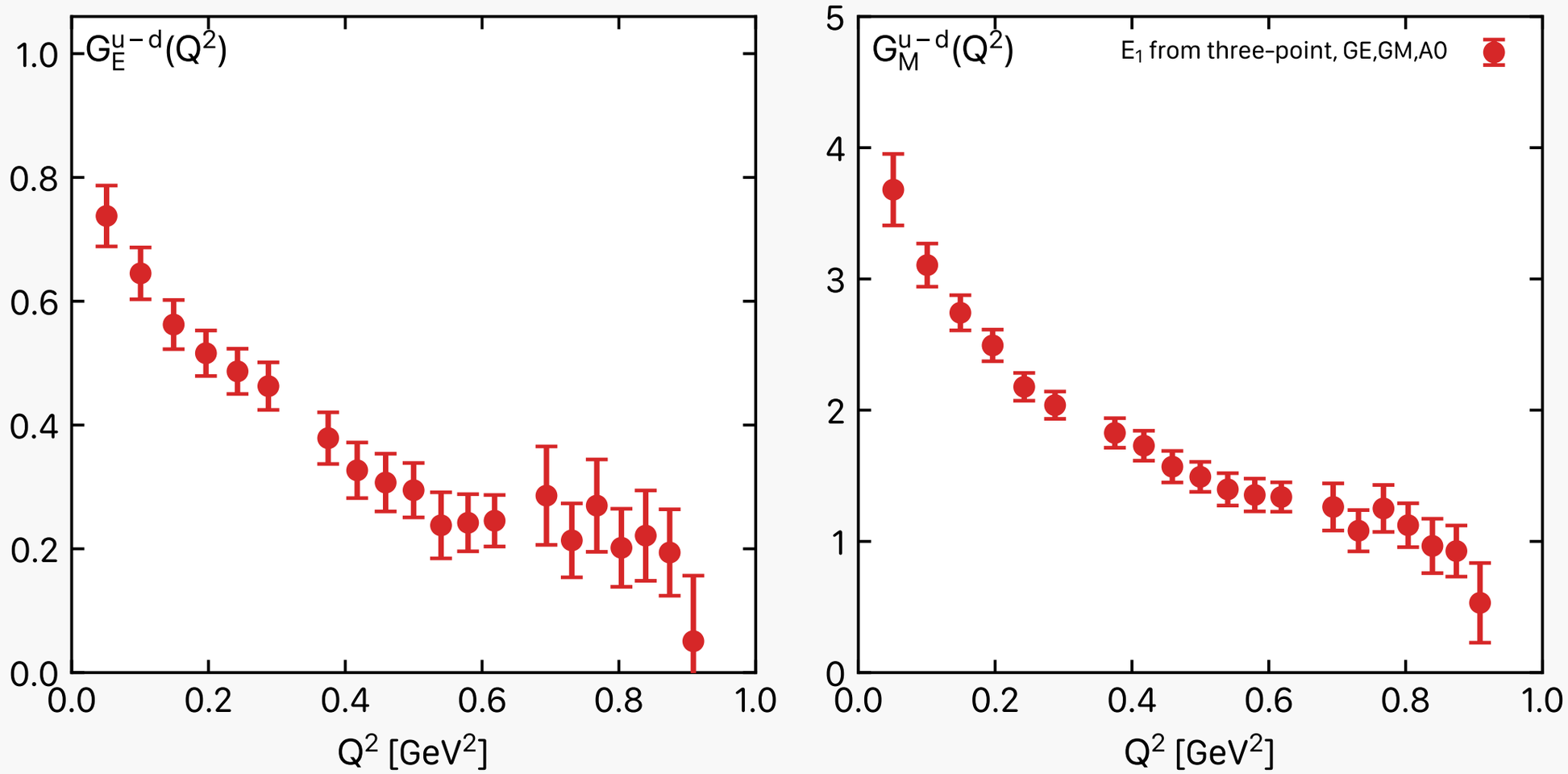


Three ensembles with: $a = 0.08,\ 0.068,\ 0.057\ \mathrm{fm}$

- ***Very preliminary*** continuum limit
  - Interpolate to Q^2 values of intermediate a
  - $G_{lat}(Q^2) = G_0(Q^2) + a^2\, G_1(Q^2)$

Isovector Form Factors

Excited states: Effect of fit choice



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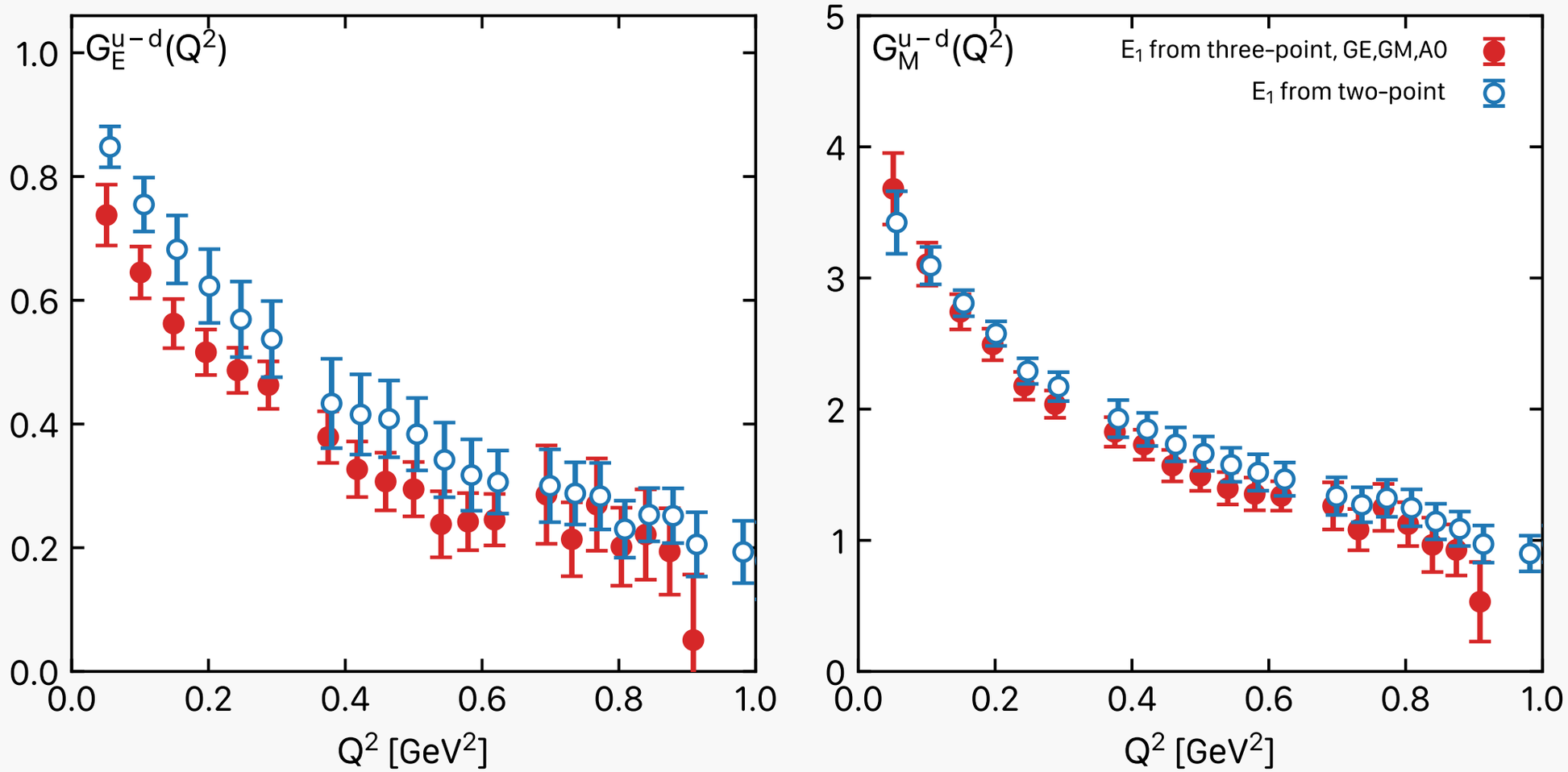

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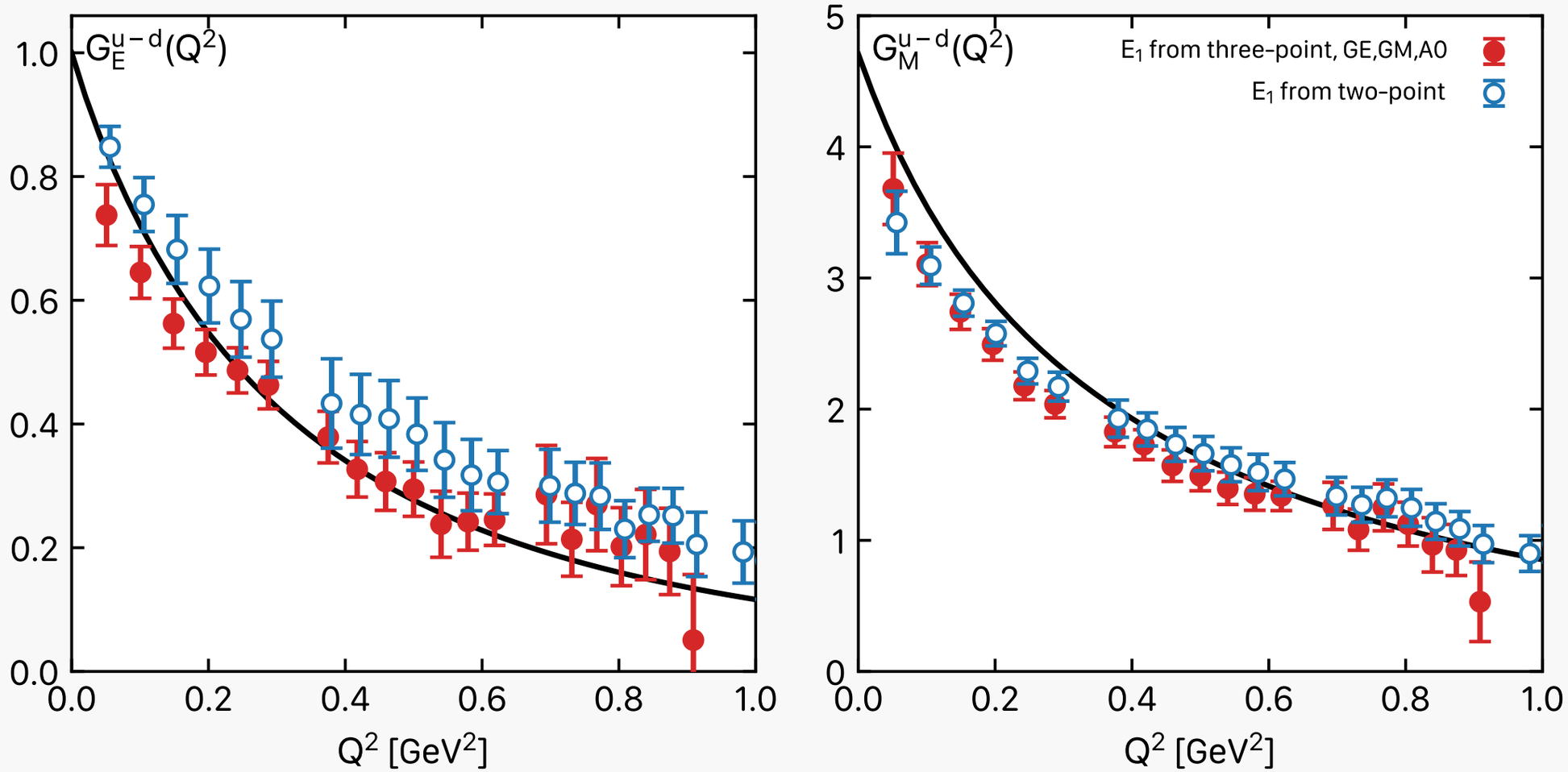
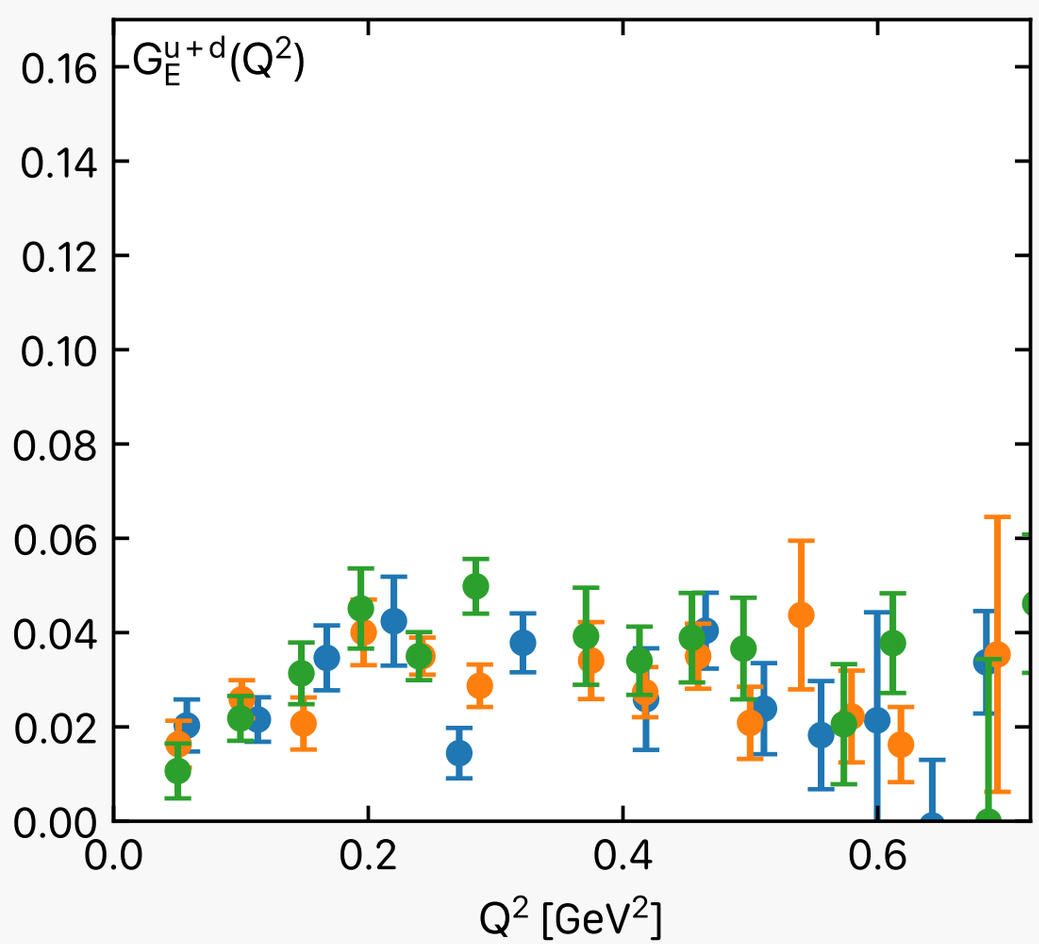
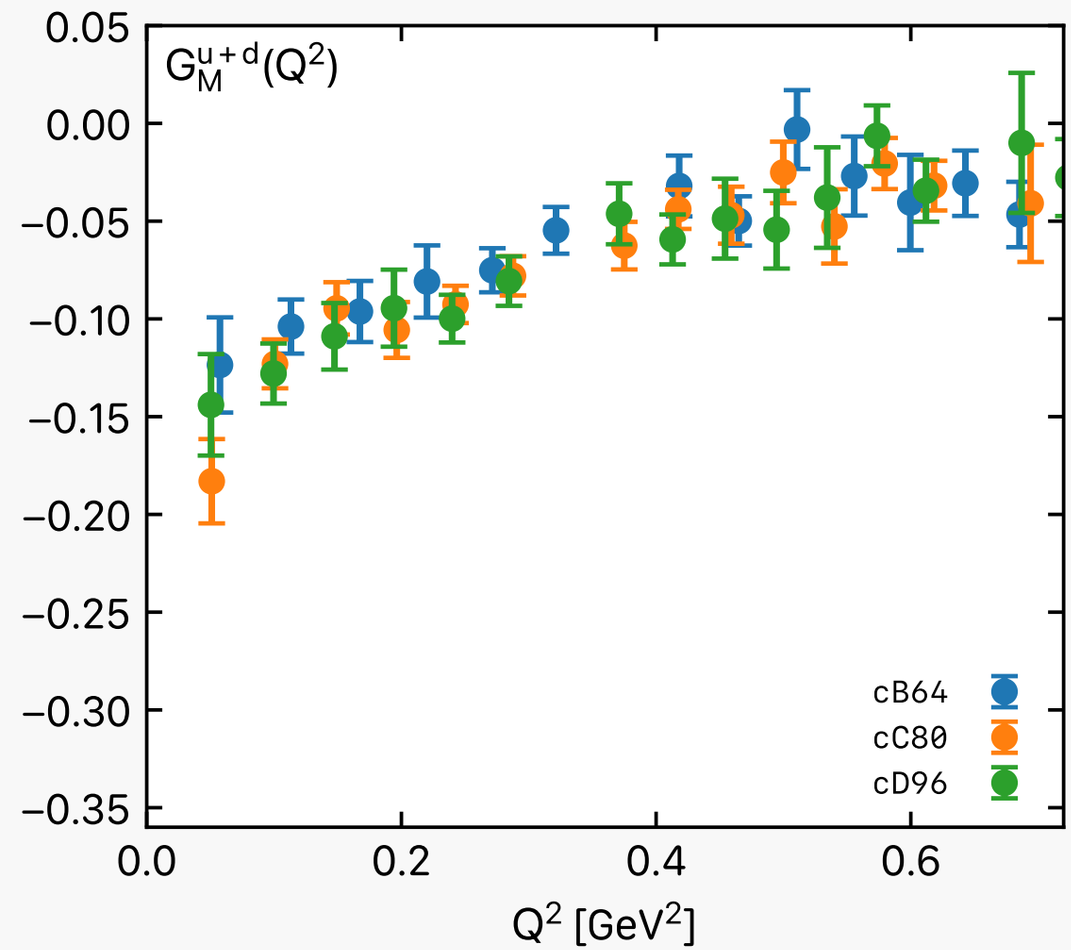


- Comparison to experiment: z-expansion fit (Yea, Arrington, Hill, Lee, Phys. Lett. B 777 (2018) 8-15 [1707.09063])
- Error dominated by coarsest lattice spacing ⇒ Statistics being increased

Disconnected contributions

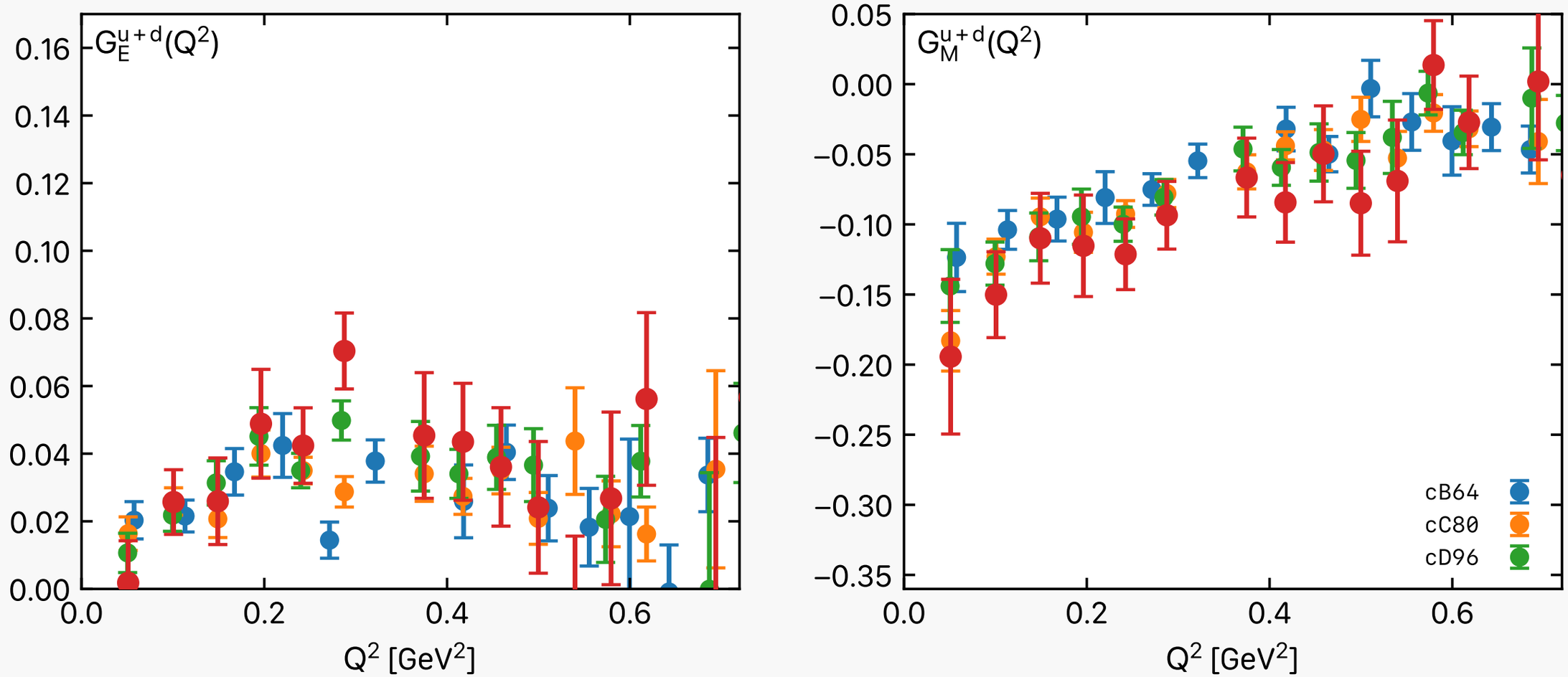




Three ensembles: $a = 0.08,\ 0.068,\ 0.057\ \mathrm{fm}$

- ***Showing:*** summation method

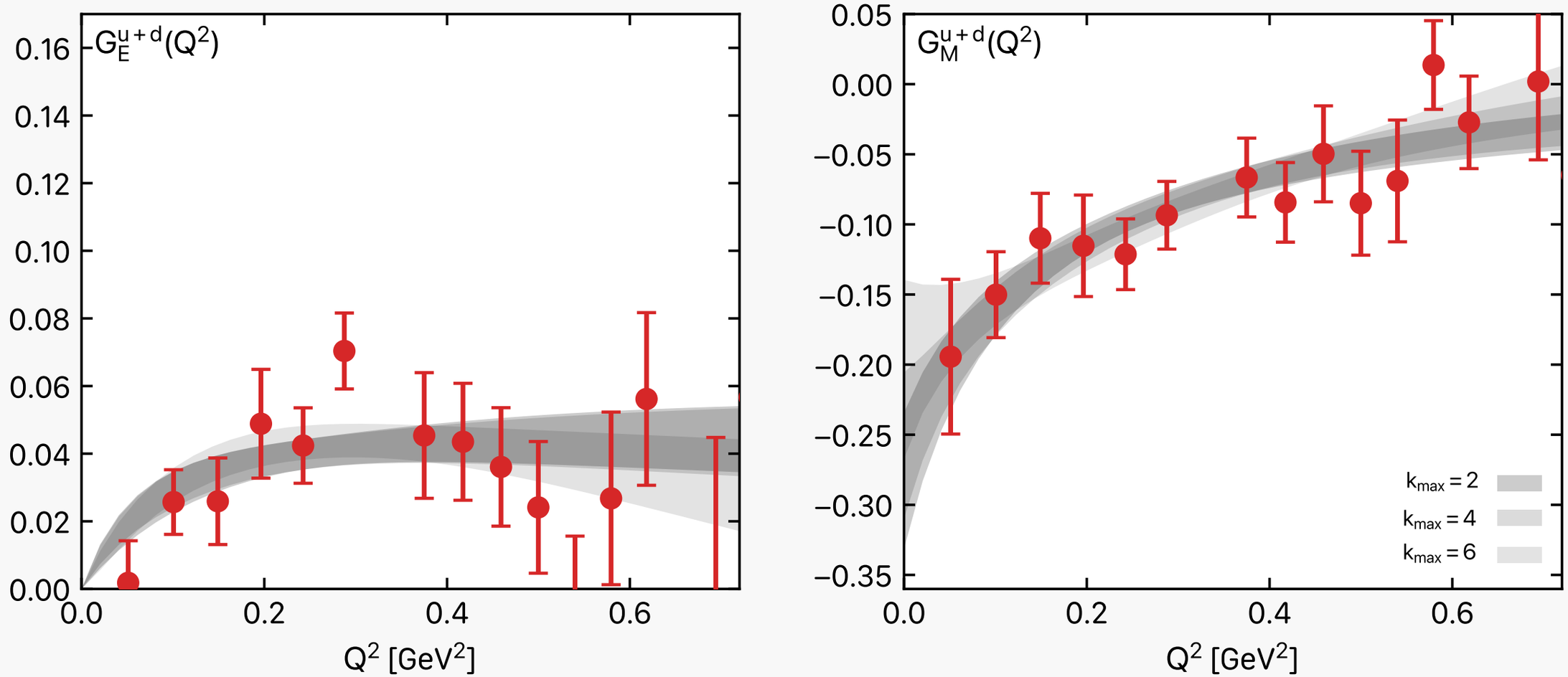
Disconnected contributions



Three ensembles: $a = 0.08,\ 0.068,\ 0.057\ \mathrm{fm}$

- ***Showing:*** summation method
  - Interpolate to Q^2 values of intermediate a
  - $G_{lat}(Q^2) = G_0(Q^2) + a^2\, G_1(Q^2)$

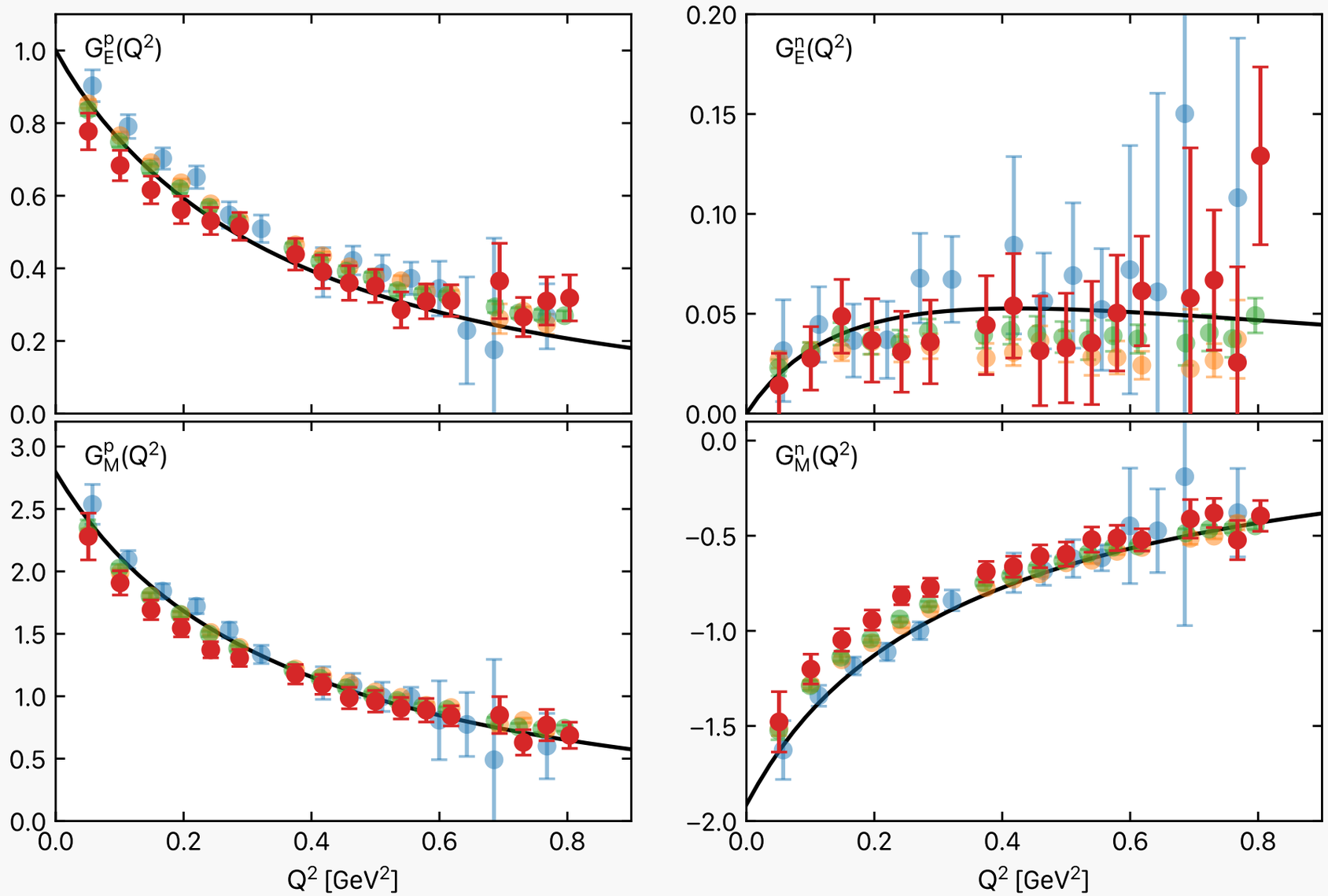
Disconnected contributions



Three ensembles: $a = 0.08,\ 0.068,\ 0.057\ \mathrm{fm}$

- ***Showing:*** summation method
  - Interpolate to Q^2 values of intermediate a
  - $G_{lat}(Q^2) = G_0(Q^2) + a^2\, G_1(Q^2)$

First Look: Proton and Neutron (combining disc.)



- Same interpolation as before
  - Interpolate to Q^2 values of intermediate a
  - $G_{lat}(Q^2) = G_0(Q^2) + a^2\, G_1(Q^2)$

Summary & Outlook

Nucleon EM form factors at the physical point

- Multiple sink-source separation with increasing statistics
- Three lattice spacings at the physical pion mass value
- Preliminary continuum extrapolation
  - Some tension ($1-2\sigma$) at the continuum limit for low-Q^2 values
  - Analysis for systematics on excited state treatment pending
- Ongoing
  - Increase in statistics of coarsest lattice spacing
  - Finalisation of systematic errors from excited states
  - Analysis and extraction of radii and moments
  - Larger volume ensembles
- Outlook
  - Complete $V \to \infty$ and $a \to 0$ extrapolation using $N_f=2+1+1$ ensembles all at the physical point

Acknowledgements

RESEARCH & INNOVATION FOUNDATION

ΕΥΡΩΠΑΪΚΗ ΕΝΩΣΗ

2016-2020 ReSTART SEARCH
ΠΡΟΓΡΑΜΜΑΤΑ ΕΡΕΥΝΑΣ, ΤΕΧΝΟΛΟΓΙΚΗΣ ΑΝΑΠΤΥΞΗΣ ΚΑΙ ΚΑΙΝΟΤΟΜΙΑΣ

ΚΥΠΡΙΑΚΗ ΔΗΜΟΚΡΑΤΙΑ

ΔΙΑΡΘΡΩΤΙΚΑ ΤΑΜΕΙΑ
οι ιδέες μας, πρόξη και ανάπτυξη

EXCELLENCE/0421/0195 POST-DOC/0718/0100

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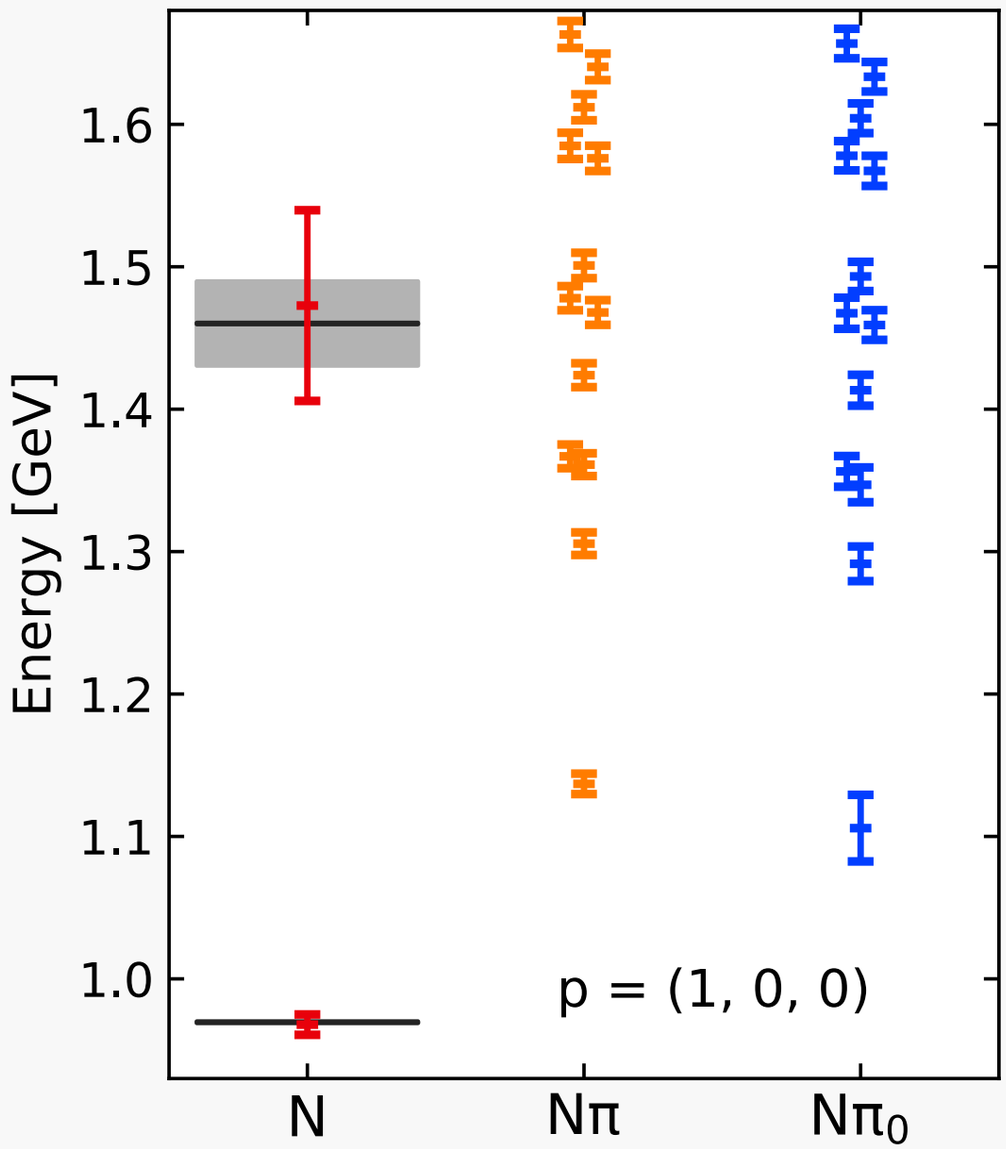
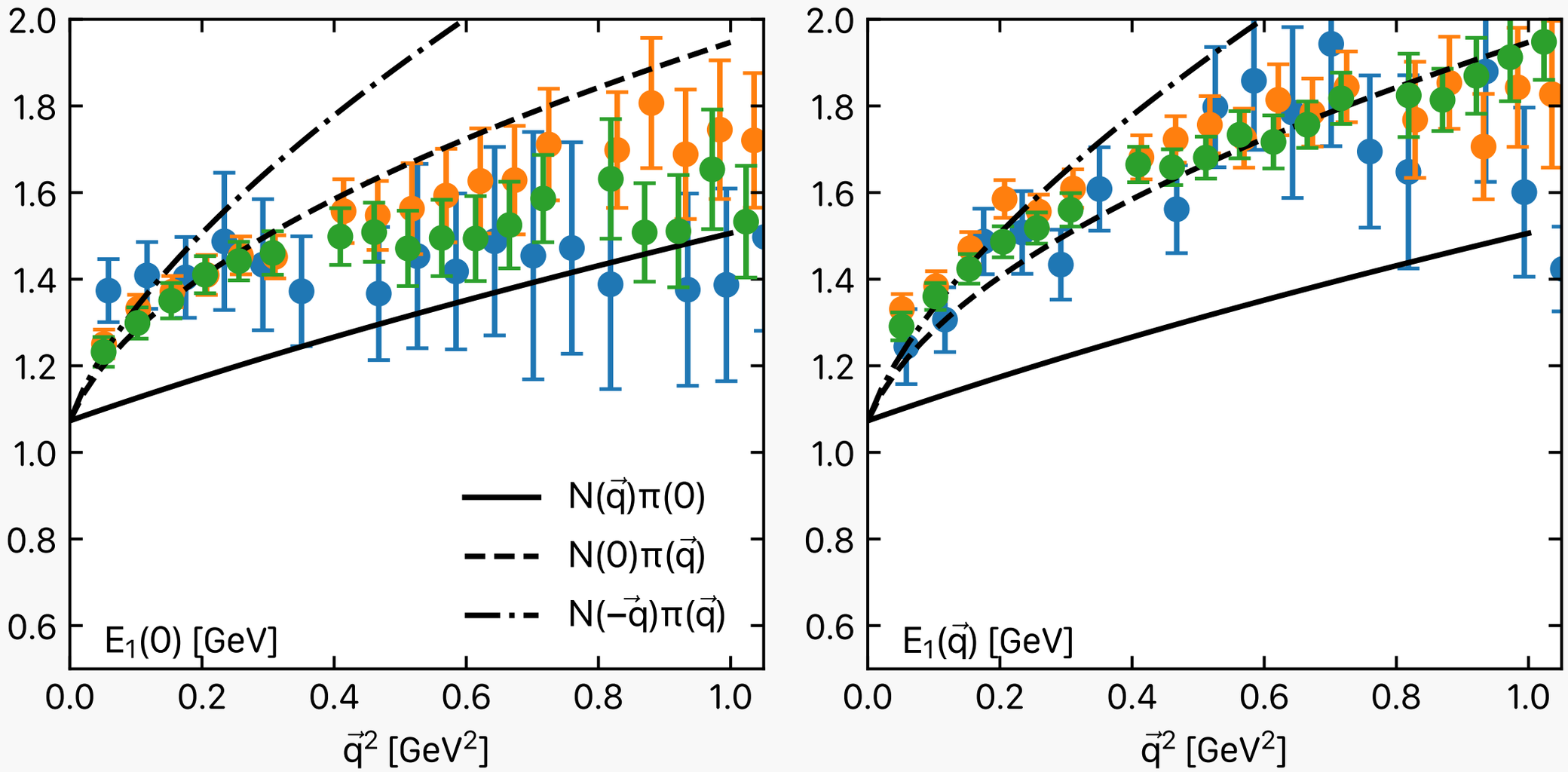
BACKUP



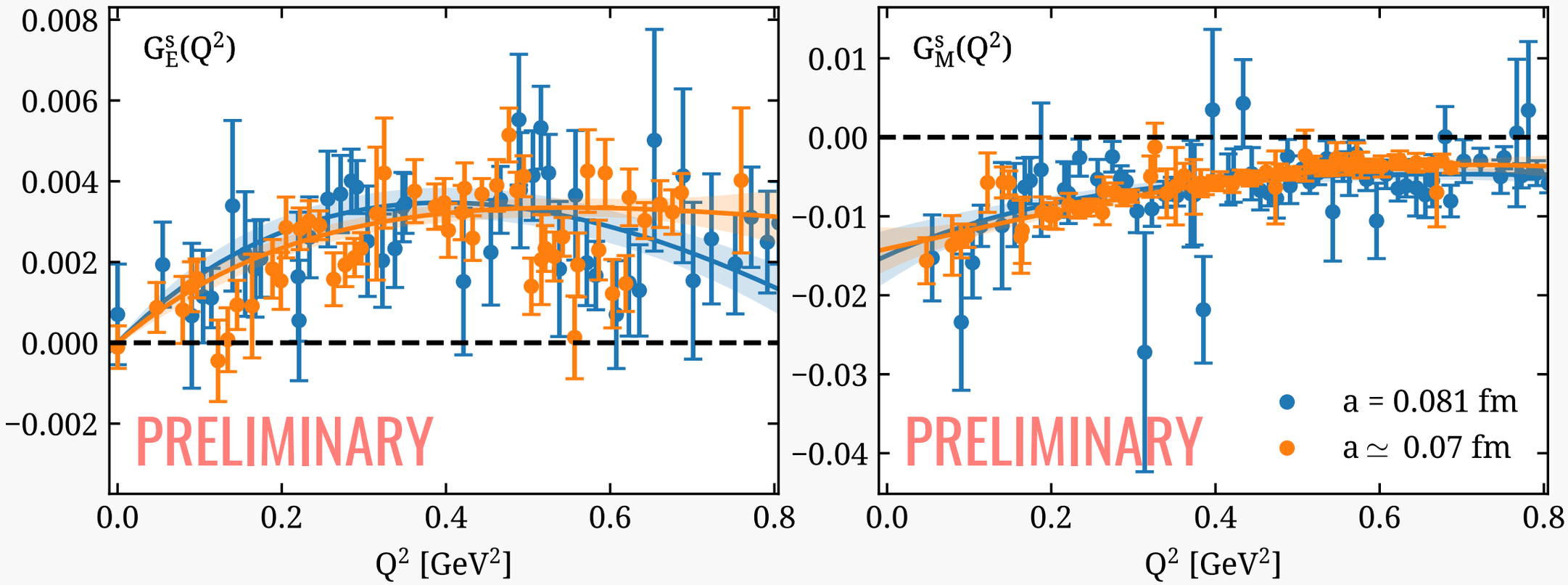
Nucleon states obtained from two-point function (here $\mathtt{cB64}$**)**

- $\mathrm{N}\pi$: energies of non-interacting states with m_N and m_π from two-point function
- Nucleon excited state from two-point function consistent with Roper

BACKUP


2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.0
0.2
0.4
0.6
0.8
1.0
$E_1(0)$ [GeV]
$\vec{q}^2$ [GeV^2]
$N(\vec{q})\pi(0)$
$N(0)\pi(\vec{q})$
$N(-\vec{q})\pi(\vec{q})$
$E_1(\vec{q})$ [GeV]
$\vec{q}^2$ [GeV^2]

BACKUP



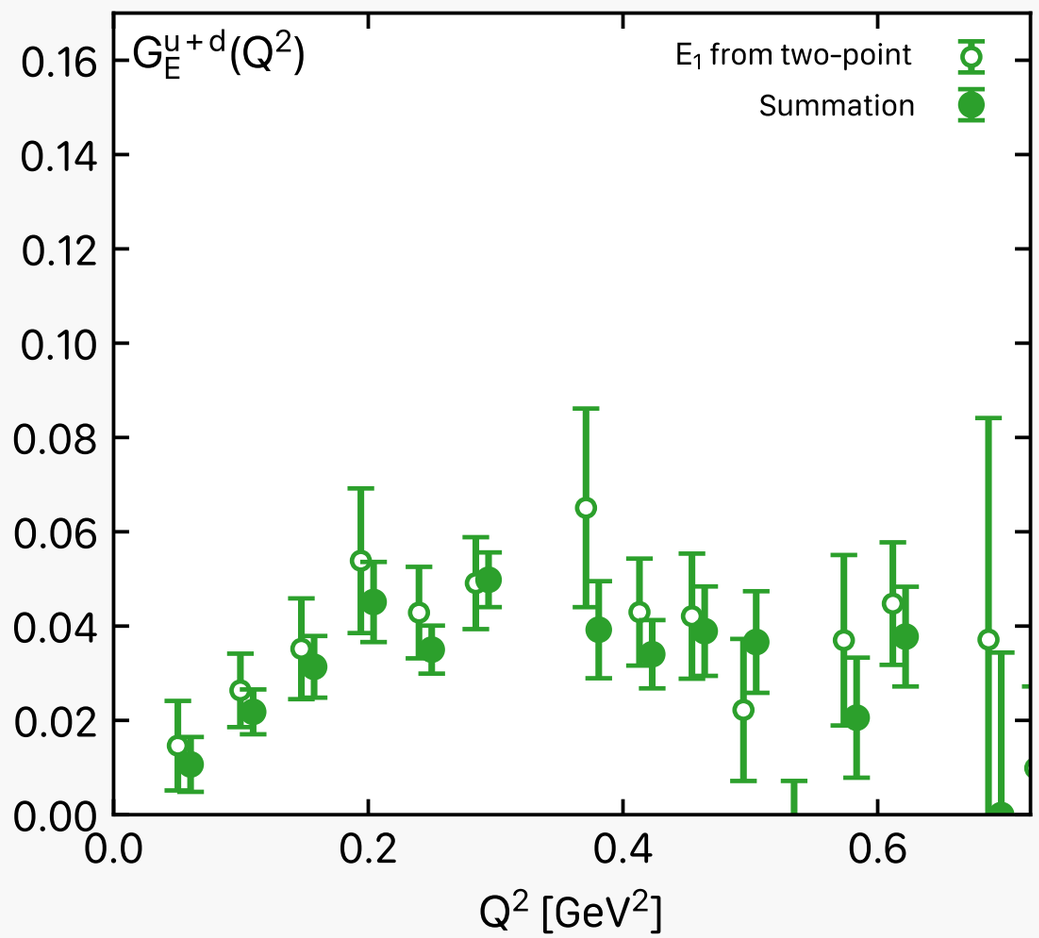
Strange EM form factors: not well known experimentally

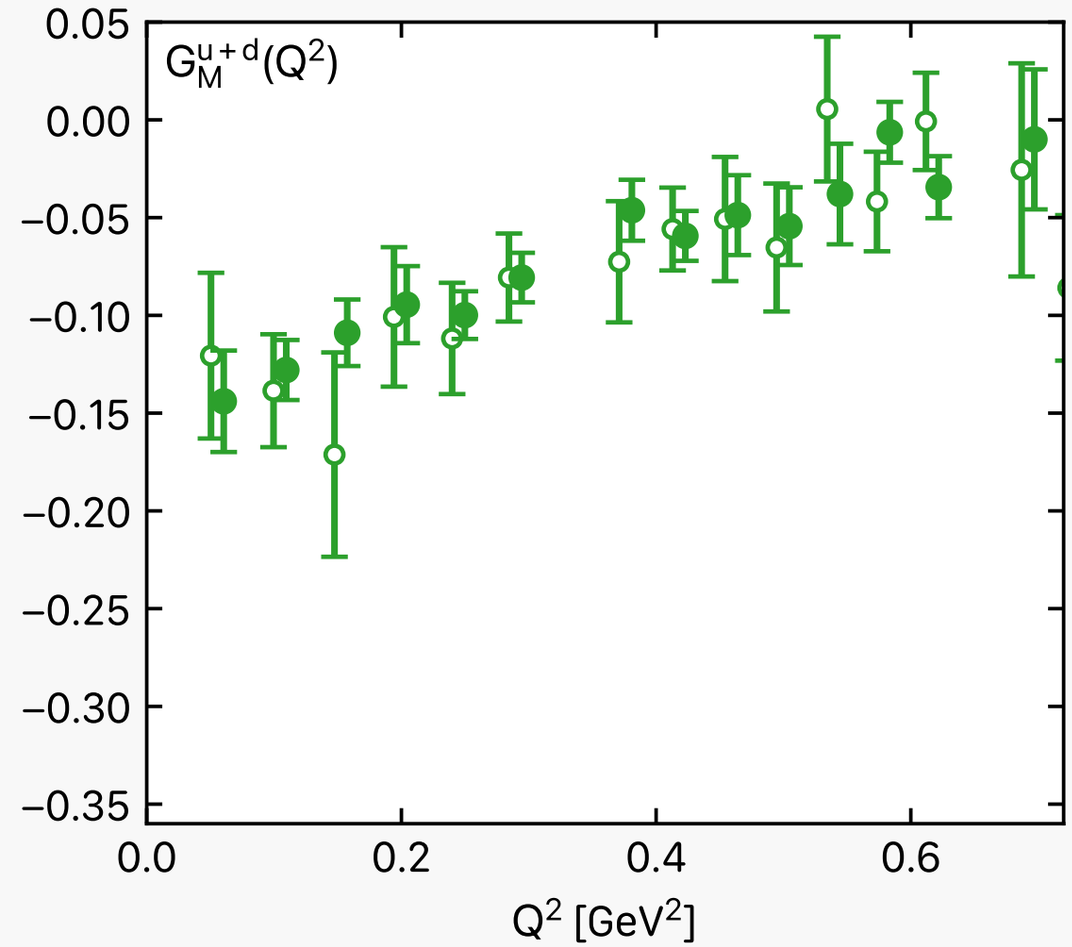
- Preliminary finer lattice spacing $a = 0.07\ \mathrm{fm}$
- Compare to best experimental results so far:
  - HAPPEX: $G_M^s(Q^2 \sim 0.62\ \mathrm{GeV}^2) = -0.070(67)$
  - A4: $G_E^s(Q^2 \sim 0.22\ \mathrm{GeV}^2) = 0.050(38)(19)$
    $G_M^s(Q^2 \sim 0.22\ \mathrm{GeV}^2) = 0.14(11)(11)$

Phys. Rev. **D101** (2020) 031501, `arXiv:1909.10744 [hep-lat]`

BACKUP

Disconnected contributions





Showing: Example from cD96 (finest a)

Statistics

$$R_\Gamma(P;\vec{q};t_s;t_{ins}) = \frac{G_\Gamma(P;\vec{q};t_s;t_{ins})}{G(\vec{0};t_s)}\sqrt{\frac{G(\vec{p};t_s-t_{ins})G(\vec{0};t_{ins})G(\vec{0};t_s)}{G(\vec{0};t_s-t_{ins})G(\vec{p};t_{ins})G(\vec{p};t_s)}}$$

Disconnected: High number of sources per config. + hierarchical probing, color/spin dilution and exact of low-mode estimation of loops