Chiral-even twist-3 GPDs for the proton

Jack Dodson
S. Bhattacharya, K. Cichy, M. Constantinou, A. Metz, A. Scapellato, F. Steffens

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Outline

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Twist classification

- All types of distribution functions can be expanded in terms of their twist (dimension minus spin), which is also the order that they enter into QCD factorization formulas

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q_o} + \frac{f_i^{(2)}}{Q_o^2} + \ldots$$

- twist-2 contribution: $\mathcal{O}(Q_o^0)$ (e.g., unpolarized and helicity)
- twist-3 contribution: $\mathcal{O}(Q_o^{-1})$
- Q_0 is the large energy scale of the process.

Generalized Parton Distributions (GPDs)

- Necessary for studying the three dimensional structure of the hadrons.
- Provide extensive information on the hadron properties (e.g., spin and mass decomposition, orbital angular momentum).
- Their Mellin moments (e.g., electromagnetic and axial form factors) have physical interpretation and are extracted experimentally.
- Experimentally accessed through exclusive processes.
- GPD extraction poses several challenges with limited information available compared to PDFs.

The above motivate dedicated calculations of GPDs from lattice QCD

Twist-3 distributions

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q_o} + \frac{f_i^{(2)}}{Q_o^2} + \ldots$$

- Twist-3 contributions in the cross section may be sizeable.
- Lack density interpretation; but have physical interpretation (g_T: $F_\perp$)
- Twist-3 GPDs relevant for spin-orbit correlations.[Lorce,PLB(2014),arXiv:1401.7784]
- Contain information on multi-parton correlators (q-g-q).
- Knowledge of twist-3 GPDs is necessary to reliably disentangle twist-2 GPDs.
- PDFs: twist-2 case has been extensively studied [K. Cichy, PoS LATTICE2021 (2022) 017, arXiv: 2110.07440]; little is known about twist-3. [S. Bhattacharya et al., PRD 104 (2021) 11, 114510; PRD 102 (2020) 11, 111501]
- GPDs: limited information on twist-2; almost nothing available for twist-3.

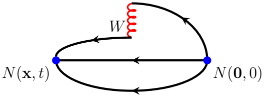
Outline

Quasi-PDf method (LaMET)

- Boosted hadrons with nonlocal operators
- Extraction of matrix elements from two and three point function
- Nonperturbative renormalization in RI′ scheme
- Reconstruct x-dependence using the Backus-Gilbert method
- Matching with only 2-parton correlators
  S. Bhattacharya et al., PRD 102 (2020) 11, arXiv:2004.04130
- Matching for qgq-correlation has been discussed
  [V. Braun et al., JHEP 05 (2021) 086; JHEP 10 (2021) 087]

Extraction of Matrix Elements

Matrix elements calculated in symmetric frame:



$$h_{\mathcal{O}}(\Gamma_\kappa, z, P_f, P_i, \mu) = Z_{\mathcal{O}}(z,\mu)\,\langle N(P+\frac{Q}{2})|\,\overline{\psi}(z)\,\mathcal{O}\,\mathcal{W}(z,0)\psi(0)\,|N(P-\frac{Q}{2})\rangle$$

- The indices of $\mathcal{O}$ are transverse to the boost (for twist-3): γ^j (vector) and $\gamma^5\,\gamma^j$ (axial), $j = 1, 2$.
- $\mathbf{P} = (0, 0, P_3)$ is the proton momentum boost.
- Wilson line in the same direction as the momentum boost.
- Γ_κ is the parity projector with $\kappa = 0,\ 1,\ 2,\ 3$ **(to disentangle GPDs)**
- $\Gamma_0 = \frac{1+\gamma_0}{2}$ and $\Gamma_j = \frac{1}{4}(1+\gamma^0)i\gamma^5\gamma^j$.

F_X Function Disentanglement

- For matrix element parameterization we use Kiptily and Polyakov [Kiptily et al., Eur. Phys. J. C(2002), arXiv:0212372]

$$\mathrm{h}_{\gamma^j} = \langle\langle \frac{g_\perp^{j\rho}\Delta_\rho}{2m} \rangle\rangle [F_E + F_{G_1}] + \langle\langle g_\perp^{j\rho}\gamma_\rho \rangle\rangle [F_H + F_{G_2}] + \langle\langle \frac{g_\perp^{j\rho}\Delta_\rho\gamma^+}{P^+} \rangle\rangle F_{G_3} + \langle\langle \frac{i\epsilon_\perp^{j\rho}\Delta_\rho\gamma^+\gamma_5}{P^+} \rangle\rangle F_{G_4}$$

$$\mathrm{h}_{\gamma^j\gamma_5} = \langle\langle \frac{g_\perp^{j\rho}\Delta_\rho\gamma_5}{2m} \rangle\rangle [F_{\widetilde{E}} + F_{\widetilde{G}_1}] + \langle\langle g_\perp^{j\rho}\gamma_\rho\gamma_5 \rangle\rangle [F_{\widetilde{H}} + F_{\widetilde{G}_2}] + \langle\langle \frac{g_\perp^{j\rho}\Delta_\rho\gamma^+\gamma_5}{P^+} \rangle\rangle F_{\widetilde{G}_3} + \langle\langle \frac{i\epsilon_\perp^{j\rho}\Delta_\rho\gamma^+}{P^+} \rangle\rangle F_{\widetilde{G}_4}$$

$\widetilde{H}$, $\widetilde{E}$: twist-2, $\widetilde{G}_i$: twist-3

- Matrix elements lead to independent equations depending on the index of the operator and parity projector.

Computational Challenges

- Due to the momentum transfer, there are increased statistical uncertainties compared to the PDFs case.
- Values of momentum transfer controlled by the spatial extent of the lattice $(\frac{2\pi}{L})$
- Increased statistical uncertainties in the twist-3 contributions compared to twist-2 case
- Mixing from the qgq-correlators.
- There is a need for as many independent matrix elements as there are GPDs, so that we can disentangle them

$$\Pi^1(\Gamma_0) = C\left(-F_{\widetilde{G}_4}\frac{Q_y(E+m)}{2m^2} - [F_{\widetilde{H}} + F_{\widetilde{G}_2}]\frac{P_3 Q_y}{4m^2} \right),$$

$$\Pi^1(\Gamma^1) = i\,C\left(-[F_{\widetilde{E}} + F_{\widetilde{G}_1}]\frac{Q_x^2(E+m)}{8m^3} + [F_{\widetilde{H}} + F_{\widetilde{G}_2}]\frac{(4m(E+m)+Q_y^2)}{8m^2} + F_{\widetilde{G}_4}\frac{Q_y^2(E+m)}{4m^2P_3} \right)$$

$$\Pi^1(\Gamma^2) = i\,C\left(-[F_{\widetilde{E}} + F_{\widetilde{G}_1}]\frac{Q_xQ_y(E+m)}{8m^3} - F_{\widetilde{G}_4}\frac{Q_xQ_y(E+m)}{4m^2P_3} - [F_{\widetilde{H}} + F_{\widetilde{G}_2}]\frac{Q_xQ_y}{8m^2} \right),$$

$$\Pi^1(\Gamma^3) = i\,C\left(F_{\widetilde{G}_3}\frac{EQ_x(E+m)}{2m^2P_3} \right),$$

$$\Pi^2(\Gamma_0) = C\left(F_{\widetilde{G}_4}\frac{Q_x(E+m)}{2m^2} + [F_{\widetilde{H}} + F_{\widetilde{G}_2}]\frac{P_3 Q_x}{4m^2} \right),$$

$$\Pi^2(\Gamma^1) = i\,C\left(-[F_{\widetilde{E}} + F_{\widetilde{G}_1}]\frac{Q_xQ_y(E+m)}{8m^3} - F_{\widetilde{G}_4}\frac{Q_xQ_y(E+m)}{4m^2P_3} - [F_{\widetilde{H}} + F_{\widetilde{G}_2}]\frac{Q_xQ_y}{8m^2} \right),$$

$$\Pi^2(\Gamma^2) = i\,C\left(-[F_{\widetilde{E}} + F_{\widetilde{G}_1}]\frac{Q_y^2(E+m)}{8m^3} + [F_{\widetilde{H}} + F_{\widetilde{G}_2}]\frac{(4m(E+m)+Q_x^2)}{8m^2} + F_{\widetilde{G}_4}\frac{Q_x^2(E+m)}{4m^2P_3} \right)$$

$$\Pi^2(\Gamma^3) = i\,C\left(F_{\widetilde{G}_3}\frac{EQ_y(E+m)}{2m^2P_3} \right),$$

Parameters of Calculation

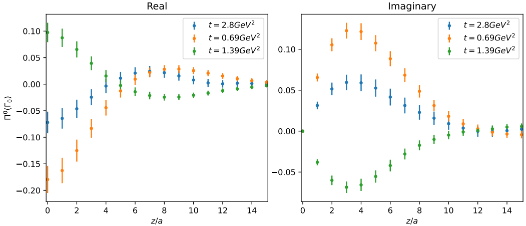
- $N_f = 2+1+1$ ensemble of maximally **twisted mass fermions**.
- pion mass $M_\pi = 260$ MeV,
- Lattice spacing $a \simeq 0.093$ fm and volume $V = 32^3 \times 64$.
- The nucleon boost is nonzero along the z-direction, $\vec{P} = (0, 0, \pm P_3)$.
- The source-sink time separation is chosen as $t_s = 10a$ (0.93 fm), due to the increased uncertainties
- Results available for $\xi = 0$

$P_3\,[\mathrm{GeV}]$	$q\,[\frac{2\pi}{L}]$	$-t\,[\mathrm{GeV}^2]$	N_{confs}	N_{src}	N_{total}
± 0.83	$(\pm 2, 0, 0), (0, \pm 2, 0)$	0.69	67	8	4288
± 1.25	$(\pm 2, 0, 0), (0, \pm 2, 0)$	0.69	249	8	15936
± 1.25	$(\pm 2, \pm 2, 0)$	1.39	223	8	28544
± 1.67	$(\pm 2, 0, 0), (0, \pm 2, 0)$	0.69	294	32	75264
± 1.25	$(\pm 4, 0, 0), (0, \pm 4, 0)$	2.76	329	32	84224

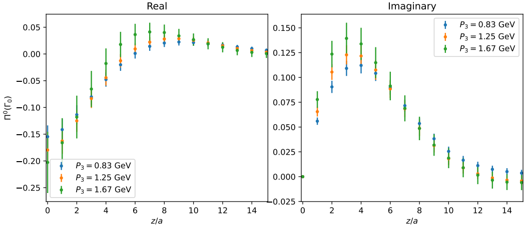
Outline

Matrix Elements

- matrix elements at $P_3 = 1.25$ GeV for various t.



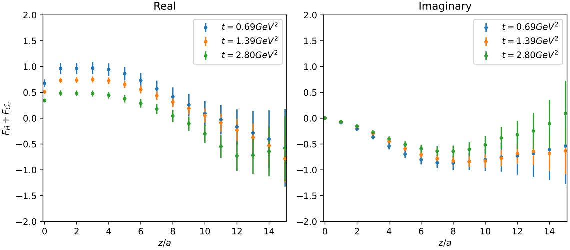
- matrix elements at $t = 0.69 GeV^2$ for various P_3.



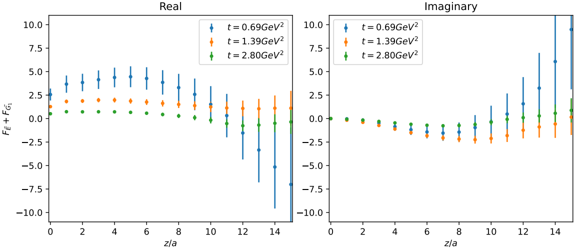
- We find a good signal, and we observe an hierarchy between the different matrix elements with respect to changes in t.
- $\Pi(\Gamma_0)$ at $t = -0.69$ GeV^2 is dominant in magnitude.
- P_3 dependence mild and within uncertainties.

F_X Functions

- $F_{\tilde{H}} + F_{\tilde{G}_2}$ at $P_3 = 1.25$ GeV for various t.



- $F_{\tilde{E}} + F_{\tilde{G}_1}$ at $P_3 = 1.25 GeV^2$ for various t.



Decomposed functions F_X, $X = \widetilde{H} + \widetilde{G}_2,\ \widetilde{E} + \widetilde{G}_1,\ \widetilde{G}_3,\ \widetilde{G}_4$

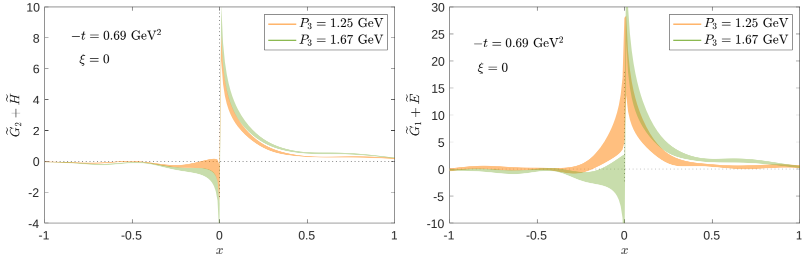
- F_X decreases with increase of t (standard behavior)
- $F_{\widetilde{E}} + F_{\widetilde{G}_1}$ has the largest magnitude (expected from axial and induced pseudoscalar form factors).

Not shown:

- $F_{\widetilde{G}_3}$ is found to be exactly zero, due to the fact $\int dx\ x\widetilde{G}_3 = \frac{\xi}{4} G_E(t)$
- $F_{\widetilde{G}_4}$ is noisy and very small: $\int dx \widetilde{G}_i = 0$ $(i = 1, 2, 3, 4)$, could possibly be the reason.

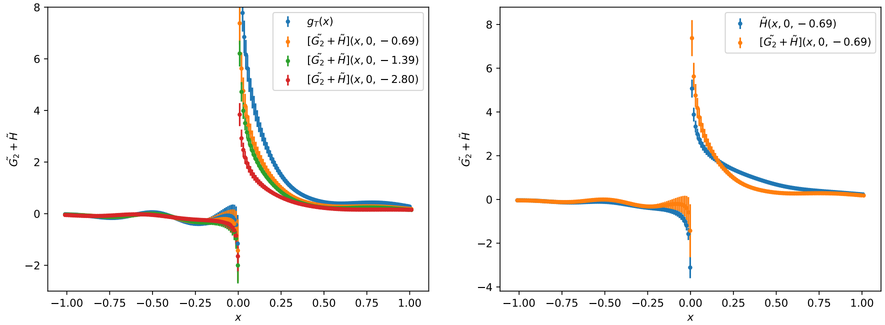
GPDs: Momentum Boost Dependence

- Reconstruction of x dependence not unique (Naive FT, Backus-Gilbert Method, etc.).
  We use Backus-Gilbert Backus and Gilbert, Geophysical Journal International, 1968
- After x-dependence reconstruction and matching.



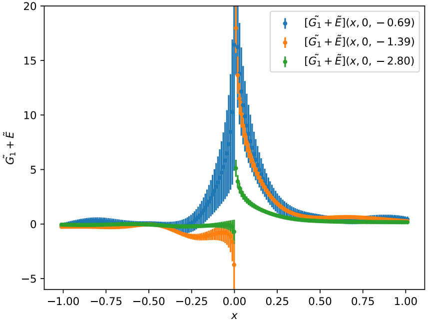
- We find mild P_3 dependence with a marginal agreement in the small-x region.

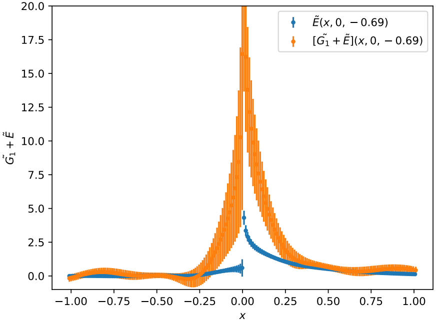
GPDs: Momentum transfer Dependence



- $\widetilde{H} + \widetilde{G}_2$ compared with the forward limit, g_T []Bhattacharya et al., PRD (2020)]
- $g_T(x)$ is the dominant distribution in magnitude
- Noticeable dependence on t for both $\widetilde{H} + \widetilde{G}_2$.
- For $t = -0.69$, 1.39 GeV^2 $\widetilde{H} + \widetilde{G}_2$ approach zero at $x \sim 0.4$; for $t = -2.8$ GeV^2 decay is faster
- Right: difference between $\widetilde{H} + \widetilde{G}_2$ and $\widetilde{H}$ gives an estimate for $\widetilde{G}_2$

GPDs (Momentum Boost Dependence)





- Similar hierarchy with respect to t with a tendency for $t = -0.69$ GeV^2 to be the largest (compatible with $t = -1.39$ GeV^2 within uncertainties)).
- $\widetilde{G_1} + \widetilde{E}$ at $t = -2.8$ GeV^2 very suppressed
- Right: $\tilde{E}$ is much smaller than $\widetilde{G_1} + \widetilde{E}$, indicating large $\widetilde{G_1}$ (unlike $\widetilde{G_2}$).

Outline

Summary

Conclusions:

- There is a reasonable path to access twist-3 GPDs from lattice.
- Good signal for twist-3 GPDs.

Future work

- Extend calculation to nonzero skewness (matching must be calculated)
- Address systematic effect (e.g., discretization effects, volume effects).
- Explore difference renormalization schemes (Hybrid, Improved RI) [X. Ji et al., 964 (2021) 115311][M. Constantinou et al., arXiv:2207.09977]
- Complete analysis for the vector twist-3 GPD
- Study of chiral-odd twist-3 GPDs.
- Include matching with quark-gluon-quark mixing [Braun et al., arXiv:2103.12105]
- Study twist-3 GPDs in alternative frames [talks by Bhattacharrya & Constantinou]

Thank you!

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