Non-perturbative renormalization of quark and gluon operators using a gauge-invariant scheme

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Outline

A. Introduction:

- Gauge-invariant renormalization scheme (GIRS)
- Energy-momentum tensor (EMT) and its nucleon matrix elements. Renormalization

B. Application of GIRS to the renormalization of non-singlet vector derivative quark bilinear operator

$$\mathcal{O}_{\mathrm{DV}_{\mu\nu}}(x) = \bar{d}(x)\gamma_{\{\mu}\overleftrightarrow{D}_{\nu\}}u(x)$$

C. Application of GIRS to the renormalization of QCD traceless EMT

$$\overline{T}^{g}_{\mu\nu}(x) = -2\ \mathrm{Tr}[F_{\rho\{\mu}(x)\ F_{\nu\}\rho}(x)], \qquad \overline{T}^{q}_{\mu\nu}(x) = \sum_{f=1}^{N_f} \bar{q}_f(x)\gamma_{\{\mu}\overleftrightarrow{D}_{\nu\}}q_f(x)$$

D. Conclusions and future prospects

Introduction

- **Renormalization** removes divergences from "bare" correlation functions in order to make contact with physical observables.
- Reference renormalization scheme: $\overline{\mathbf{MS}}$, which is defined perturbatively in dimensional regularization (DR).
- In non-perturbative studies on the lattice (L), one employs an **intermediate scheme**, which is both applicable in DR and L, e.g. RI/MOM.
- The matching to $\overline{\text{MS}}$ is done by using **perturbative conversion factors** (regularization independent, calculable in DR).
- In this work, we study a **gauge-invariant renormalization scheme (GIRS)**, by extending older investigations of **coordinate-space renormalization prescriptions** (see e.g., [V. Gimenez et al., PLB598 (2004) 227], [K. G. Chetyrkin et al., NPB844 (2011) 266], [K. Cichy, et al., NPB865 (2012) 268, NPB913 (2016) 278], [M. Tomii et al., PRD99 (2019) 014515])

Gauge-invariant renormalization scheme (GIRS)

- Consider on-shell **Green's functions** of gauge-invariant operators at different spacetime points in coordinate space, e.g.,

$$\langle \mathcal{O}_1(x)\mathcal{O}_2(y)\rangle, \quad (x \neq y)$$

- Impose **renormalization conditions** of the following form ($\mathcal{O}_i^{\rm R} = Z_{\mathcal{O}_i}\mathcal{O}_i$):

$$\langle \mathcal{O}_1^R(x)\mathcal{O}_2^R(y)\rangle = |_{x-y=z} = \langle \mathcal{O}_1(x)\mathcal{O}_2(y)\rangle^{\rm tree}|_{x-y=z} \Rightarrow$$

$$Z_{\mathcal{O}_1}\ Z_{\mathcal{O}_2}\ \langle \mathcal{O}_1(x)\mathcal{O}_2(y)\rangle|_{x-y=z} = \langle \mathcal{O}_1(x)\mathcal{O}_2(y)\rangle^{\rm tree}|_{x-y=z},$$

where $z \neq (0,0,0,0)$ and $a \ll |z| \ll \Lambda_{\rm QCD}^{-1}$ (renormalization window).

- **Good features:** No need for gauge-fixing, No contact terms, No need to consider mixing with gauge-non-invariant operators.

- **In our work:** Summations over different time slices of the operators' positions:

$$\sum_{\vec{x},\vec{y}} \langle \mathcal{O}_1(\vec{x},x_4)\mathcal{O}_2(\vec{y},y_4)\rangle|_{x_4-y_4=t}, \qquad t \neq 0$$

Energy-Momentum Tensor (EMT)

- **QCD gauge-invariant symmetric traceless EMT:**

  $\overline{T}^g_{\mu\nu}(x) = -2\ \mathrm{Tr}[F_{\rho\{\mu}(x)\ F_{\nu\}\rho}(x)]$, $\qquad \overline{T}^q_{\mu\nu}(x) = \sum_{f=1}^{N_f} \bar{q}_f \gamma_{\{\mu} \overleftrightarrow{D}_{\nu\}} q_f$,

  $\{\ldots\}$ = symmetrization over Lorentz indices μ, ν and subtraction of the trace.

- **Nucleon matrix elements of traceless EMT:**
  [X. Ji, J. Phys.G24, (1998) 1181]

$$\langle N(\vec{p}', s')|\overline{T}^{g(q)}_{\mu\nu}|N(\vec{p}, s)\rangle =$$

$$\overline{u}_N(\vec{p}', s')\left[A^{g(q)}_{20}(q^2)\ i\gamma_{\{\mu}P_{\nu\}} + B^{g(q)}_{20}(q^2)\frac{iP_{\{\mu}\sigma_{\nu\}\rho}q_\rho}{2m_N} + C^{g(q)}_{20}(q^2)\frac{q_{\{\mu}q_{\nu\}}}{m_N}\right]u_N(\vec{p}, s),$$

where $\qquad P \equiv (p' + p)/2, \qquad q \equiv p' - p.$

$\boxed{\langle x\rangle^{g(q)} = A^{g(q)}_{20}(0)}$ $\qquad \boxed{J^{g(q)} = \frac{1}{2}\left[A^{g(q)}_{20}(0) + B^{g(q)}_{20}(0)\right]}$

average momentum fraction $\qquad$ **nucleon spin**

$\Rightarrow$ **Goal:** To confirm: $\quad \langle x\rangle^g + \langle x\rangle^q = 1$, $\qquad J = J^g + J^q = \frac{1}{2}$

momentum sum rule $\qquad$ **spin sum rule**

- **Renormalization of traceless EMT:**
  [S. Caracciolo et al., NPB 375 (1992) 195]
  Mixing among $\quad \left\{\overline{T}^g_{\mu\nu}, \overline{T}^q_{\mu\nu},\ \text{gauge} - \text{non} - \text{invariant operators}\right\}$

Application of GIRS to vector derivative operator

- **Non-singlet vector one-derivative quark bilinear operator:**

$$\mathcal{O}_{\mathrm{DV}_{\mu\nu}}(x) = \bar{d}(x)\gamma_{\{\mu}\overleftrightarrow{D}_{\nu\}}u(x)$$

We focus on the non-diagonal (nd) case $\mu \neq \nu$.

- **Strategy:**

1. Calculate the following Green's function in the coordinate space:

$$G(t_s/a - t_0/a) \equiv \frac{1}{6}\sum_{\vec{x},\vec{y}}\sum_{i\neq j}\langle\mathcal{O}_{\mathrm{DV}_{ij}}(\vec{x},t_s/a)\mathcal{O}^{\dagger}_{\mathrm{DV}_{ij}}(\vec{y},t_0/a)\rangle.$$

2. Reduce discretization errors by calculating tree-level artifacts in lattice perturbation theory.
3. Impose the following renormalization condition:

$$(Z^{\mathrm{GIRS}}_{\mathrm{DV}_{\mathrm{nd}}})^2 \times G(t_s/a - t_0/a)|_{t_s-t_0=t} = G^{\mathrm{tree}}(t_s/a - t_0/a)|_{t_s-t_0=t}.$$

4. Convert to $\overline{\mathrm{MS}}$ at reference scale 2 GeV by calculating perturbative conversion factors/evolution functions in the continuum:

$$C^{\mathrm{GIRS}\to\overline{\mathrm{MS}}} \equiv Z^{\overline{\mathrm{MS}}}_{\mathrm{DV}_{\mathrm{nd}}}/Z^{\mathrm{GIRS}}_{\mathrm{DV}_{\mathrm{nd}}}, \qquad R^{\overline{\mathrm{MS}}}(\mu_1,\mu_2) = \exp\left(-\int_{g^{\overline{\mathrm{MS}}}(\mu_1)}^{g^{\overline{\mathrm{MS}}}(\mu_2)} dg\,\frac{\gamma^{\overline{\mathrm{MS}}}_{\mathcal{O}_{DV}}(g)}{\beta^{\overline{\mathrm{MS}}}(g)}\right).$$

5. Take plateau fit.

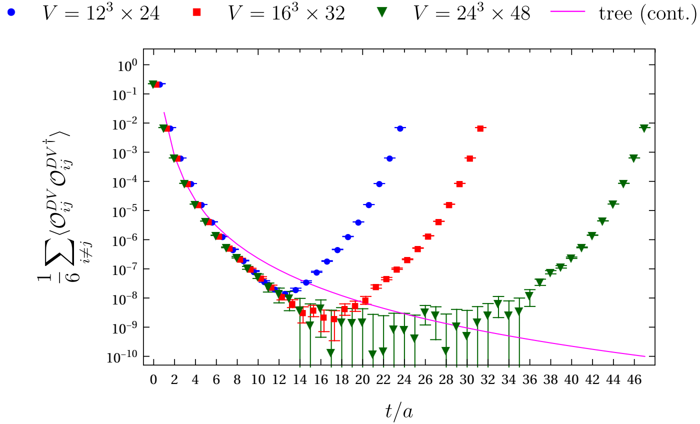
Application of GIRS to vector derivative operator

- **Ensembles:**

Ensemble	β	a (fm)	$L^3 \times T$	$a\mu$	N_{confs}	$N_{\text{stoch. src.}}$
cB4	1.778	0.080	$24^3 \times 48$	0.006	200	10
cC4	1.836	0.069	$32^3 \times 64$	0.005	132	12
cD4	1.900	0.058	$48^3 \times 96$	0.004	102	05

We exemplify the procedure by showing step-by-step results for **cB4**.

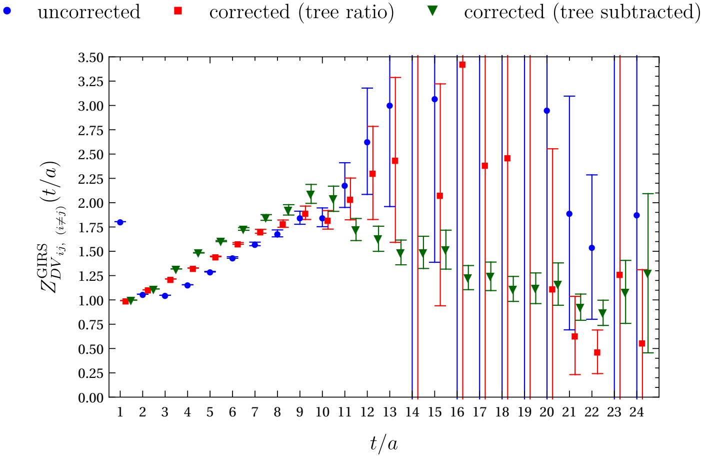
- **Step 1: Green's function (cB4)**



Application of GIRS to vector derivative operator

- **Step 2: Reduce tree-level discretization errors**
  1. **Ratio method:** $G(t/a) \to G(t/a) \times R(t/a)$, $\quad R(t/a) \equiv \frac{G^{\text{tree, cont.}}(t/a)}{G^{\text{tree, lat.}}(t/a)}$
  2. **Subtraction method:** $G(t/a) \to G(t/a) - D(t/a)$, $\quad D(t/a) \equiv G^{\text{tree, lat.}}(t/a) - G^{\text{tree, cont.}}(t/a)$

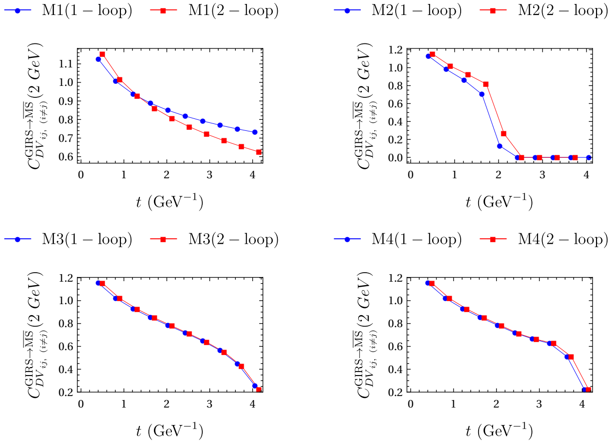
- **Step 3: GIRS renormalization factor (cB4)**



Application of GIRS to vector derivative operator

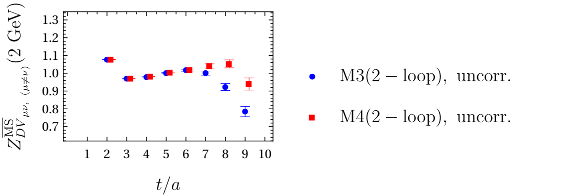
- **Step 4: Conversion to $\overline{\text{MS}}$ scheme at 2 GeV**

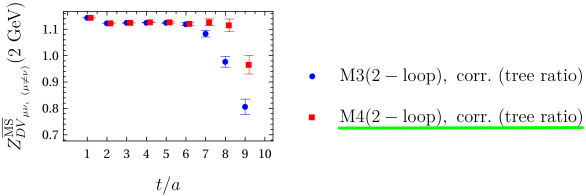
1. **M1:** (Conversion to $\overline{\text{MS}}$ scheme at 2 GeV) [M. Costa et al., PRD 103 (2021) 9]
2. **M2:** (Conversion to $\overline{\text{MS}}$ scheme at scale $1/t$) + (Evolution to 2 GeV)
3. **M3:** (Conversion to $\overline{\text{MS}}$ scheme at scale $\bar{\mu} = c/t$) + (Evolution to 2 GeV) [optimal value: $c = 2.18$]
4. **M4:** (Conversion to $\overline{\text{MS}}$ scheme at scale $\bar{\mu} = c/t$) + (Evolution of $g^{\overline{\text{MS}}}$ to scale c'/t) + (Evolution to 2 GeV) [JLQCD, PRD 94 (2016) 5] [optimal values: $c = 2.18, c' = 1.5$]

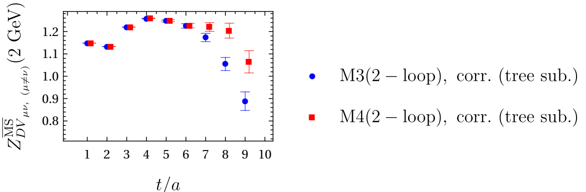


Application of GIRS to vector derivative operator

- $\overline{\text{MS}}$ **renormalization factor (cB4)**

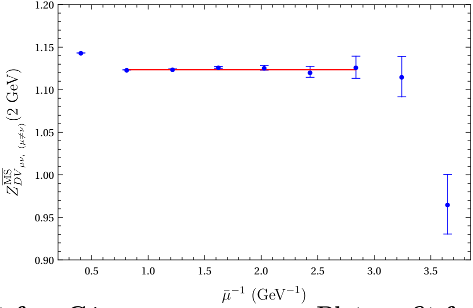






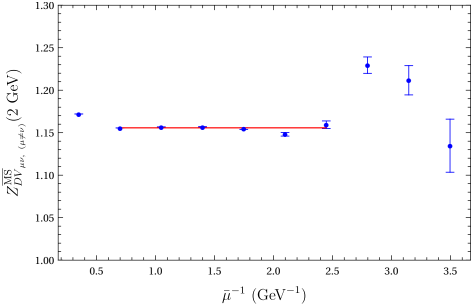
Application of GIRS to vector derivative operator

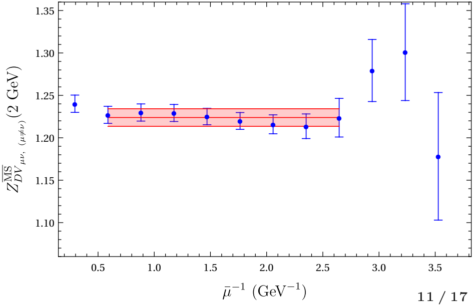
- **Step 5: Plateau fit (cB4)**



- **Plateau fit for cC4**

Plateau fit for cD4

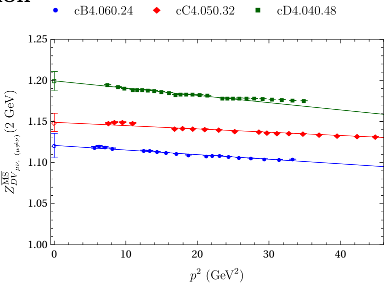




Application of GIRS to vector derivative operator

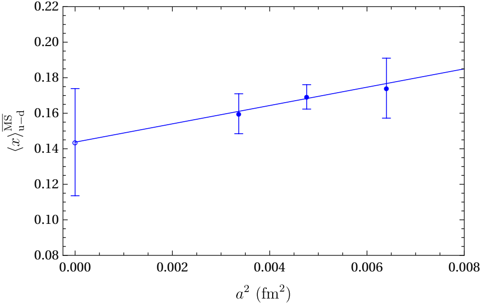
- **Comparison with RI/MOM prescription**

Ensemble	GIRS	RI/MOM
cB4	1.1234(1)(46)	1.1209(5)(142)
cC4	1.1558(1)(43)	1.1490(3)(111)
cD4	1.2239(104)(131)	1.1995(4)(113)



- **Application to isovector average momentum fraction**

Continuum limit $a \to 0$:



Comparison with other studies:

	$\langle x\rangle^{\overline{\rm MS}}_{u-d}$
NNPDF3.1	0.152(3)
CT14	0.158(4)
MMHT2014	0.151(4)
ABMP2016	0.167(4)
CJ15	0.152(2)
HERAPDF2.0	0.188(3)
χQCD18	0.151(28)(29)
ETMC22	0.126(32)
This work	0.144(30)

Application of GIRS to EMT

- **Mixing matrix:** (non-diagonal operators $\mu \neq \nu$)

$$\begin{pmatrix} \bar{T}^{g,R}_{\mu\nu} \\ \bar{T}^{q,R}_{\mu\nu} \end{pmatrix} = \begin{pmatrix} Z_{gg} & Z_{gq} \\ Z_{qg} & Z_{qq} \end{pmatrix} \begin{pmatrix} \bar{T}^{g}_{\mu\nu} \\ \bar{T}^{q}_{\mu\nu} \end{pmatrix}$$

- **Renormalization conditions: (in terms of renormalized operators)**

$$\frac{1}{6} \sum_{\vec{x},\vec{y}} \sum_{i\neq j} \left\langle \overline{T}^{g}_{ij}{}^{\rm GIRS}(\vec{x}, t_s/a)\ \overline{T}^{g}_{ij}{}^{\rm GIRS}(\vec{y}, t_0/a) \right\rangle |_{t_s - t_0 = t} = \text{tree},$$

$$\frac{1}{6} \sum_{\vec{x},\vec{y}} \sum_{i\neq j} \left\langle \overline{T}^{q}_{ij}{}^{\rm GIRS}(\vec{x}, t_s/a)\ \overline{T}^{q}_{ij}{}^{\rm GIRS}(\vec{y}, t_0/a) \right\rangle |_{t_s - t_0 = t} = \text{tree},$$

$$\frac{1}{6} \sum_{\vec{x},\vec{y}} \sum_{i\neq j} \left\langle \overline{T}^{g}_{ij}{}^{\rm GIRS}(\vec{x}, t_s/a)\ \overline{T}^{q}_{ij}{}^{\rm GIRS}(\vec{y}, t_0/a) \right\rangle |_{t_s - t_0 = t} = \text{tree} = 0,$$

$$\frac{1}{6} \sum_{\vec{x},\vec{y},\vec{z}} \sum_{i\neq j} \left\langle \mathcal{O}^{\rm GIRS}_{\gamma_i}(\vec{x}, t_s/a) \overline{T}^{g}_{ij}{}^{\rm GIRS}(\vec{y}, t_i/a)\ \mathcal{O}^{\rm GIRS\,\dagger}_{\gamma_j}(\vec{z}, t_0/a) \right\rangle |_{\substack{t_s - t_0 = 2t, \\ t_i - t_0 = t}} = \text{tree} = 0,$$

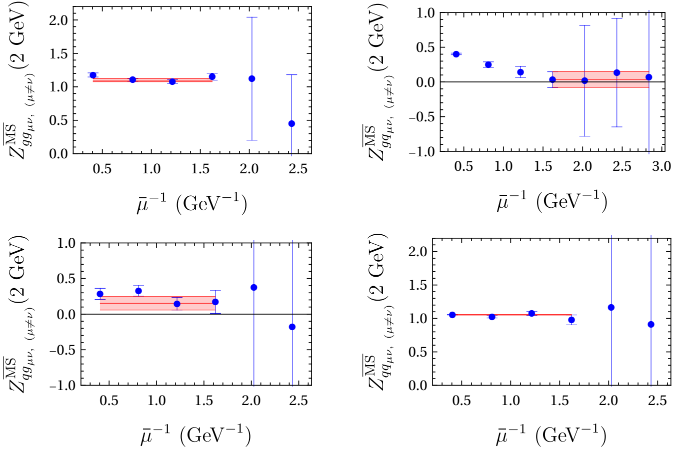
where $\mathcal{O}_{\gamma_i} = \bar{d}\gamma_i u$.

Application of GIRS to EMT

- **Ensemble:**

Ensemble	β	a (fm)	$L^3 \times T$	$a\mu$
cB4	1.778	0.080	$12^3 \times 24$	0.006

- $\overline{\text{MS}}$ **mixing matrix:** (non-diagonal operators $\mu \neq \nu$)



Application of GIRS to EMT

- **Table of Z functions:**

Ensemble	$Z_{gg}^{\overline{\text{MS}}}$ (2 GeV)	$Z_{gq}^{\overline{\text{MS}}}$ (2 GeV)	$Z_{qg}^{\overline{\text{MS}}}$ (2 GeV)	$Z_{qq}^{\overline{\text{MS}}}$ (2 GeV)
cB4	1.100(23)(125)	0.035(114)(34)	0.152(95)(68)	1.054(4)(5)

- **Application to average momentum fraction:**

$$\langle x\rangle_q^{\overline{\text{MS}}} = Z_{qq(ns)}^{\overline{\text{MS}}}\langle x\rangle_q^B + \frac{\delta Z_{qq}^{\overline{\text{MS}}}}{N_f}\sum_{q=u,d,s,c}\langle x\rangle_q^B + \frac{Z_{qg}^{\overline{\text{MS}}}}{N_f}\langle x\rangle_g^B, \qquad \delta Z_{qq}^{\overline{\text{MS}}} = Z_{qq(s)}^{\overline{\text{MS}}} - Z_{qq(ns)}^{\overline{\text{MS}}}$$

$$\langle x\rangle_g^{\overline{\text{MS}}} = Z_{gg}^{\overline{\text{MS}}}\langle x\rangle_g^B + Z_{gq}^{\overline{\text{MS}}}\sum_{q=u,d,s,c}\langle x\rangle_q^B$$

Comparison with other studies:

	$\langle x\rangle_u^{\overline{\text{MS}}}$	$\langle x\rangle_d^{\overline{\text{MS}}}$	$\langle x\rangle_s^{\overline{\text{MS}}}$	$\langle x\rangle_c^{\overline{\text{MS}}}$	$\langle x\rangle_g^{\overline{\text{MS}}}$
NNPDF3.1	0.348(4)	0.196(3)	0.039(3)	—	0.410(4)
CT14	0.348(3)	0.190(3)	0.035(5)	—	0.416(5)
MMHT2014	0.348(5)	0.197(5)	0.035(9)	—	0.411(9)
ABMP2016	0.353(3)	0.186(3)	0.041(2)	—	0.412(4)
CJ15	0.348(1)	0.196(1)	—	—	0.416(1)
HERAPDF2.0	0.372(4)	0.185(7)	0.035(11)	—	0.401(10)
χQCD18	0.307(30)(18)	0.160(27)(40)	0.051(26)(5)	—	0.482(69)(48)
ETMC20	0.359(30)	0.188(19)	0.052(12)	0.019(9)	0.427(92)
This work	0.348(27)	0.181(27)	0.049(16)	0.016(15)	0.465(99)

$\Rightarrow$ Total sum: $\langle x\rangle_g^{\overline{\text{MS}}} + \sum_{q=u,d,s,c}\langle x\rangle_q^{\overline{\text{MS}}} = 1.060(109)$

Conclusions and future prospects

- We employ GIRS in the renormalization of non-singlet vector one-derivative quark bilinear operator, using $Nf = 4$ degenerate twisted mass/clover fermion ensembles of 3 different lattice spacings.

	cB4	cC4	cD4
$Z^{\overline{\mathrm{MS}}}_{\mathrm{DV}_{\mu\neq\nu}}$	1.1234(1)(46)	1.1558(1)(43)	1.2239(104)(131)

- We use the extracted values of Z factors for calculating the isovector average momentum fraction of the nucleon, by taking the continuum limit.

$$\boxed{\langle x\rangle^{\overline{\mathrm{MS}}}_{\mathrm{u-d}} = 0.144(30)}$$

- We employ GIRS in the renormalization of traceless EMT, using $Nf = 4$ degenerate twisted mass/clover fermion ensemble of lattice spacing $a = 0.080$ fm.

$Z^{\overline{\mathrm{MS}}}_{gg}$ (2 GeV)	$Z^{\overline{\mathrm{MS}}}_{gq}$ (2 GeV)	$Z^{\overline{\mathrm{MS}}}_{qg}$ (2 GeV)	$Z^{\overline{\mathrm{MS}}}_{qq}$ (2 GeV)
1.100(23)(125)	0.035(114)(34)	0.152(95)(68)	1.054(4)(5)

- We use the extracted values of Z factors for calculating the up, down, strange, charm and gluon average momentum fractions of the nucleon.

$\langle x\rangle^{\overline{\mathrm{MS}}}_{u}$	$\langle x\rangle^{\overline{\mathrm{MS}}}_{d}$	$\langle x\rangle^{\overline{\mathrm{MS}}}_{s}$	$\langle x\rangle^{\overline{\mathrm{MS}}}_{c}$	$\langle x\rangle^{\overline{\mathrm{MS}}}_{g}$
0.348(27)	0.181(27)	0.049(16)	0.016(15)	0.465(99)

- We confirm the momentum sum rule (within error):

$$\boxed{\langle x\rangle^{\overline{\mathrm{MS}}}_{g} + \sum_{q=u,d,s,c} \langle x\rangle^{\overline{\mathrm{MS}}}_{q} = 1.060(109)}$$

Conclusions and future prospects

- **Future plans:**
  - Extract EMT mixing matrix (using GIRS) for C and D ensembles.
  - Check for dependence on quark mass. Take chiral extrapolation by using ensembles of the same lattice spacing and different values of the quark mass.
  - Extend our calculation to diagonal EMT operators $\mu = \nu$.
  - Study of renormalization for the trace part of EMT: Additional mixing with lower dimensional operators and operators allowed by hypercubic invariance [S. Caracciolo et al., Annals Phys. 197 (1990) 119])

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