Pion nucleon excited state effects in nucleon observables

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Precision study of nucleon matrix elements with local light currents

Definition

$$\mathcal{M}_{\Gamma,\tau}(Q^2) = \langle N, \vec{p}', \sigma' | \mathcal{J} | N, \vec{p}, \sigma \rangle,$$

$$Q^2 = -(P' - P)^2, P'^2 = P^2 = M_N^2, \mathcal{J} = Z_{\mathcal{J}} \bar{\psi} \Gamma \otimes \tau \psi, \psi = \begin{pmatrix} u \\ d \end{pmatrix}$$

Targeting matrix elements with their current insertion

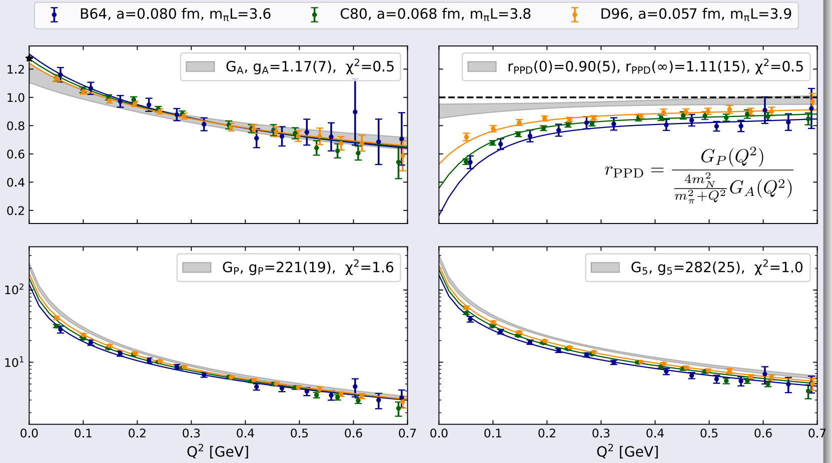
- Sigma term $\mathcal{J} = m_q(\bar{u}u + \bar{d}d)\delta_{\sigma,\sigma'}$ $\quad \mathcal{M} = \sigma_{\pi N}$
- Nucleon electromagnetic form factors

  $\mathcal{J} = Z_V(2/3\bar{u}\gamma_\mu u - 1/3\bar{d}\gamma_\mu d)$ $\quad \mathcal{M} = \bar{u}_{\sigma'}(\vec{p}') \left[\gamma_\mu G_1(Q^2) + \frac{\sigma_{\mu\nu} Q_\nu}{2m_N} G_2(Q^2)\right] u_\sigma(\vec{p})$
- Axial Pseudoscalar form factors

  $\mathcal{J} = Z_A(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d)$ $\quad \mathcal{M} = \bar{u}_{\sigma'}(\vec{p}') \left[\gamma_\mu G_A(Q^2) - \frac{Q_\nu}{2m_N} G_P(Q^2)\right] u_\sigma(\vec{p})$

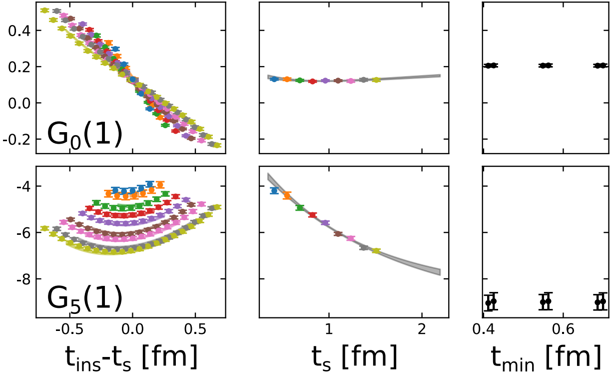
State of the art lattice results (ETMC)

Axial and Pseudoscalar form factor at the physical point (Preliminary)



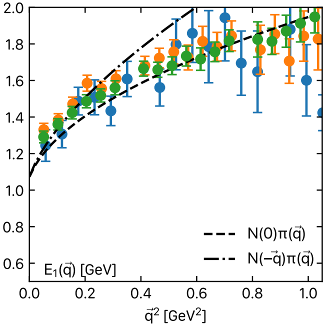
Excited states (Plenary talk by Aaron Meyer (Wed. 09:20-09:50))

- Excited states not removed for G_5 at low momentum
- Plateau method Residual time dependence due to excited states
- Goal: To account for these excited states



Excited states: πN scattering states, Roper resonance

- Easiest way: Fitting the excited states explicitely
- Excited state extracted from the 2pt or 3pt function
- Overlaps of the excited states in 2 and 3pt can be different



- $\pi - N$ states volume suppressed in 2pt
- Extreme case: excited states cannot even detected using single hadron operator

Investigating excited state effects

Including two hadron interpolating operators

- Extracting matrix elements from ratio of 3pt and 2pt

$$C(\Gamma_0,\vec{p};t_s,t_0) = \sum_{\vec{x}_s} \mathrm{Tr}\left[\Gamma_0 \langle O_N(t_s,\vec{x}_s)\,\bar{O}_N(t_0,\vec{x}_0)\rangle\right] \mathrm{e}^{-i\vec{p}(\vec{x}_s-\vec{x}_0)}, \quad \Gamma_0=\frac{1}{2}(\mathbb{I}+\gamma_0)$$

$$C_{\mathscr{J}}(\Gamma_{\mathscr{J}},\vec{q},\vec{p}\,';t_s,t_{\mathrm{ins}},t_0) = \sum_{\vec{x}_s,\vec{x}_{\mathrm{ins}}} \mathrm{Tr}\left[\Gamma_{\mathscr{J}} \langle O_N(t_s,\vec{x}_s)\,\mathscr{J}(t_{\mathrm{ins}},\vec{x}_{\mathrm{ins}})\,\bar{O}_N(t_0,\vec{x}_0)\rangle\right] \mathrm{e}^{-i(\vec{x}_s-\vec{x}_0)\vec{p}\,'}\,\mathrm{e}^{-i(\vec{x}_{\mathrm{ins}}-\vec{x}_0)\vec{q}}$$

Spectral decomposition of the 3-pt function

$$C_{\mathscr{J}}(\Gamma_{\mathscr{J}},\vec{q},\vec{p}\,';t_s,t_{\mathrm{ins}}) =$$

$$\mathrm{Tr}\left[\Gamma_{\mathscr{J}} Z_{0,N}\bar{Z}_{N,0}\,\langle \boldsymbol{N},0|\,\mathscr{J}(0)\,|\boldsymbol{N},-\vec{\boldsymbol{q}}\rangle\right] \mathrm{e}^{-m_N\Delta t_{s,\mathrm{ins}}}\,\mathrm{e}^{-E_N\Delta t_{\mathrm{ins},0}}$$

$$+\mathrm{Tr}\left[\Gamma_{\mathscr{J}} Z_{0,N}\bar{Z}_{\pi N,0}\,\langle \boldsymbol{N},0|\,\mathscr{J}(0)\,|\boldsymbol{\pi N},-\vec{\boldsymbol{q}}\rangle\right] \mathrm{e}^{-m_N\Delta t_{s,\mathrm{ins}}}\,\mathrm{e}^{-\boldsymbol{E}_{\pi N}\Delta t_{\mathrm{ins},0}}$$

$$+\mathrm{Tr}\left[\Gamma_{\mathscr{J}} Z_{0,\pi N}\bar{Z}_{N,0}\,\langle \boldsymbol{\pi N},0|\,\mathscr{J}(0)\,|\boldsymbol{N},-\vec{\boldsymbol{q}}\rangle\right] \mathrm{e}^{-\boldsymbol{E}_{\pi N}\Delta t_{s,\mathrm{ins}}}\,\mathrm{e}^{-E_N\Delta t_{\mathrm{ins},0}}$$

$$+\mathrm{Tr}\left[\Gamma_{\mathscr{J}} Z_{0,\pi N}\bar{Z}_{\pi N,0}\times\langle \boldsymbol{\pi N},0,|\,\mathscr{J}(0)\,|\boldsymbol{\pi N},-\vec{\boldsymbol{q}}\rangle\right] \mathrm{e}^{-\boldsymbol{E}_{\pi N}\Delta t_{s,\mathrm{ins}}}\,\mathrm{e}^{-\boldsymbol{E}_{\pi N}\Delta t_{\mathrm{ins},0}}$$

$$+\ldots$$

Simulation details

Project aim:

- Excited states in the 2pt,3pt function using $N, \pi N$ interpolating fields
- Two-hadron spectrum in the $N(I = 1/2, I_3 = +1/2)$ channel
- Performing a GEVP analysis for the 3pt function using $N, N\pi$ operator basis

Interpolating fields

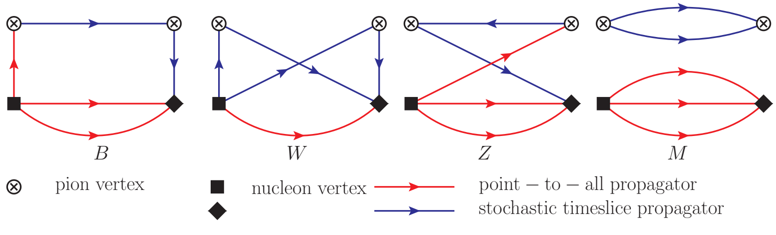
- $\mathcal{O}_{N^+} = (u \mathrm{C}\gamma_5 d)\, u$
- $\mathcal{O}_{N^0} = (d \mathrm{C}\gamma_5 u)\, d$
- $\mathcal{O}_{\pi^0} = \frac{1}{\sqrt{2}}(\bar{u}\mathrm{i}\gamma_5 u - \bar{d}\mathrm{i}\gamma_5 d)$
- $\mathcal{O}_{\pi^+} = (\bar{d}\mathrm{i}\gamma_5)\, u$

Parameters

- Confs: 2 flavour Twisted mass Clover,
- $M_\pi = 134\mathrm{MeV}, \quad a = 0.0913\mathrm{fm}$
- $L = 4.3\mathrm{fm}, \quad M_\pi \cdot L = 3, \quad N_s = 48$
- $\#conf = 600{,}48$ *sp* with Gauss-smearing at source,sink.

Contractions

Typical diagrams in πN 2-pt function



- Fermion lines through the pion vertex are estimated with the one-end trick
- Method easily generalizable to 3pt functions
- Implementation on GPU-s PLEGMA software package

Consequences of finite volume: Projections

- Instead of spin we have the degrees of freedom:
  - irrep, irrep row(μ), $\#$ occurances

Irreps in this work

$\vec{p}_{\mathrm{tot}}$, irrep name	ℓ	N_{dim}
$\vec{p}=(0,0,0), \mathrm{G1_g}$	s	8x8
$\vec{p}=(0,0,0), \mathrm{G1_u}$	s	8x8
$\vec{p}=(0,0,1), \mathrm{G1}$	s,p,d	22x22
$\vec{p}=(1,1,0), \mathrm{(2)G}$	s,p,d	26x26
$\vec{p}=(1,1,1), \mathrm{(3)G}$	s,p,d	14x14

$G1g$ irrep $\vec{p}_{\mathrm{tot}}=(0,0,0), p_N = 1, p_\pi = 1, \mu = 0$

- Occurance a

$$\frac{\sqrt{6}}{6}\left(N_{-1,0,0}(0)\pi_{1,0,0} - iN_{0,-1,0}(0)\pi_{0,1,0} + iN_{0,1,0}(0)\pi_{0,-1,0} - N_{1,0,0}(0)\pi_{-1,0,0} - N_{0,0,1}(0)\pi_{0,0,-1}N_{0,0,-1}(0)\pi_{0,0,1}\right)$$

- Occurance b

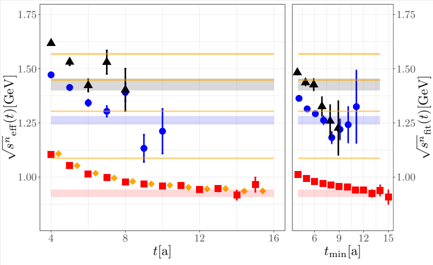
$$\frac{\sqrt{6}}{6}\left(N_{-1,0,0}(2)\pi_{1,0,0} - iN_{0,-1,0}(2)\pi_{0,1,0} + iN_{0,1,0}(2)\pi_{0,-1,0} - N_{1,0,0}(2)\pi_{-1,0,0} - N_{0,0,1}(2)\pi_{0,0,-1}N_{0,0,-1}(2)\pi_{0,0,1}\right)$$

Generalized EigenValue Problem (GEVP)

Energy levels

- Using correlation matrices we form GEVP:
  $C_{i,j}(t)v_j^n(t_0) = \lambda_n(t,t_0)C_{i,j}(t_0)v_j^n(t_0)$

$G1_g$: Effective mass and stability plot



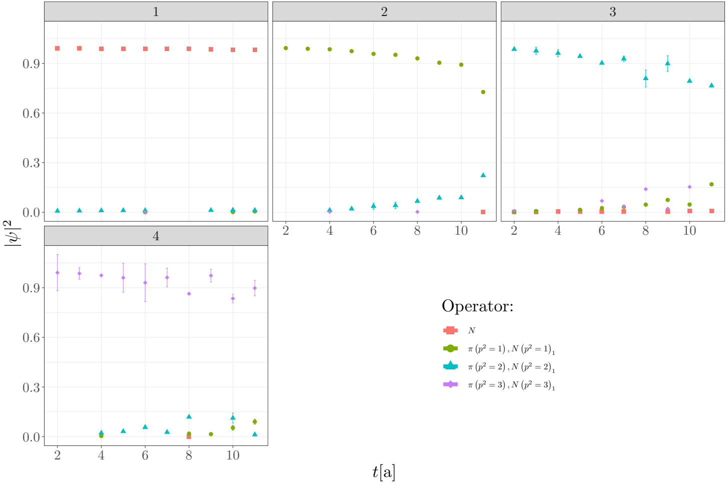
- No level at $N(0)\pi(0)$ due to parity

Conclusion from 2pt functions

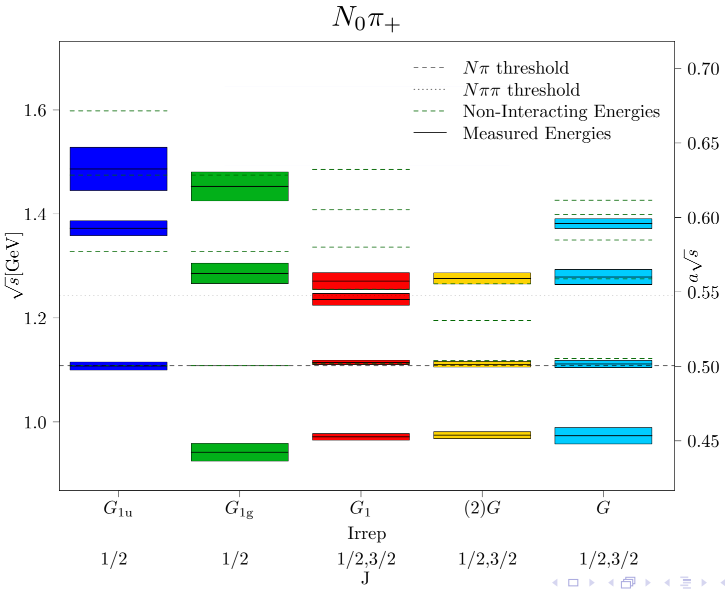
- First level compatible with the physical nucleon mass
- Single Hadron 2pt compatible with GEVP ground state
- Excited states in single hadron *2pt* are not resolved by our 8x8 GEVP.

Eigenvectors from the $\mathcal{O}(N), \mathcal{O}(\pi N)$ GEVP

- Ground state of the GEVP is dominated by single nucleon interpolator



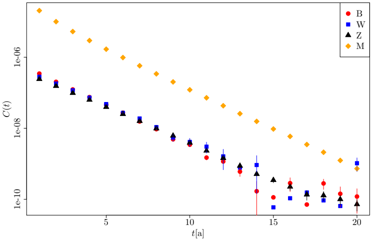
Two hadron spectrum

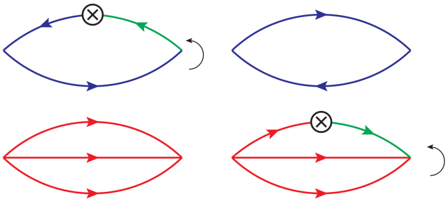


3pt-function

Local current insertion

- Only M-type of diagram is used
- Justification (as a preliminary investigation)
  - Two point function most of the signal originates from the M diagram
  - M diagram still involves the interaction via gluon exchange between pion and nucleon



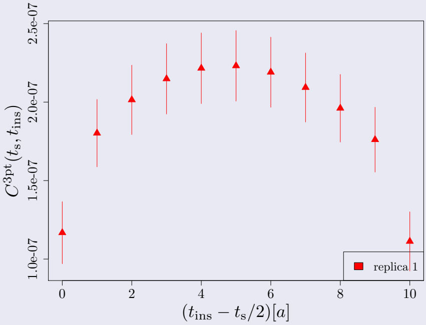


Signal checks from 3pt function πN source and sink (Work in progress)

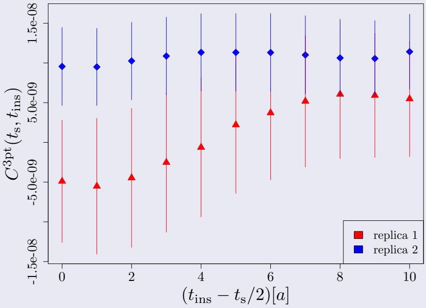
Checking signal

- Scalar insertion in $G1_g$ (CMF) $\Delta t = 10$:

Zero momentum



One unit back-to-back momentum



Conclusion

- We have determined the spectrum in the nucleon channel including two hadron interpolating fields
- For both isospin: neutron and proton
- We found that single hadron dominates the ground state
- We started to look at three pt between πN states
- Stay tuned: Talk by Lorenzo Barca Today 16:40
- Thank you very much for your attention