Accessing proton GPDs in non-symmetric frames:

Numerical implementation

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in collaboration with:

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The 39th International Symposium on Lattice Field Theory (Lattice 2022)

August 11, 2022

Generalized Parton Distributions

★ Crucial in understanding hadron tomography; accessed via exclusive reactions (DVCS, DVMP)

★ Provide a correlation between the transverse position and the longitudinal momentum of the quarks in the hadron and its mechanical properties (OAM, pressure, etc.)
[M. Burkardt, PRD62 071503 (2000), hep-ph/0005108] [M. V. Polyakov, PLB555 (2003) 57, hep-ph/0210165]

★ GPDs are not well-constrained experimentally:

- x-dependence extraction is challenging
- independent measurements to disentangle GPDs
- limited coverage of kinematic region
- data on certain GPDs
- indirectly related to GPDs through the Compton FFs
- GPDs phenomenology more complicated than PDFs (multi-dimensionality)

★ GPD results from lattice QCD can be incorporated in global analysis of experimental data

Light-cone GPDs

★ Off-forward matrix elements of non-local light-cone operators

$$F^{[\gamma^+]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int\frac{dz^-}{2\pi}e^{ik\cdot z}\langle p';\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\gamma^+\,\mathscr{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})\,|p;\lambda\rangle\Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

★ Parametrization in two leading twist GPDs

$$F^{[\gamma^+]}(x,\Delta;\lambda,\lambda') = \frac{1}{2P^+}\bar{u}(p',\lambda')\left[\gamma^+H(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M}E(x,\xi,t)\right]u(p,\lambda)$$

★ How can one define GPDs on a Euclidean lattice?

GPDs on the lattice

Off forward correlators with nonlocal (equal-time) operators

[X. Ji, PRL 110 (2013) 262002]

$$\tilde{q}_\mu^{\rm GPD}(x,t,\xi,P_3,\mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z}\, \langle N(P_f)\,|\,\bar{\Psi}(z)\,\gamma^\mu\,\mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$

$$\Delta = P_f - P_i$$

$$t = \Delta^2 = -\,Q^2$$

$$\xi = \frac{Q_3}{2P_3}$$

Variables of the calculation:

- length of the Wilson line (z)
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→ reduction of power corrections in fwd limit [Radyushkin, PLB 767, 314, 2017]

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→ finite mixing with scalar [Constantinou & Panagopoulos (2017)]

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reduction of power corrections in fwd limit
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- Lorentz non-invariant parametrization
- Typically used in symmetric frame
- A non-symmetric setup may result to different functional form for GPDs compared to the symmetric one

Motivation - Outline of work

- ★ Calculation expected to be performed in symmetric frame to extract the “standard” GPDs
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Extraction of GPDs in the symmetric frame using lattice correlators calculated in non-symmetric frames

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★ Calculation expected to be performed in symmetric frame to extract the “standard” GPDs

★ Symmetric frame requires separate calculations at each t

1st goal:

Extraction of GPDs in the symmetric frame using lattice correlators calculated in non-symmetric frames

2nd goal:

New definition of Lorentz covariant quasi-GPDs that may have faster convergence to light-cone GPDs (elimination of kinematic corrections)

Theoretical setup

★ Parametrization of matrix elements in Lorentz invariant amplitudes

[see S. Bhattacharya talk]

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^{\mu}}{M}A_1 + z^{\mu}\,MA_2 + \frac{\Delta^{\mu}}{M}A_3 + i\sigma^{\mu z}\,MA_4 + \frac{i\sigma^{\mu\Delta}}{M}A_5 + \frac{P^{\mu}i\sigma^{z\Delta}}{M}A_6 + \frac{z^{\mu}i\sigma^{z\Delta}}{M}A_7 + \frac{\Delta^{\mu}i\sigma^{z\Delta}}{M}A_8\right]u(p,\lambda)$$

Advantages

- Applicable to any kinematic frame and have definite symmetries
- Lorentz invariant amplitudes A_i can be related to the standard H, E GPDs
- Quasi H, E may be redefined (Lorentz covariant) to eliminate $1/P_3$ contributions:

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$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = A_1 + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}A_3$$

$$E(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = -A_1 - \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}A_3 + 2A_5 + 2P_{avg,s/a}\cdot zA_6 + 2\Delta_{s/a}\cdot zA_8$$

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★ Proof-of-concept calculation (zero quasi-skewness):

- symmetric frame: $\vec{p}^{\,s}_f = \vec{P} + \frac{\vec{Q}}{2}$, $\quad \vec{p}^{\,s}_i = \vec{P} - \frac{\vec{Q}}{2} \quad t^s = -\vec{Q}^2$

- asymmetric frame: $\vec{p}^{\,a}_f = \vec{P}$, $\quad \vec{p}^{\,a}_i = \vec{P} - \vec{Q} \quad t^a = -\vec{Q}^2 + (E_f - E_i)^2$

Matrix element decomposition

Symmetric

$$C_s = \frac{2m^2}{E(E+m)}$$

$$\Gamma_0 = \frac{1}{2}(1+\gamma^0)$$

$$\Gamma_j = \frac{i}{4}(1+\gamma^0)\gamma^5\gamma^j \quad (j=1,2,3)$$

$$\Pi^0_s(\Gamma_0) = C_s\left(\frac{E\left(E(E+m)-P_3^2\right)}{2m^3}A_1 + \frac{(E+m)\left(-E^2+m^2+P_3^2\right)}{m^3}A_5 + \frac{EP_3\left(-E^2+m^2+P_3^2\right)z}{m^3}A_6\right)$$

$$\Pi^0_s(\Gamma_1) = i\,C_s\left(\frac{EP_3Q_2}{4m^3}A_1 - \frac{(E+m)P_3Q_2}{2m^3}A_5 - \frac{E\left(P_3^2+m(E+m)\right)zQ_2}{2m^3}A_6\right)$$

$$\Pi^0_s(\Gamma_2) = i\,C_s\left(-\frac{EP_3Q_1}{4m^3}A_1 + \frac{(E+m)P_3Q_1}{2m^3}A_5 + \frac{E\left(P_3^2+m(E+m)\right)zQ_1}{2m^3}A_6\right)$$

Asymmetric

$$C_a = \frac{2m^2}{\sqrt{E_iE_f(E_i+m)(E_f+m)}}$$

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Novel feature: z-dependence

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No definite symmetries for Π_μ^a

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$$\Gamma_j = \frac{i}{4}(1+\gamma^0)\gamma^5\gamma^j \quad (j = 1,2,3)$$

$$\Pi_s^1(\Gamma_0) = i\,C_s\left(-\frac{\left(E(E+m)-P_3^2\right)Q_1}{2m^3}A_3 + \frac{P_3Q_1z}{4m}A_4 - \frac{P_3\left(-E^2+m^2+P_3^2\right)zQ_1}{m^3}A_8\right)$$

$$\Pi_s^1(\Gamma_1) = C_s\left(\frac{P_3Q_1Q_2}{4m^3}A_3 + \frac{Q_1Q_2z}{8m}A_4 - \frac{\left(P_3^2+m(E+m)\right)Q_1Q_2z}{2m^3}A_8\right)$$

$$\Pi_s^1(\Gamma_2) = C_s\left(-\frac{P_3Q_1^2}{4m^3}A_3 + \frac{\left(4E(E+m)-Q_1^2\right)z}{8m}A_4 + \frac{\left(P_3^2+m(E+m)\right)Q_1^2z}{2m^3}A_8\right)$$

$$\Pi_s^1(\Gamma_3) = C_s\,\frac{(E+m)Q_2}{2m^2}A_5$$

Asymmetric

$$C_a = \frac{2m^2}{\sqrt{E_iE_f(E_i+m)(E_f+m)}}$$

$$\Pi_1^a(\Gamma_0) = i\,C_a\Bigg(-\frac{(E_f-E_i-2m)(E_f+m)Q_1}{8m^3}A_1 + \frac{(E_f-E_i-2m)(E_f+m)Q_1}{4m^3}A_3 + \frac{P_3Q_1z}{4m}A_4$$
$$+\frac{(E_f-E_i)(E_f+m)Q_1}{4m^3}A_5 + \frac{E_f(E_f-E_i)P_3Q_1z}{4m^3}A_6 + \frac{E_f(E_i-E_f)P_3Q_1z}{2m^3}A_8\Bigg)$$

$$\Pi_1^a(\Gamma_1) = C_a\Bigg(-\frac{P_3Q_1Q_2}{8m^3}A_1 + \frac{P_3Q_1Q_2}{4m^3}A_3 + \frac{P_3Q_1Q_2}{4m^3}A_5 + \frac{E_f(E_f+m)Q_1Q_2z}{4m^3}A_6$$
$$-\frac{E_f(E_f+m)Q_1Q_2z}{2m^3}A_8\Bigg)$$

No definite symmetries for Π_μ^a

$$\Pi_1^a(\Gamma_2) = C_a\Bigg(\frac{P_3Q_1^2}{8m^3}A_1 - \frac{P_3Q_1^2}{4m^3}A_3 + \frac{(E_f+E_i)(E_f+m)z}{4m}A_4 + \frac{P_3\left(2(E_f-E_i)m-Q_1^2\right)}{4m^3}A_5$$
$$-\frac{E_f(E_f+m)Q_1^2z}{4m^3}A_6 + \frac{E_f(E_f+m)Q_1^2z}{2m^3}A_8\Bigg)$$

$$\Pi_1^a(\Gamma_3) = C_a\left(\frac{P_3zQ_2}{4m}A_4 + \frac{(E_f+m)Q_2}{2m^2}A_5\right)$$

Lorentz-Invariant amplitudes

Symmetric

$$A_1 = \frac{\left(m(E+m)+P_3^2\right)}{E(E+m)}\Pi_0^s(\Gamma_0) - i\,\frac{P_3 Q_1}{2E(E+m)}\Pi_0^s(\Gamma_2) - \frac{Q_1}{2E}\Pi_2^s(\Gamma_3)$$

$$A_5 = -\frac{E}{Q_1}\Pi_2^s(\Gamma_3)$$

$$A_6 = \frac{P_3}{2Ez(E+m)}\Pi_0^s(\Gamma_0) + i\,\frac{\left(P_3^2 - E(E+m)\right)}{EQ_1 z(E+m)}\Pi_0^s(\Gamma_2) + \frac{P_3}{EQ_1 z}\Pi_2^s(\Gamma_3)$$

Asymmetric

$$A_1 = \frac{2m^2}{E_f(E_i+m)}\frac{\Pi_0^a(\Gamma_0)}{C_a} + i\,\frac{2(E_f-E_i)P_3 m^2}{E_f(E_f+m)(E_i+m)Q_1}\frac{\Pi_0^a(\Gamma_2)}{C_a} + \frac{2(E_i-E_f)P_3 m^2}{E_f(E_f+E_i)(E_f+m)(E_i+m)}\frac{\Pi_1^a(\Gamma_2)}{C_a}$$
$$+i\,\frac{2(E_i-E_f)m^2}{E_f(E_i+m)Q_1}\frac{\Pi_1^a(\Gamma_0)}{C_a} + \frac{2(E_i-E_f)P_3 m^2}{E_f(E_f+E_i)(E_f+m)(E_i+m)}\frac{\Pi_2^a(\Gamma_1)}{C_a} + \frac{2(E_f-E_i)m^2}{E_f(E_i+m)Q_1}\frac{\Pi_2^a(\Gamma_3)}{C_a}$$

$$A_5 = \frac{m^2 P_3}{E_f(E_f+m)(E_i+m)}\frac{\Pi_2^a(\Gamma_1)}{C_a} - \frac{(E_f+E_i)m^2}{E_f(E_i+m)Q_1}\frac{\Pi_2^a(\Gamma_3)}{C_a}$$

$$A_6 = \frac{P_3 m^2}{E_f^2(E_f+m)(E_i+m)z}\frac{\Pi_0^a(\Gamma_0)}{C_a} + i\,\frac{(E_f-E_i-2m)m^2}{E_f^2(E_i+m)Q_1 z}\frac{\Pi_0^a(\Gamma_2)}{C_a} + i\,\frac{(E_i-E_f)P_3 m^2}{E_f^2(E_f+m)(E_i+m)Q_1 z}\frac{\Pi_1^a(\Gamma_0)}{C_a}$$
$$+\frac{(-E_f+E_i+2m)m^2}{E_f^2(E_f+E_i)(E_i+m)z}\frac{\Pi_1^a(\Gamma_2)}{C_a} + \frac{2(m-E_f)m^2}{E_f^2(E_f+E_i)(E_i+m)z}\frac{\Pi_2^a(\Gamma_1)}{C_a} + \frac{2P_3 m^2}{E_f^2(E_i+m)Q_1 z}\frac{\Pi_2^a(\Gamma_3)}{C_a}$$

- ★ Asymmetric frame equations more complex
- ★ A_i have definite symmetries
- ★ System of 8 independent matrix elements to disentangle the A_i

Lorentz-Invariant amplitudes

Symmetric

$$A_1 = \frac{(m(E+m)+P_3^2)}{E(E+m)}\Pi_0^s(\Gamma_0) - i\,\frac{P_3Q_1}{2E(E+m)}\Pi_0^s(\Gamma_2) - \frac{Q_1}{2E}\Pi_2^s(\Gamma_3)$$

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$$=$$

$$=$$

$$=$$

$$A_1 = \frac{2m^2}{E_f(E_i+m)}\frac{\Pi_0^a(\Gamma_0)}{C_a} + i\,\frac{2(E_f-E_i)P_3m^2}{E_f(E_f+m)(E_i+m)Q_1}\frac{\Pi_0^a(\Gamma_2)}{C_a} + \frac{2(E_i-E_f)P_3m^2}{E_f(E_f+E_i)(E_f+m)(E_i+m)}\frac{\Pi_1^a(\Gamma_2)}{C_a}$$
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$$A_6 = \frac{P_3m^2}{E_f^2(E_f+m)(E_i+m)z}\frac{\Pi_0^a(\Gamma_0)}{C_a} + i\,\frac{(E_f-E_i-2m)m^2}{E_f^2(E_i+m)Q_1z}\frac{\Pi_0^a(\Gamma_2)}{C_a} + i\,\frac{(E_i-E_f)P_3m^2}{E_f^2(E_f+m)(E_i+m)Q_1z}\frac{\Pi_1^a(\Gamma_0)}{C_a}$$
$$+\frac{(-E_f+E_i+2m)m^2}{E_f^2(E_f+E_i)(E_i+m)z}\frac{\Pi_1^a(\Gamma_2)}{C_a} + \frac{2(m-E_f)m^2}{E_f^2(E_f+E_i)(E_i+m)z}\frac{\Pi_2^a(\Gamma_1)}{C_a} + \frac{2P_3m^2}{E_f^2(E_i+m)Q_1z}\frac{\Pi_2^a(\Gamma_3)}{C_a}$$

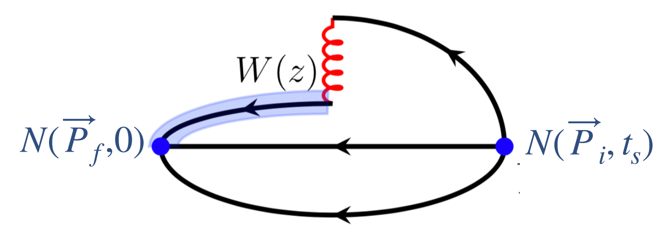
- ★ Asymmetric frame equations more complex
- ★ A_i have definite symmetries
- ★ System of 8 independent matrix elements to disentangle the A_i

Parameters of calculation

★ Nf=2+1+1 twisted mass (TM) fermions & clover improvement

Pion mass: 260 MeV
Lattice spacing: 0.093 fm
Volume: 32^3 x 64
Spatial extent: 3 fm

★ Calculation:
- isovector combination
- zero skewness
- T_{sink}=1 fm



frame	P_3 [GeV]	$\mathbf{Q}$ $[\frac{2\pi}{L}]$	$-t$ [GeV^2]	ξ	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm tot}$
symm	1.25	(±2,0,0), (0,±2,0)	0.69	0	8	249	8	15936
non-symm	1.25	(±2,0,0), (0,±2,0)	0.64	0	8	269	8	17216

★ Computational cost:
- symmetric frame 4 times more expensive than asymmetric frame for same set of $\vec{Q}$ (requires separate calculations at each t)

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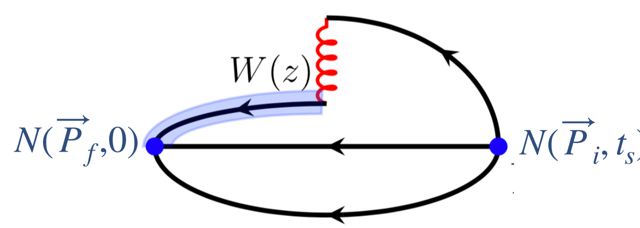

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Small difference: $t^s = -\vec{Q}^2$ $\quad t^a = -\vec{Q}^2 + (E_f - E_i)^2$

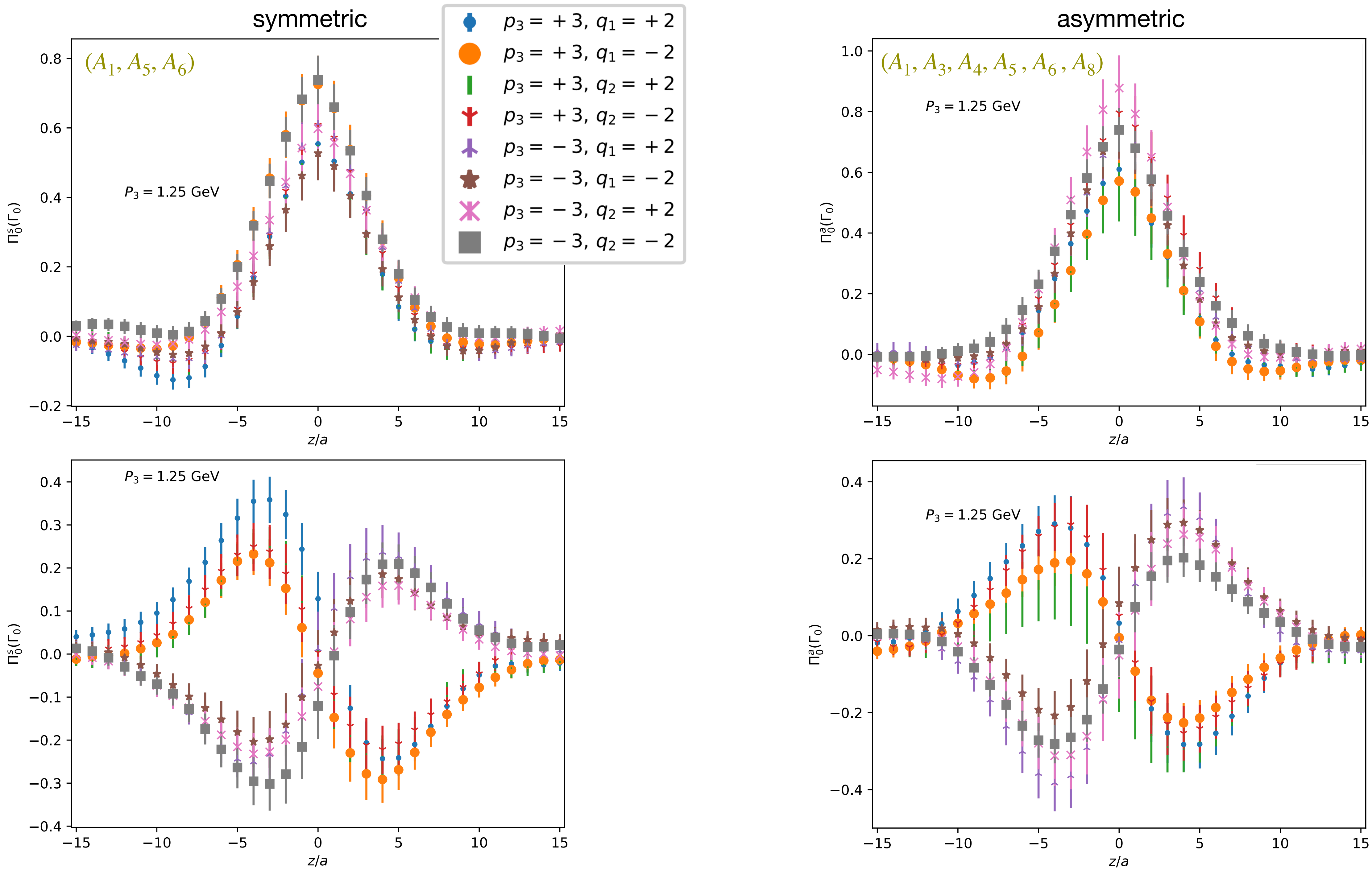
$$A(-0.64\text{GeV}^2) \sim A(-0.69\text{GeV}^2)$$

★ **Computational cost:**
- symmetric frame 4 times more expensive than asymmetric frame for same set of $\vec{Q}$ (requires separate calculations at each t)

Results: matrix elements

Real

Imag

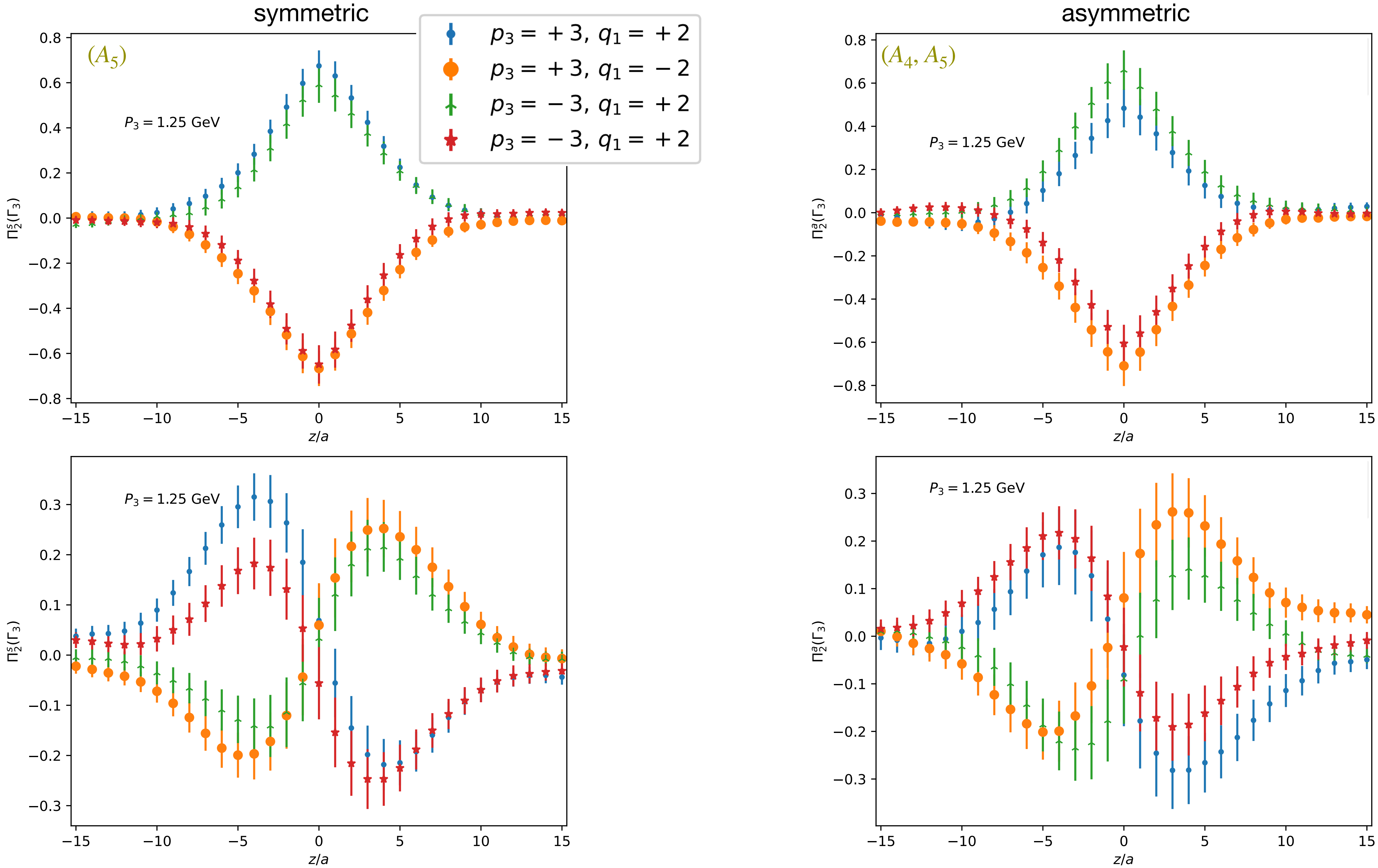


- ★ Lattice data confirm symmetries where applicable **(e.g., $\Pi^s_0(\Gamma_0)$ in $\pm P_3, \pm Q, \pm z$)**
- ★ ME decompose to different A_i
- ★ Multiple ME contribute to the same quantity

Results: matrix elements

Real

Imag

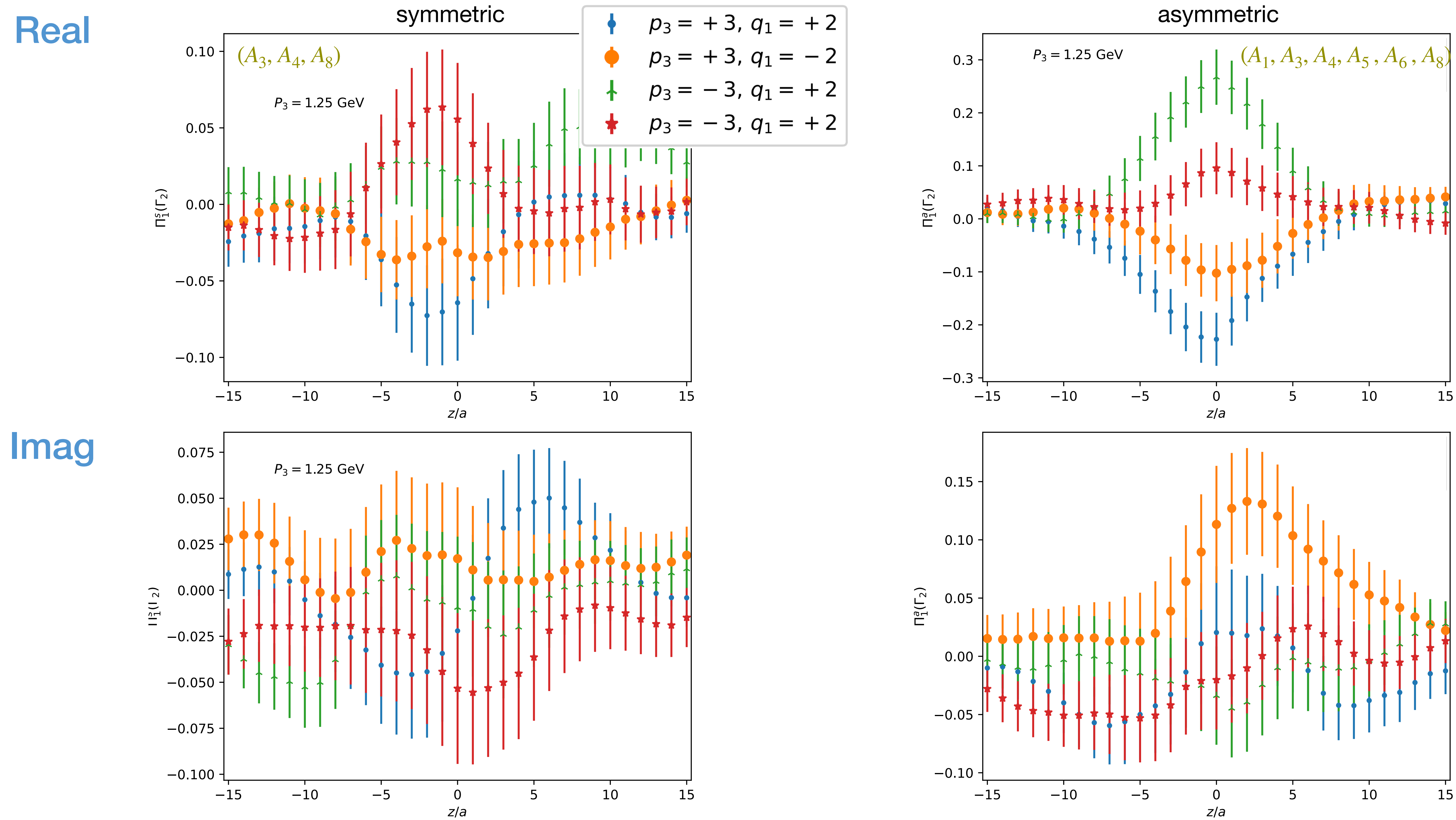


★ Matrix elements depend on frame (comparison pedagogical)

★ ME in asymmetric frame do not have definite symmetries in $\pm P_3, \pm Q, \pm z$

Frame comparison and symmetries applied on Lorentz-invariant amplitudes

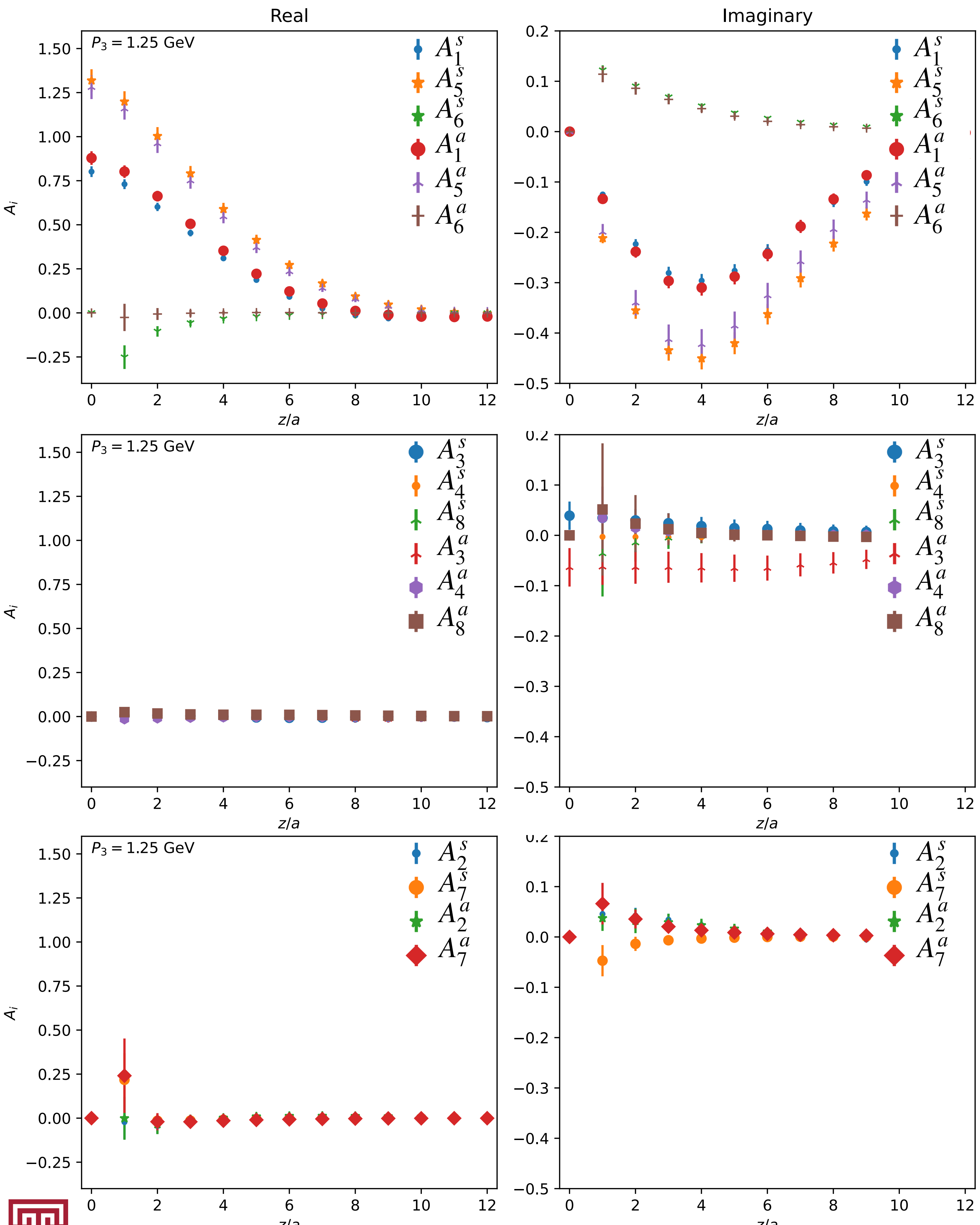
Results: matrix elements



★ $\Pi_1(\Gamma_2)$ theoretically nonzero

★ Noisy contributions lead to challenges in extracting A_i of sub-leading magnitude

Results: A_i



- ★ A_1, A_5 dominant contributions
- ★ Full agreement in two frames for both Re and Im parts of A_1, A_5
- ★ A_6 small but non-negligible. Tension between frames is statistical effect
- ★ A_3, A_4, A_8 negligible
- ★ A_2, A_7 appear only in Π_3 and are negligible

Π_H, Π_E in terms of A_i

★ Mapping of $\{\Pi_H, \Pi_E\}$ to A_i using $F^{[\gamma^0]} \sim \left[\gamma^0 H_{Q(0)}(x,\xi,t;P^3) + \frac{i\sigma^{0\mu}\Delta_\mu}{2M} E_{Q(0)}(x,\xi,t;P^3)\right]$ in each frame leading to frame dependent relations: [see S. Bhattacharya talk]

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[see S. Bhattacharya talk]

$(\xi = 0)$

$$\Pi_H^s = A_1 + \frac{zQ_1^2}{2P_3} A_6$$

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$$\Pi_H^a = A_1 + \frac{Q_0}{P_0} A_3 + \frac{m^2 z Q_0}{2P_0 P_3} A_4 + \frac{z(Q_0^2 + Q_\perp^2)}{2P_3} A_6 + \frac{z(Q_0^3 + Q_0 Q_\perp^2)}{2P_0 P_3} A_8$$

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★ Definition of Lorentz invariant Π_H & Π_E

$(\xi = 0)$

$$\Pi_H^{\text{impr}} = A_1$$

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1st approach: extraction of $\{\Pi_H^s, \Pi_E^s\}$ using A_i from any frame (universal)

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2nd approach: extraction of $\{\Pi_H, \Pi_E\}$ from a purely asymmetric frame; GPDs may differ in functional form from $\{\Pi_H^s, \Pi_E^s\}$

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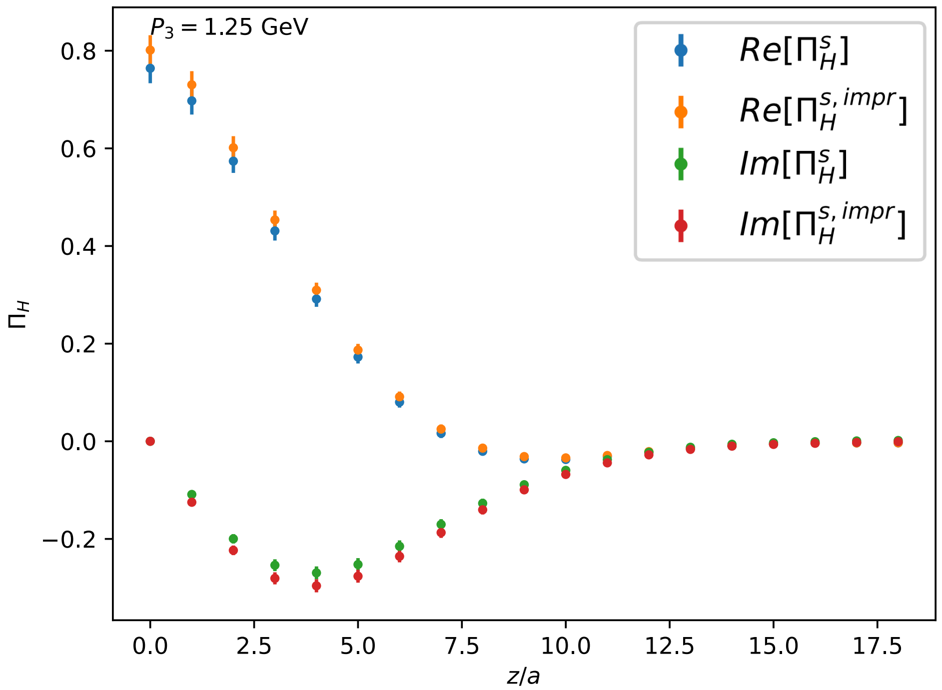
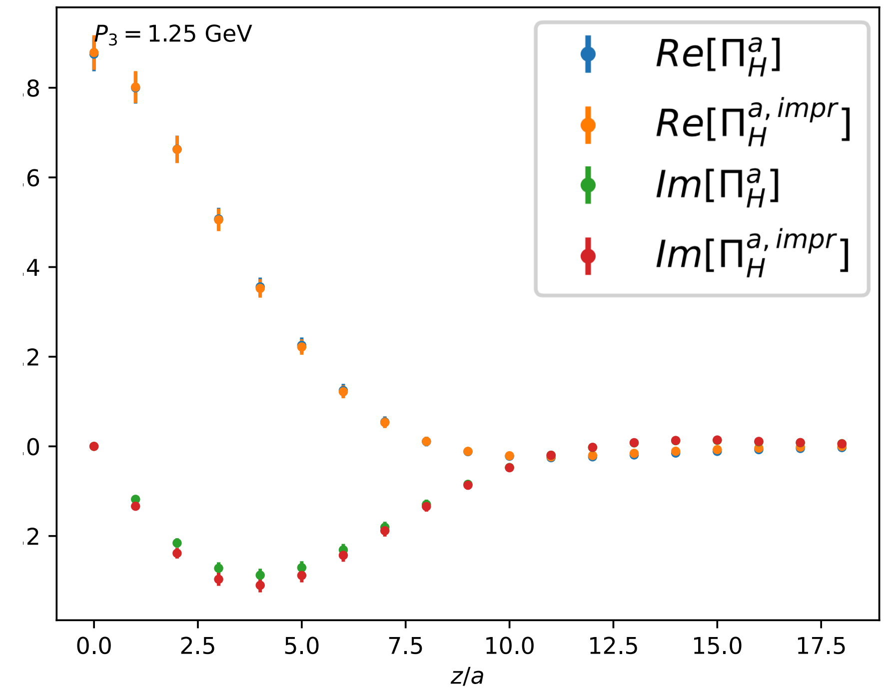
3rd approach: use redefined Lorentz covariant $\{\Pi_H, \Pi_E\}$ in desired frame

Results: H − GPD

Π_H^s vs $\Pi_H^{s,impr}$

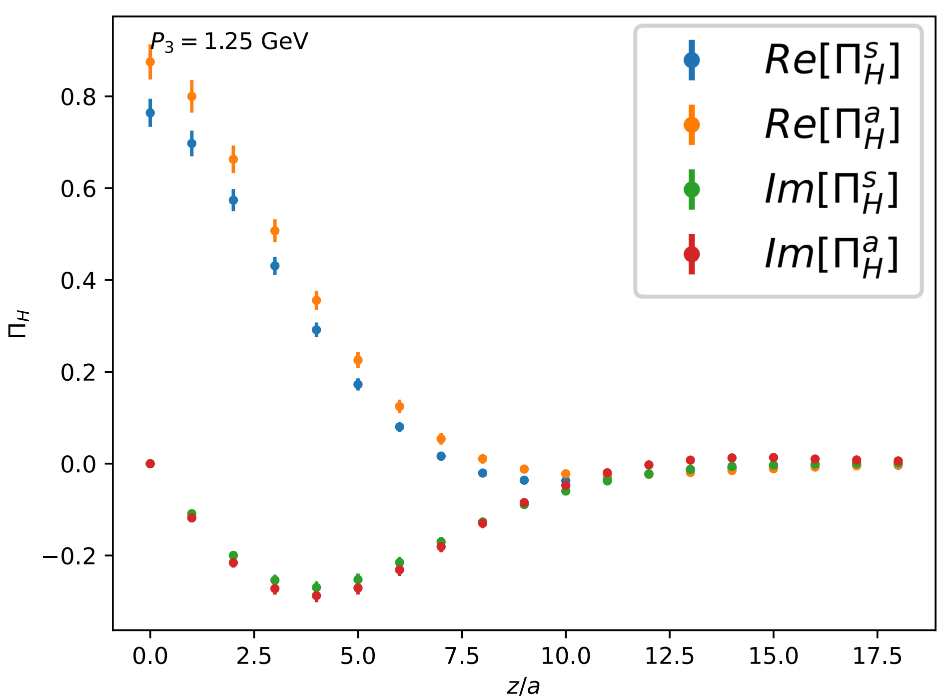


Π_H^a vs $\Pi_H^{a,impr}$

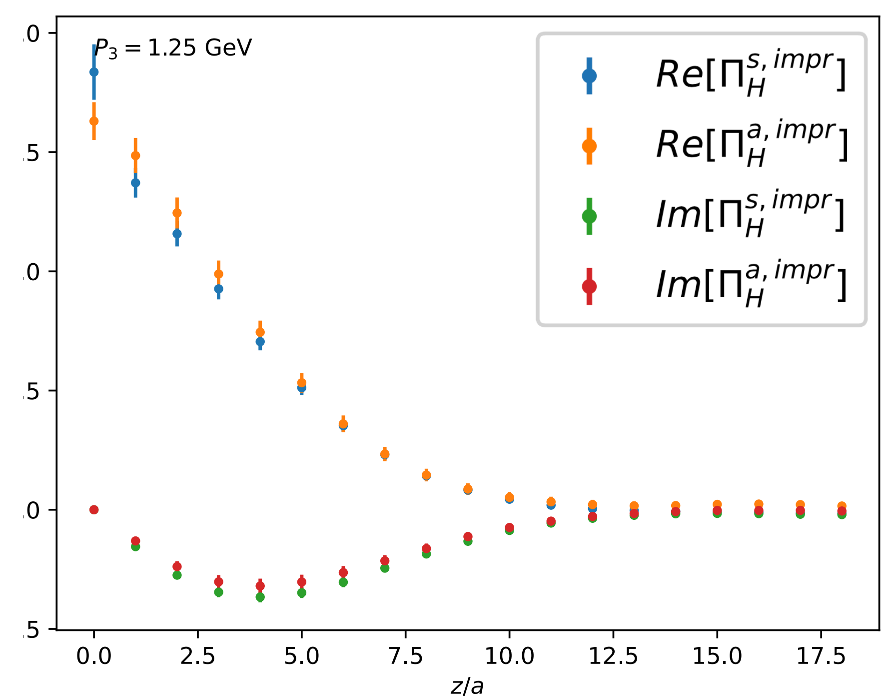


Π_H agree with Π_H^{impr} for both frames despite different definitions (agreement not by construction)

Π_H^s vs Π_H^a



$\Pi_H^{s,\,impr}$ vs $\Pi_H^{a,\,impr}$

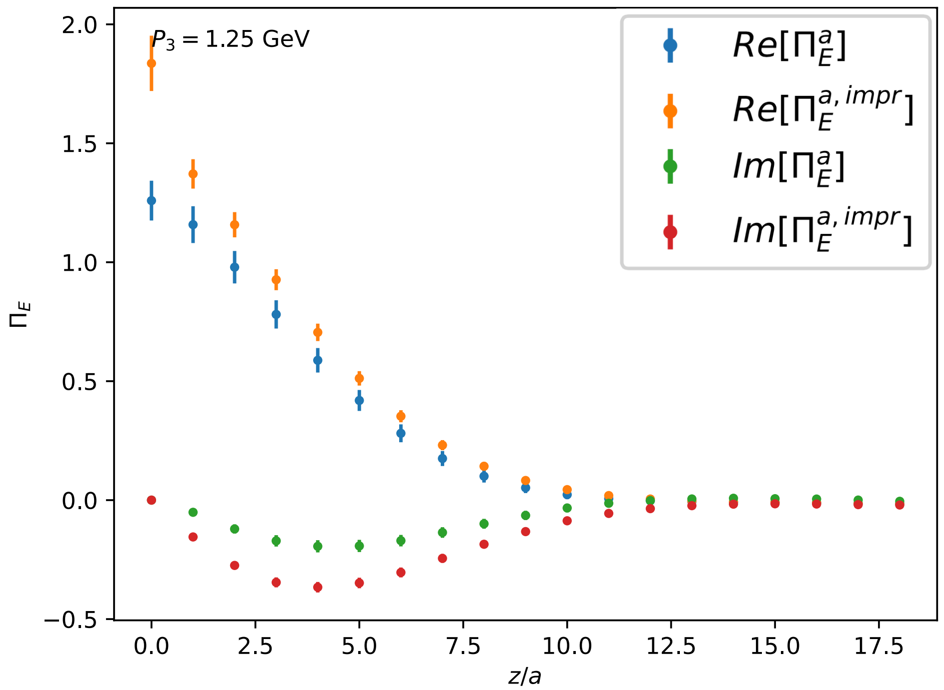


Agreement between Π_H^s and Π_H^a also not required theoretically

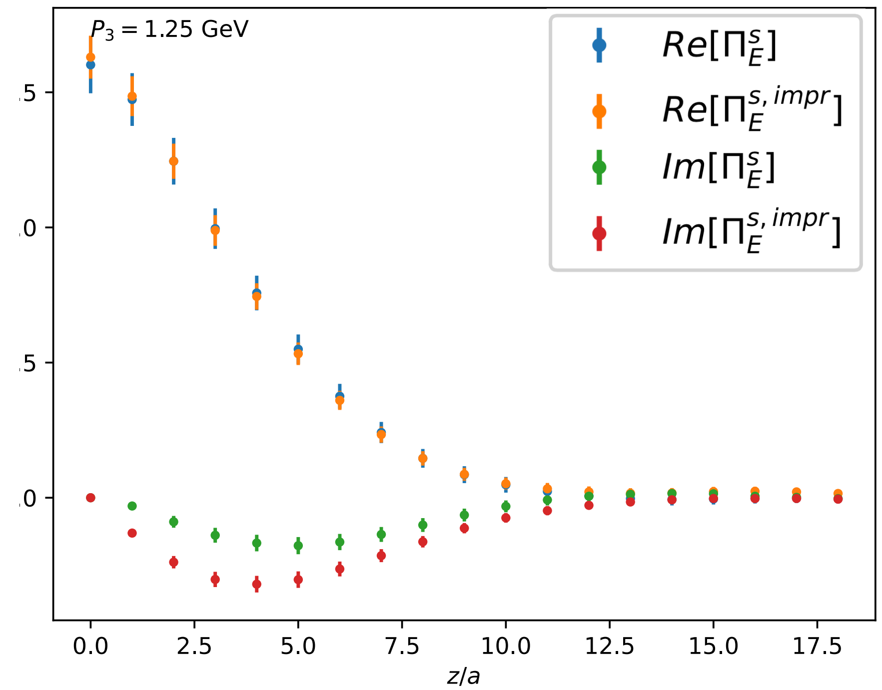
Π_H^s & Π_H^a agreement achieved for improved definition, as expected from Lorentz invariance

Results: Π_E – GPD

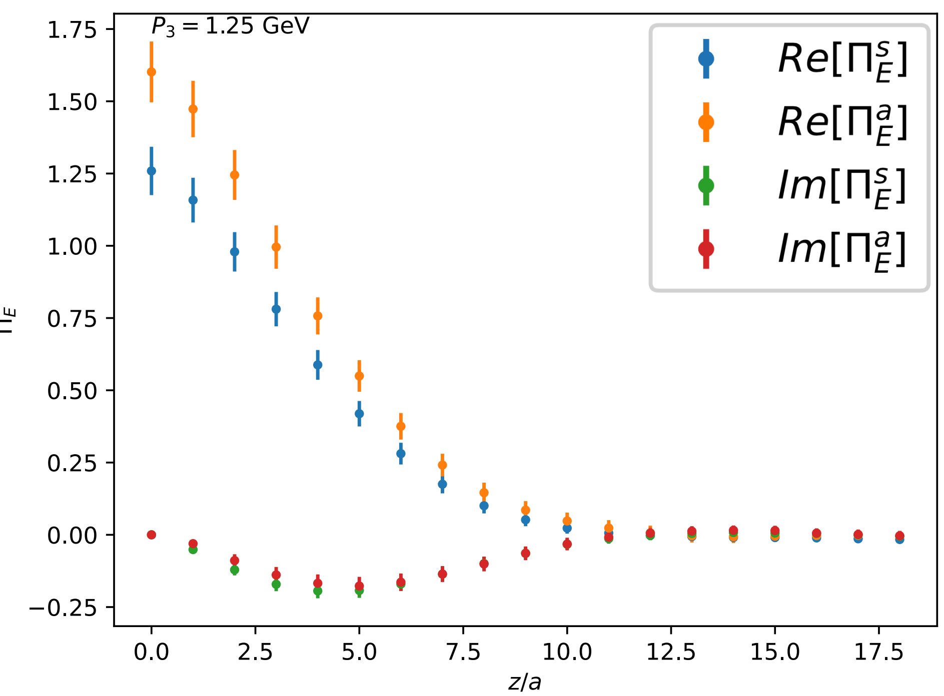
Π_E^s vs $\Pi_E^{s,impr}$



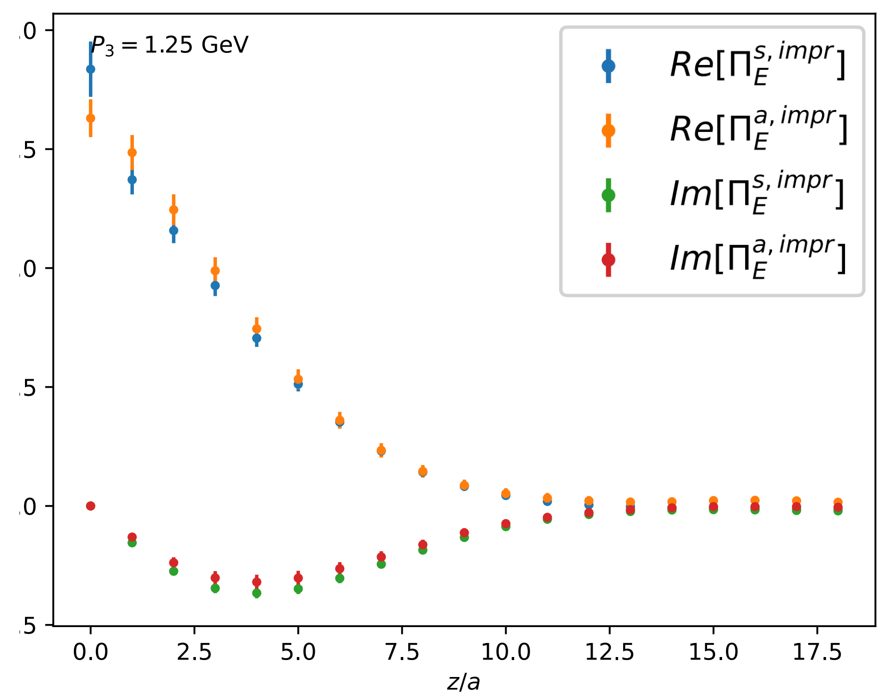
Π_E^a vs $\Pi_E^{a,impr}$



Π_E^s vs Π_E^a



$\Pi_E^{s,\,impr}$ vs $\Pi_E^{a,\,impr}$



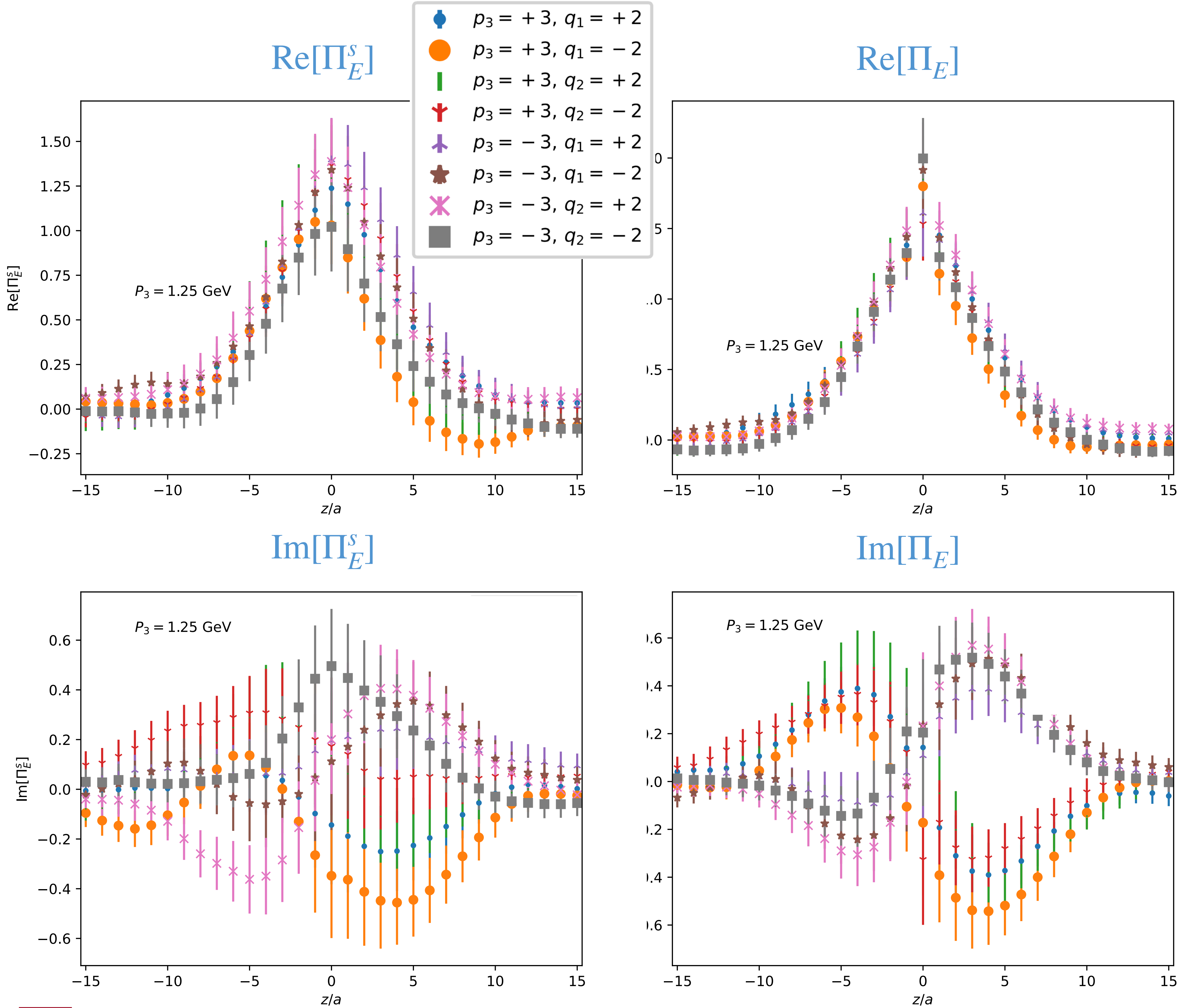
Both frames: $\mathrm{Im}[\Pi_E^{impr}]$ **enhanced compared to** $\mathrm{Im}[\Pi_E]$.

$\mathrm{Re}[\Pi_E^{s,impr}]$ **larger than other** $\mathrm{Re}[\Pi_E^s]$, $\mathrm{Re}[\Pi_E^a]$ **and** $\mathrm{Re}[\Pi_E^{a,impr}]$

Agreement reached between frames for improved definition (expected theoretically)

A comment on Lorentz covariant definitions

Example: symmetric frame



Lorentz covariant definition leads to more precise results for Π_E

Same effect of improvement also for asymmetric frame

Numerical indications that using Π_E leads to better converge to light-cone GPDs with respect to P_3

Signal quality in Π_H same across all cases (not shown)

Summary

★ quasi-GPDs are intrinsically frame dependent

★ Widely used symmetric frame is computationally very expensive

★ Novel Lorentz invariant decomposition has great advantages:
- access to symmetric-frame GPDs from matrix elements in any frame
- Lorentz covariant quasi-GPDs eliminate power corrections
- Level of P_3 convergence to light-cone for Lorentz covariant definition will be addressed with lattice and models

★ Numerical results demonstrate the validity of the approach

★ Computational cost decreased at a minimum of 4 times

★ Potential to extract more than one t within the same computational cost (different levels of signal quality)

★ Generalized for mesons, and all types of GPDs including twist-3

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Thank you

U.S. DEPARTMENT OF ENERGY INCITE LEADERSHIP COMPUTING
U.S. DEPARTMENT OF ENERGY | Office of Science
DOE Early Career Award (NP)
Grant No. DE-SC0020405