Lattice QCD Determination of the Bjorken-x Dependence of PDFs at NNLO

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In collaboration with A. D. Hanlon, S. Mukherjee, P. Petreczky, P. Scior, S. Syritsyn and Y. Zhao

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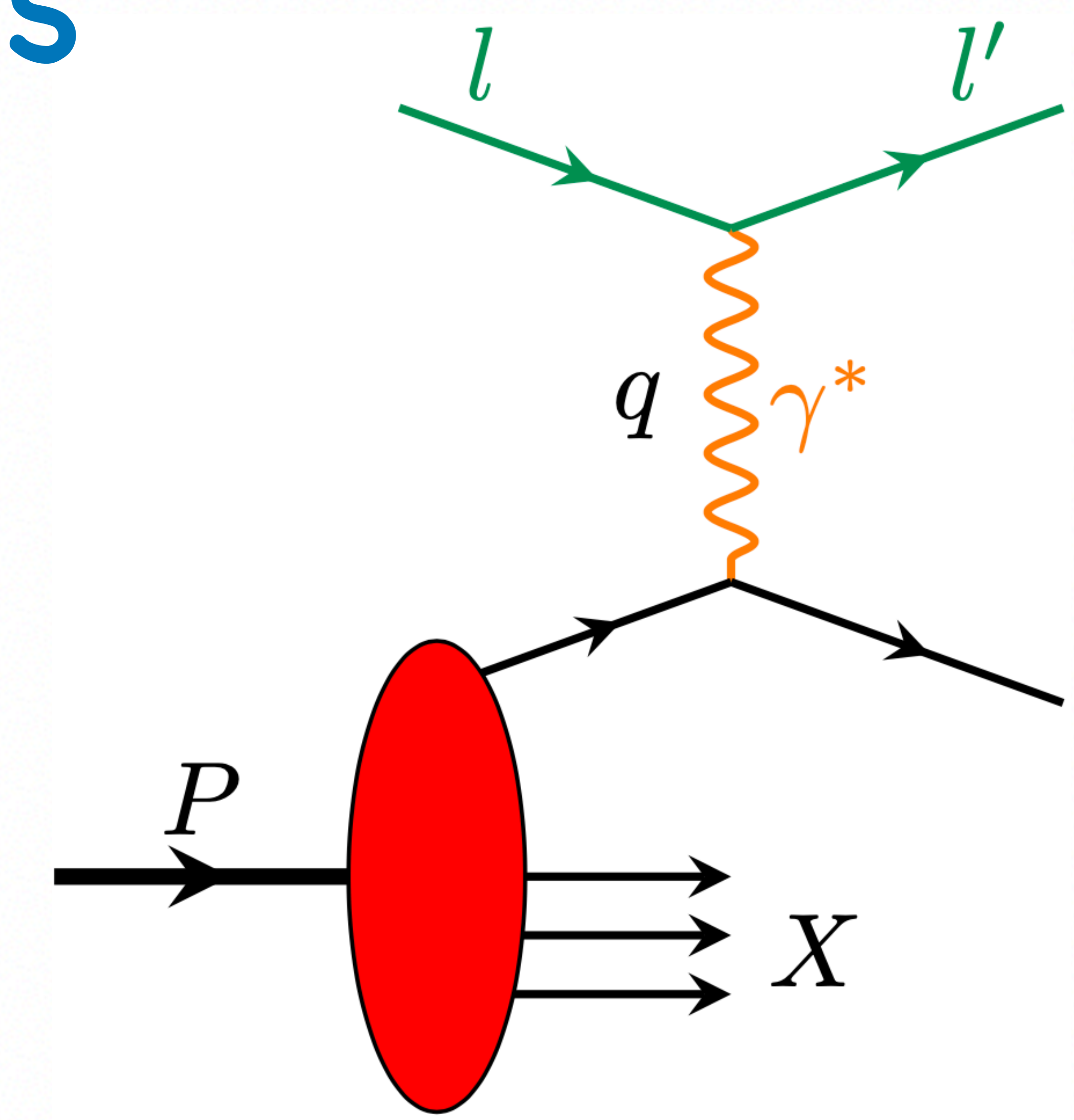
Parton distribution functions

DIS



Non-perturbative PDFs

$$\sigma = \sum_i f_i(x, Q^2) \circledast \sigma \{eq_i(xP) \to eq_i(xP+q)\}$$

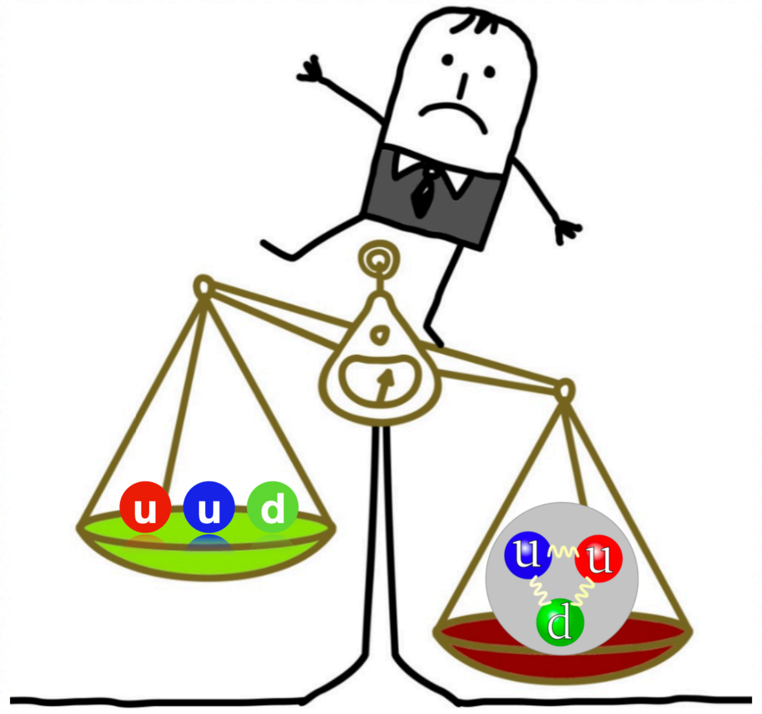
Perturbative parton process

Hadron Structure and Tomography:

- How hadrons are built.
- Mass and spin decomposition of hadron.

High-energy phenomenology:

- Standard Model backgrounds.
- Higgs physics and search for physics beyond the Standard Model.



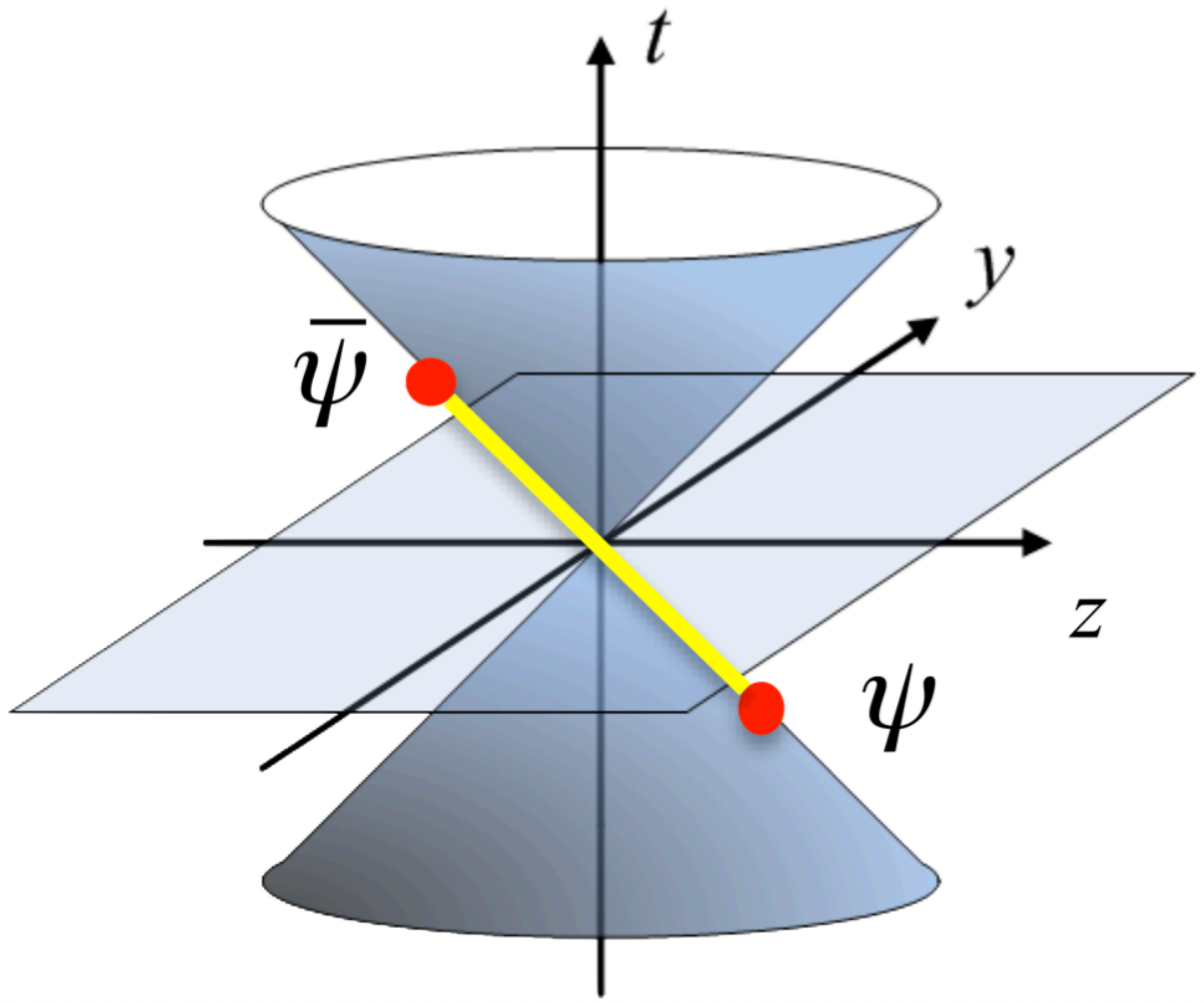
Parton distribution functions

Field theoretic Gauge-invariant and Lorentz invariant construction. (Soper ’77)

$$q(x) \equiv \int \frac{d\xi_-}{4\pi} e^{-ixP^+\xi_-} \langle P \,|\, O_\Gamma(\xi_-, \epsilon) \,|\, P \rangle, \; O_\Gamma(\xi_-, \epsilon) = \overline{\psi}(0)\Gamma W_-(0,\xi_-)\psi(\xi_-)$$

$$z + ct = 0, \;\; z - ct \neq 0$$

Not implementable in Monte-Carlo methodology!



Projecting to hadron state is easy on lattice, but presence of unequal time separation between $\psi(0)$ and $\psi(\xi_-)$ sandwiched between hadron states is a sign problem for Euclidean lattice.

Parton distribution functions from Lattice

Lattice computation of PDF：

- Mellin or Gegenbauer Moments from leading-twist local operators.
- Operator product expansion (OPE) of current-current matrix elements.

W. Detmold and C. Lin, PRD 2006
V. Braun and D. Müller, EPJC 2008
A. J. Chambers, et al, PRL 2017

- Large-momentum effective theory: x -space matching of quasi-PDF.

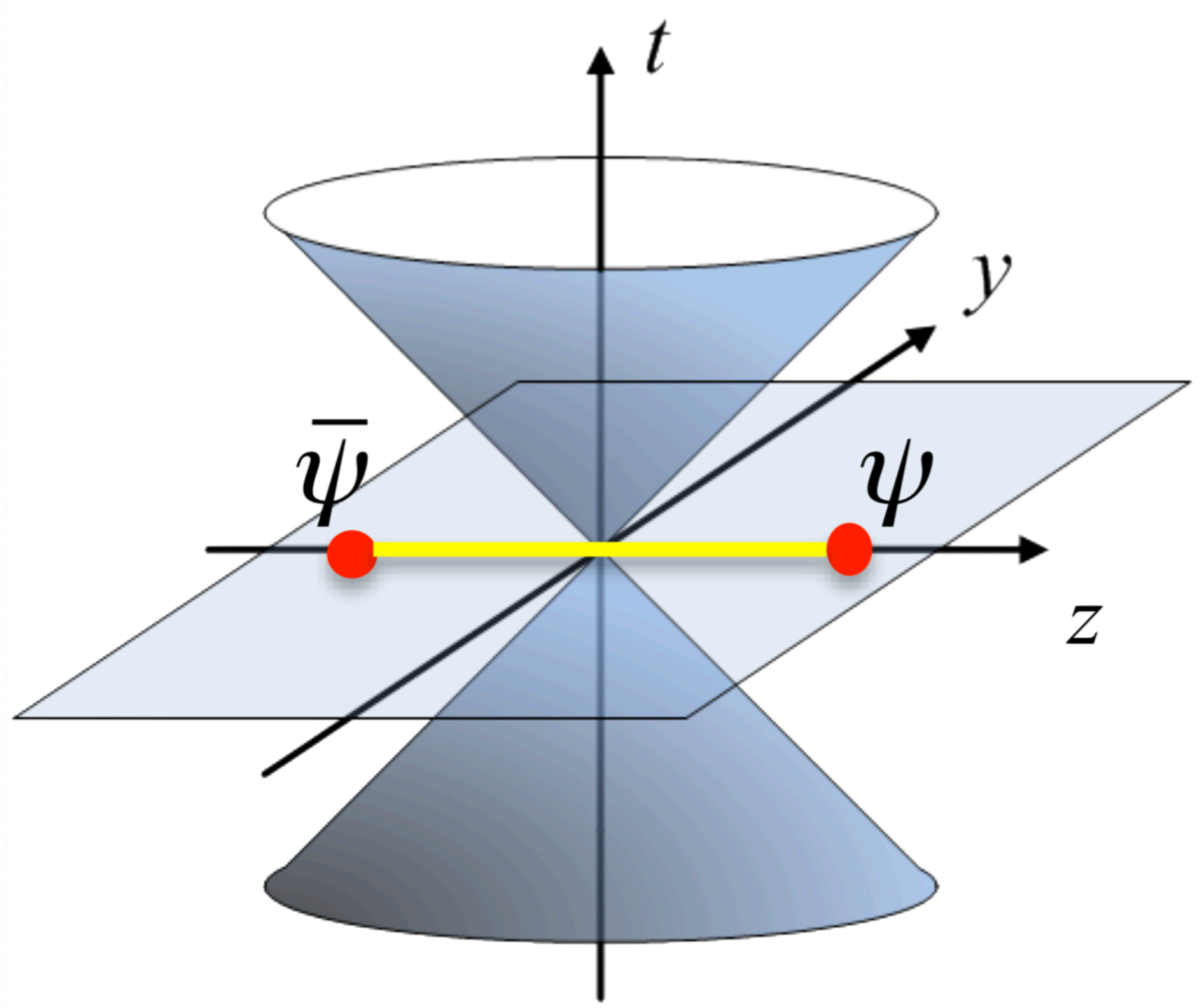
X. Ji, PRL 2013
X. Ji, et al, RevModPhys 2021

- Short distance factorization of the quasi-PDF matrix elements in position space or the pseudo-PDF approach.

A. V. Radyushkin, PRD 2017
A. V. Radyushkin, Int.J.Mod.Phys.A 2020

- …

$t = 0, \quad z \neq 0$



quasi-PDF matrix elements

$$iP_z h(z, P_z) = \langle \pi^+; P \,|\, \bar{d}(-z/2)\gamma_0 W_z u(z/2) \,|\, \pi^+; P \rangle$$

Parton distribution functions from Lattice

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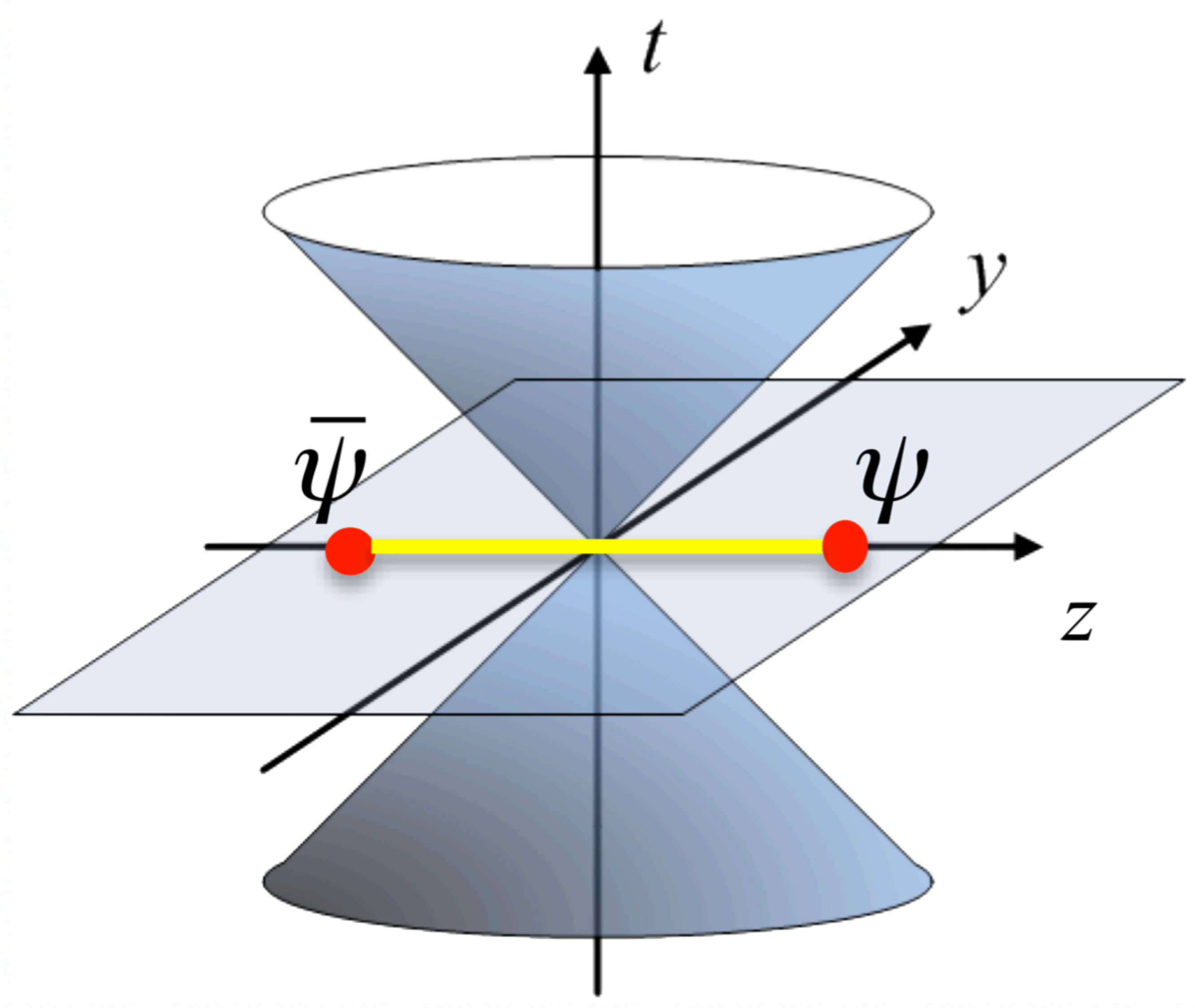

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$$iP_z h(z, P_z) = \langle \pi^+; P \,|\, \bar{d}(-z/2)\gamma_0 W_z u(z/2) \,|\, \pi^+; P\rangle$$

Large momentum effective theory

IR must cancel

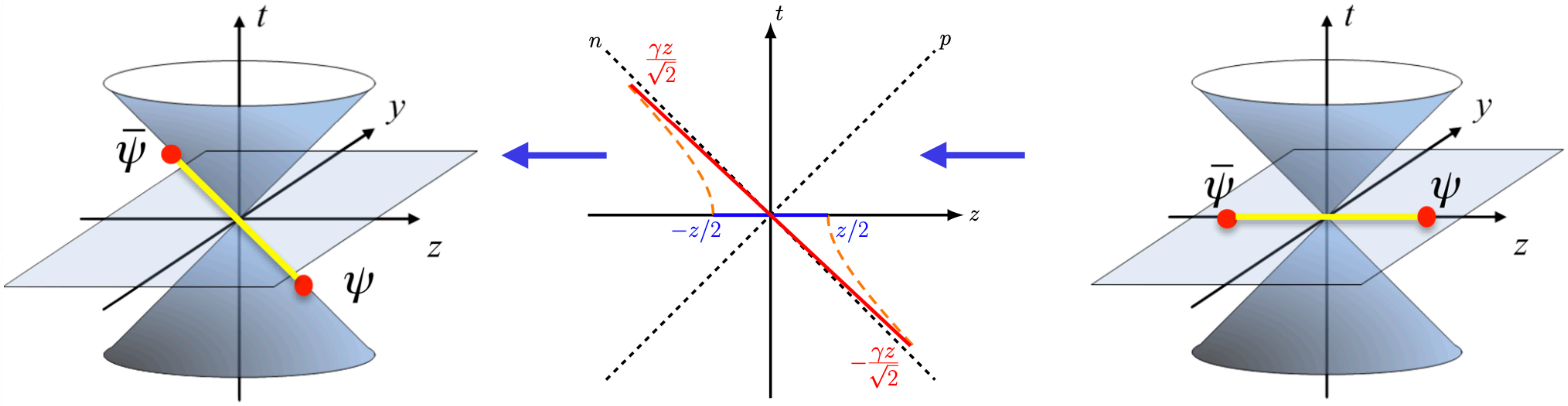
Quasi-PDFs
Factorization

$$\tilde{q}(x, P_z, \mu) = q(x, \mu) + \alpha_s(\mu)(\tilde{q}^{(1)}(x, P_z, \mu) - q^{(1)}(x, \mu))$$

$$= \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP_z}\right) q(y, \mu) + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{x^2 P_z^2}, \frac{\Lambda^2_{QCD}}{(1-x)^2 P_z^2}\right)$$

- X. Ji, PRL 110 (2013); SCPMA57 (2014);
- X. Xiong, X. Ji, et al, PRD 90 (2014);
- Y.-Q. Ma, et al, PRD98 (2018), PRL 120 (2018);
- T. Izubuchi, X. Ji, et al PRD98 (2018).
- X. Ji, Y. Zhao, et al, RMP 93 (2021).

large P_z is essential



$z + ct = 0, \;\; z - ct \neq 0$ Related by Lorentz boost $t = 0, \;\; z \neq 0$

Large momentum effective theory

Quasi-PDFs Factorization

IR must cancel

$$\tilde{q}(x, P_z, \mu) = q(x, \mu) + \alpha_s(\mu)(\tilde{q}^{(1)}(x, P_z, \mu) - q^{(1)}(x, \mu))$$

$$= \int \frac{dy}{|y|} C(\frac{x}{y}, \frac{\mu}{yP_z}) q(y, \mu) + \mathcal{O}(\frac{\Lambda_{QCD}^2}{x^2 P_z^2}, \frac{\Lambda_{QCD}^2}{(1-x)^2 P_z^2})$$

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large P_z is the key

Precision is enssential = reliability within $[x_{\min}, x_{\max}]$ for given P_z

Systematics

- Lattice artifacts: spacing a→0, physical m_π, lattice size L→∞ …;
- Perturbative matching (currently at NNLO) and resummation at small and large x;
- Power corrections.

- L.-B. Chen, R.-L. Zhu and W. Wang, PRL126 (2021);
- Z.-Y. Li, Y.-Q. Ma and J.-W. Qiu, PRL126 (2021);
- X. Gao, K. Lee, S. Mukherjee, C. Shugert and YZ, PRD103 (2021).

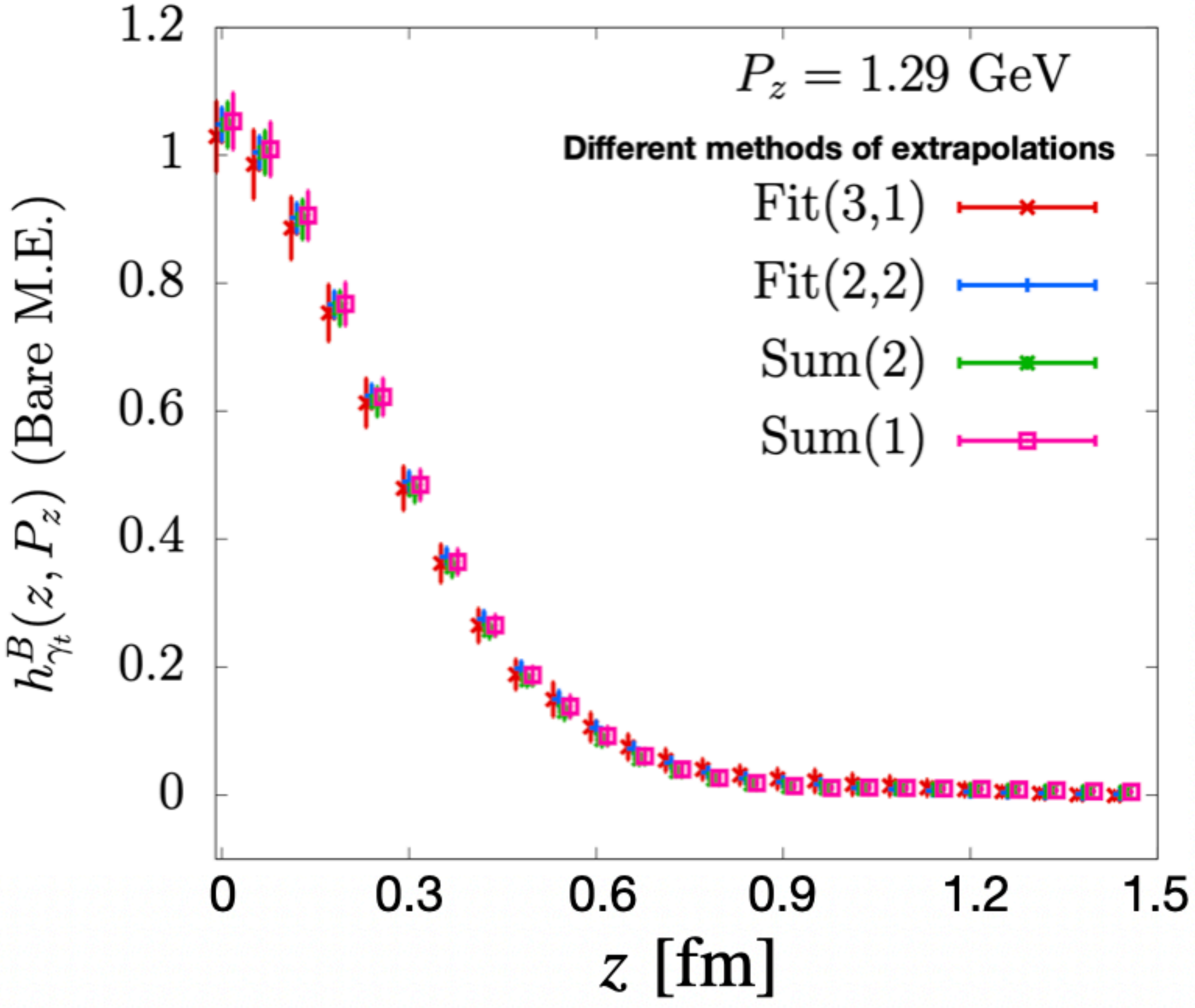
Lattice setup and bare matrix elements

Wilson-clover fermion on 2+1 flavor HISQ configurations.

n_z	P_z (GeV) $a = 0.06$ fm	P_z (GeV) $a = 0.04$ fm	ζ
0	0	0	0
1	0.43	0.48	0
2	0.86	0.97	1
3	1.29	1.45	2/3
4	1.72	1.93	3/4
5	2.15	2.42	3/5
	$48^3 \times 64$	$64^3 \times 64$	

$m_\pi = 300$ MeV

Bare matrix elements of **boosted** pion state

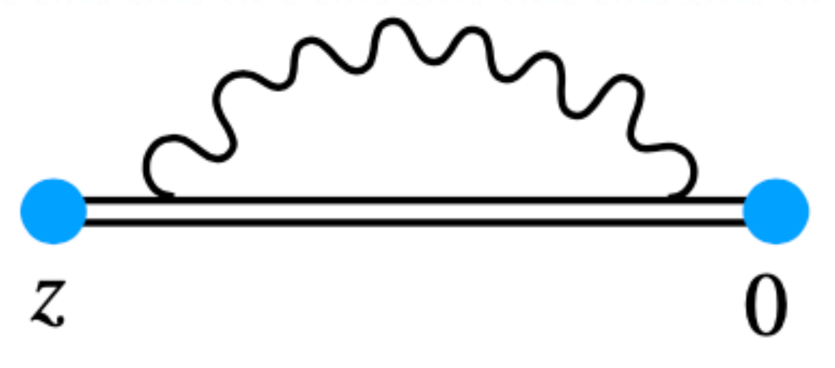


- BNL, PRD 102 (2020) 9, 094513
- BNL-ANL, PRL 128 (2022) 14, 142003

Bare matrix elements and renormalization

The operator can be multiplicatively renormalized:

- X. Ji, J. H. Zhang and Y. Zhao, PRL120 (2018)
- J. Green, K. Jansen and F. Steffens, PRL121 (2018)
- T. Ishikawa, et al, PRD 96 (2017)

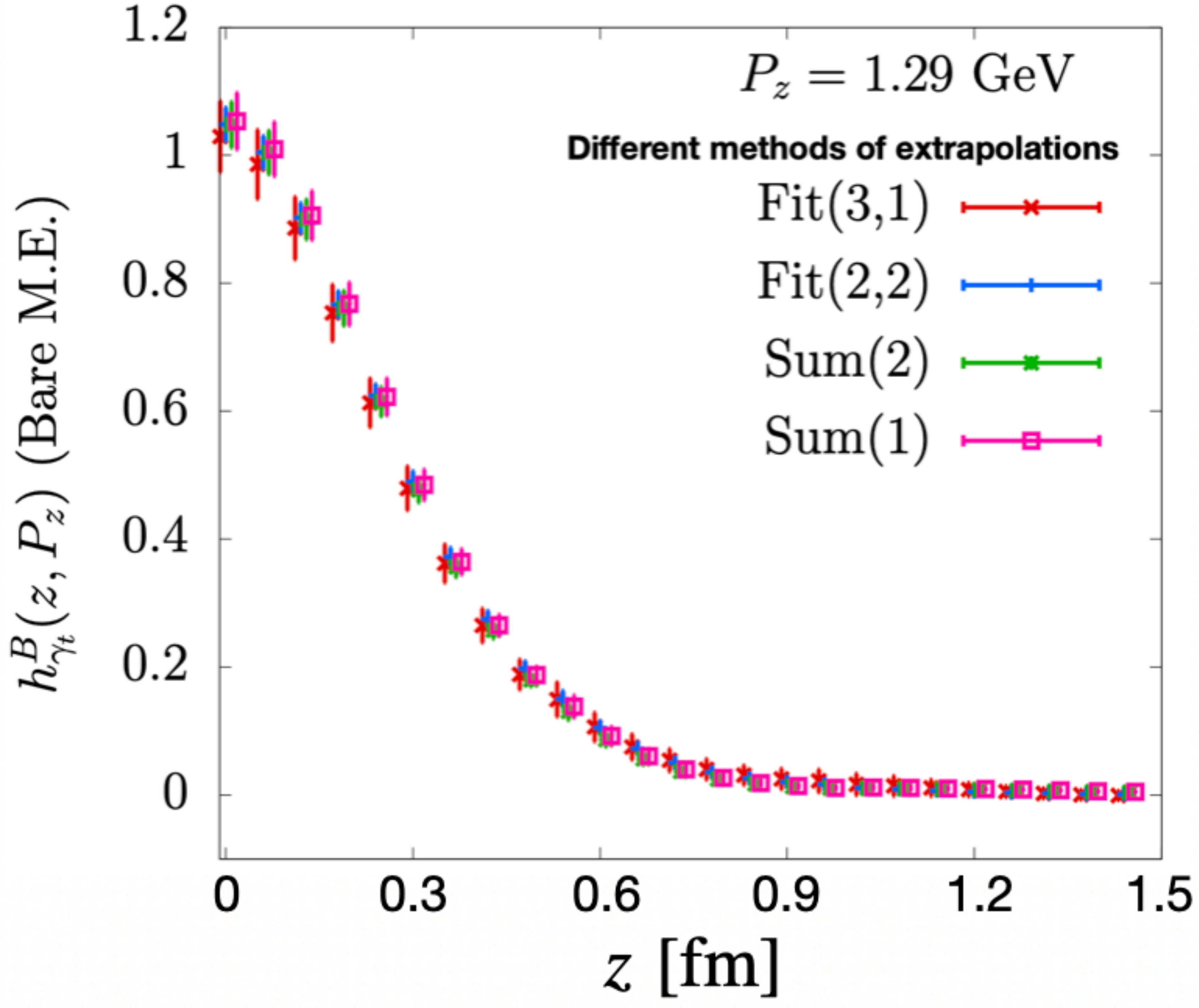


$$= \delta m(a)|z| \propto \frac{|z|}{a}$$

$$[\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_B$$

$$= e^{-\delta m|z|}Z(a)[\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_R$$

Bare matrix elements of **boosted** pion state



$$\delta m(a) = m_{-1}(a)/a + m_0$$

Wilson-line self energy + renormalon ambiguity

Hybrid renormalization

Hybrid renormalization:

- X. Ji, et al., NPB 964 (2021).

- Short distance $z \in [0, z_s],\ z_s \ll \Lambda_{\mathrm{QCD}}$:

$$h^R = \frac{\tilde{h}(z, P_z, a)}{\tilde{h}(z, 0, a)}$$

Ratio scheme

- Long distance $z \in [z_s, +\infty]$:

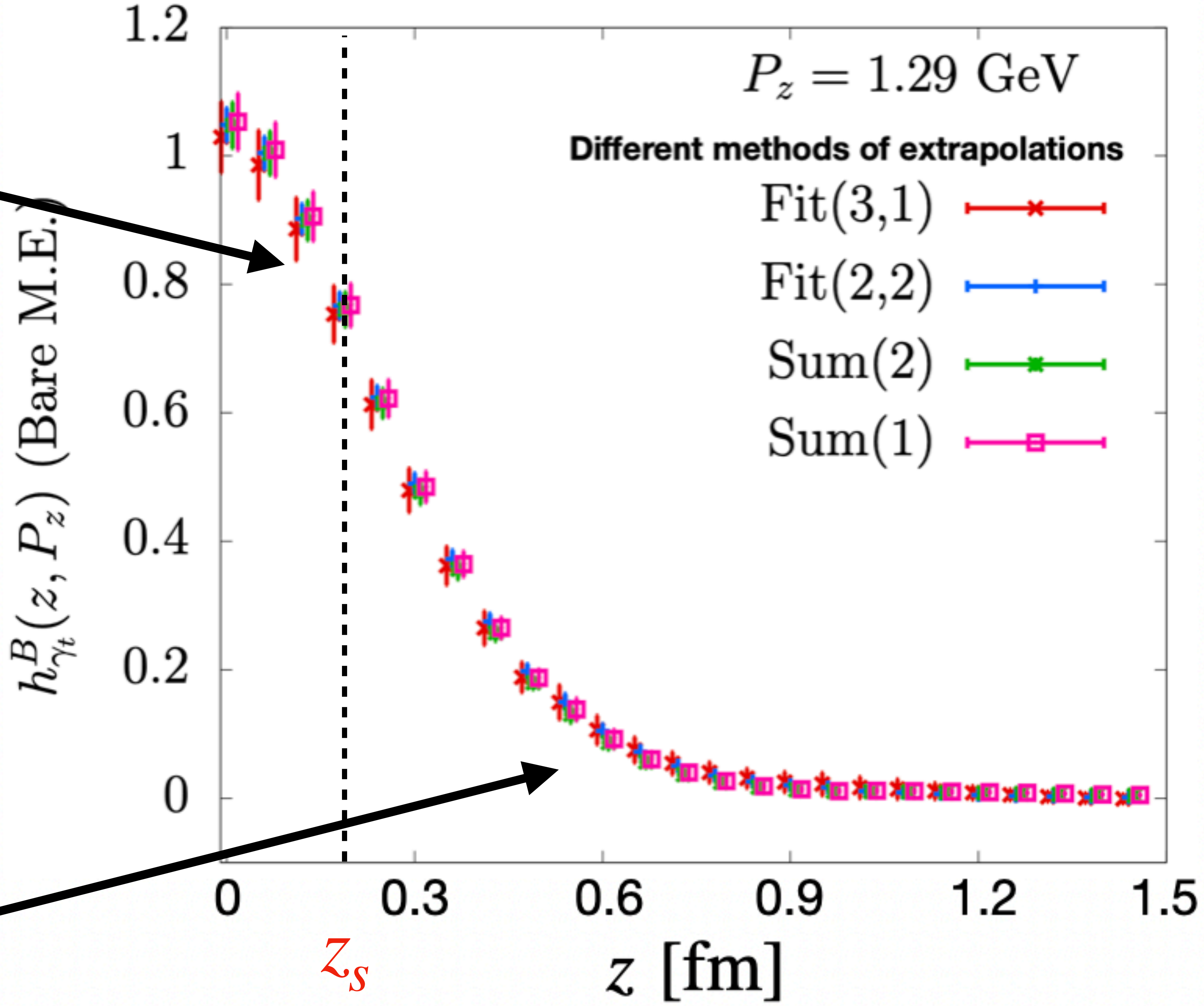
A "minimal" subtraction

$$h^R = e^{\delta m |z - z_s|} \frac{\tilde{h}(z, P_z, a)}{\tilde{h}(z_s, 0, a)}$$

$$\delta m = m_{-1}/a + m_0$$

$$[\bar{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_B = e^{-\delta m|z|} Z(a) [\bar{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_R$$

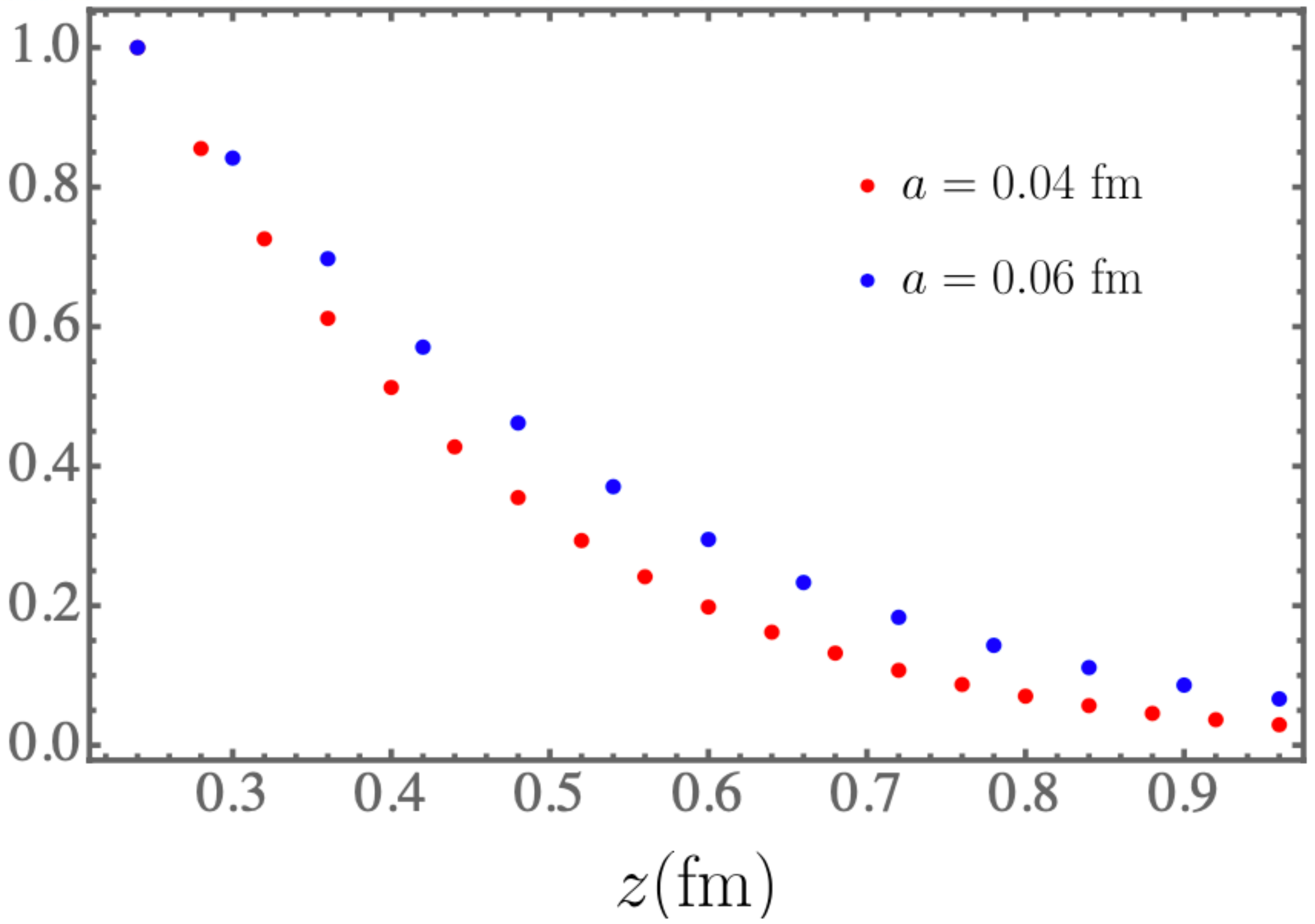
Bare matrix elements of **boosted** pion state



Hybrid renormalization: Wilson-line mass

matrix elements before mass subtraction

$$\frac{\tilde{h}(z, P_z = 0, a)}{\tilde{h}(z_s, P_z = 0, a)}$$



- Clear lattice dependence before subtract $\delta m(a)$

$$[\bar{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_B$$
$$= e^{-\delta m|z|} Z(a) [\bar{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_R$$

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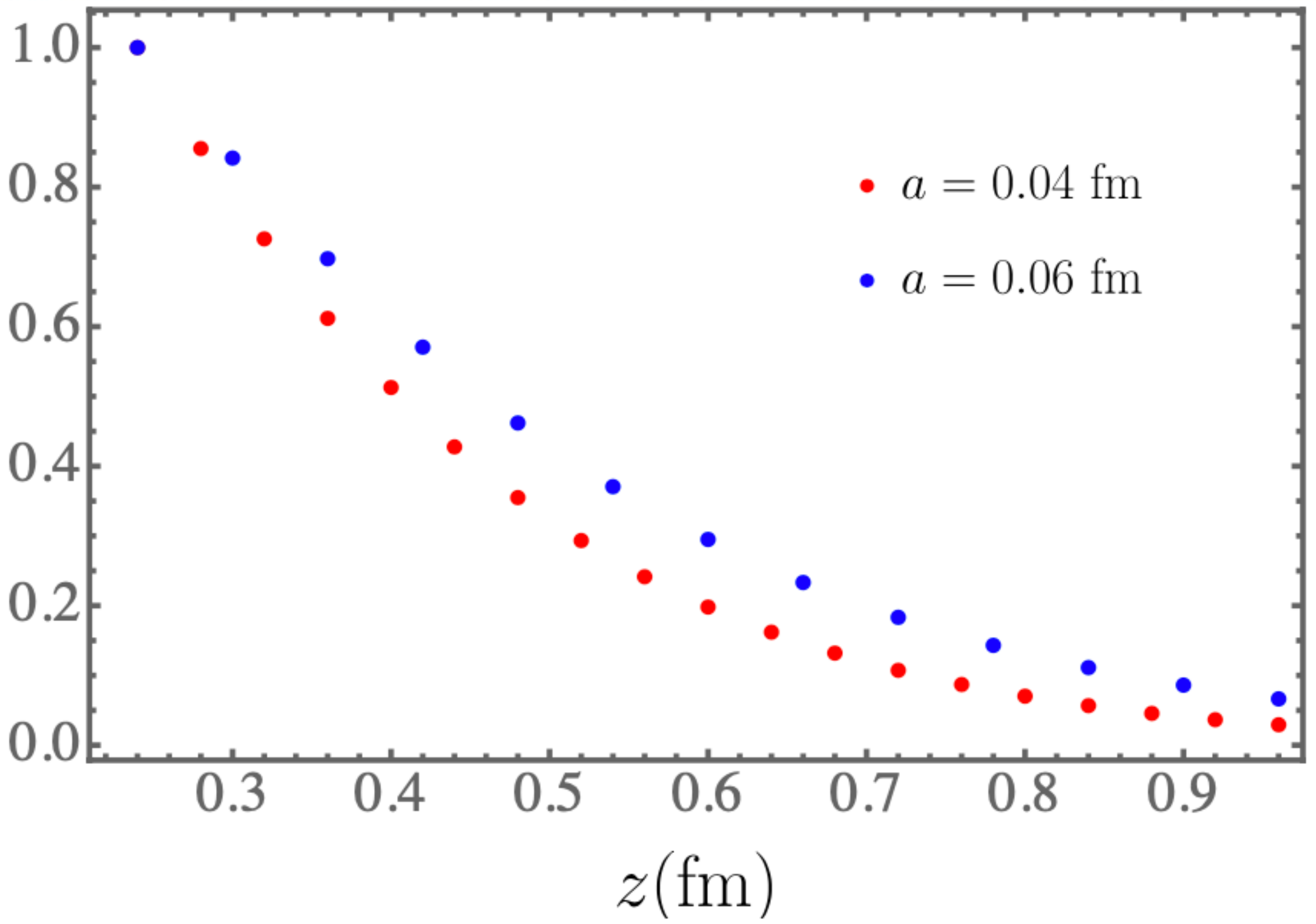

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$$= e^{-\delta m|z|} Z(a) [\bar{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_R$$

- $\delta m(a)$ from static quark-antiquark potential with renormalization condition

$$V^{\text{lat}}(r,a)\Big|_{r=r_0} + 2\delta m(a) = 0.95/r_0$$

- C. Bauer, G. Bali and A. Pineda, PRL108 (2012).
- A. Bazavov et al., TUMQCD, PRD98 (2018).

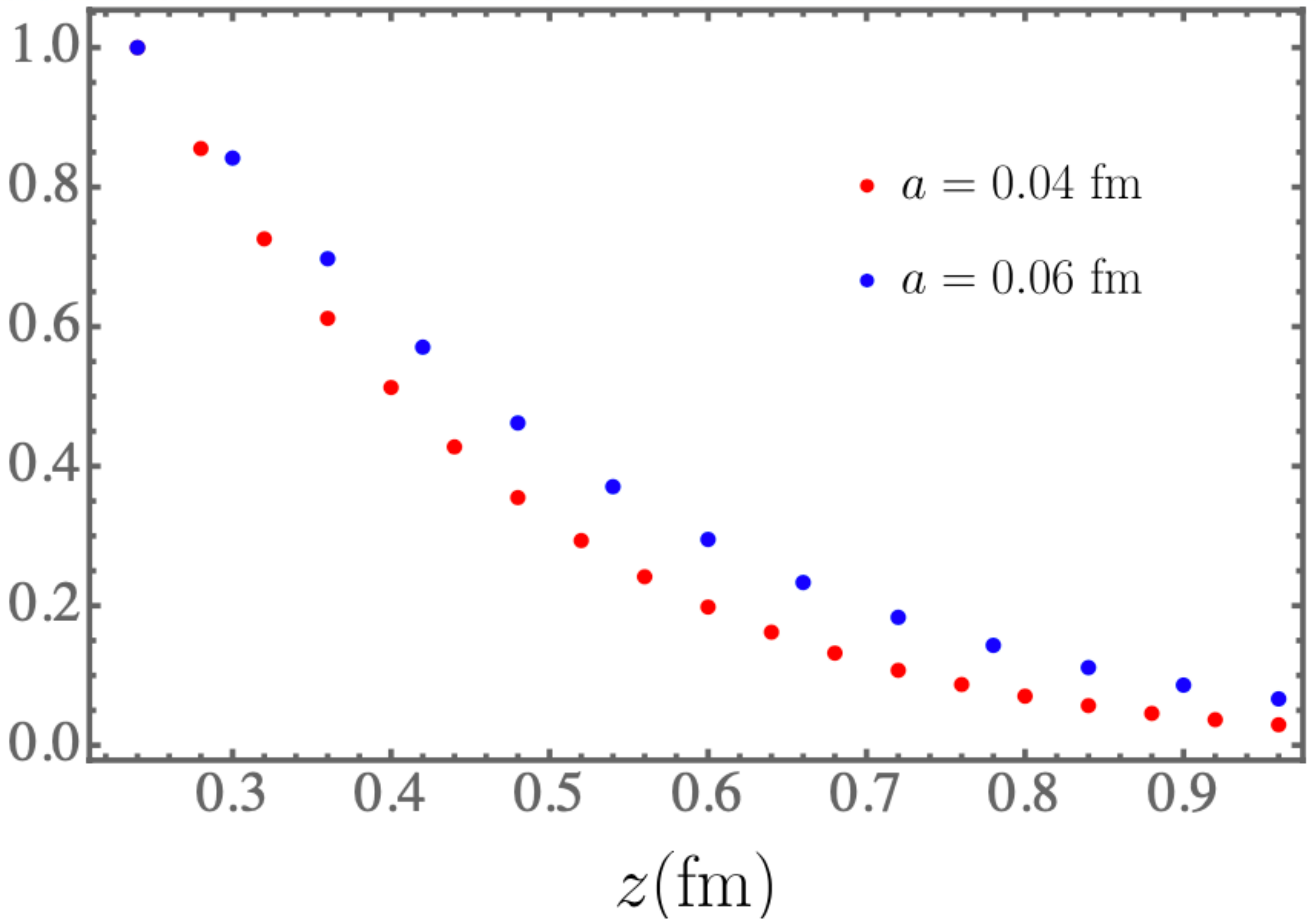
$$a\delta m(a = 0.04 \text{ fm}) = 0.1508(12)$$

$$a\delta m(a = 0.06 \text{ fm}) = 0.1586(8)$$

Hybrid renormalization: Wilson-line mass

matrix elements before mass subtraction

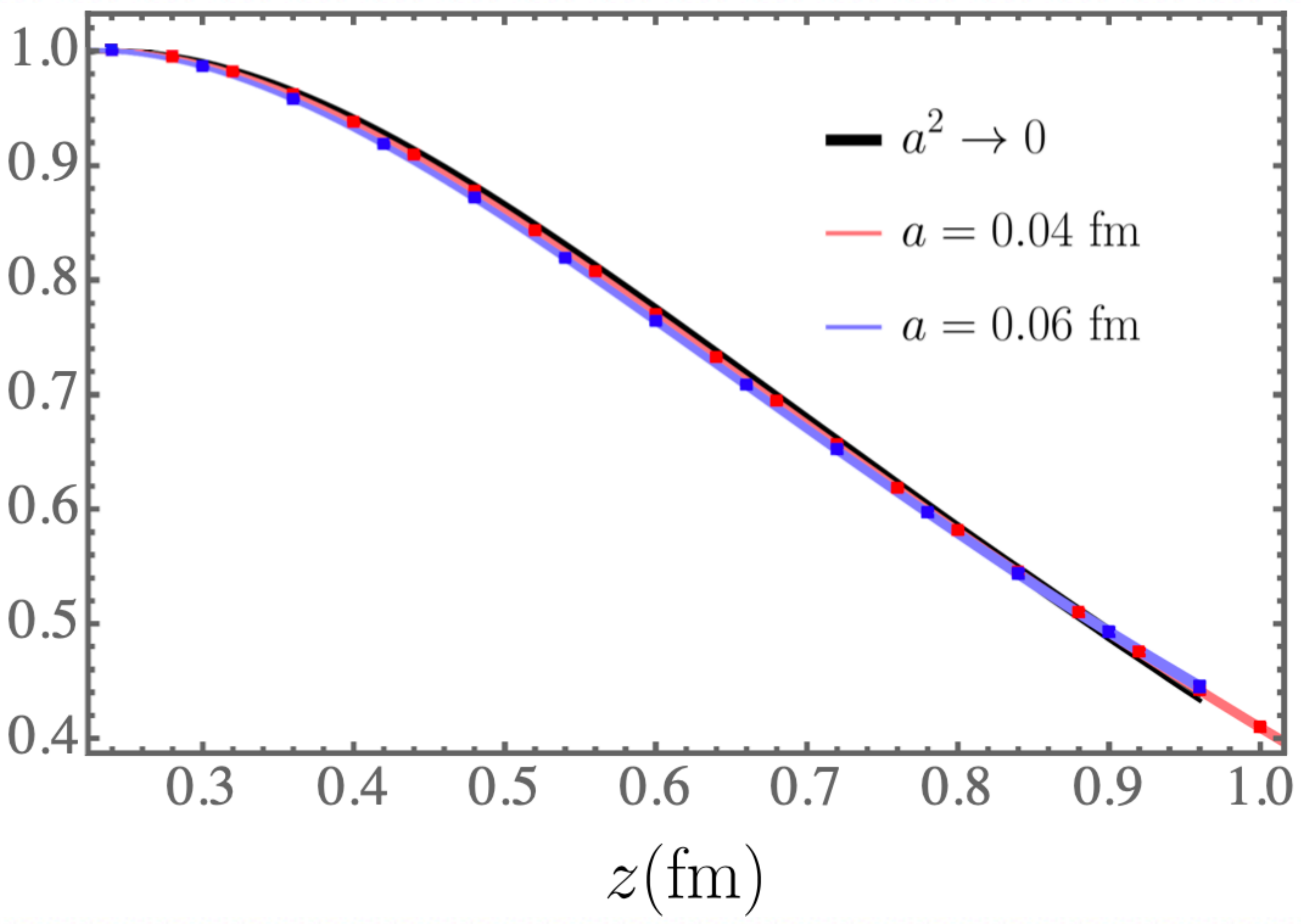
$$\frac{\tilde{h}(z, P_z = 0, a)}{\tilde{h}(z_s, P_z = 0, a)}$$



- Clear lattice dependence before subtract $\delta m(a)$

$\delta m(a)$ subtracted matrix elements :

$$e^{\delta m(a)(z-z_s)} \frac{\tilde{h}(z, P_z = 0, a)}{\tilde{h}(z_s, P_z = 0, a)}$$



- Good continuum condition
- The linear divergences have been sufficiently subtracted by $\delta m(a)$

$$[\bar{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_B$$
$$= e^{-\delta m|z|} Z(a) [\bar{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_R$$

Hybrid renormalization: UV renormalon

OPE of $\overline{\text{MS}}$ matrix elements

- L.-B. Chen, R.-L. Zhu and W. Wang, PRL126 (2021);
- Z.-Y. Li, Y.-Q. Ma and J.-W. Qiu, PRL126 (2021);
- V. Braun and K. G. Chetyrkin, JHEP 07 (2020).

Perturbative:

Known to NNLO

$$\tilde{h}^{\overline{\text{MS}}}(z, P^z = 0, \mu) = e^{-m_0^{\overline{\text{MS}}}(z-z_0)}\left[C_0(\alpha_s(\mu), z^2\mu^2) + \mathcal{O}(z^2\Lambda^2_{\text{QCD}})\right]$$

UV renormalon,

- M. Beneke and V. Braun, NPB 426 (1994)

IR renormalon:

Leading IR renormalon

- V. Braun, A. Vladimirov and J.-H. Zhang, PRD99 (2019)

Matching the mass-subtracted ratio to the $\overline{\text{MS}}$ OPE ratio

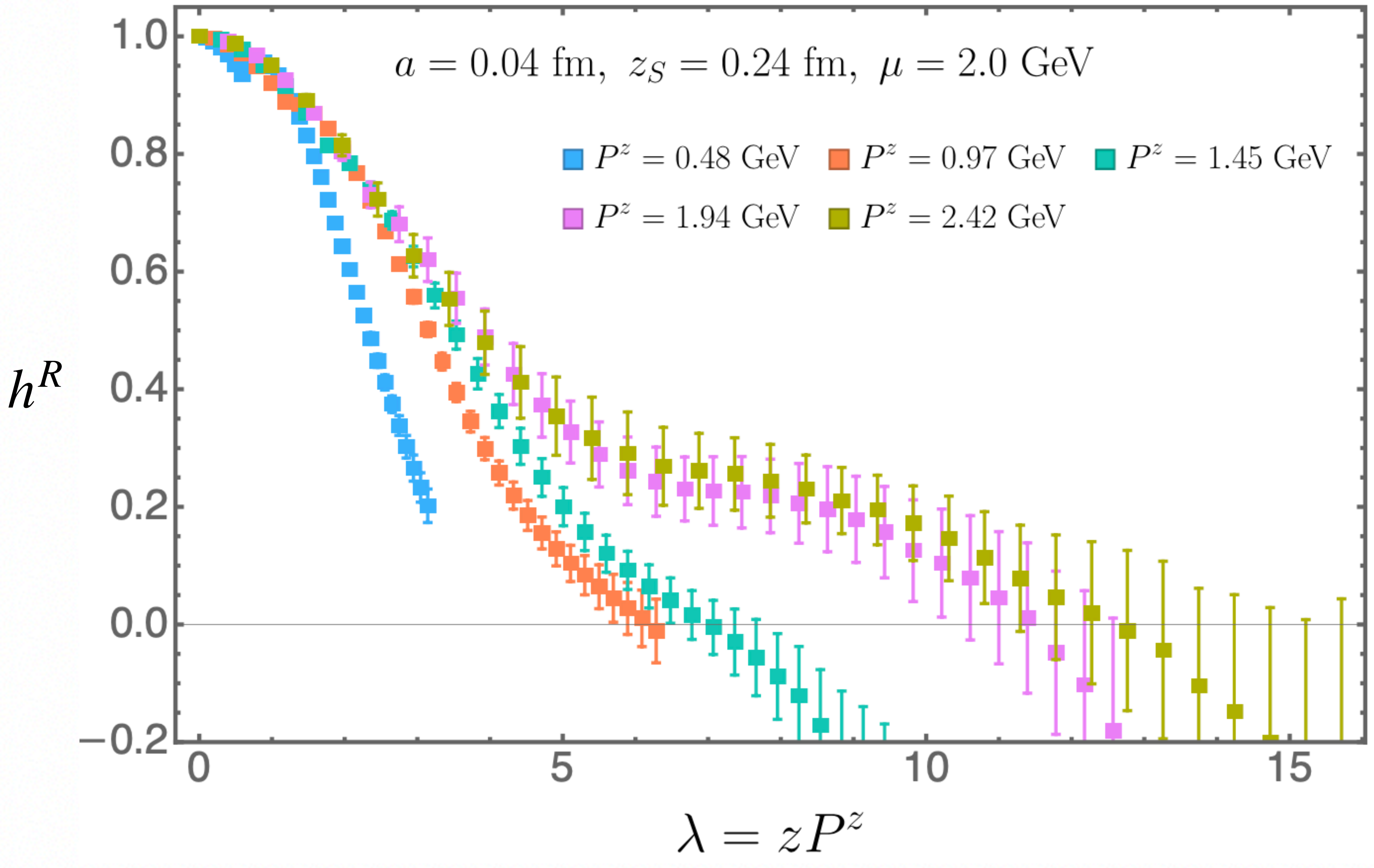
$$\lim_{a\to 0} e^{\delta m(a)(z-z_s)}\frac{\tilde{h}(z, P_z = 0, a)}{\tilde{h}(z_s, P_z = 0, a)} = e^{-\bar{m}_0(z-z_s)}\frac{C_0(\alpha_s(\mu), z^2\mu^2) + \Lambda z^2}{C_0(\alpha_s(\mu), z_s^2\mu^2) + \Lambda z_s^2}$$

$$[\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_B = e^{-\delta m|z|}Z(a)[\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_R$$

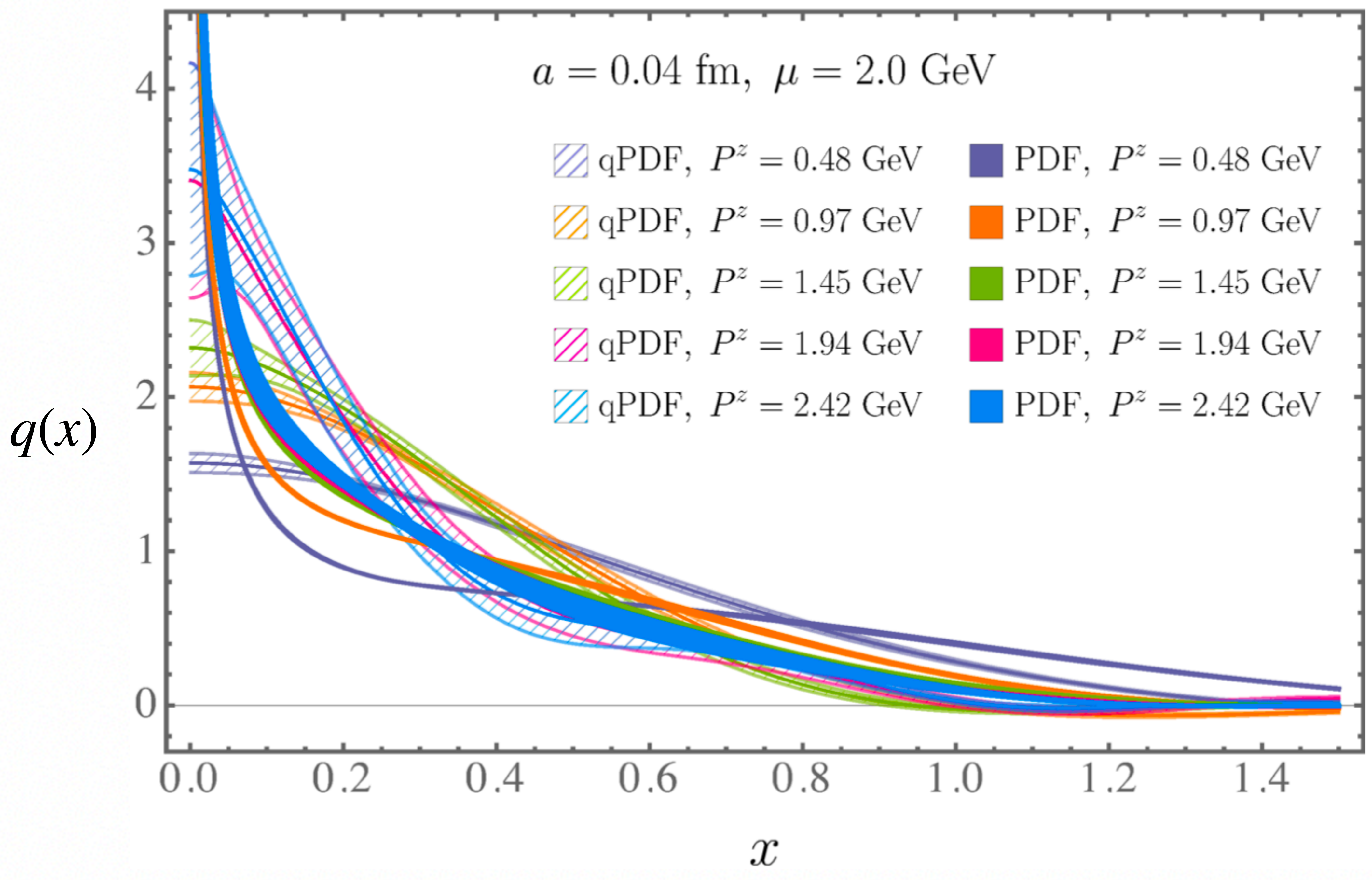
$$\bar{m}_0 = m_0^{\overline{\text{MS}}} + m_0^{\text{Lat}/\overline{\text{MS}}}$$

Hybrid renormalization and quasi-PDF

Renormalized matrix elements

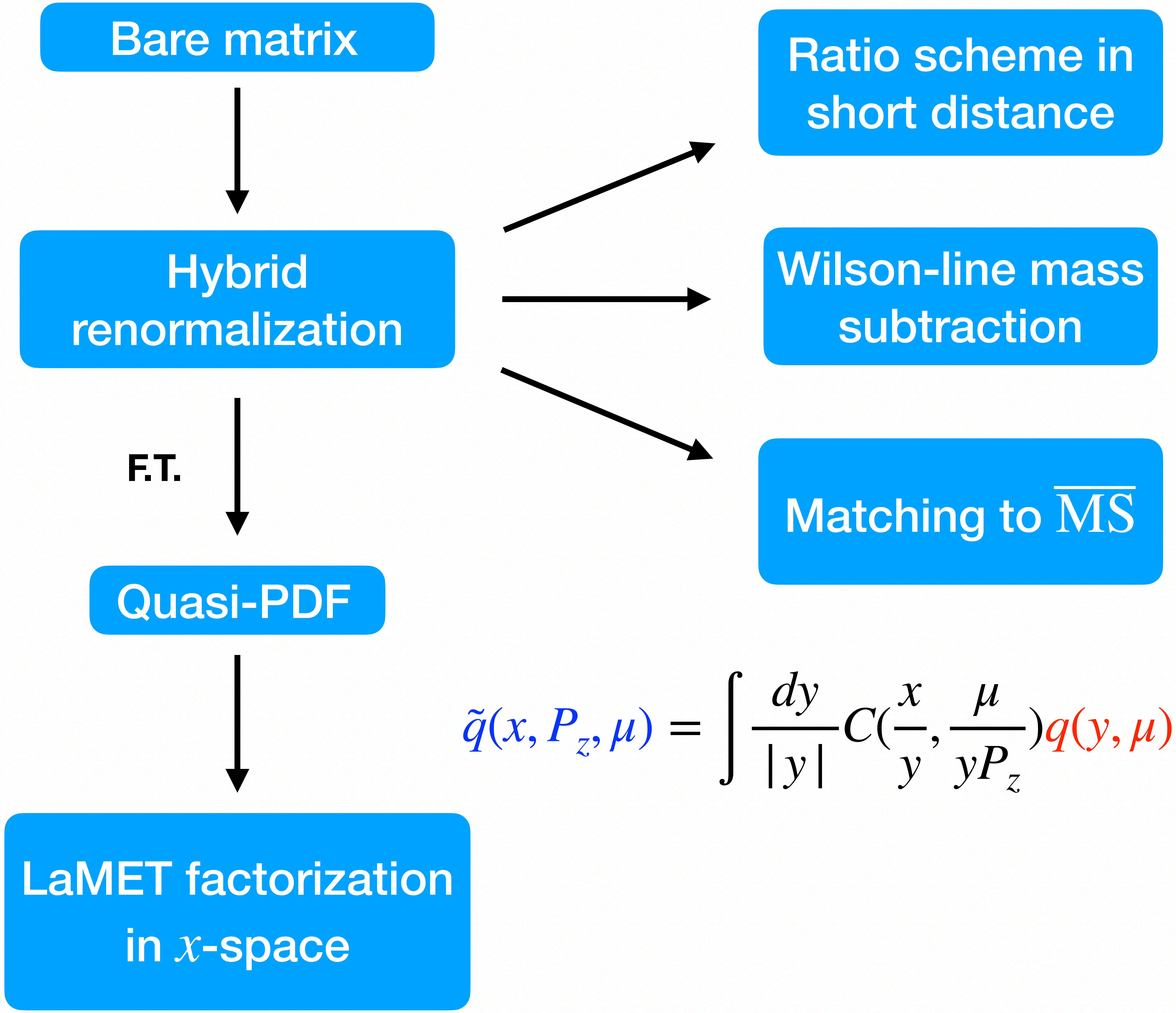


qPDF and PDF after matching



- Matrix elements and qPDFs start to converge for large P_z (perturbation region).
- Matching makes the convergence faster and drives the quasi-PDF to smaller x.

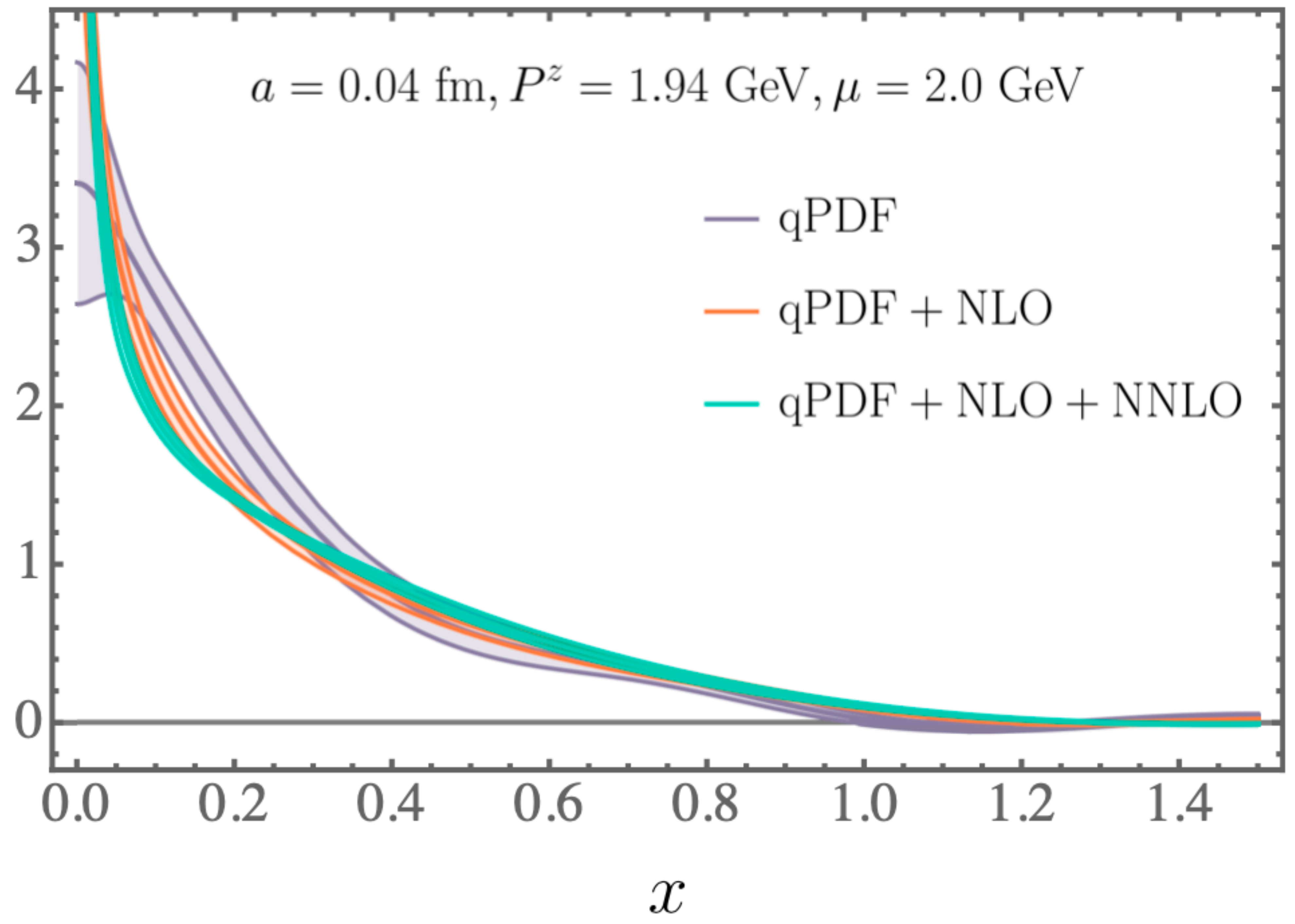
Hybrid renormalization and quasi-PDF



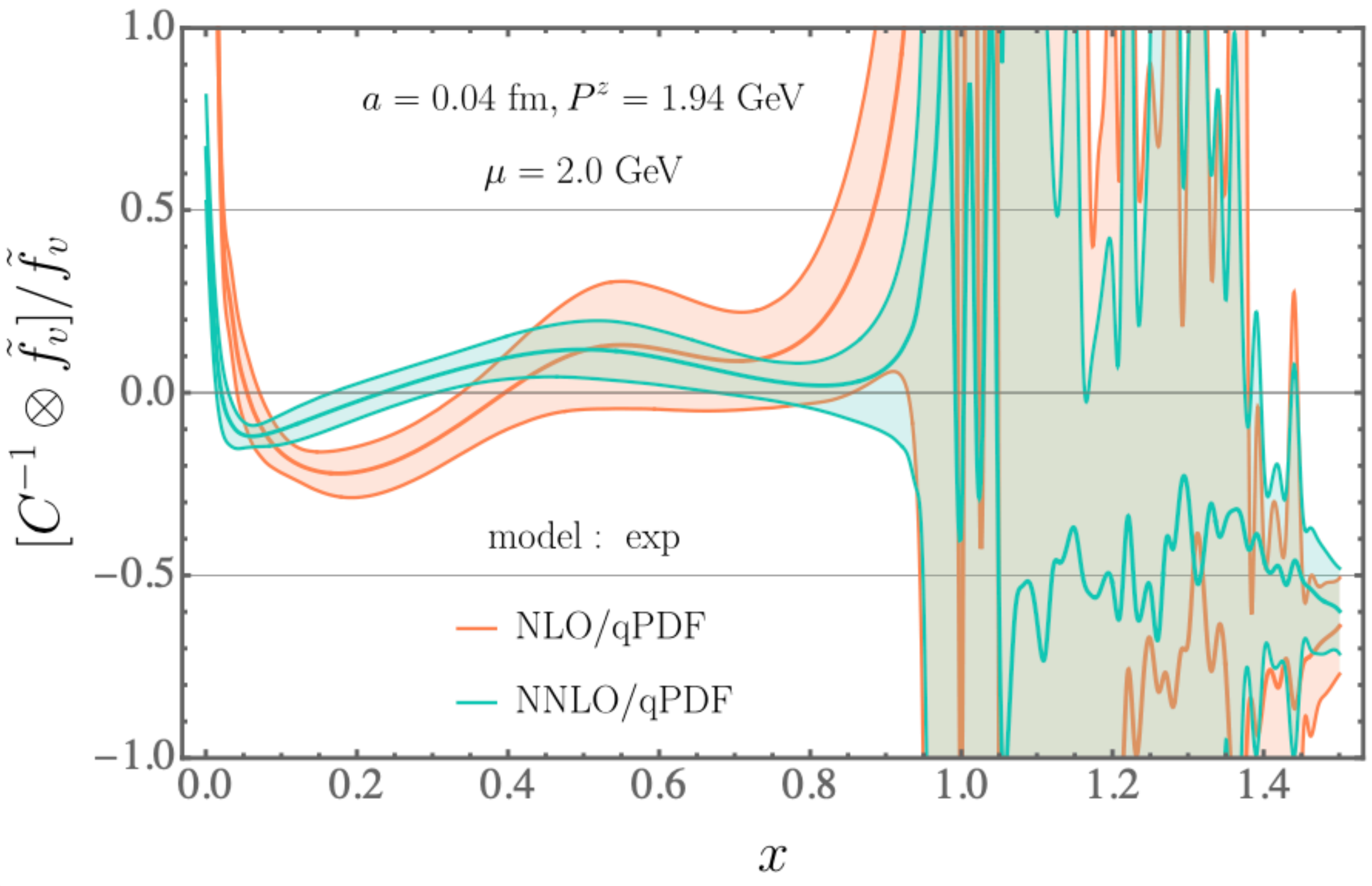
$$\tilde{q}(x, P_z, \mu) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP_z}\right) q(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{x^2 P_z^2}, \frac{\Lambda_{QCD}^2}{(1-x)^2 P_z^2}\right)$$

Systematics

LO → NLO → NNLO



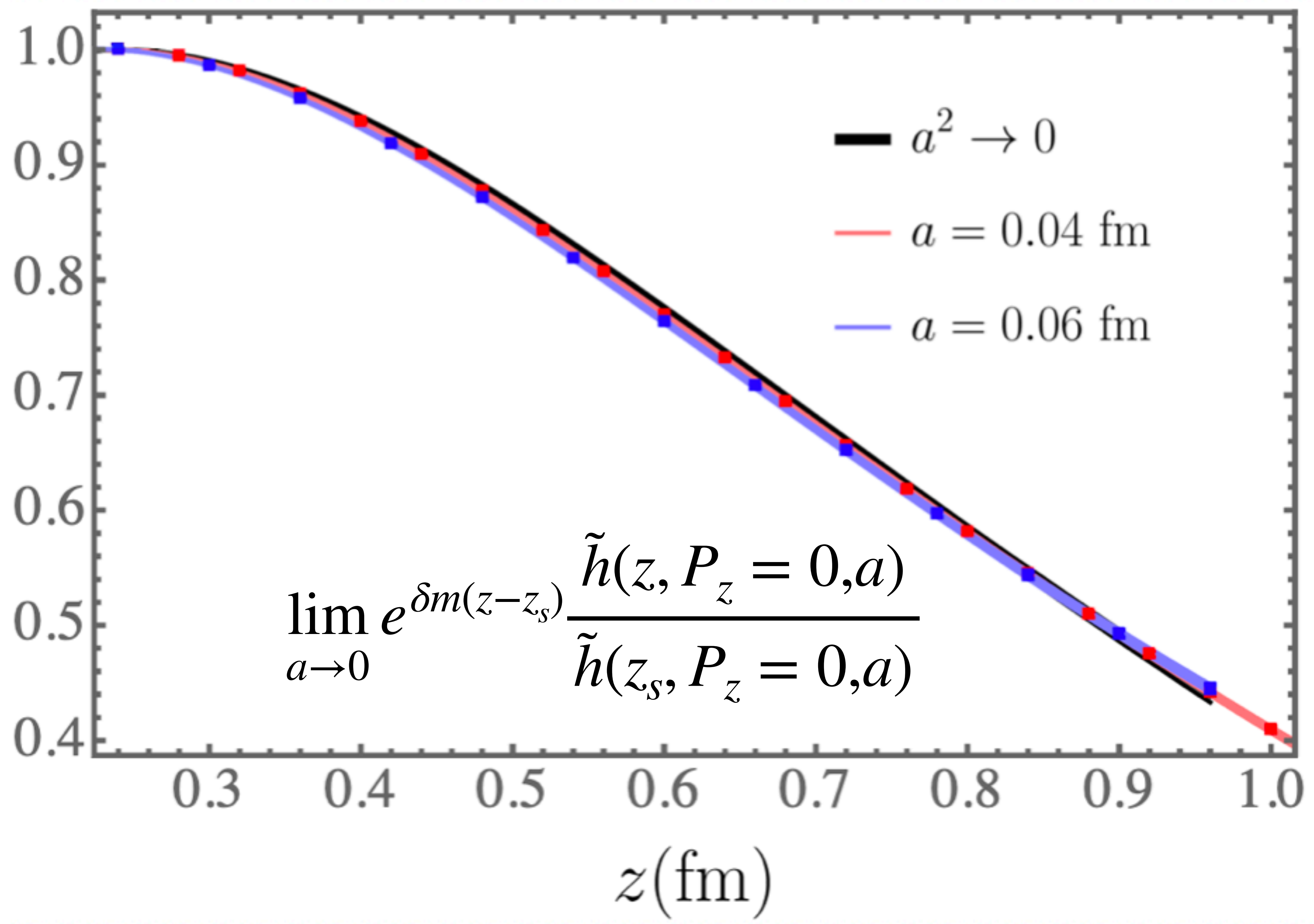
pQCD correction to the qPDF



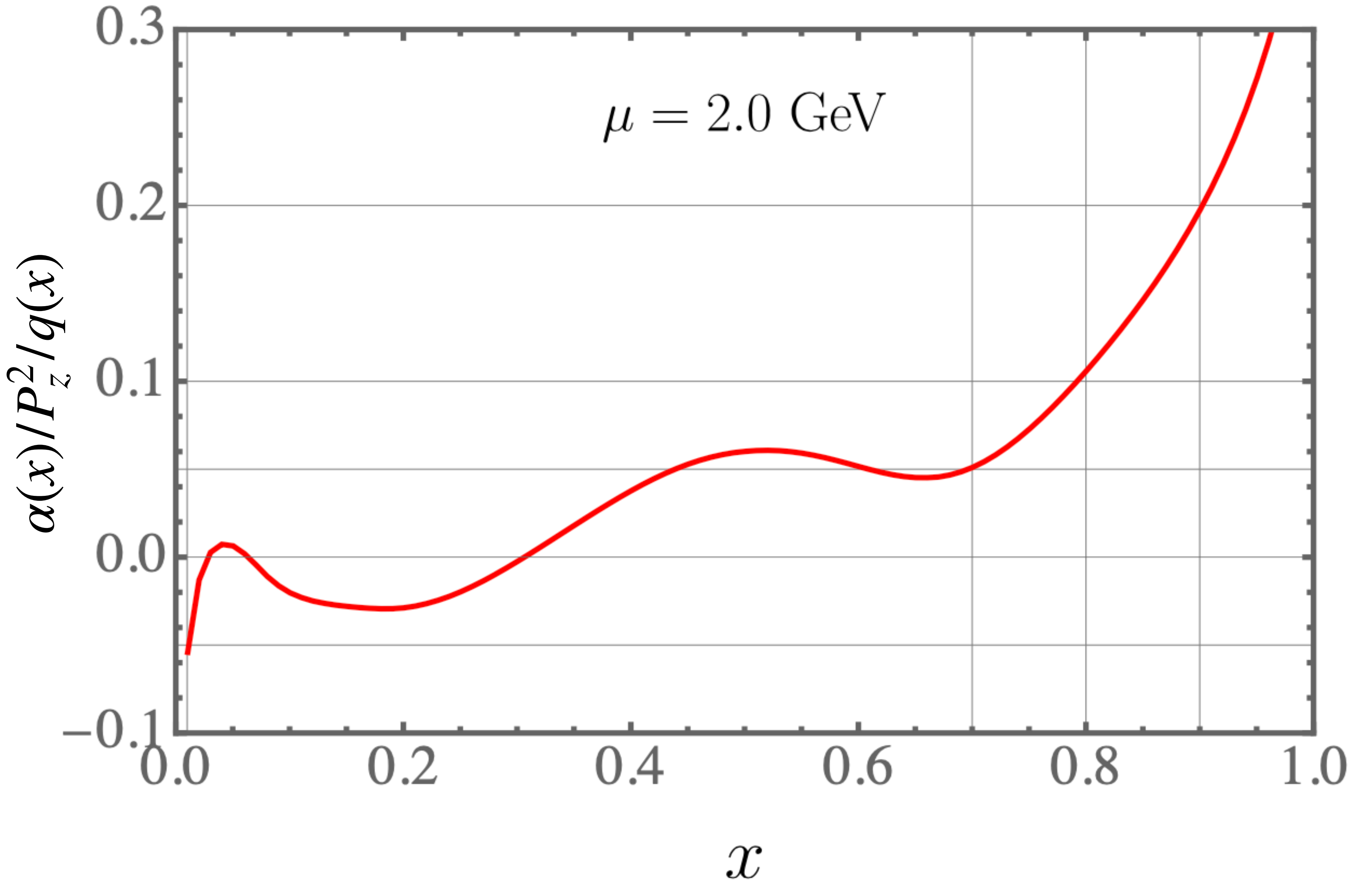
- Good convergence at moderate x.
- Large corrections in end-point regions, need resummation.

Systematics

Wilson-line mass subtraction



Power correction $q(x, P_z) = q(x) + \alpha(x)/P_z^2$

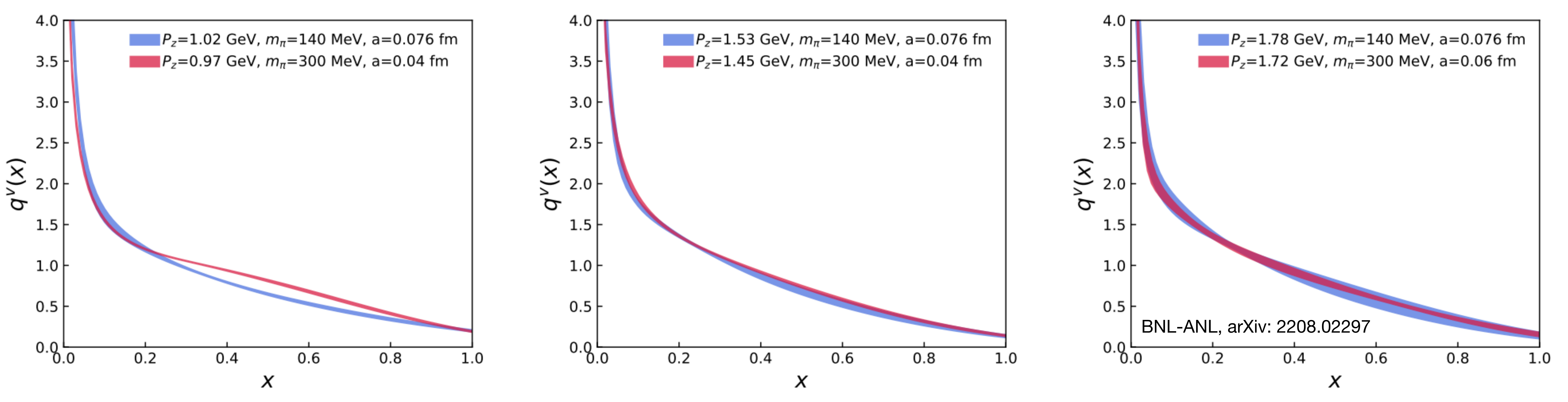


- Mild lattice spacing dependence

- Small P_z dependence in middle x

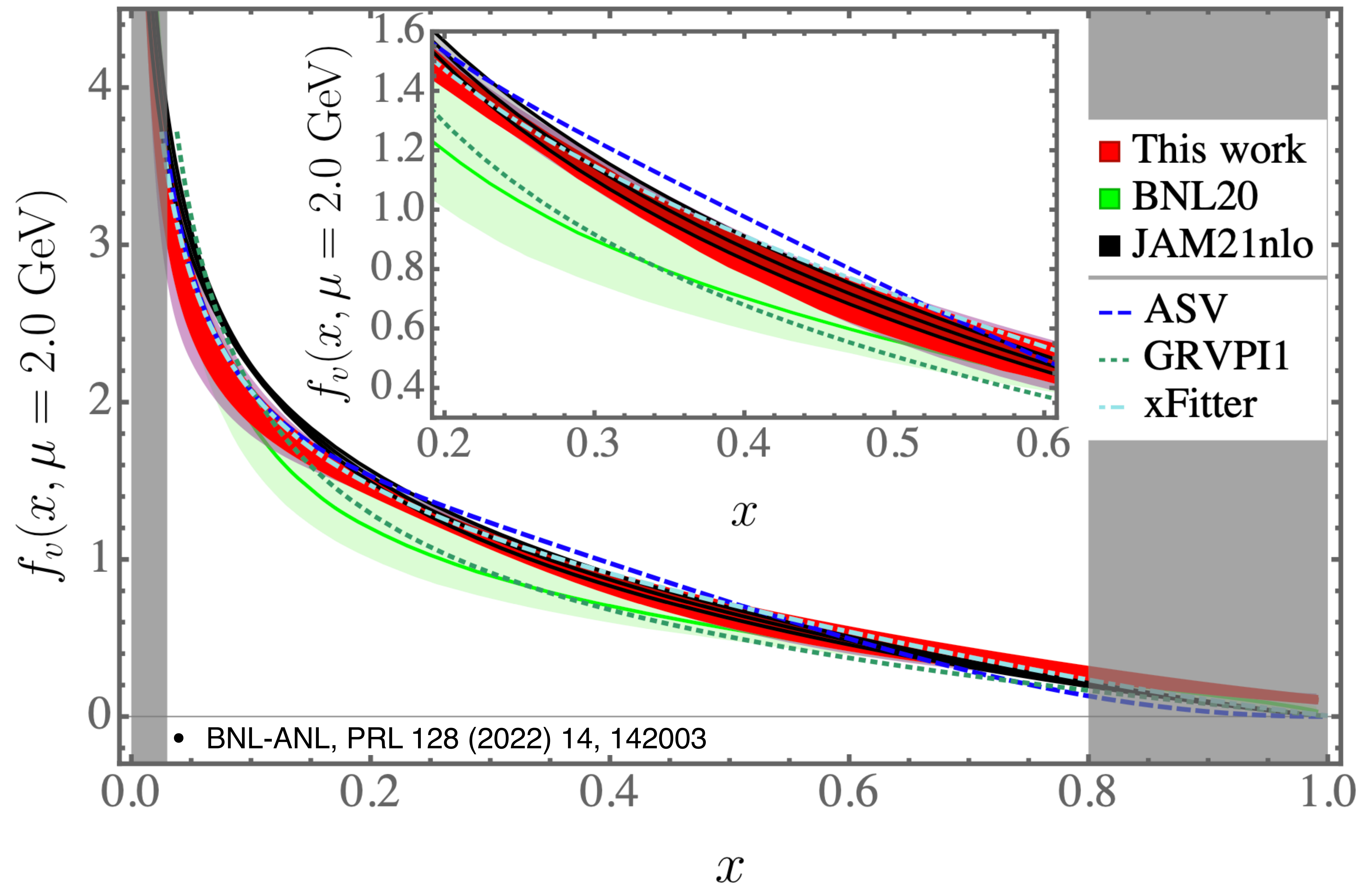
Systematics

qPDF from different lattice spacing and pion mass



- Pion mass dependence absent in large P_z.
- Lattice spacing dependence is small.

Final predictions on x dependence



The shaded regions x < 0.03 and x > 0.8 are excluded by requiring that estimates of $O(\alpha_s^3)$ and power corrections be smaller than 5% and 10%, respectively.

- Lattice prediction show good agreement with Global analysis from JAM, xFitter in moderate-x region.
- Reduced uncertainty and model dependence compared to the short-distance factorization (BNL20).

x	Statistical	Scale	$\mathcal{O}(\alpha_s^3)$	Power corrections	$\mathcal{O}(a^2 P_z^2)$
0.03	0.10	0.04	< 0.05	< 0.01	< 0.01
0.40	0.07	< 0.01	< 0.05	0.04	< 0.01
0.80	0.15	0.03	< 0.05	0.10	< 0.01

Summary

- We carried out lattice calculation of the quasi-PDF matrix elements of pion with large momentum and renormalized them using hybrid scheme.

- We predict x-dependence of pion valence PDF by applying NNLO matching formula, which show excellent agreement with recent global analysis.

- We demonstrate that we can predict the x-dependence with controlled systematic uncertainties within a subregion of x.

Thanks for your attention

Bare matrix elements and Renormalization

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- J. Green, K. Jansen and F. Steffens, PRL121 (2018)
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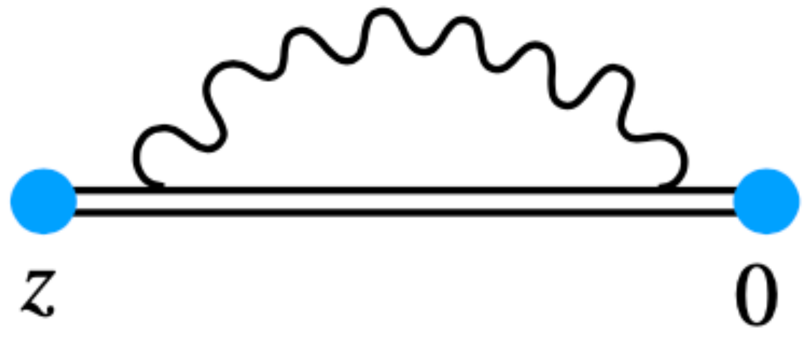

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$$= e^{-\delta m(a)|z|} Z(a) [\bar{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_R$$

$$\delta m = m_{-1}/a + m_0$$

Ratio scheme

A. V. Radyushkin, PRD 2017
K. Orginos, et al, PRD 96, 2017
Bálint Joó, et al, PRL125, 2020
X. Gao, et al, PRD 102, 2020
X. Gao, et al, arXiv: 2208.02297

$$h^R = \frac{\tilde{h}(z, P_z, a)}{\tilde{h}(z,0,a)}$$

to get RG-invariant ratio, then apply the short-distance factorization in coordinate-space.

$$\tilde{h}(z, P_z, \mu)$$

$$= \sum_n C_n(\mu^2 z^2) \frac{(-izP_z)^n}{n!} \int_{-1}^{1} dy\, y^n q(y,\mu) + \mathcal{O}(z^2\Lambda_{QCD}^2)$$

- Reducing the discretization effect and higher-twist effect by the ratio.
- Predicting the moments model independently.
- Limited in small z.

Physical extrapolation and Fourier transform

Extrapolation with model featuring an exponential decay $\langle \pi | j(x) j(0) | \pi \rangle \xrightarrow{|x|\to\infty} e^{-m_\rho |x|}$

- Burkardt, Grandy and Negele, Annals of Physics 238 (1995).

- exp: $Ae^{-m_{\text{eff}}|z|}/|\lambda|^d$
- 2p-exp: $A\mathrm{Re}\left[\frac{\Gamma(1+a)}{(-i|\lambda|)^{a+1}} + e^{i\lambda}\frac{\Gamma(1+b)}{(i|\lambda|)^{b+1}}\right] e^{-m_{\text{eff}}|z|}$