Mass and isovector matrix elements of the nucleon at zero-momentum transfer

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in collaboration with

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Observables

We study **isovector** nucleon matrix elements (NMEs)

$$\left\langle N(p',s')\,|\mathcal{O}|\,N(p,s)\right\rangle$$

at zero-momentum transfer $p' = p = 0$ for a set of six different operator insertions $\mathcal{O}$, i.e.

- $\mathcal{O}^A_\mu = \bar{q}\gamma_\mu\gamma_5 q, \quad \mathcal{O}^S = \bar{q}q, \quad \mathcal{O}^T_{\mu\nu} = \bar{q}i\sigma_{\mu\nu}q$
- $\mathcal{O}^{vD}_{\mu\nu} = \bar{q}\gamma_{\{\mu}\overleftrightarrow{D}_{\nu\}}\,q\,, \quad \mathcal{O}^{aD}_{\mu\nu} = \bar{q}\gamma_{\{\mu}\gamma_5\overleftrightarrow{D}_{\nu\}}\,q\,, \quad \mathcal{O}^{tD}_{\mu\nu\rho} = \bar{q}\sigma_{[\mu\{\nu]}\overleftrightarrow{D}_{\rho\}}\,q$

$\rightarrow$ g_A^{u-d}, g_T^{u-d}, g_S^{u-d} from local operators and $\langle x\rangle_{u-d}$, $\langle x\rangle_{\Delta u-\Delta d}$, $\langle x\rangle_{\delta u-\delta d}$ from twist-2 operators

Analysis requires:

- Computation of two- and three-point functions.
- Extraction of ground state NMEs from a dedicated method to tame excited states.
- Chiral, continuum and finite volume (CCF) extrapolation to obtain physical results.

Statistically **very precise data** for the nucleon mass M_N on the same set of two-point functions:

- Physical extrapolation of m_N provides a cross-check for scale setting.

Ensembles and setup

ID	a/fm	T/a	L/a	M_π / MeV	$M_\pi L$	N_{conf}	N_{meas}	$t_{\mathrm{sep}}^{\mathrm{lo}}$ / fm	$t_{\mathrm{sep}}^{\mathrm{hi}}$ / fm	$N_{t_{\mathrm{sep}}}$
C101	0.086	96	48	0.225	4.73	2000	64000	0.35	1.47	14
N101		128	48	0.282	5.91	1595	51040			
H105		96	32	0.281	3.93	1027	49296			
H102		96	32	0.354	4.96	2005	32080			
D450	0.076	128	64	0.216	5.35	500	64000	0.31	1.53	17
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- $N_f = 2+1$ flavors of non-perturbatively improved Wilson clover fermions. *JHEP 1502 (2015) 043*
- Lüscher-Weisz gauge action *Commun.Math.Phys. 97 (1985)*
- Twisted mass regulator to suppress exceptional configurations. *PoS LATTICE2008 (2008) 049*
- Production of correlators is complete / available statistics now fully included in analysis.

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  $\rightarrow$ continuum extrapolation
- Many different physical volumes with $L \approx 2 \ldots 6\,\mathrm{fm}$, typically $M_\pi L > 4$.
  $\rightarrow$ extrapolation to infinite volume / check for finite size effects.
- Pion masses from $\sim 130\,\mathrm{MeV}$ to $\sim 350\,\mathrm{MeV}$
  $\rightarrow$ chiral extrapolation and checking its convergence
- Two very large and fine boxes at (near) physical quark mass.

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- **Large number of source-sink separations available, typically $t_{\mathrm{sep}} \approx 0.3 \ldots 1.5\,\mathrm{fm}$.**
- N_{meas} reduced by factor of two in steps of $\Delta t_{\mathrm{sep}} \approx 0.2\,\mathrm{fm}$ for $t_{\mathrm{sep}} < 1\,\mathrm{fm}$.
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Further details

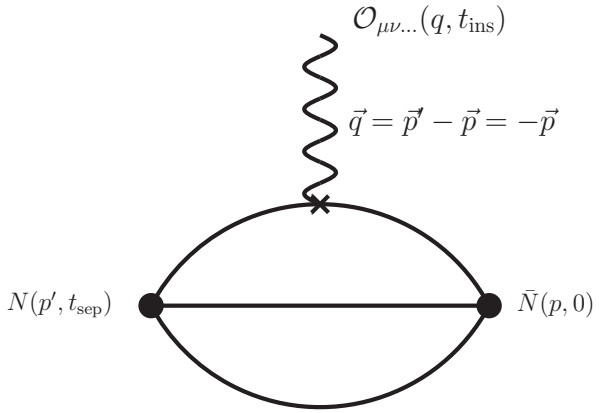
- NMEs are computed from the usual ratio with projector $\Gamma_z = \frac{1}{2}(1+\gamma_0)(1+i\gamma_5\gamma_3)$

$$R^{\mathcal{O}}_{\mu_1,\ldots,\mu_n}(t_{\rm sep}, t_{\rm ins}) \equiv \frac{C^{\mathcal{O},3{\rm pt}}_{\mu_1,\ldots\mu_n}(\vec{q}=0, t_{\rm sep}, t_{\rm ins}; \Gamma_z)}{C^{2{\rm pt}}(\vec{q}=0, t_{\rm sep}; \Gamma_z)} \,. \tag{1}$$

- For the nucleon mass we use $C^{2{\rm pt}}(\vec{q}=0, t_{\rm sep}; \Gamma_0)$ with $\Gamma_0 = \frac{1}{2}(1+\gamma_0)$ to improve statistics.

- For 3pt functions we use sequential inversions through the sink, setting $p' = 0$.
- Only quark-connected 3pt functions for isovector NMEs.
- Truncated solver method gives speedup of 2-5:

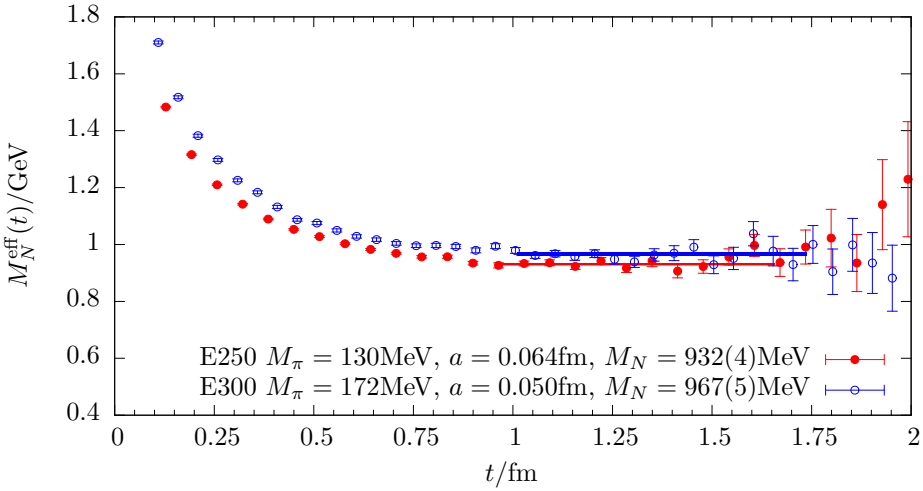
Comput.Phys.Commun. 181 (2010) 1570-1583
Phys.Rev. D91 (2015) no.11, 114511

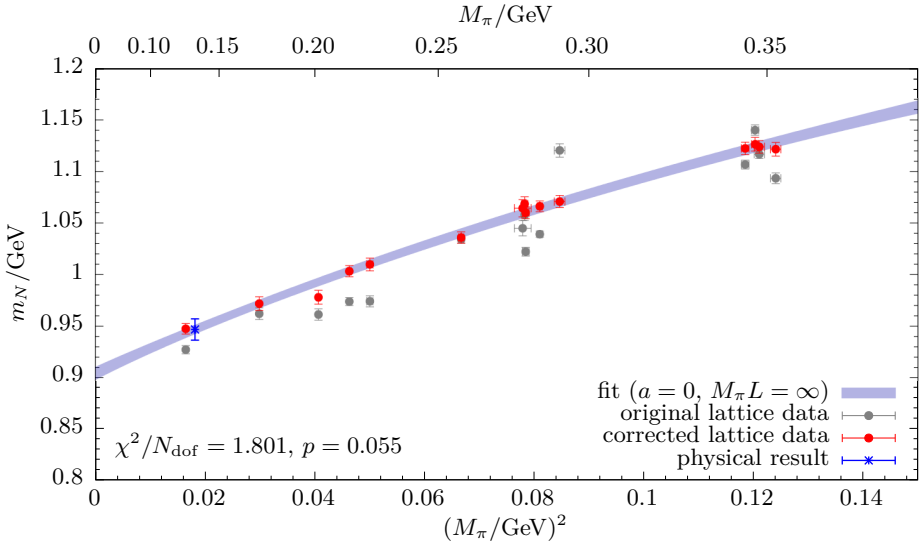


$$\langle \mathcal{O} \rangle = \langle \frac{1}{N_{LP}} \sum_{i=1}^{N_{LP}} \mathcal{O}_n^{LP} \rangle + \langle \mathcal{O}_{\rm bias} \rangle \,, \quad \mathcal{O}_{\rm bias} = \frac{1}{N_{HP}} \sum_{i=1}^{N_{HP}} (\mathcal{O}_n^{HP} - \mathcal{O}_n^{LP}) \,.$$

- Full non-perturbative renormalization (SF) available for g_A. Eur.Phys.J.C 79 (2019) 1, 23
- For other observables non-perturbative renormalization (RI'-MOM) at $\beta = 3.40, 3.46, 355$; Extrapolation for $\beta = 3.7$ as in 2019 paper. Phys.Rev.D 100 (2019) 3, 034513

M_N analysis





- Statistical error of M_N lattice data typically at a few per mille.
- Chiral, continuum and finite volume extrapolation from χPT-inspired fit model up to $\mathcal{O}(M_\pi^3)$

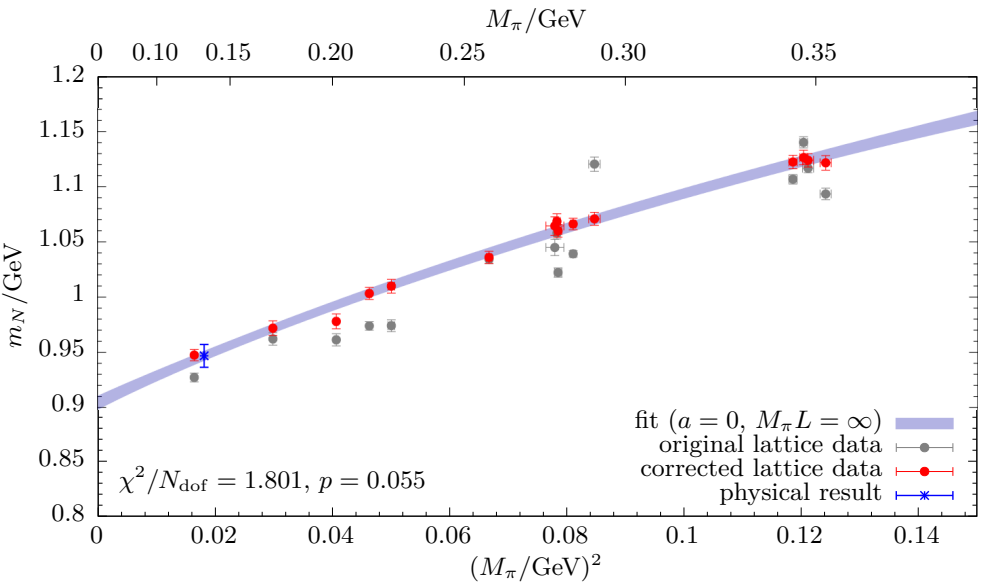
$$m_N(M_\pi, a, L) = \mathring{m}_N + BM_\pi^2 + CM_\pi^3 + Da^2 + E\frac{M_\pi^3}{(M_\pi L)}e^{-M_\pi L}.$$

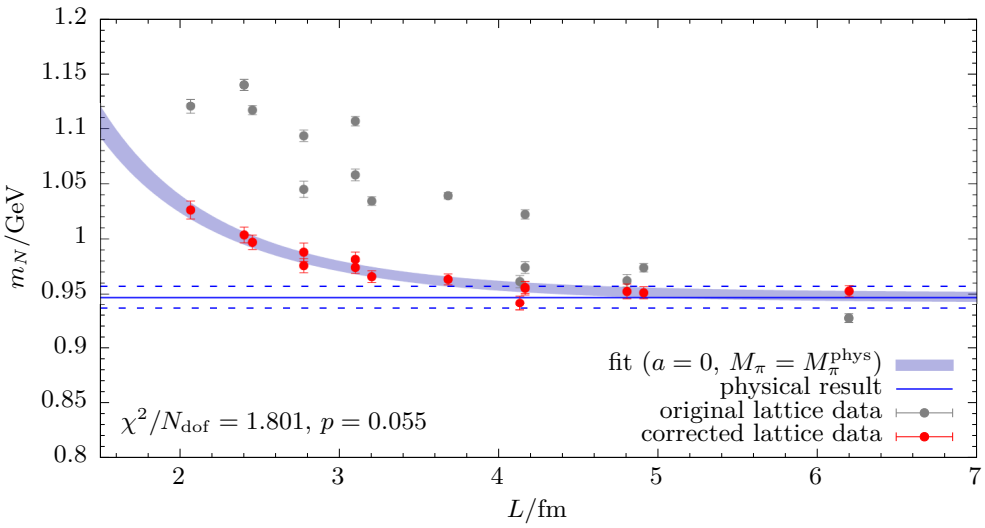
Phys. Lett. B 649, 390 (2007)

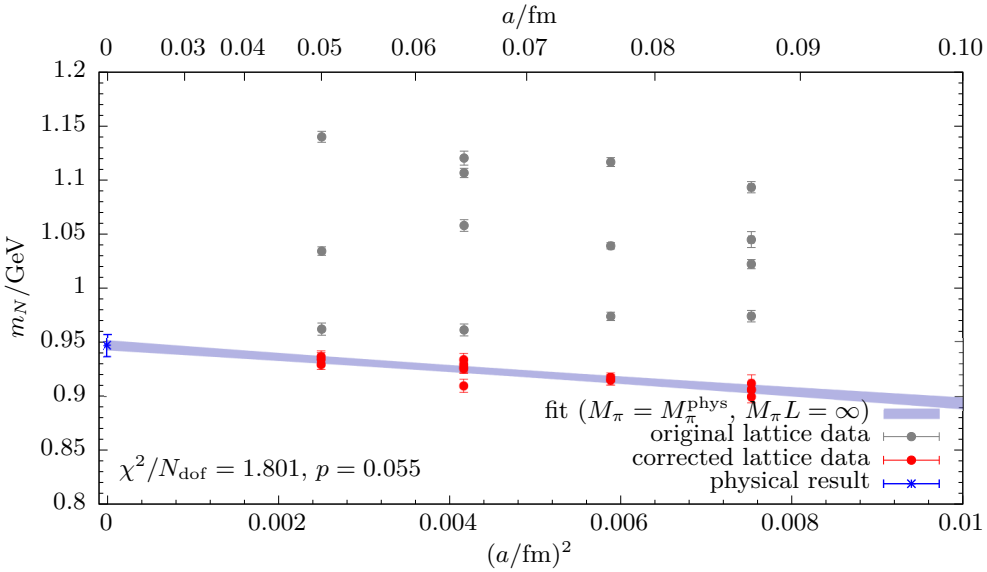
with $\mathring{m}_N$, B, C, D and E free parameters of the fit.

- **Physical result** $M_N = 947(10)\mathrm{MeV}$ dominated by scale setting error.
- In agreement with experimental value $\rightarrow$ Xcheck for scale setting.
- Large corrections for individual data points ...

Effects from finite volume and continuum extrapolation



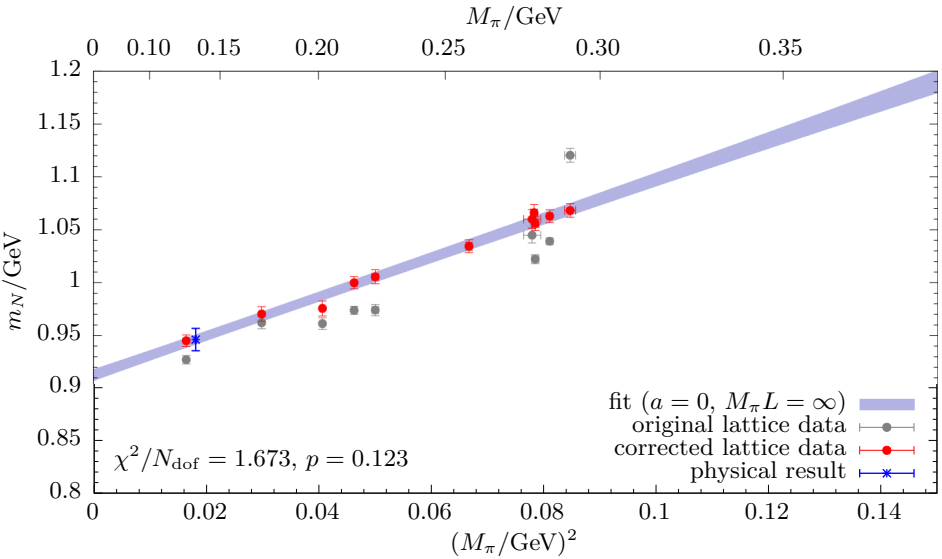


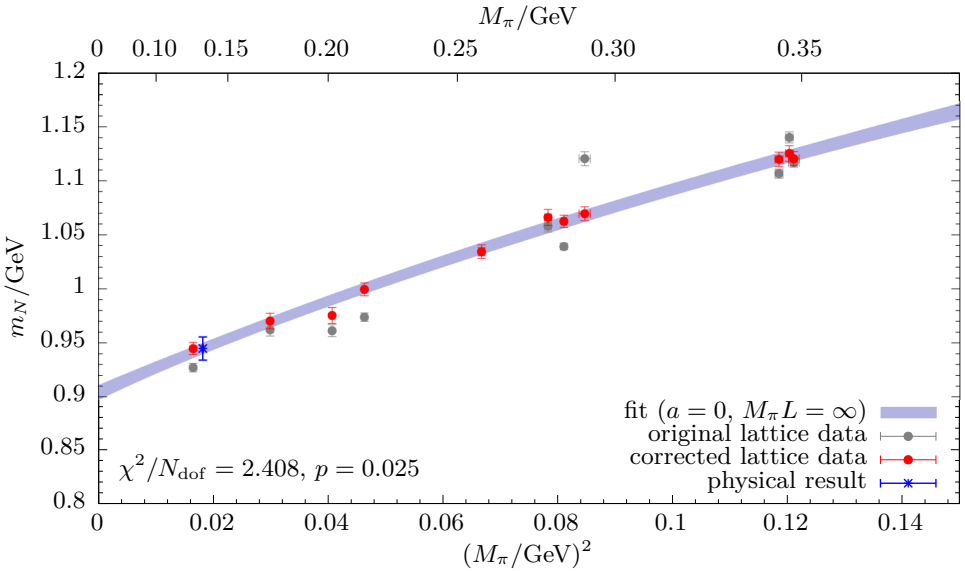


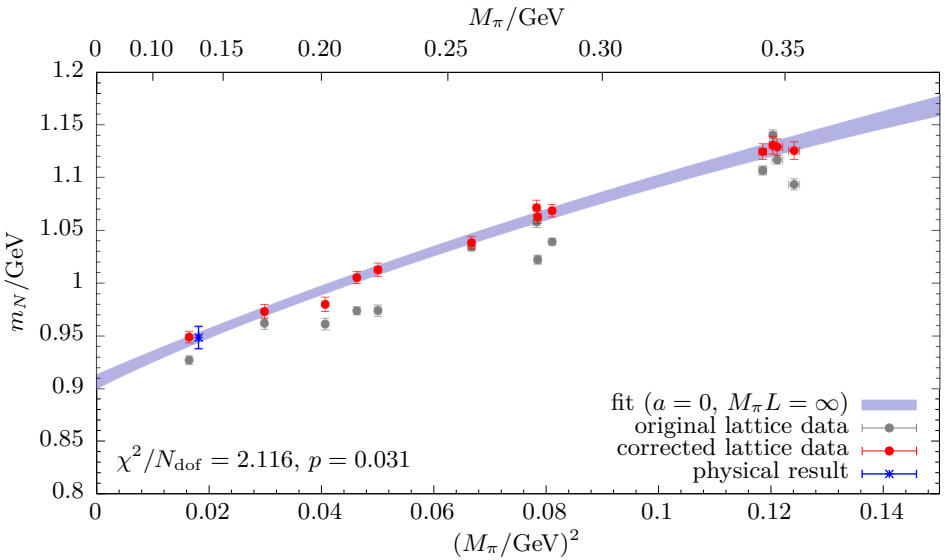
- Steep chiral extrapolation $\rightarrow$ largest corrections.
- Finite volume corrections significant on top of chiral extrapolation.
- Effect from continuum extrapolation not negligible.

$\Rightarrow$ **All three extrapolations are required s.t. corrected data aligns with the fitted curve.**

Fit stability / systematics







Perform cuts to study systematics:

1. $M_\pi < 300\mathrm{MeV}$: $M_N = 946(11)\,\mathrm{MeV}$
2. $a < 0.08\mathrm{fm}$: $M_N = 945(11)\,\mathrm{MeV}$
3. $M_\pi L > 4$: $M_N = 948(10)\,\mathrm{MeV}$

Physical result is very stable.

<u>However</u>: M_π-cut affects slope of extrapolation.

Excited states

We consider two fits models for the summed ratio $S(t_{\rm sep}) = \sum_{t_{\rm ins}=a}^{t_{\rm sep}-a} R(t_{\rm ins}, t_{\rm sep})$:

1. Plain summation method fits to individual observables:

$$\boxed{S(t_{\rm sep}) = {\rm const} + M_{00}(t_{\rm sep} - a)\,.}$$

2. Two-state truncation

$$S(t_{\rm sep}) = M_{00}(t_{\rm sep} - a) + 2\tilde{M}_{01}\frac{e^{-\Delta a} - \left(1 + \frac{|A_1|^2}{|A_0|^2}e^{-\Delta a}\right)e^{-\Delta t_{\rm sep}}}{1 - e^{-\Delta a}} + \tilde{M}_{11}e^{-\Delta t_{\rm sep}}(t_{\rm sep} - a) + \mathcal{O}(e^{-2\Delta t_{\rm sep}})\,.$$

Terms $\sim \frac{|A_1|^2}{|A_0|^2}$ ($\tilde{M}_{11} = M_{11}\frac{|A_1|^2}{|A_0|^2}$) not constrained at our level of statistics and excluded from final fits:

$$\boxed{S(t_{\rm sep}) = M_{00}(t_{\rm sep} - a) + 2\tilde{M}_{01}\frac{e^{-\Delta a} - e^{-\Delta t_{\rm sep}}}{1 - e^{-\Delta a}}}\,.$$

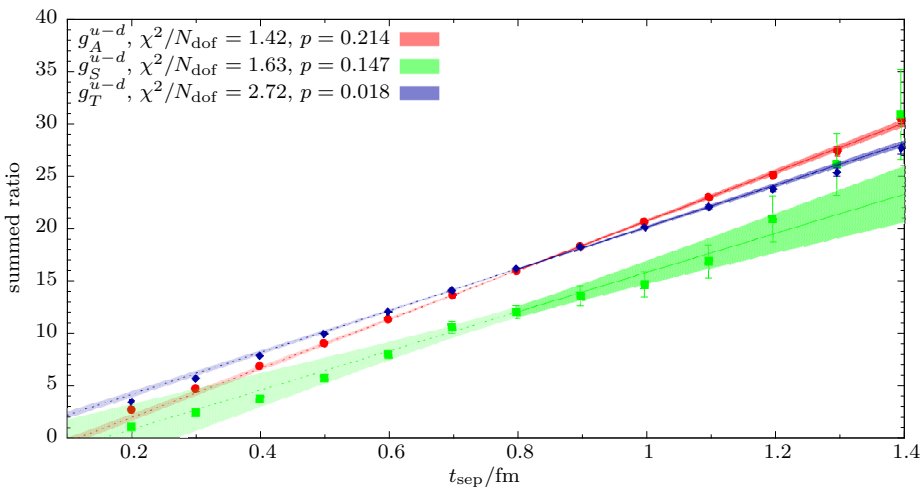
- Fits are carried out simultaneously for $g_{A,S,T}^{u-d}$ and $\langle x\rangle_{u-d}$, $\langle x\rangle_{\Delta u-\Delta d}$, $\langle x\rangle_{\delta u-\delta d}$.

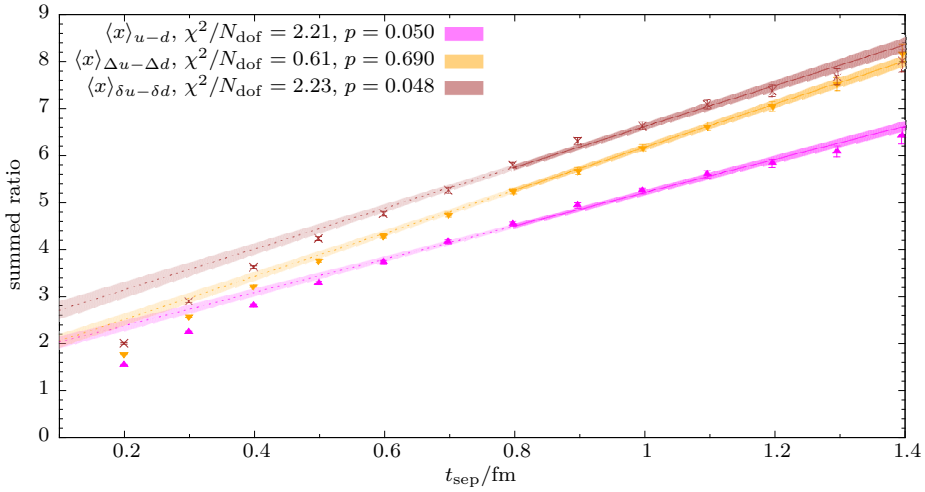
  $\Rightarrow$ **Correlation helps to reduce errors.**

- (Much) smaller covariance matrices than for (simultaneous) ratio based fits.

  $\Rightarrow$ **Simultanous two-state summation fits are more stable than ratio fits (no priors).**

Plain vs simultaneous two-state summation method (local NMEs)

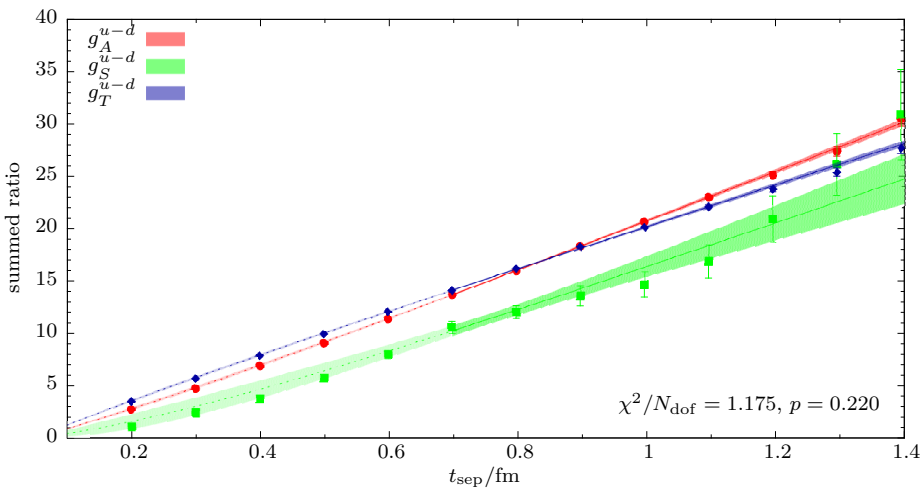


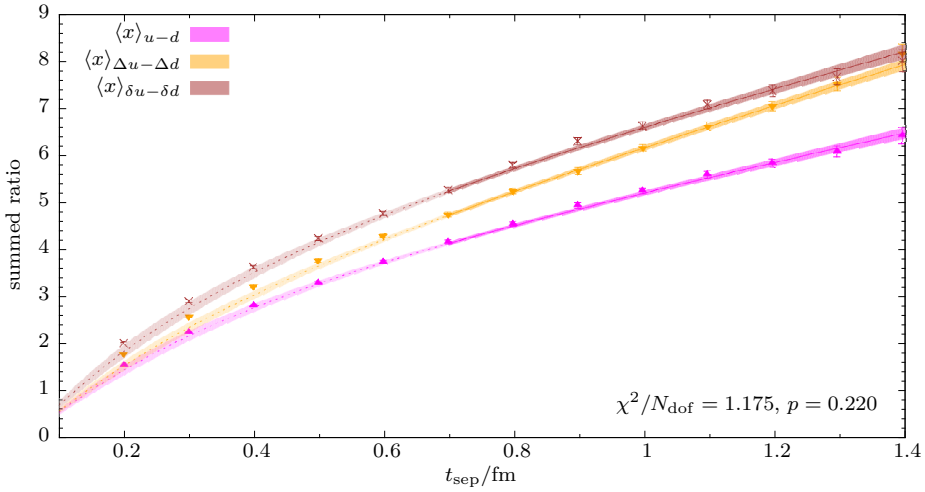


Plain summation method fits for local operator insertions on E300 ensemble ($M_\pi = 173\,\mathrm{MeV}$, $a \approx 0.050\,\mathrm{fm}$).

- Deviation from linear behavior at small values of $t_{\rm sep}$.
- Observables are fitted independently.

Plain vs simultaneous two-state summation method (local NMEs)

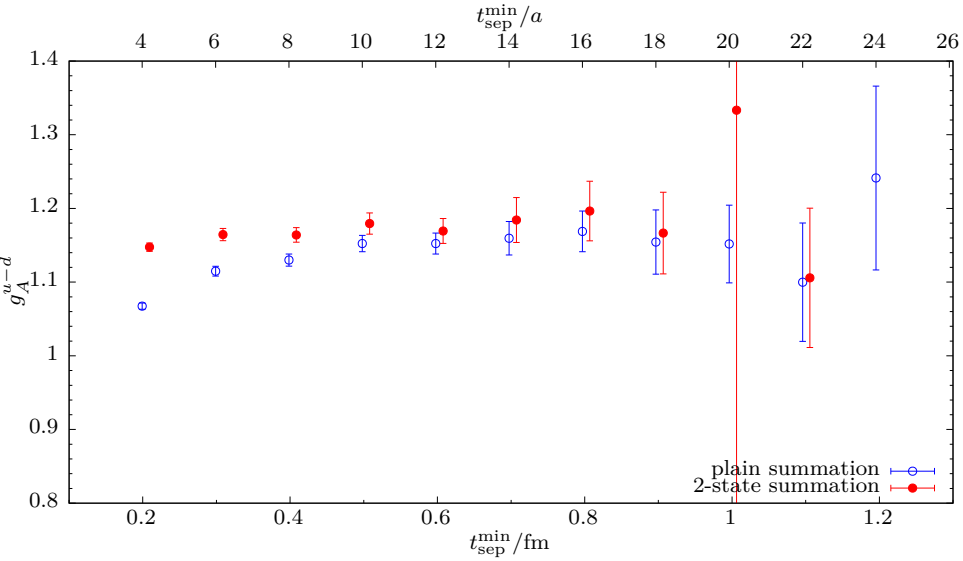




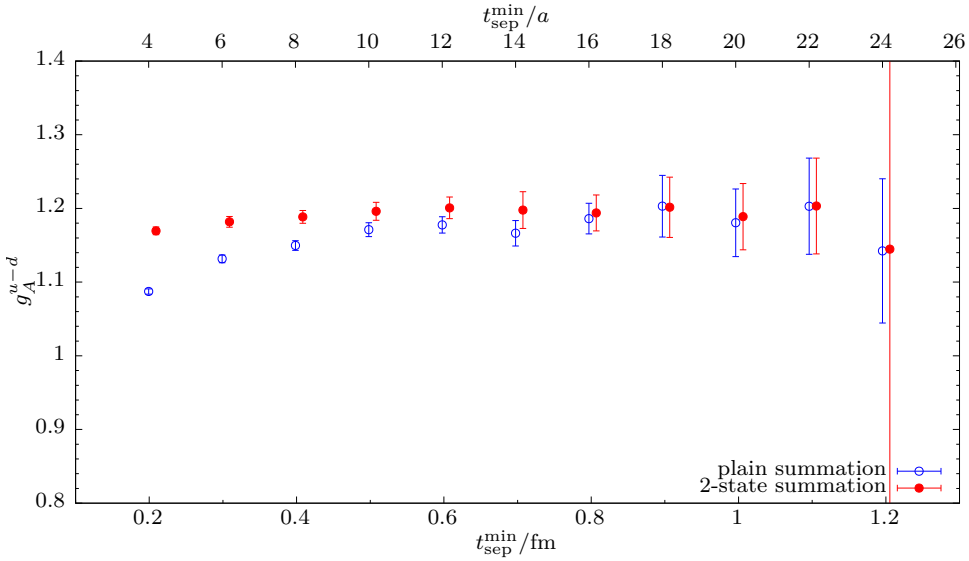
Simultaneous two-state summation method fits for local operator insertions on E300 ensemble ($M_\pi = 173\,\mathrm{MeV}$, $a \approx 0.050\,\mathrm{fm}$).

- Data described well by two-state fit to much smaller $t_{\rm sep}$.
- **All six observables are fitted simultaneously.**

Convergence of plain and two-state summation method for g_A^{u-d}

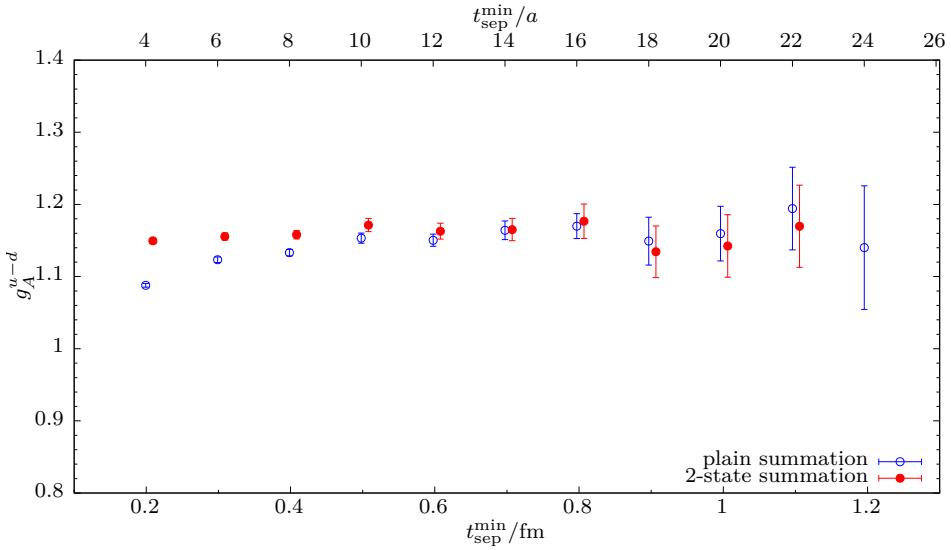


E300 $M_\pi = 173\,\mathrm{MeV}$, $a = 0.050\,\mathrm{fm}$



J303 $M_\pi = 260\,\mathrm{MeV}$, $a = 0.050\,\mathrm{fm}$

- Plain summation and two-state fits converge.
- Two-state fit allows to include smaller $t_{\rm sep}$.
- **Plain summation fits:**
  Choose $M_\pi t_{\rm sep}^{\rm min} \gtrsim 0.7$ and $t_{\rm sep}^{\rm min} \gtrsim 0.5\,\mathrm{fm}$.
- **Two-state fits:**
  Choose $M_\pi t_{\rm sep}^{\rm min} \gtrsim 0.5$.



N302 $M_\pi = 349\,\mathrm{MeV}$, $a = 0.050\,\mathrm{fm}$

Physical extrapolation – CCF fit models

We consider the following ansatz for the chiral, continuum and finite volume extrapolation of any observable $O(M_\pi, a, L)$ inspired by the NNLO chiral expansion of g_A *JHEP 04 (1999) 031*

$$O(M_\pi, a, L) = A_O + B_O M_\pi^2 + C_O M_\pi^2 \log M_\pi + D_O M_\pi^3 + E_O a^{n(O)} + F_O \frac{M_\pi^2}{\sqrt{M_\pi L}} e^{-M_\pi L},$$

where

- $n(O) = 2$ for $O = g_{A,S}^{u-d}$ and $n(O) = 1$ otherwise.
- A_O, B_O, D_O E_O and F_O are free fit parameters.
- The C_O are known analytically, e.g. $C_{g_A} = \frac{-\mathring{g}_A}{(2\pi f_\pi)^2}\left(1 + 2\mathring{g}_A^2\right)$.

Remarks:

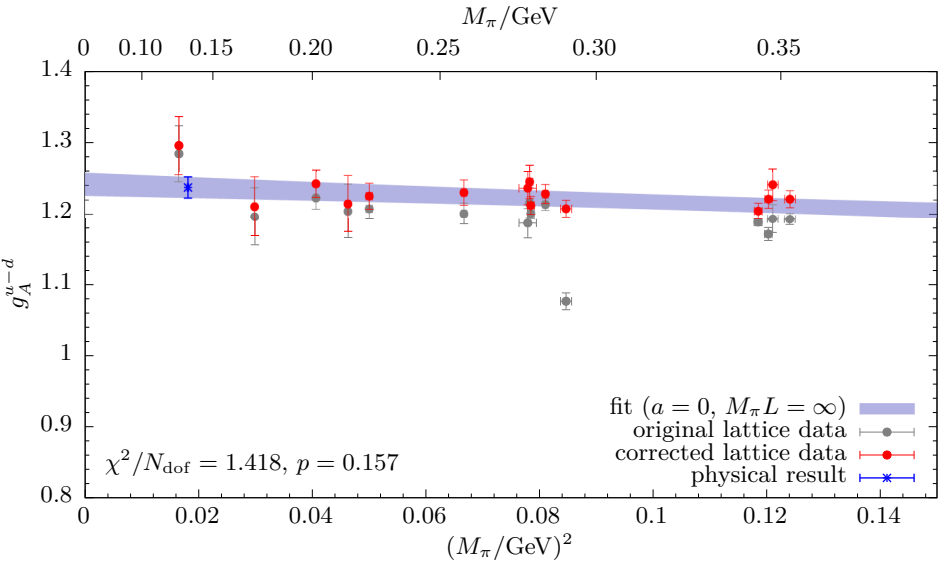
- An NLO g_A^{u-d} fit including the chiral log imposes a curvature not observed in the data.
- An NLO g_A^{u-d} fit with a free parameter C gives the "wrong" sign.

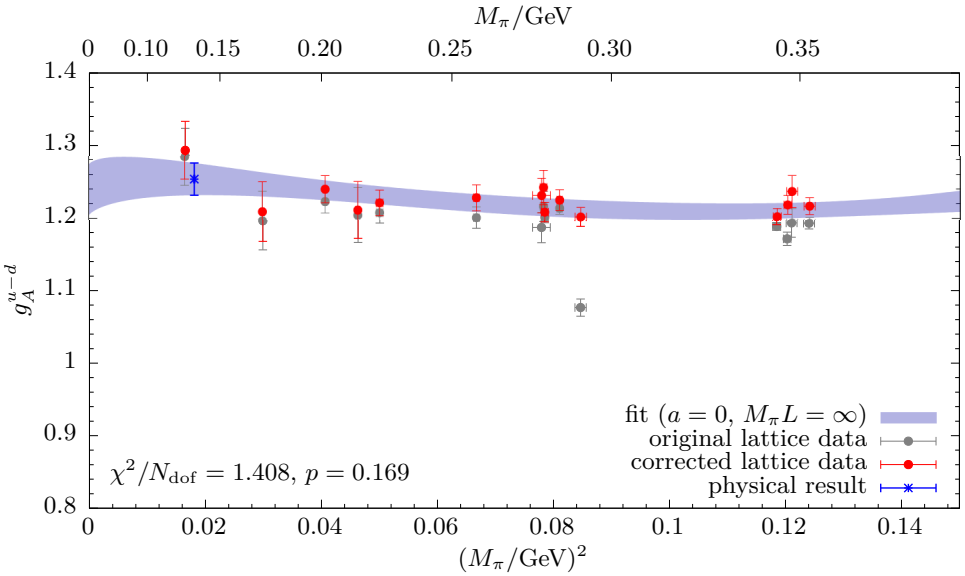
We employ two fit models:

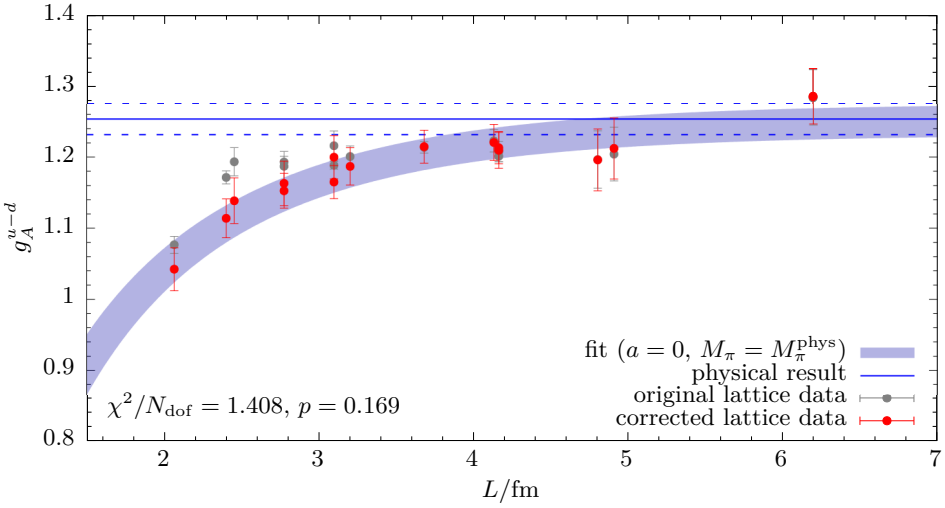
1. NLO fit without a chiral log: $g_A^{u-d}(M_\pi, a, L) = A + BM_\pi^2 + Ea^2 + F\frac{M_\pi^2}{\sqrt{M_\pi L}} e^{-M_\pi L}$.
2. **Full NNLO fit as given above. (currently only for g_A^{u-d}!)**

We use t_0 to set the scale, with $\sqrt{8t_0^{\rm phys}} = 0.415(4)_{\rm stat}(2)_{\rm sys}\,{\rm fm}$. *JHEP 08 (2010) 071* *PRD 95 (2017) 074504*

Physical extrapolation for g_A^{u-d} (two-state summation)







- Chiral and continuum extrapolations are mild.
- Finite volume corrections can be sizable for g_A^{u-d} (already seen in 2019 analysis).
- Physical results from both fit models agree

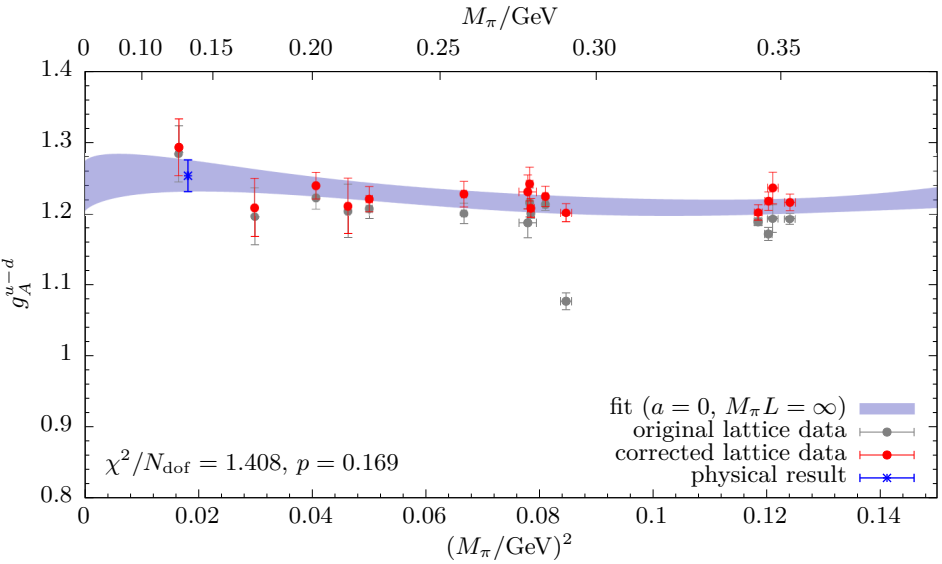
  $g_A^{u-d} = 1.237(15)_{\rm stat}$ (fit 1)

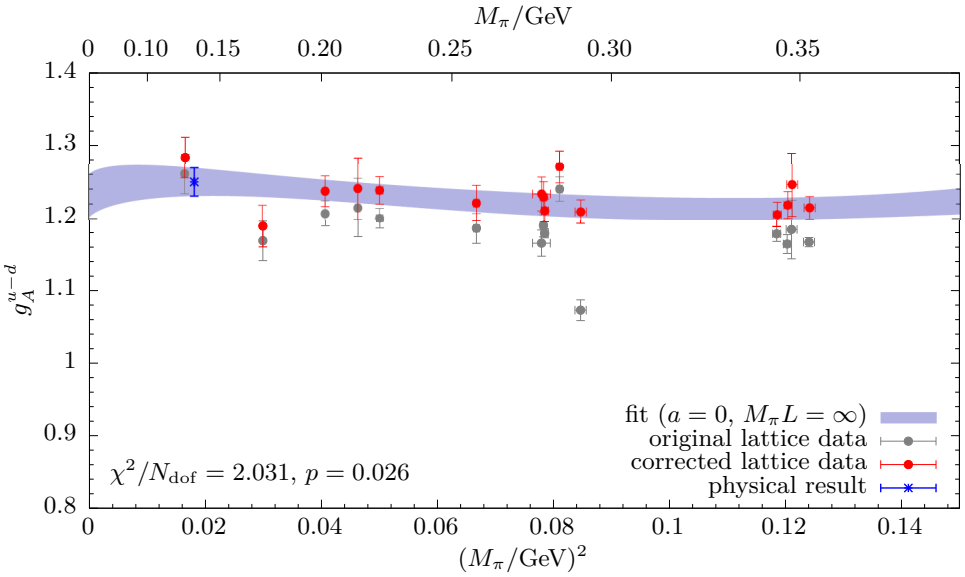
  $g_A^{u-d} = 1.250(25)_{\rm stat}$ (fit 2)

  but only NNLO fit in agreement with result on E250 and with experiment.

All results are preliminary!

Systematics





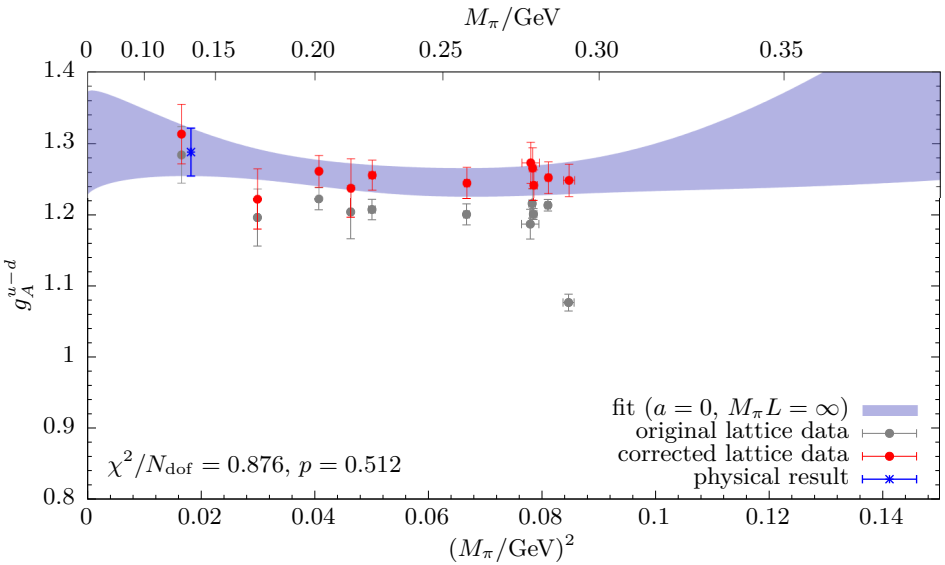
- Compatible NNLO results from two-state and plain summation method

$$g_A^{u-d} = 1.250(25)_{\rm stat}$$
$$g_A^{u-d} = 1.247(22)_{\rm stat}$$

- $M_\pi < 300\,\mathrm{MeV}$-cut prefers larger values

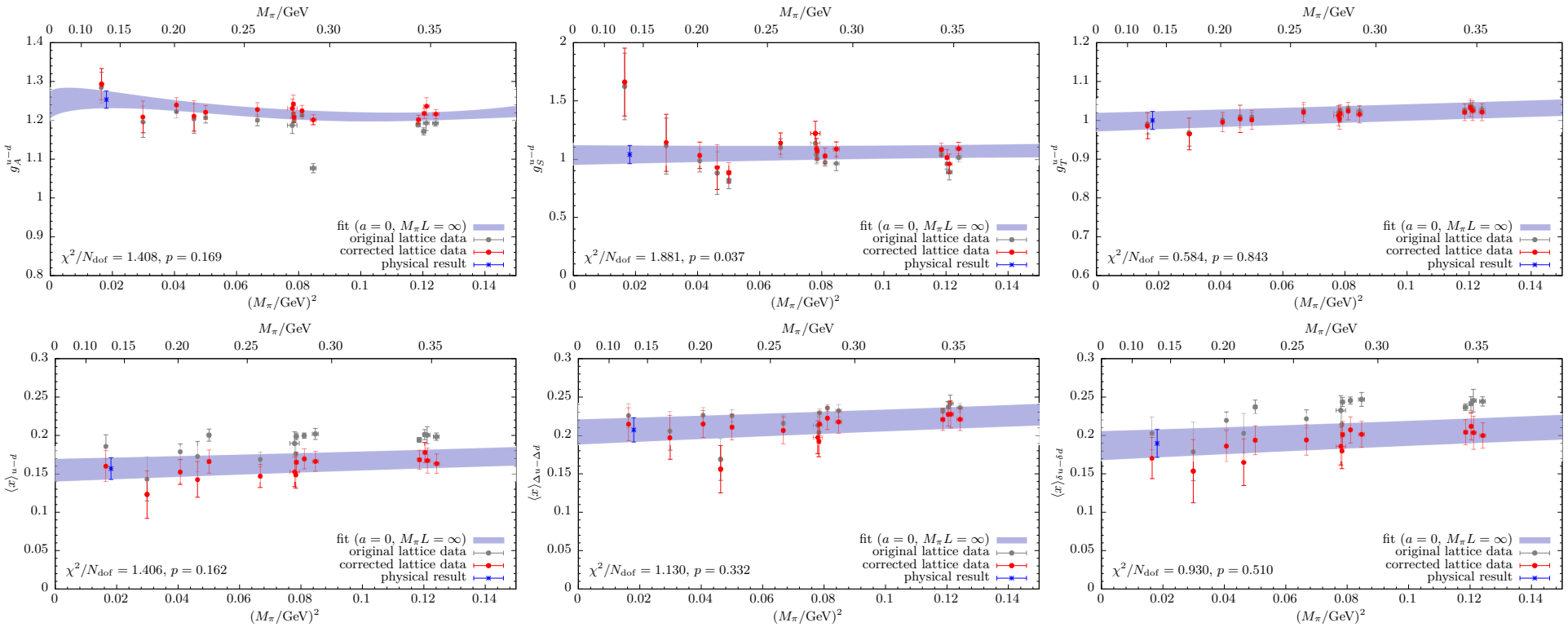
$$g_A^{u-d} = 1.264(20)_{\rm stat} \text{ (fit 1)}$$
$$g_A^{u-d} = 1.286(36)_{\rm stat} \text{ (fit 2)}$$

- Use cuts (M_π, a, volume) and fit model variations for model average $\rightarrow$ systematic error.



All results are preliminary!

Overview of chiral extrapolations for all six NMEs



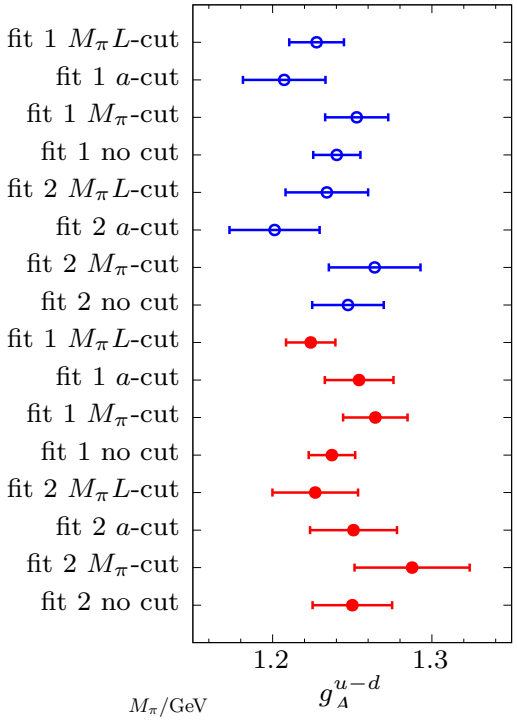
- Data for other five NMEs are well described by fit 1 $\sim M_\pi^2$.
- Large finite volume corrections only seen for g_A^{u-d} (and M_N).
- Typical rel. stat. errors of physical results:

$$g_{A,T}^{u-d}\colon \sim 1\% - 3\% \qquad g_S^{u-d} \text{ and } \langle x \rangle_{\dots}^{u-d}\colon \sim 5\% - 10\%$$

- Simultaneous fit ($\mathring{g}_A$ as common parameter) exploiting correlations might further improve results (or allow to fit full NNLO expressions).

Summary and outlook

- Calculation of $g_{A,S,T}^{u-d}$ and $\langle x \rangle_{u-d}$, $\langle x \rangle_{\Delta u - \Delta d}$, $\langle x \rangle_{\delta u - \delta d}$:
  - Excited states tamed by (2-state truncated) summation method.
  - Full, chiral, continuum finite size extrapolations to obtain physical results.
  - Results for g_A^{u-d} in agreement with ensemble at physical quark mass.
  - Systematics may be assessed from model averaging.
  - Simulatenous CCF fits may further stabilize / improve results.



- Calculation of physical M_N with controlled systematics:
  - Physical result $M_N^{\text{phys}} = 947(10)\text{MeV}$ in good agreement with experimental value.
  - Small systematics / result stable under fit variations.
  - Statistically very precise, error dominated by scale setting $\rightarrow$ Xcheck for scale setting.