GPDs in non-symmetric frames

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Andreas Metz (Temple U.)
Swagato Mukherjee (BNL)
Aurora Scapellato (Temple U.)
Fernanda Steffens (Bonn U.)
Yong Zhao (ANL)

The 39th International Symposium on Lattice Field Theory

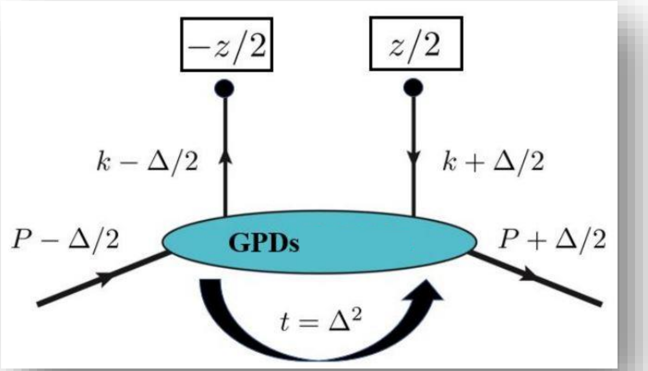
Bonn, Germany

Background

What? Why? How?

Background

What? Why? How?



Generalized Parton Distributions (GPDs): (See Diehl, arXiv: 0307382)

$$F^{[\Gamma]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \langle p';\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p;\lambda\rangle\Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

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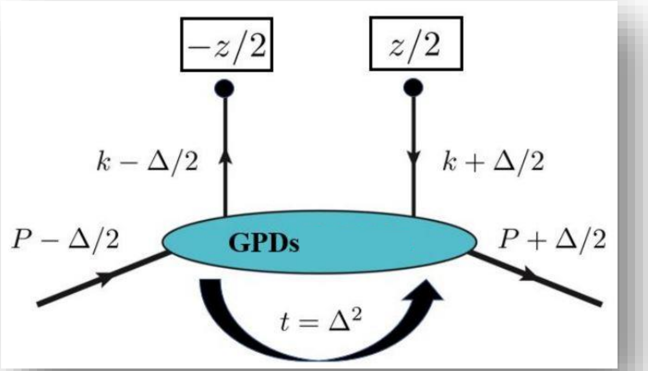

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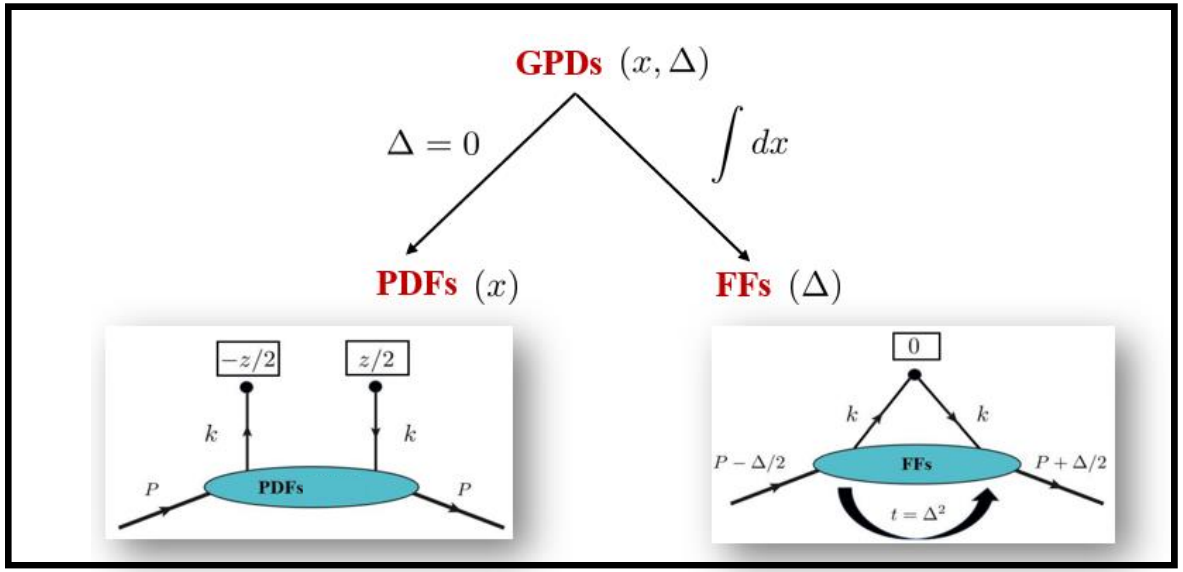
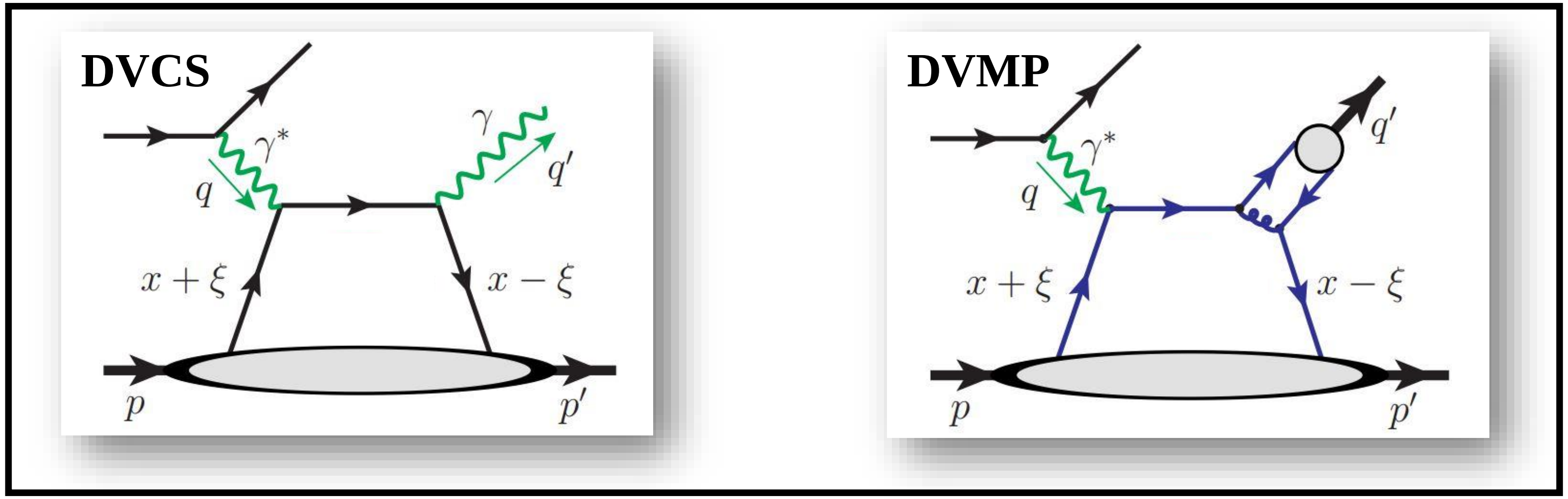
Relation with PDFs & FFs:



Background

What? **Why?** How?

Physical processes giving access to GPDs:



Amplitude:

$$\mathcal{M} \propto \int_{-1}^{1} dx \frac{F(x,\xi,t)}{x \pm \xi + i\epsilon}$$

Background

What? **Why?** How?

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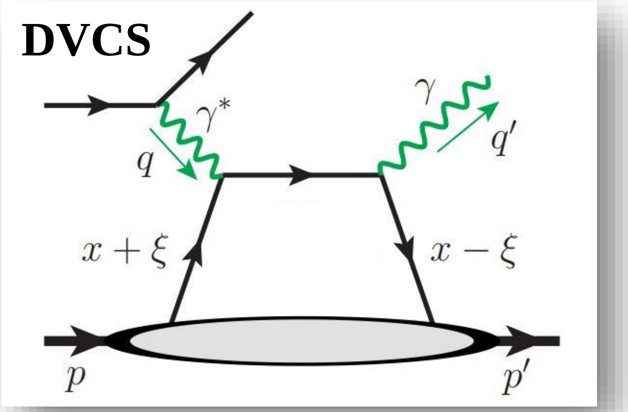

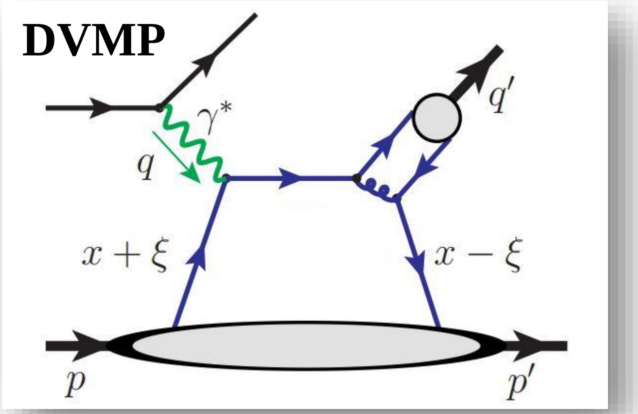

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x-dependence lost!

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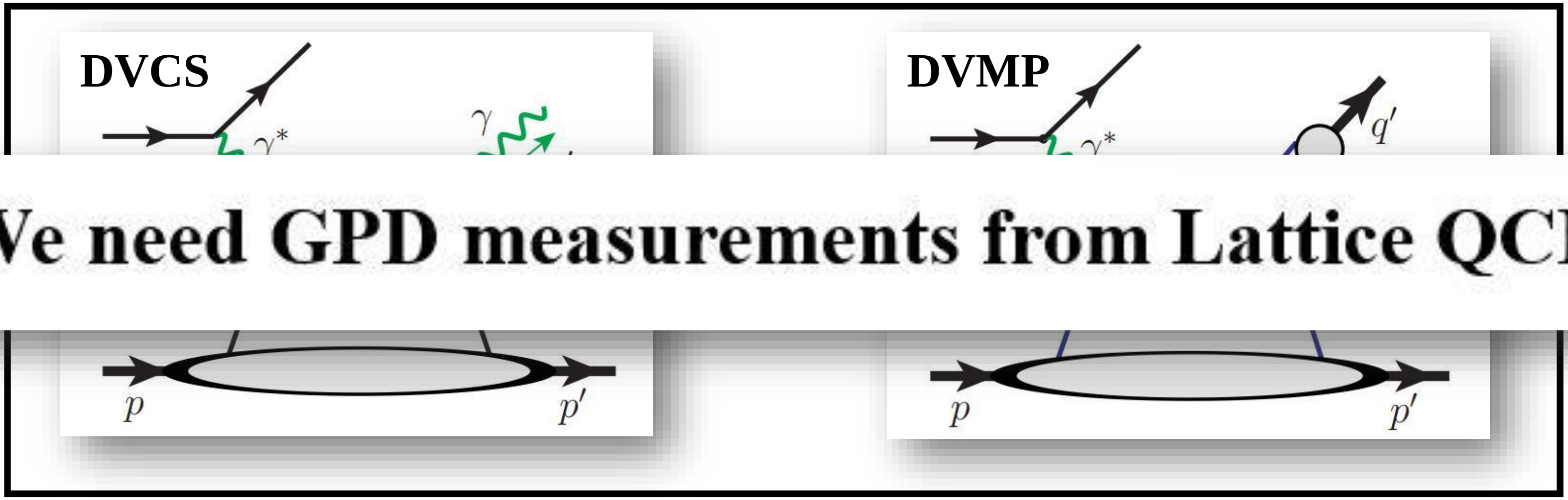

We need GPD measurements from Lattice QCD

Amplitude:

$$\mathcal{M} \propto \int_{-1}^{1} dx \frac{F(x,\xi,t)}{x \pm \xi + i\epsilon}$$

x-dependence lost!

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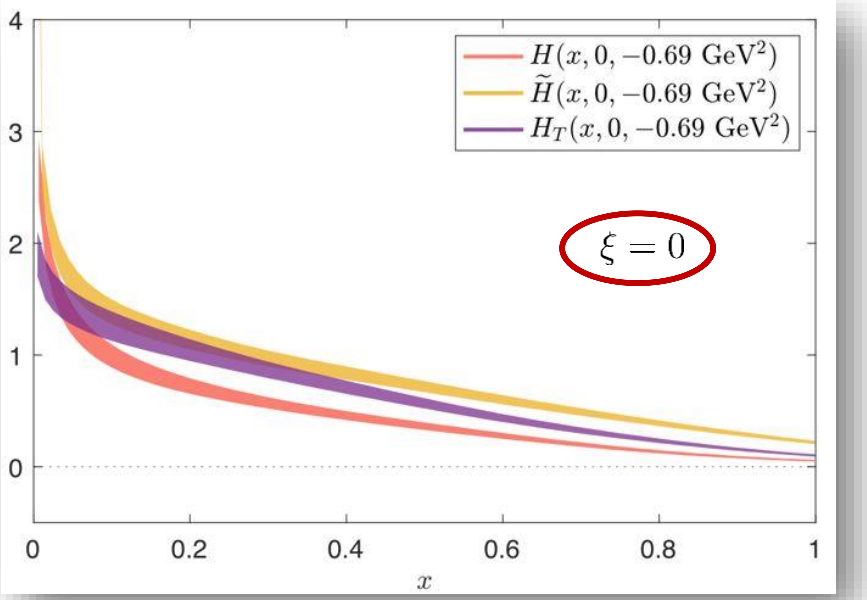
Pioneering Lattice QCD calculations of GPDs:



C. Alexandrou et. al. (PRL 125 (2020) 26, 262001)

Quasi-distribution formalism

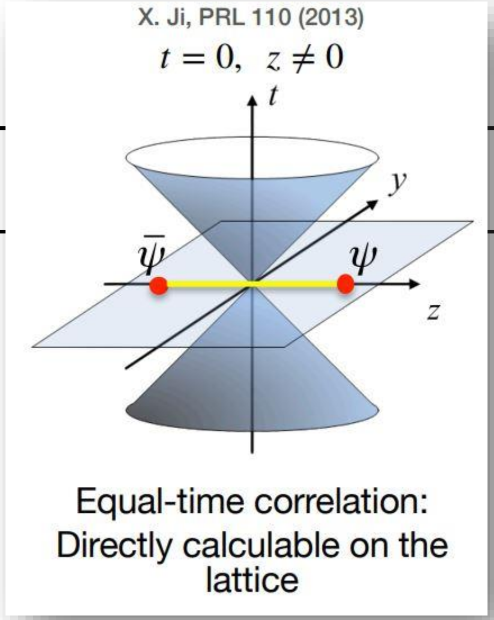
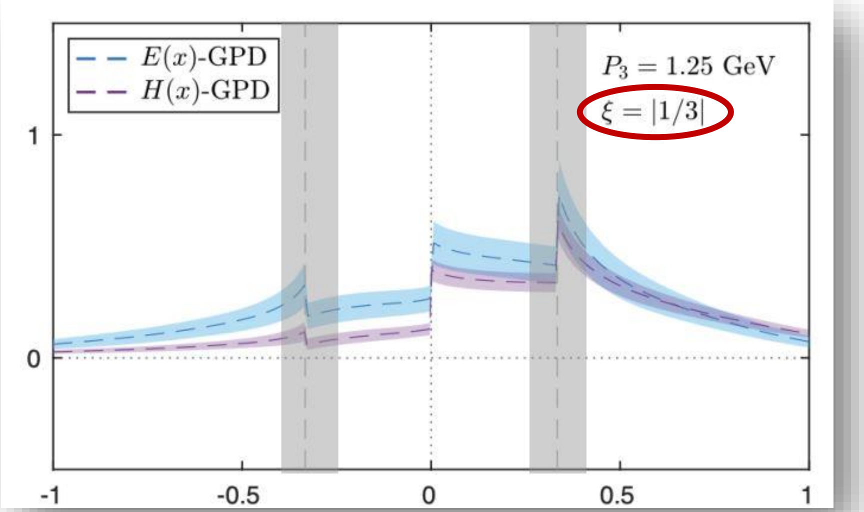


Figure courtesy: Yong Zhao



C. Alexandrou et. al. (arXiv: 2008.10573)

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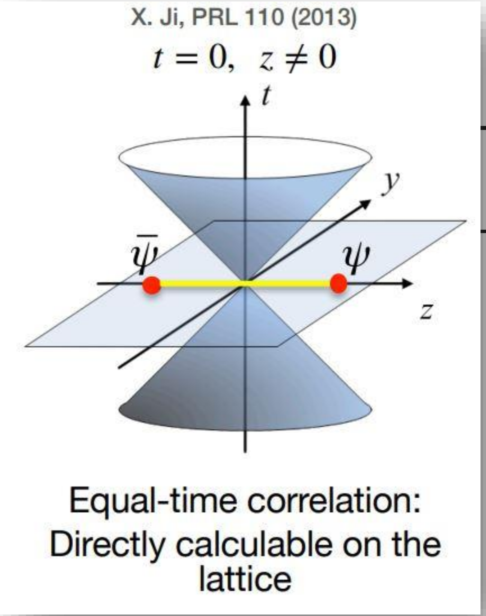
Pioneering Lattice QCD calculations of GPDs:

Quasi-distribution formalism



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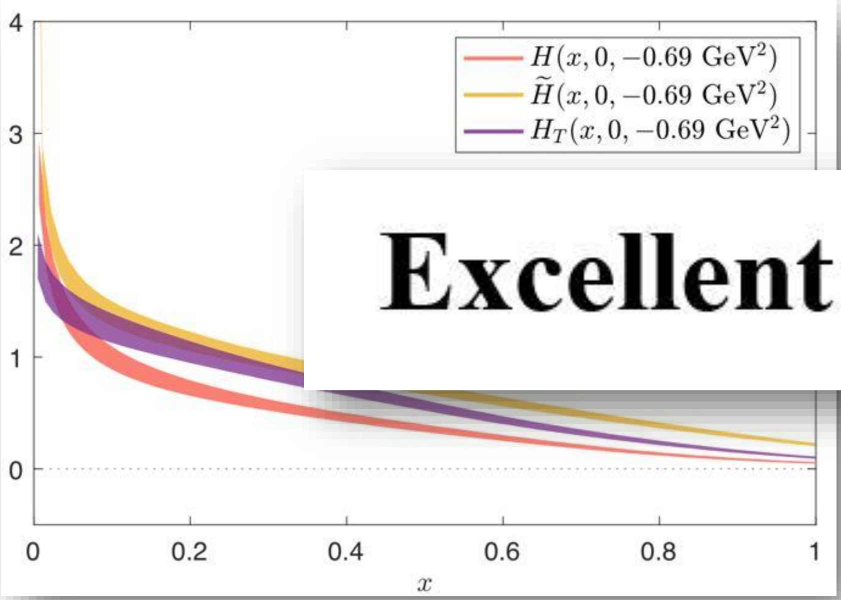
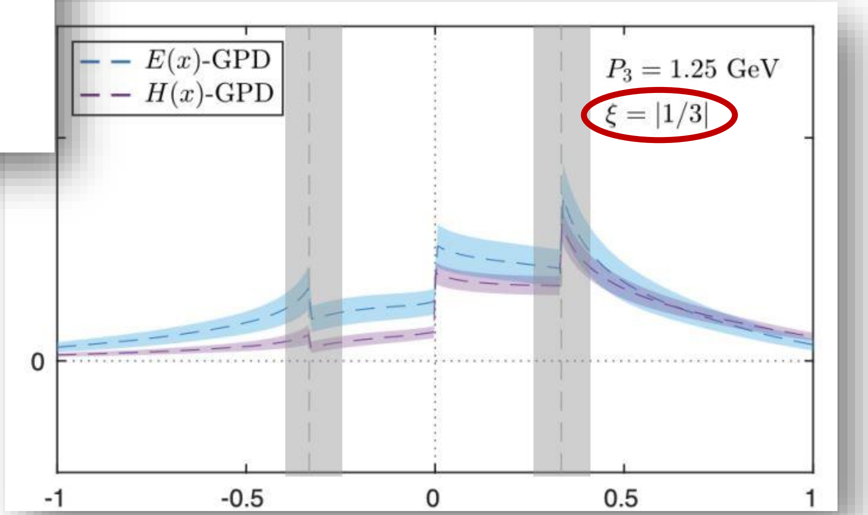


Excellent progress!!!

C. Alexandrou et. al. (PRL 125 (2020) 26, 262001)



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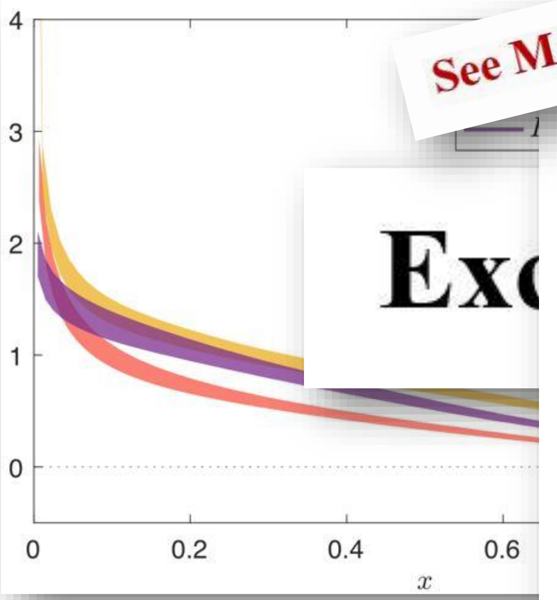
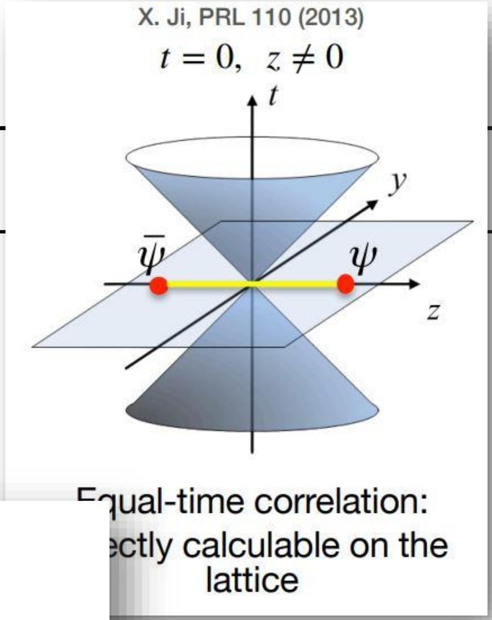
Pioneering Lattice QCD calculations of GPDs:



See Martha's talk

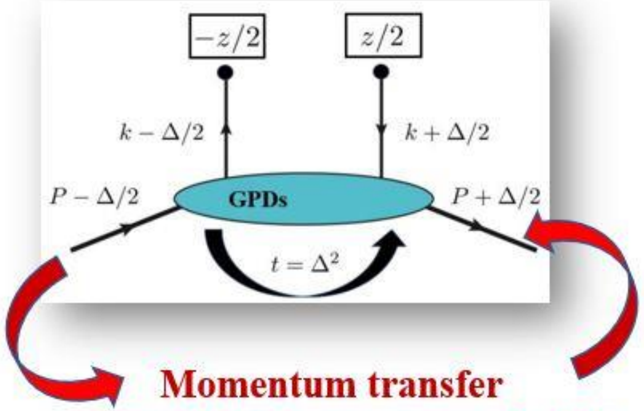
Exc

C. Alexandrou et. al. (PRL 1



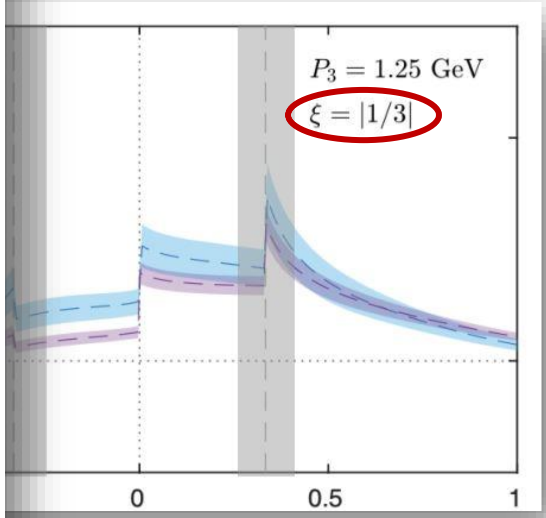
e courtesy: Yong Zhao

Practical drawback



Momentum transfer symmetric between source & sink

Lattice QCD calculations in symmetric frames are expensive



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Background

What? Why? How?

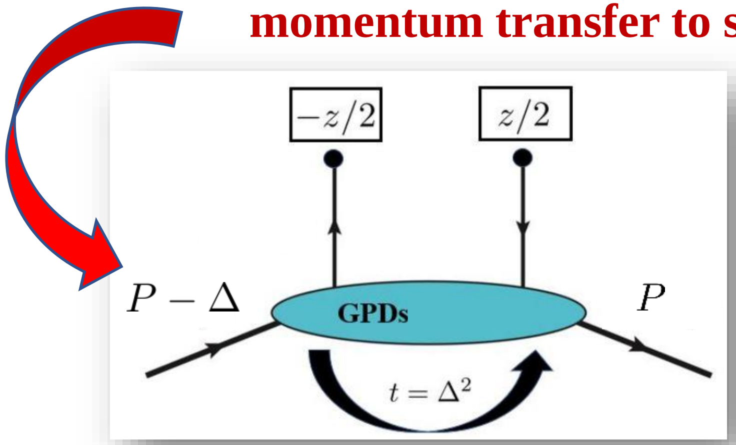
Resolution:

Background

What? Why? **How?**

Resolution:

All momentum transfer to source



- **Perform Lattice QCD calculations of GPDs in asymmetric frames**

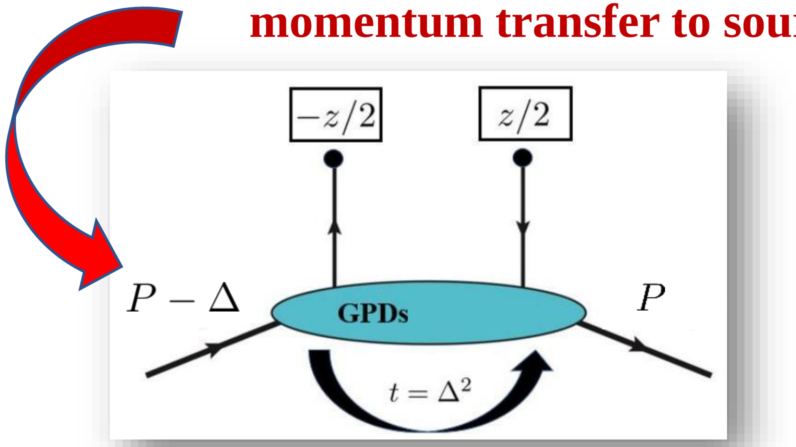
See Martha's talk

Background

What? Why? How?

Our contribution in a nutshell:

All
momentum transfer to source



Key findings: QCD calculations of GPDs in asymmetric frames

- **Lorentz covariant formalism for calculating quasi-GPDs in any frame**

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What? Why? How?

<u>Our contribution in a nutshell</u>:

All
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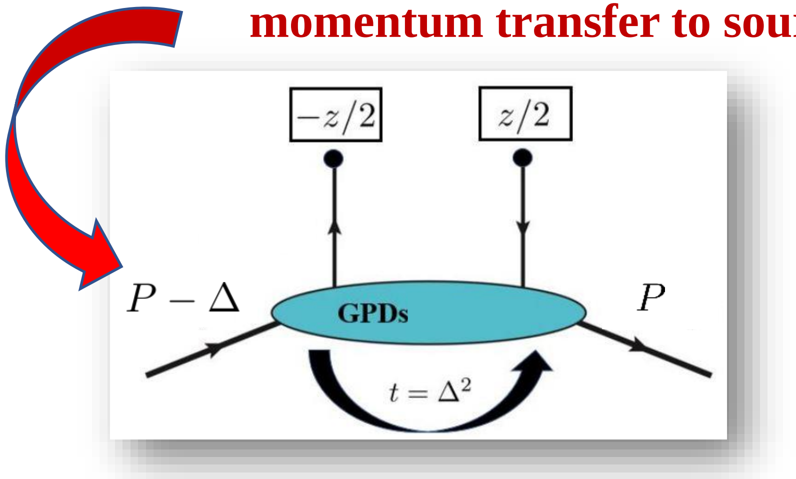
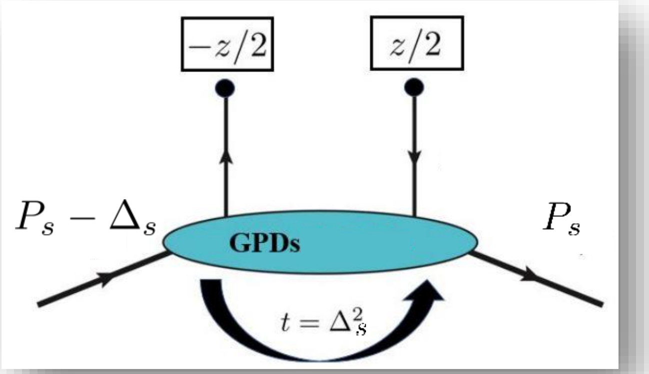


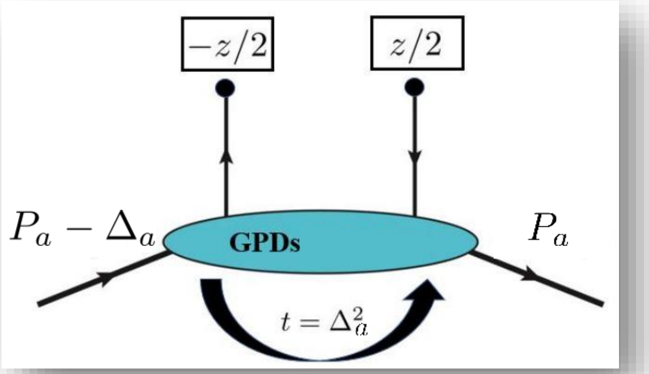
Key findings: QCD calculations of GPDs in asymmetric frames

- **Lorentz covariant formalism for calculating quasi-GPDs in any frame**
- **Elimination of (frame-dependent) power corrections allowing faster convergence to light-cone GPDs at LO**

Preamble

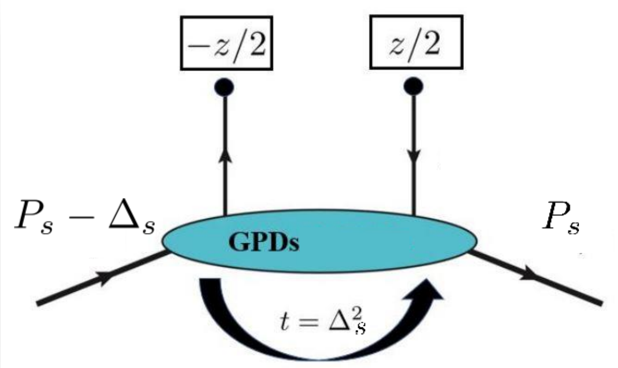
Symmetric & asymmetric frames

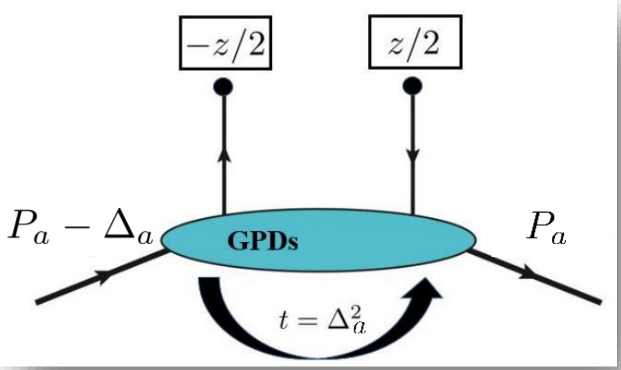




Preamble

Symmetric & asymmetric frames

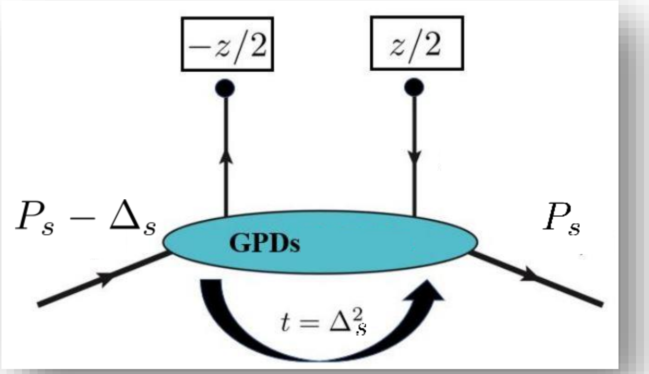




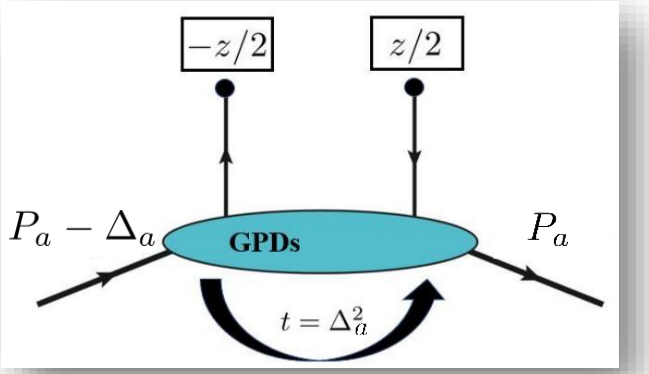
Approach 1: Can we calculate a quasi-GPD in symmetric frame through an asymmetric frame?

Preamble

Symmetric & asymmetric frames



Related via
Lorentz transformation?



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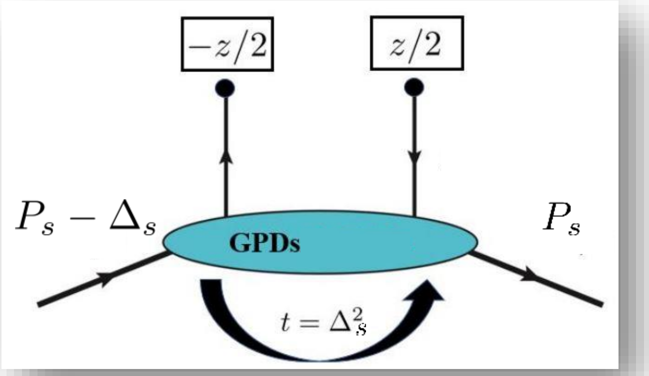

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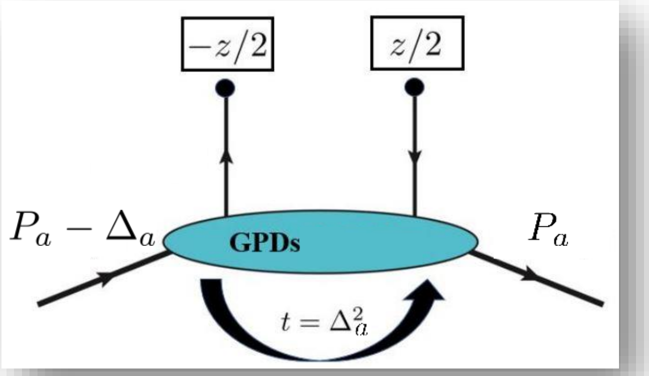
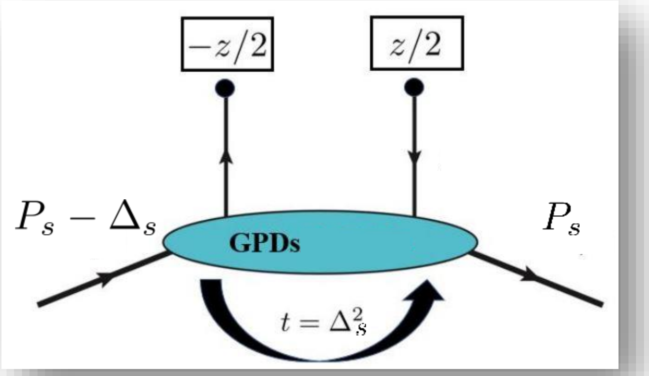


Approach 1: Can we calculate a quasi-GPD in symmetric frame through an asymmetric frame?

Yes, since symmetric & asymmetric frames are connected via Lorentz transformation

Preamble

Symmetric & asymmetric frames



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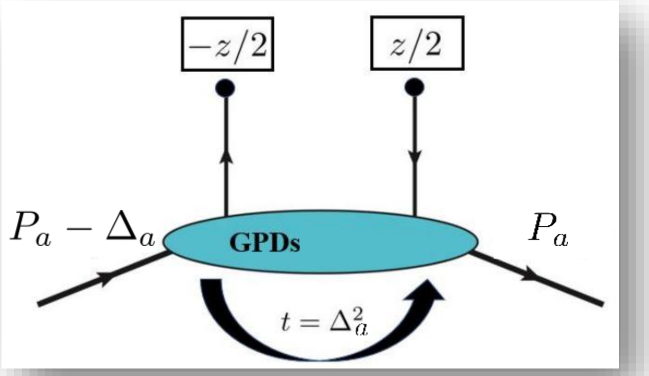
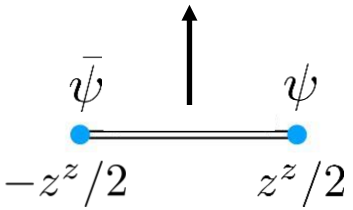


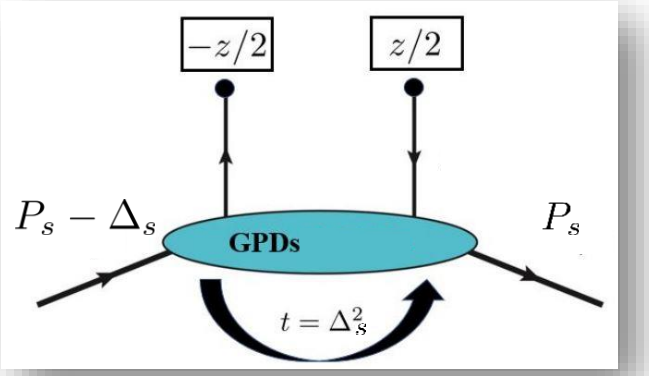
Case 1: Lorentz transformation in the z-direction

$$\begin{pmatrix} z_s^0 \\ z_s^x \\ z_s^z \end{pmatrix} = \begin{pmatrix} \gamma & 0 & -\gamma\beta \\ 0 & 1 & 0 \\ -\gamma\beta & 0 & \gamma \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ z_a^z \end{pmatrix}$$



Preamble

Symmetric & asymmetric frames



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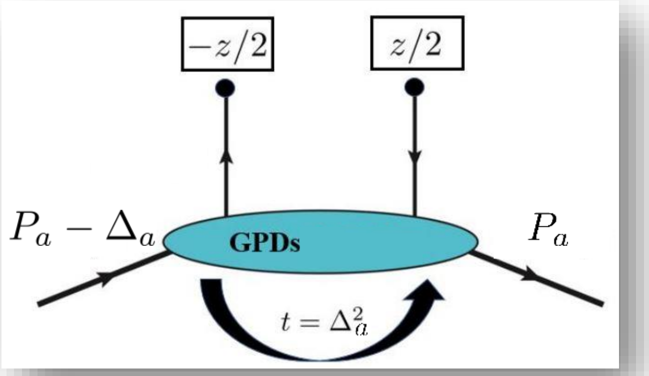

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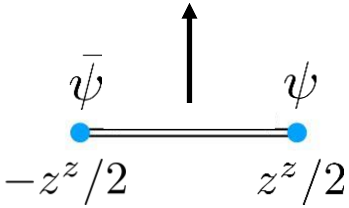


Results:

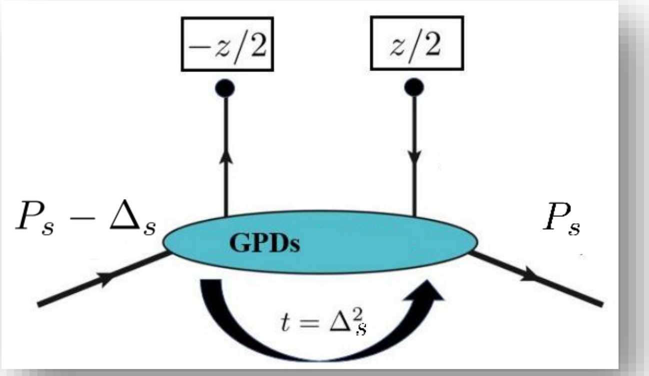
$$z_s^0 = -\gamma\beta z_a^z$$

$$z_s^z = \gamma z_a^z$$

Operator distance develops a non-zero temporal component

Preamble

Symmetric & asymmetric frames



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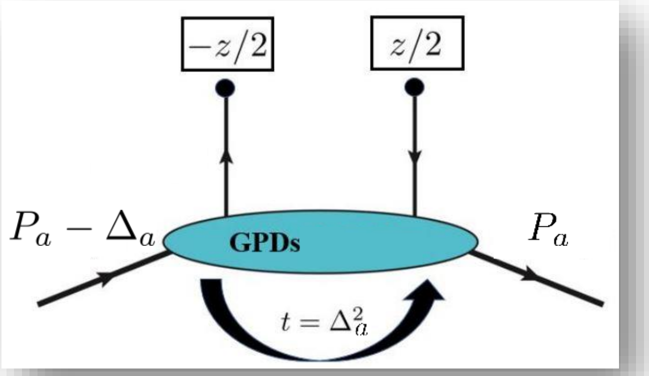
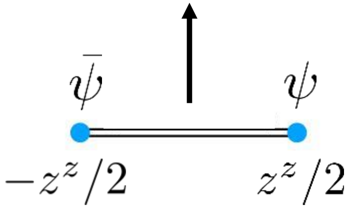


Case 2: Transverse boost in the x-direction

$$\begin{pmatrix} z_s^0 \\ z_s^x \\ z_s^z \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 \\ -\gamma\beta & \gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ z_a^z \end{pmatrix}$$



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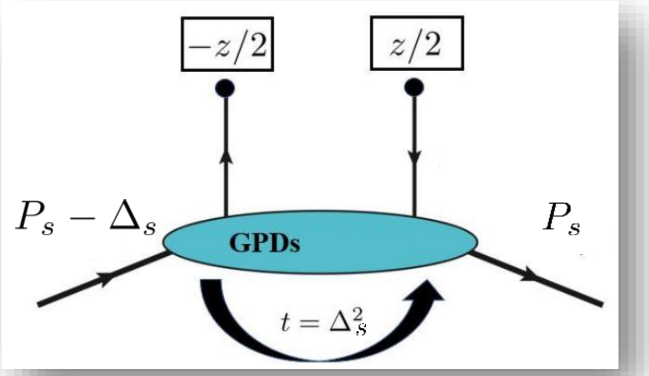
Symmetric & asymmetric frames



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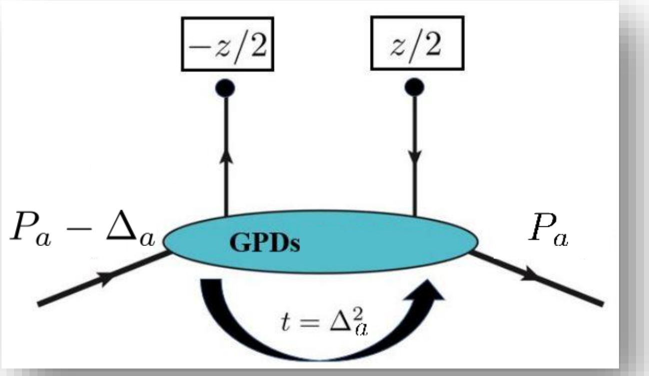

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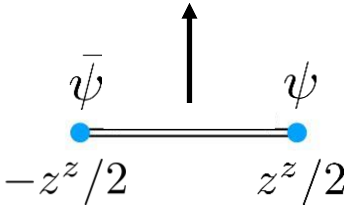


Results:

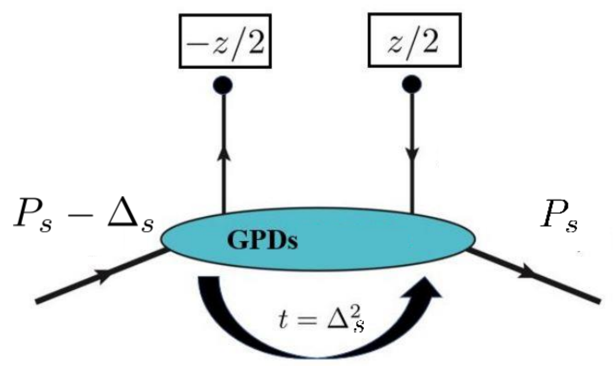
$$z_s^0 = 0$$

$$z_s^z = z_a^z$$

Operator distance remains spatial (& same)

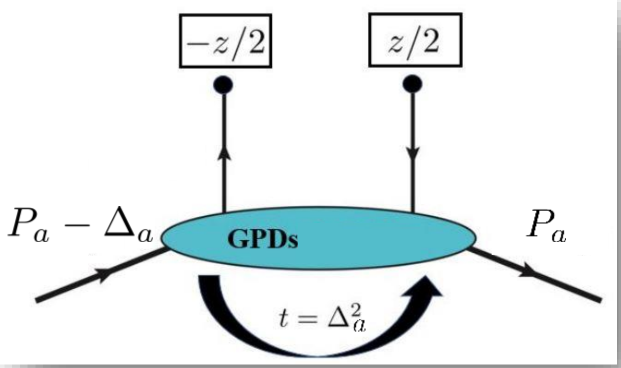
Preamble

Symmetric & asymmetric frames



Related via Lorentz transformation?

What kind?



Case 2: Transver

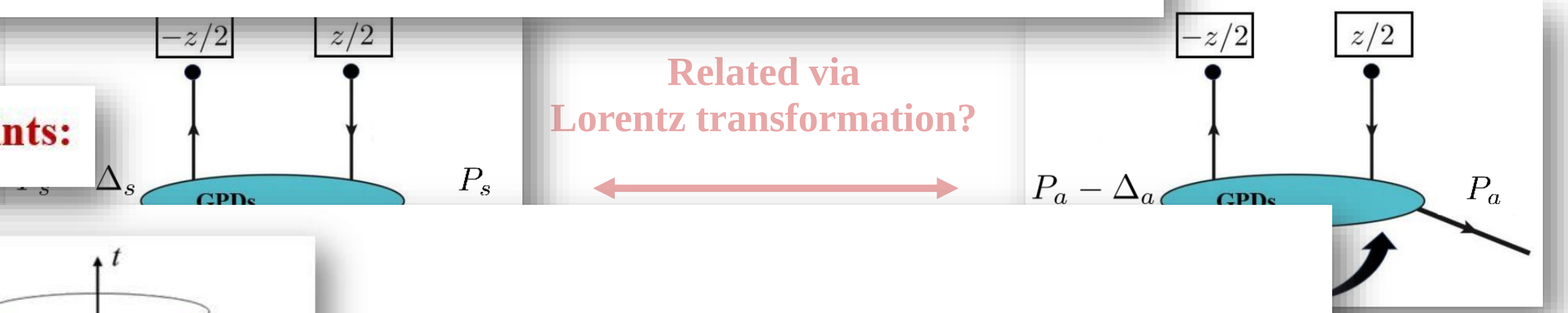
Approach 1: Can we calculate a quasi-GPD in symmetric frame through an asymmetric frame?

Transverse boost: This Lorentz transformation allows for an exact calculation of quasi-GPDs in symmetric frame through matrix elements of asymmetric frame

Approach 2: Why go through a calculation in asymmetric frame?

Why not calculate directly in asymmetric frame?

Key points:



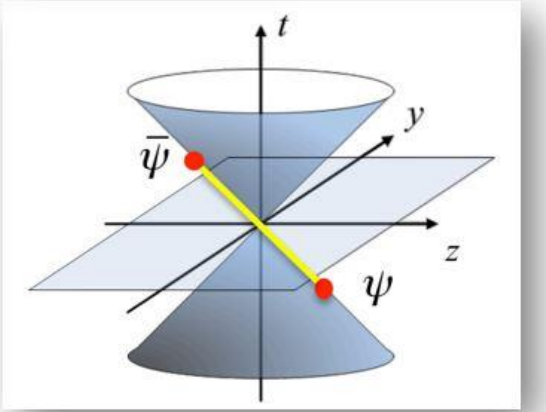


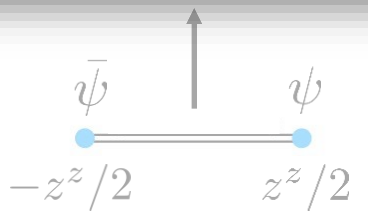
Figure courtesy: Yong Zhao

GPDs on the light-cone are Lorentz covariant

(It doesn't matter in which frame I do the calculation)

tor distance remains

patial (& same)



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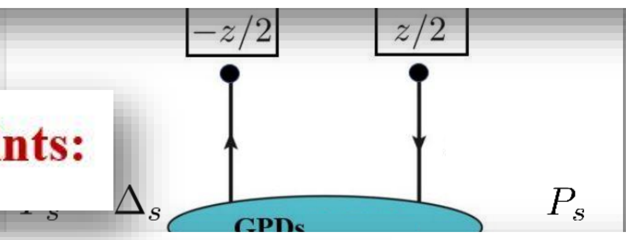
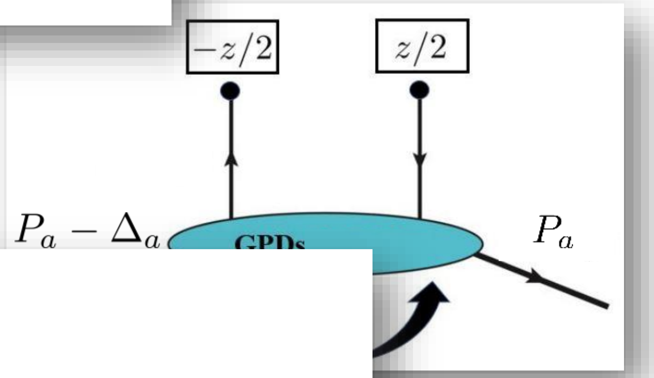
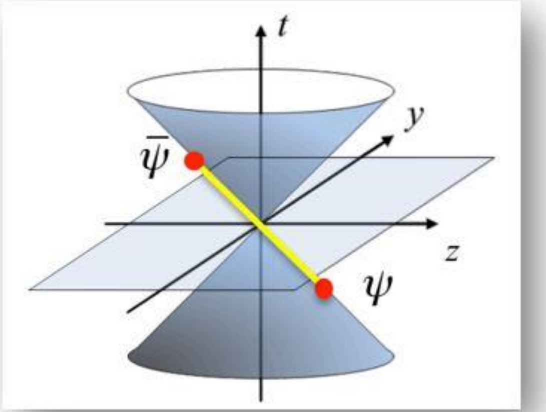


Related via Lorentz transformation?





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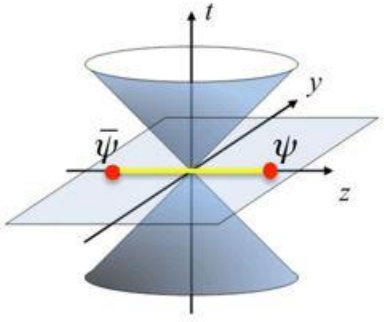
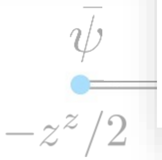


Figure courtesy: Yong Zhao

Are quasi-GPDs Lorentz covariant?



Main results

Definitions of quasi-GPDs

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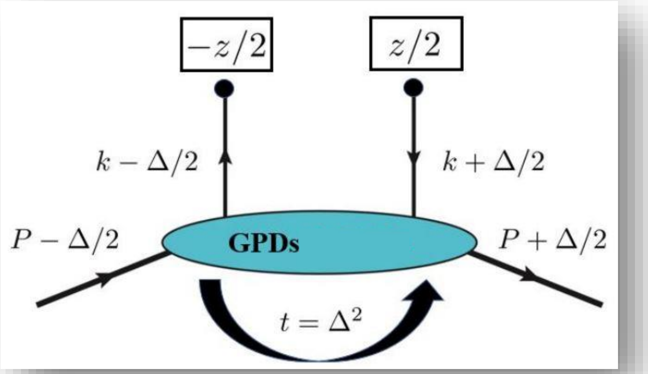


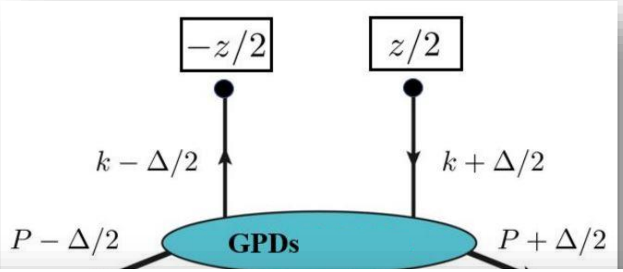
Definition of quasi-GPDs in symmetric frames: (Historical)

$$F^0_{\lambda,\lambda'}\Big|_s = \langle p'_s,\lambda'|\bar{q}(-z/2)\gamma^0 q(z/2)|p_s,\lambda\rangle\Big|_{z=0,\vec{z}_\perp=\vec{0}_\perp}$$

$$= \bar{u}_s(p'_s,\lambda')\left[\gamma^0 H_{Q(0)}(z,P_s,\Delta_s)\Big|_s + \frac{i\sigma^{0\mu}\Delta_{\mu,s}}{2M}E_{Q(0)}(z,P_s,\Delta_s)\Big|_s\right]u_s(p_s,\lambda)$$

Main results

Definitions of quasi-GPDs



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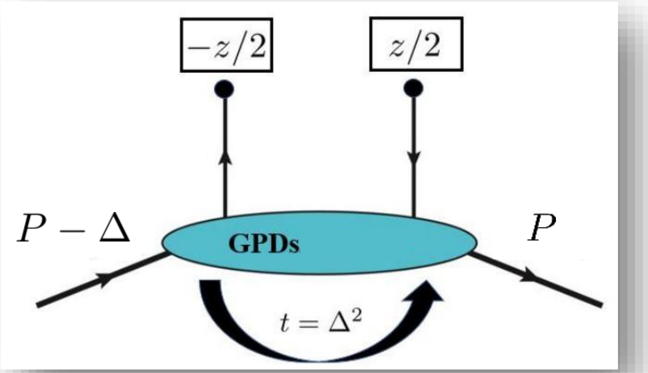
If quasi-GPDs are Lorentz covariant then:

Use γ^0

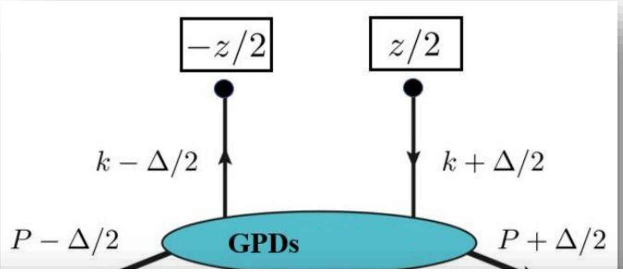


Definition of quasi-GPDs in asymmetric frames:

$$F^0_{\lambda,\lambda'}\Big|_a = \langle p'_a,\lambda'|\bar{q}(-z/2)\gamma^0 q(z/2)|p_a,\lambda\rangle\Big|_{z=0,\vec{z}_\perp=\vec{0}_\perp}$$
$$= \bar{u}_a(p'_a,\lambda')\left[\gamma^0 H_{Q(0)}(z,P_a,\Delta_a)\Big|_a + \frac{i\sigma^{0\mu}\Delta_{\mu,a}}{2M}E_{Q(0)}(z,P_a,\Delta_a)\Big|_a\right]u_a(p_a,\lambda)$$

Main results

Definitions of quasi-GPDs



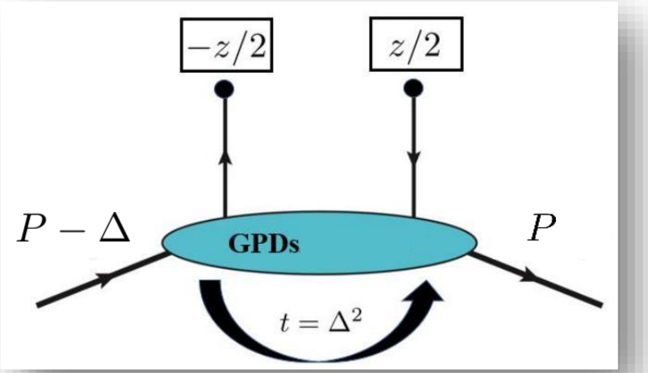
If quasi-GPDs are Lorentz covariant then:

Definition of quasi-GPDs in symmetric frames: (Historical)

$$F^0_{\lambda,\lambda'}\Big|_s = \langle p'_s,\lambda'|\bar q(-z/2)\gamma^0 q(z/2)|p_s,\lambda\rangle\Big|_{z=0,\vec z_\perp=\vec 0_\perp}$$
$$=\bar u_s(p'_s,\lambda')\left[\gamma^0 H_{Q(0)}(z,P_s,\Delta_s)\Big|_s + \frac{i\sigma^{0\mu}\Delta_{\mu,s}}{2M}E_{Q(0)}(z,P_s,\Delta_s)\Big|_s\right]u_s(p_s,\lambda)$$

Use γ^0

Use kinematics of asymmetric frame

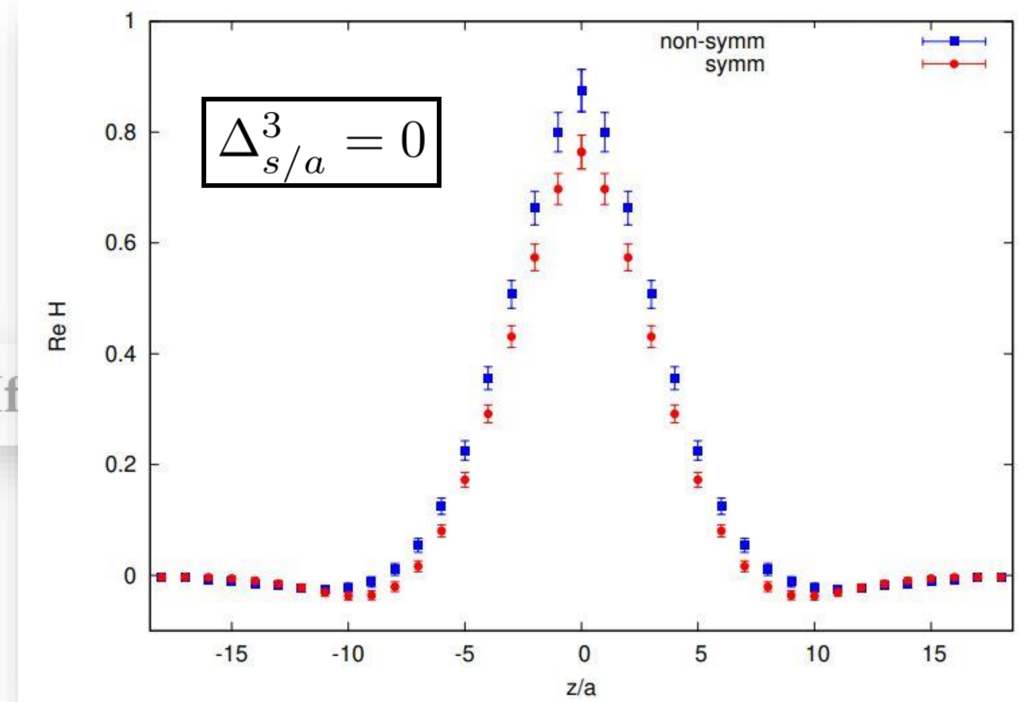


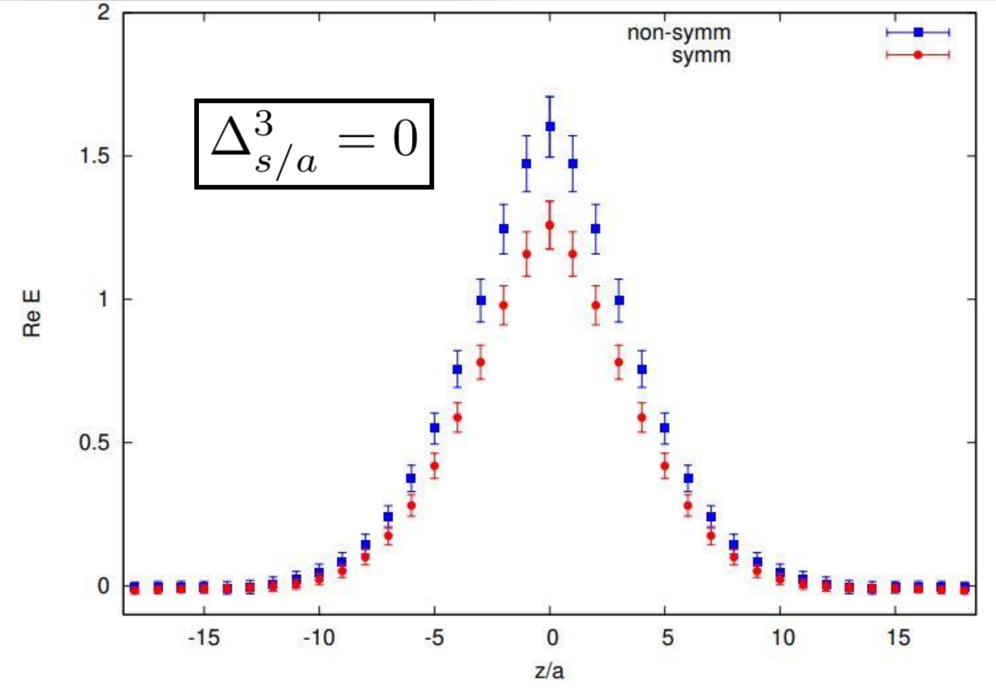
Definition of quasi-GPDs in asymmetric frames:

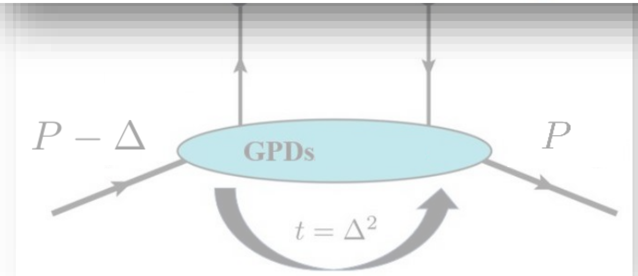
$$F^0_{\lambda,\lambda'}\Big|_a = \langle p'_a,\lambda'|\bar q(-z/2)\gamma^0 q(z/2)|p_a,\lambda\rangle\Big|_{z=0,\vec z_\perp=\vec 0_\perp}$$
$$=\bar u_a(p'_a,\lambda')\left[\gamma^0 H_{Q(0)}(z,P_a,\Delta_a)\Big|_a + \frac{i\sigma^{0\mu}\Delta_{\mu,a}}{2M}E_{Q(0)}(z,P_a,\Delta_a)\Big|_a\right]u_a(p_a,\lambda)$$

Main results

Lattice QCD results: (See Martha's talk for details)





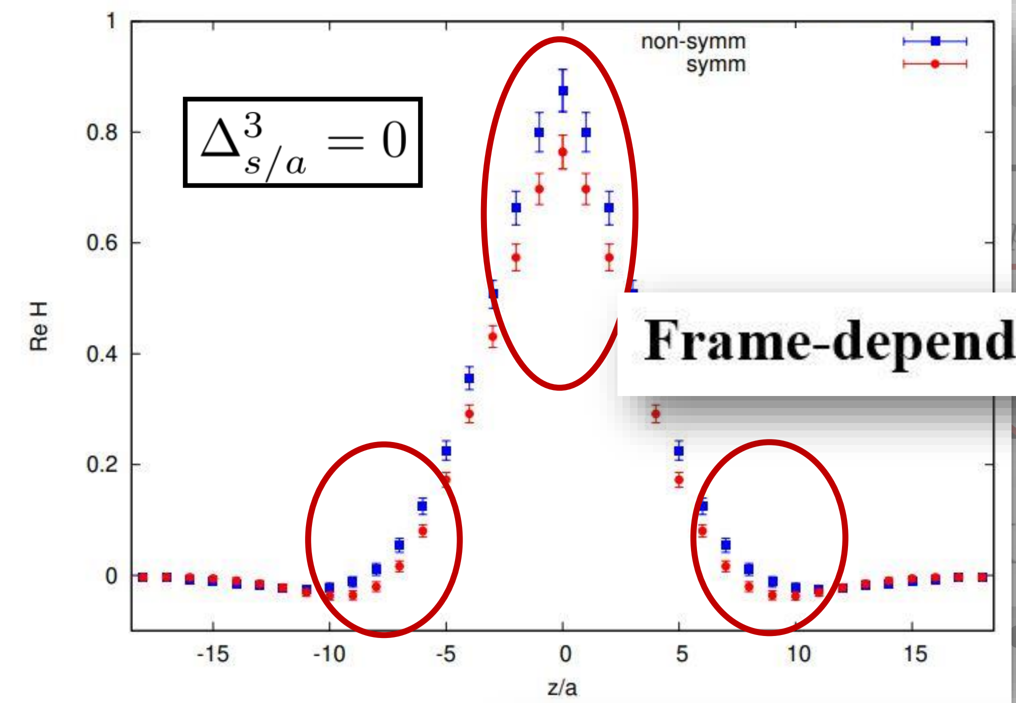


$$F^0_{\lambda,\lambda'}\Big|_a = \langle p'_a, \lambda' | \bar{q}(-z/2)\gamma^0 q(z/2) | p_a, \lambda\rangle \Big|_{z=0,\vec{z}_\perp=\vec{0}_\perp}$$

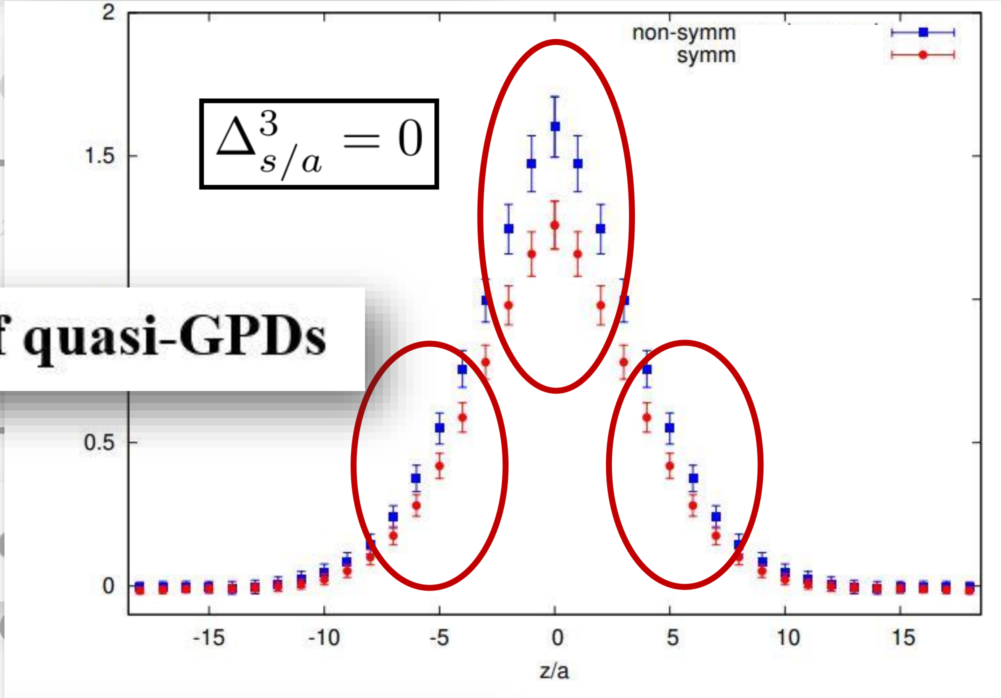
$$= \bar{u}_a(p'_a,\lambda')\left[\gamma^0 H_{Q(0)}(z,P_a,\Delta_a)\Big|_a + \frac{i\sigma^{0\mu}\Delta_{\mu,a}}{2M} E_{Q(0)}(z,P_a,\Delta_a)\Big|_a\right] u_a(p_a,\lambda)$$

Main results

Lattice QCD results: (See Martha's talk for details)



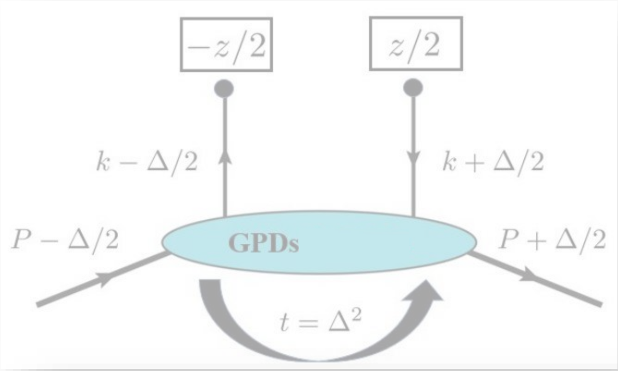
Frame-dependence of quasi-GPDs



Qualitatively similar results for imaginary parts

Main results

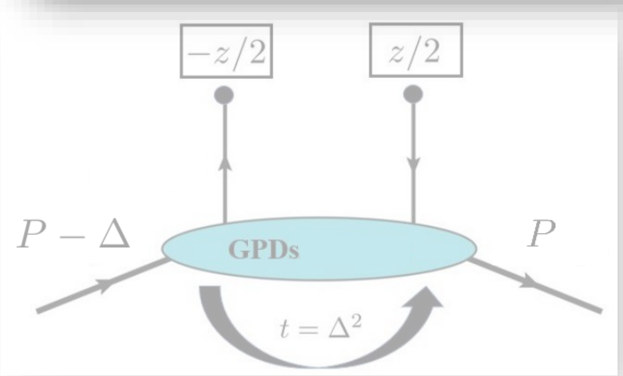
Definitions of quasi-GPDs



Definition of quasi-GPDs in symmetric frames: (Historical)

$$F^0_{\lambda,\lambda'}\Big|_s = \langle p'_s,\lambda'|\bar{q}(-z/2)\gamma^0 q(z/2)|p_s,\lambda\rangle\Big|_{z=0,\vec{z}_\perp=\vec{0}_\perp}$$
$$= \bar{u}_s(p'_s,\lambda')\left[\gamma^0 H_{Q(0)}(z,P_s,\Delta_s)\Big|_s + \frac{i\sigma^{0\mu}\Delta_{\mu,s}}{2M}E_{Q(0)}(z,P_s,\Delta_s)\Big|_s\right]u_s(p_s,\lambda)$$

Historic definitions of H & E quasi-GPDs are not manifestly Lorentz covariant

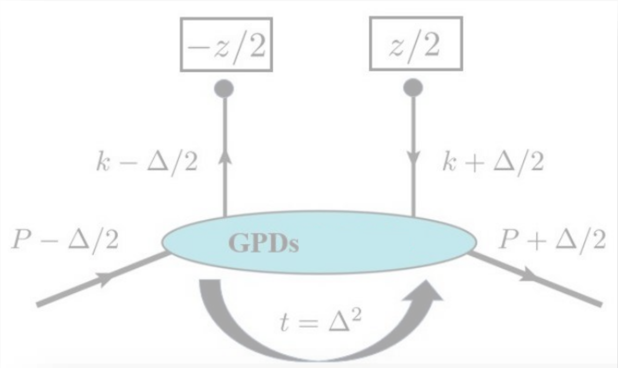


Definition of quasi-GPDs in asymmetric frames:

$$F^0_{\lambda,\lambda'}\Big|_a = \langle p'_a,\lambda'|\bar{q}(-z/2)\gamma^0 q(z/2)|p_a,\lambda\rangle\Big|_{z=0,\vec{z}_\perp=\vec{0}_\perp}$$
$$= \bar{u}_a(p'_a,\lambda')\left[\gamma^0 H_{Q(0)}(z,P_a,\Delta_a)\Big|_a + \frac{i\sigma^{0\mu}\Delta_{\mu,a}}{2M}E_{Q(0)}(z,P_a,\Delta_a)\Big|_a\right]u_a(p_a,\lambda)$$

Main results

Definitions of quasi-GPDs



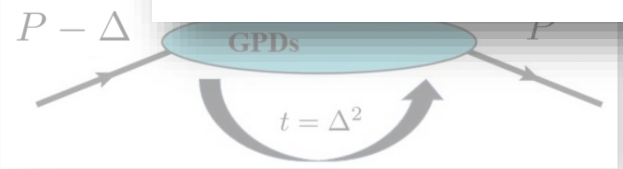
Definition of quasi-GPDs in symmetric frames: (Historical)

$$F^0_{\lambda,\lambda'}\Big|_s = \langle p'_s,\lambda'|\bar{q}(-z/2)\gamma^0 q(z/2)|p_s,\lambda\rangle\Big|_{z=0,\vec{z}_\perp=\vec{0}_\perp}$$

$$= \bar{u}_s(p'_s,\lambda')\left[\gamma^0 H_{Q(0)}(z,P_s,\Delta_s)\Big|_s + \frac{i\sigma^{0\mu}\Delta_{\mu,s}}{2M} E_{Q(0)}(z,P_s,\Delta_s)\Big|_s\right] u_s(p_s,\lambda)$$

Historic definitions of H & E quasi-GPDs are not manifestly Lorentz covariant

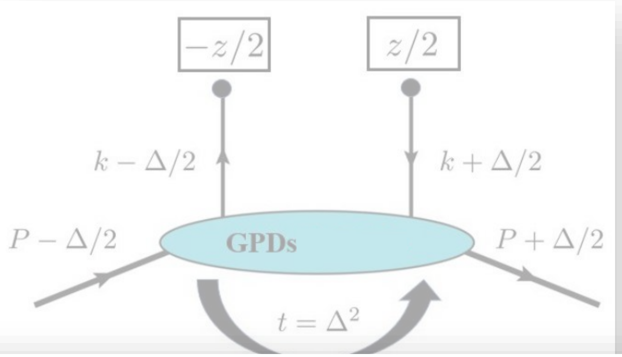
This means that the basis vectors $(\gamma^0, i\sigma^{0\Delta_{s/a}})$ do not form a complete basis for a <u>spatially-separated</u> bi-local operator at <u>finite momentum</u>



$$= \bar{u}_a(p'_a,\lambda')\left[\gamma^0 H_{Q(0)}(z,P_a,\Delta_a)\Big|_a + \frac{i\sigma^{0\mu}\Delta_{\mu,a}}{2M} E_{Q(0)}(z,P_a,\Delta_a)\Big|_a\right] u_a(p_a,\lambda)$$

Main results

Definitions of quasi-GPDs



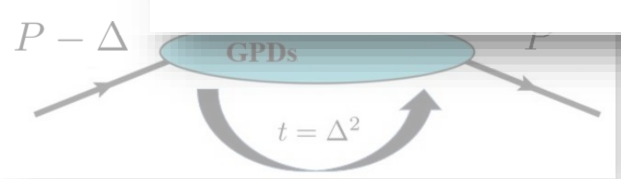
Definition of quasi-GPDs in symmetric frames: (Historical)

$$F^0_{\lambda,\lambda'}\big|_s = \langle p'_s,\lambda'|\bar{q}(-z/2)\gamma^0 q(z/2)|p_s,\lambda\rangle\Big|_{z=0,\vec{z}_\perp=\vec{0}_\perp}$$

$$= \bar{u}_s(p'_s,\lambda')\left[\gamma^0 H_{Q(0)}(z,P_s,\Delta_s)\big|_s + \frac{i\sigma^{0\mu}\Delta_{\mu,s}}{2M}E_{Q(0)}(z,P_s,\Delta_s)\big|_s\right]u_s(p_s,\lambda)$$

We do not dismiss these definitions
since they do work in the large-momentum limit (I will show this formally later)

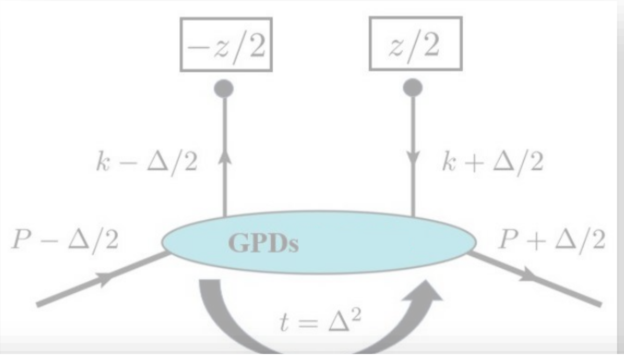
This means that the basis vectors $(\gamma^0, i\sigma^{0\Delta_{s/a}})$ do not form a complete basis for a spatially-separated bi-local operator at finite momentum



$$= \bar{u}_a(p'_a,\lambda')\left[\gamma^0 H_{Q(0)}(z,P_a,\Delta_a)\big|_a + \frac{i\sigma^{0\mu}\Delta_{\mu,a}}{2M}E_{Q(0)}(z,P_a,\Delta_a)\big|_a\right]u_a(p_a,\lambda)$$

Main results

Definitions of quasi-GPDs



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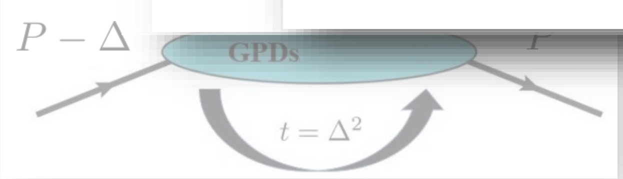
$$= \bar{u}_s(p'_s,\lambda')\left[\gamma^0 H_{Q(0)}(z,P_s,\Delta_s)\big|_s + \frac{i\sigma^{0\mu}\Delta_{\mu,s}}{2M} E_{Q(0)}(z,P_s,\Delta_s)\big|_s\right] u_s(p_s,\lambda)$$

We do not dismiss these definitions since they do work in the large-momentum limit (I will show this formally later)

Can we come up with a manifestly Lorentz covariant definition of quasi-GPDs for finite values of momentum?



$$= \bar{u}_a(p'_a,\lambda')\left[\gamma^0 H_{Q(0)}(z,P_a,\Delta_a)\big|_a + \frac{i\sigma^{0\mu}\Delta_{\mu,a}}{2M} E_{Q(0)}(z,P_a,\Delta_a)\big|_a\right] u_a(p_a,\lambda)$$

Main results

Lorentz covariant formalism

Main results

Lorentz covariant formalism

Novel parameterization of position-space matrix element: (Inspired from Meissner, Metz, Schlegel, 2009)

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^{\mu}}{M}\boldsymbol{A_1} + \frac{z^{\mu}}{M}\boldsymbol{A_2} + \frac{\Delta^{\mu}}{M}\boldsymbol{A_3} + \frac{i\sigma^{\mu z}}{M}\boldsymbol{A_4} + \frac{i\sigma^{\mu\Delta}}{M}\boldsymbol{A_5} + \frac{P^{\mu}i\sigma^{z\Delta}}{M^3}\boldsymbol{A_6} + \frac{z^{\mu}i\sigma^{z\Delta}}{M^3}\boldsymbol{A_7} + \frac{\Delta^{\mu}i\sigma^{z\Delta}}{M^3}\boldsymbol{A_8}\right]u(p,\lambda)$$

Vector operator $F^{\mu}_{\lambda,\lambda'} = \langle p',\lambda'|\bar{q}(-z/2)\gamma^{\mu}q(z/2)|p,\lambda\rangle\Big|_{z=0,\vec{z}_{\perp}=\vec{0}_{\perp}}$

Main results

Lorentz covariant formalism

Novel parameterization of position-space matrix element: (Vector operator)

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^\mu}{M}\boldsymbol{A_1} + \frac{z^\mu}{M}\boldsymbol{A_2} + \frac{\Delta^\mu}{M}\boldsymbol{A_3} + \frac{i\sigma^{\mu z}}{M}\boldsymbol{A_4} + \frac{i\sigma^{\mu\Delta}}{M}\boldsymbol{A_5} + \frac{P^\mu i\sigma^{z\Delta}}{M^3}\boldsymbol{A_6} + \frac{z^\mu i\sigma^{z\Delta}}{M^3}\boldsymbol{A_7} + \frac{\Delta^\mu i\sigma^{z\Delta}}{M^3}\boldsymbol{A_8}\right]u(p,\lambda)$$

Features:

- **General structure of matrix element based on constraints from Parity**
- **<u>8 linearly-independent Dirac structures</u> (Lorentz vectors change with frames)**
- **<u>8 Lorentz-covariant amplitudes (or Form Factors)</u>** $A_i \equiv A_i(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2)$

Main results

Lorentz covariant formalism

Novel parameterization of position-space matrix element: (Vector operator)

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^\mu}{M}\boldsymbol{A_1} + \frac{z^\mu}{M}\boldsymbol{A_2} + \frac{\Delta^\mu}{M}\boldsymbol{A_3} + \frac{i\sigma^{\mu z}}{M}\boldsymbol{A_4} + \frac{i\sigma^{\mu\Delta}}{M}\boldsymbol{A_5} + \frac{P^\mu i\sigma^{z\Delta}}{M^3}\boldsymbol{A_6} + \frac{z^\mu i\sigma^{z\Delta}}{M^3}\boldsymbol{A_7} + \frac{\Delta^\mu i\sigma^{z\Delta}}{M^3}\boldsymbol{A_8}\right]u(p,\lambda)$$

Features:

Martha's talk: Validating the frame-independence of A's

- **General structure of matrix element based on constraints from Parity**
- **<u>8 linearly-independent Dirac structures</u> (Lorentz vectors change with frames)**
- **<u>8 Lorentz-covariant amplitudes (or Form Factors)</u>** $A_i \equiv A_i(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2)$

Main results

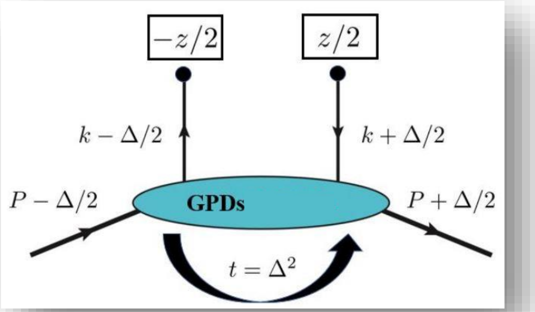
Exploring historical definitions of quasi-GPDs

Mapping Form Factors to the Lorentz non-covariant definitions of quasi-GPDs:

Main results

Exploring historical definitions of quasi-GPDs

Mapping Form Factors to the Lorentz non-covariant definitions of quasi-GPDs:



Symmetric frame:

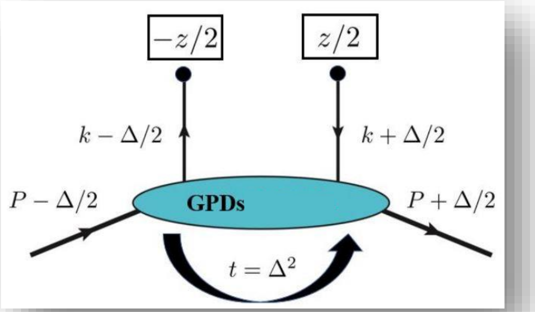
$$H_{\mathrm{Q}(0)}(z,P_s,\Delta_s)\big|_s = \boldsymbol{A_1} + \frac{\Delta_s^0}{P_s^0}\boldsymbol{A_3} - \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3}\boldsymbol{A_4} + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0 \Delta_s^3 z^3 P_s^0}{2M^2 (P_s^3)^2} - \frac{z^3 \Delta_\perp^2}{2M^2 P_s^3}\right)\boldsymbol{A_6} + \left(\frac{(\Delta_s^0)^3 z^3}{2M^2 P_s^0 P_s^3} - \frac{(\Delta_s^0)^2 \Delta_s^3 z^3}{2M^2 (P_s^3)^2} - \frac{\Delta_s^0 z^3 \Delta_\perp^2}{2M^2 P_s^0 P_s^3}\right)\boldsymbol{A_8}$$

Main results

Exploring historical definitions of quasi-GPDs

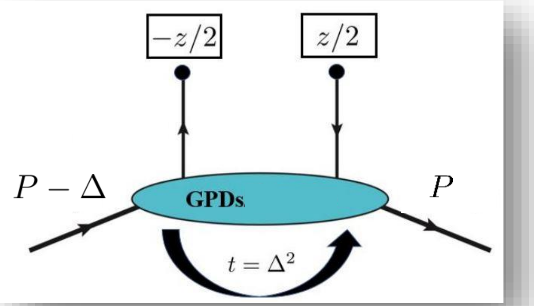
Mapping Form Factors to the Lorentz non-covariant definitions of quasi-GPDs:

Symmetric frame:



$$H_{\mathrm{Q}(0)}(z,P_s,\Delta_s)\big|_s = \boldsymbol{A_1} + \frac{\Delta_s^0}{P_s^0}\boldsymbol{A_3} - \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3}\boldsymbol{A_4} + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0\Delta_s^3 z^3 P_s^0}{2M^2(P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2P_s^3}\right)\boldsymbol{A_6} + \left(\frac{(\Delta_s^0)^3 z^3}{2M^2P_s^0P_s^3} - \frac{(\Delta_s^0)^2\Delta_s^3 z^3}{2M^2(P_s^3)^2} - \frac{\Delta_s^0 z^3\Delta_\perp^2}{2M^2P_s^0P_s^3}\right)\boldsymbol{A_8}$$

Asymmetric frame:



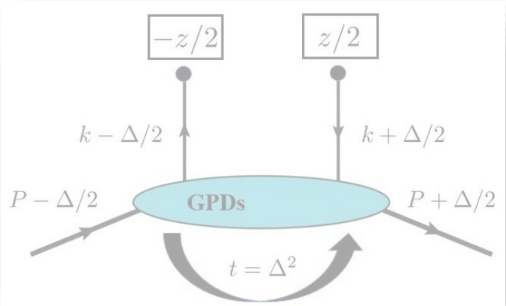
$$H_{\mathrm{Q}(0)}\big|_a(z,P_a,\Delta_a) = \boldsymbol{A_1} + \frac{\Delta_a^0}{P^0_{avg,a}}\boldsymbol{A_3} - \left(\frac{\Delta_a^0 z^3}{2P^0_{avg,a}P^3_{avg,a}} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{\Delta_a^0\Delta_a^3 z^3}{4P^0_{avg,a}(P^3_{avg,a})^2}\right)\boldsymbol{A_4}$$
$$+\left(\frac{(\Delta_a^0)^2 z^3}{2M^2P^3_{avg,a}} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{(\Delta_a^0)^2\Delta_a^3 z^3}{4M^2(P^3_{avg,a})^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{P^0_{avg,a}\Delta_a^0\Delta_a^3 z^3}{2M^2(P^3_{avg,a})^2} - \frac{z^3\Delta_\perp^2}{2M^2P^3_{avg,a}}\right)\boldsymbol{A_6}$$
$$+\left(\frac{(\Delta_a^0)^3 z^3}{2M^2P^0_{avg,a}P^3_{avg,a}} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{(\Delta_a^0)^3\Delta_a^3 z^3}{4M^2P^0_{avg,a}(P^3_{avg,a})^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{(\Delta_a^0)^2\Delta_a^3 z^3}{2M^2(P^3_{avg,a})^2} - \frac{z^3\Delta_\perp^2\Delta_a^0}{2M^2P^0_{avg,a}P^3_{avg,a}}\right)\boldsymbol{A_8}$$

Main results

Exploring historical definitions of quasi-GPDs

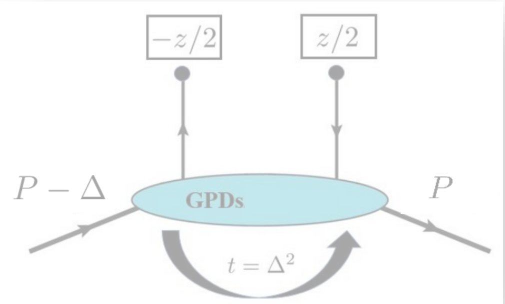
Frame-dependent expressions: Lorentz non-covariance from explicit kinematic factors

Symmetric frame:



$$H_{\mathrm{Q}(0)}(z,P_s,\Delta_s)\big|_s = \boldsymbol{A_1} + \frac{\Delta_s^0}{P_s^0}\boldsymbol{A_3} - \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3}\boldsymbol{A_4} + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0\Delta_s^3 z^3 P_s^0}{2M^2(P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2P_s^3}\right)\boldsymbol{A_6} + \left(\frac{(\Delta_s^0)^3 z^3}{2M^2P_s^0P_s^3} - \frac{(\Delta_s^0)^2\Delta_s^3 z^3}{2M^2(P_s^3)^2} - \frac{\Delta_s^0 z^3\Delta_\perp^2}{2M^2P_s^0P_s^3}\right)\boldsymbol{A_8}$$

Asymmetric frame:

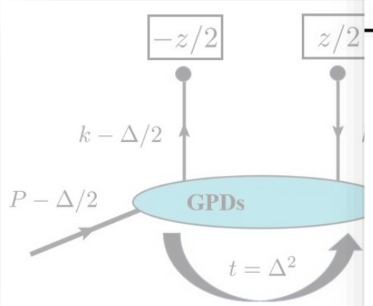


$$H_{\mathrm{Q}(0)}\big|_a(z,P_a,\Delta_a) = \boldsymbol{A_1} + \frac{\Delta_a^0}{P^0_{avg,a}}\boldsymbol{A_3} - \left(\frac{\Delta_a^0 z^3}{2P^0_{avg,a}P^3_{avg,a}} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{\Delta_a^0\Delta_a^3 z^3}{4P^0_{avg,a}(P^3_{avg,a})^2}\right)\boldsymbol{A_4}$$
$$+ \left(\frac{(\Delta_a^0)^2 z^3}{2M^2P^3_{avg,a}} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{(\Delta_a^0)^2\Delta_a^3 z^3}{4M^2(P^3_{avg,a})^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{P^0_{avg,a}\Delta_a^0\Delta_a^3 z^3}{2M^2(P^3_{avg,a})^2} - \frac{z^3\Delta_\perp^2}{2M^2P^3_{avg,a}}\right)\boldsymbol{A_6}$$
$$+ \left(\frac{(\Delta_a^0)^3 z^3}{2M^2P^0_{avg,a}P^3_{avg,a}} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{(\Delta_a^0)^3\Delta_a^3 z^3}{4M^2P^0_{avg,a}(P^3_{avg,a})^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{(\Delta_a^0)^2\Delta_a^3 z^3}{2M^2(P^3_{avg,a})^2} - \frac{z^3\Delta_\perp^2\Delta^0_a}{2M^2P^0_{avg,a}P^3_{avg,a}}\right)\boldsymbol{A_8}$$

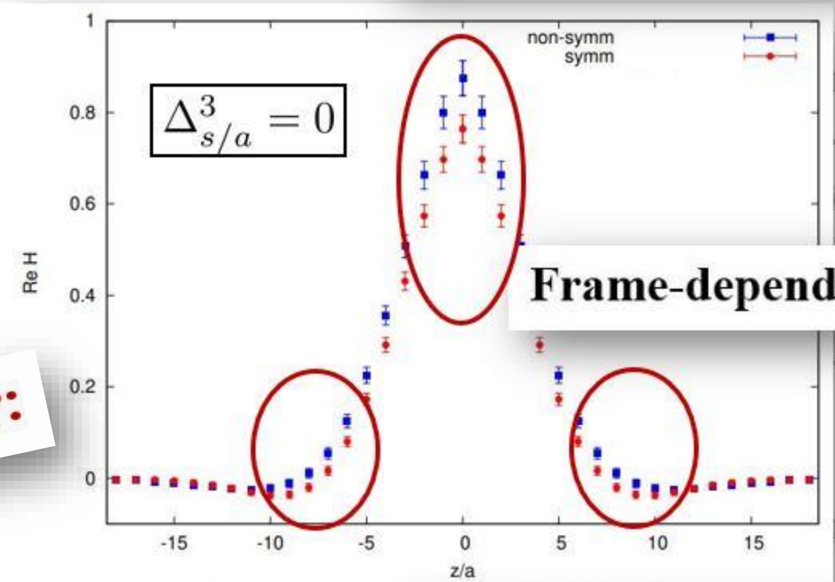
Main results

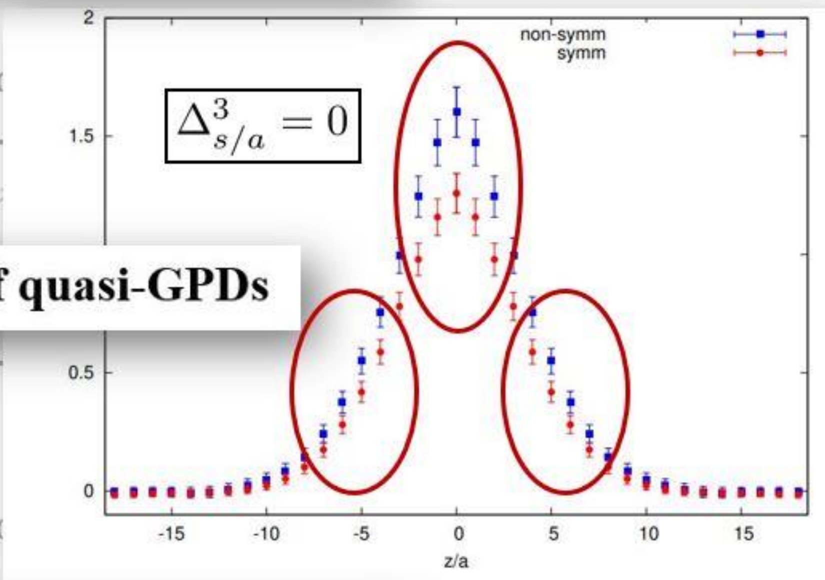
Exploring historical definitions of quasi-GPDs

Frame-dependent expressions: Lorentz non-covariance from explicit kinematic factors



Lattice QCD results: (See Martha's talk for details)

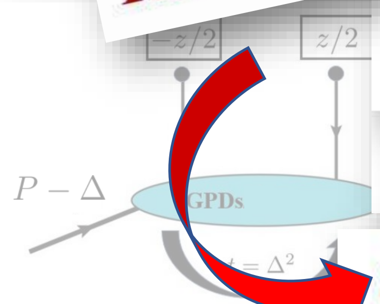




Frame-dependence of quasi-GPDs

Qualitatively similar results for imaginary parts

Reminder:

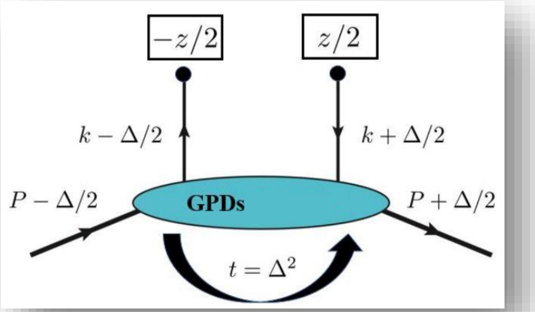


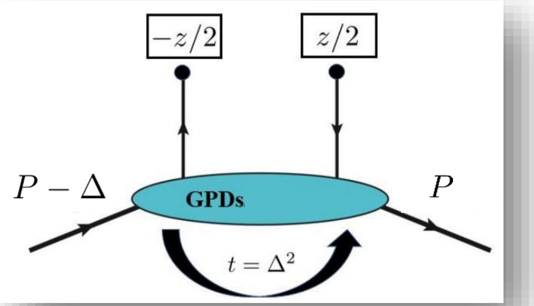
Same conclusion but now through the Form Factor approach

Main results

Light-cone GPDs

Mapping Form Factors to the light-cone GPDs: (Sample results)

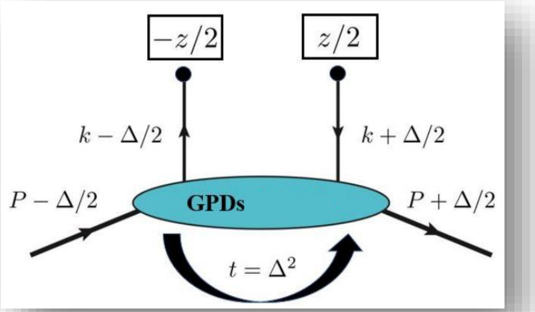




Main results

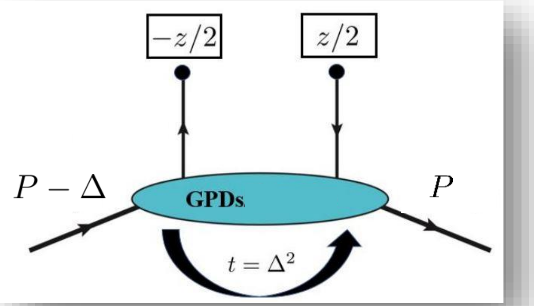
Light-cone GPDs

Mapping Form Factors to the light-cone GPDs: (Sample results)



Definition:

$$F^{+}_{\lambda,\lambda'}\Big|_{s/a} = \langle p'_{s/a},\lambda'|\bar{q}(-z/2)\gamma^{+}q(z/2)|p_{s/a},\lambda\rangle\Big|_{z=0,\vec{z}_\perp=\vec{0}_\perp}$$
$$= \bar{u}_{s/a}(p'_{s/a},\lambda')\left[\gamma^{+}H(z,P_{s/a},\Delta_{s/a}) + \frac{i\sigma^{+\mu}\Delta_{\mu,s/a}}{2M}E(z,P_{s/a},\Delta_{s/a})\right]u_{s/a}(p_{s/a},\lambda)$$



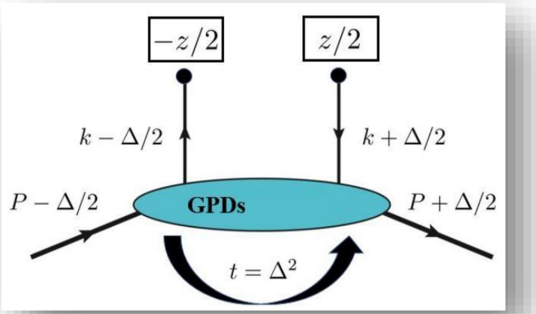
Relation between light-cone GPD H & Form Factors:

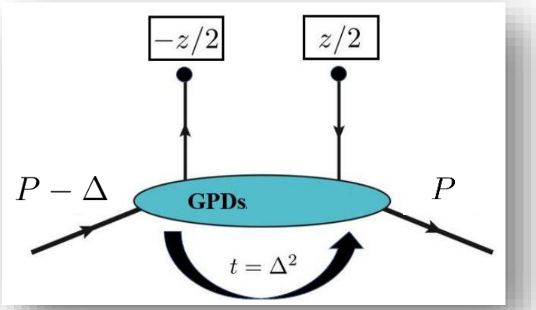
$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = \boldsymbol{A_1} + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}\boldsymbol{A_3}$$

Lorentz covariant expression!

Main results

Lorentz covariant formalism





Main results

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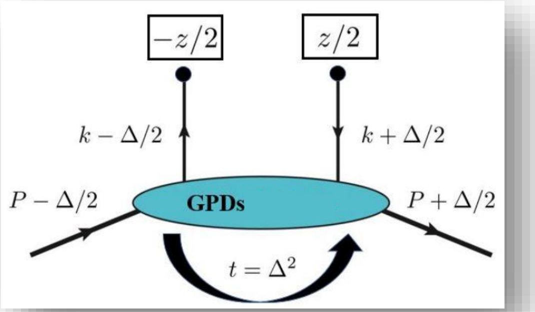
$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = A_1 + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z} A_3$$

Lorentz covariant formalism

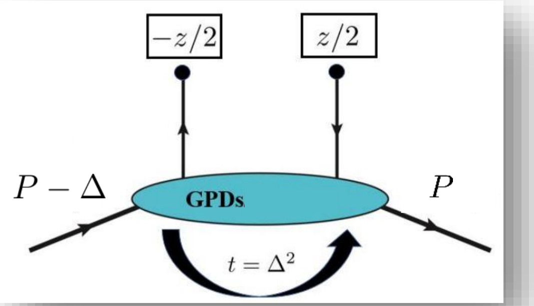
Quasi-GPDs & Form Factors: (Sample results)

Symmetric frame:



$$H_{Q(0)}(z, P_s, \Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0} A_3 - \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3} A_4 + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0 \Delta_s^3 z^3 P_s^0}{2M^2 (P_s^3)^2} - \frac{z^3 \Delta_\perp^2}{2M^2 P_s^3}\right) A_6 + \left(\frac{(\Delta_s^0)^3 z^3}{2M^2 P_s^0 P_s^3} - \frac{(\Delta_s^0)^2 \Delta_s^3 z^3}{2M^2 (P_s^3)^2} - \frac{\Delta_s^0 z^3 \Delta_\perp^2}{2M^2 P_s^0 P_s^3}\right) A_8$$

Asymmetric frame:



$$H_{Q(0)}\big|_a(z, P_a, \Delta_a) = A_1 + \frac{\Delta_a^0}{P_{avg,a}^0} A_3 - \left(\frac{\Delta_a^0 z^3}{2P_{avg,a}^0 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})} \frac{\Delta_a^0 \Delta_a^3 z^3}{4P_{avg,a}^0 (P_{avg,a}^3)^2}\right) A_4 + \left(\frac{(\Delta_a^0)^2 z^3}{2M^2 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})} \frac{(\Delta_a^0)^2 \Delta_a^3 z^3}{4M^2 (P_{avg,a}^3)^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})} \frac{P_{avg,a}^0 \Delta_a^0 \Delta_a^3 z^3}{2M^2 (P_{avg,a}^3)^2} - \frac{z^3 \Delta_\perp^2}{2M^2 P_{avg,a}^3}\right) A_6 + \left(\frac{(\Delta_a^0)^3 z^3}{2M^2 P_{avg,a}^0 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})} \frac{(\Delta_a^0)^3 \Delta_a^3 z^3}{4M^2 P_{avg,a}^0 (P_{avg,a}^3)^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})} \frac{(\Delta_a^0)^2 \Delta_a^3 z^3}{2M^2 (P_{avg,a}^3)^2} - \frac{z^3 \Delta_\perp^2 \Delta_a^0}{2M^2 P_{avg,a}^0 P_{avg,a}^3}\right) A_8$$

Main results

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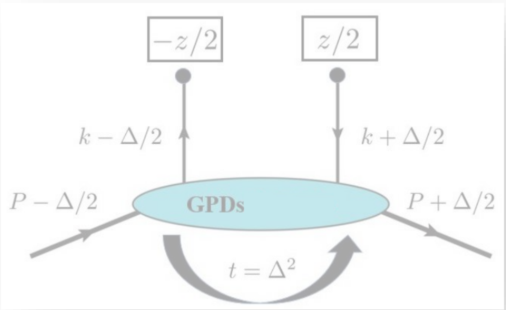
Lorentz covariant formalism

Quasi-GPDs & Form Factors: (Sample results)

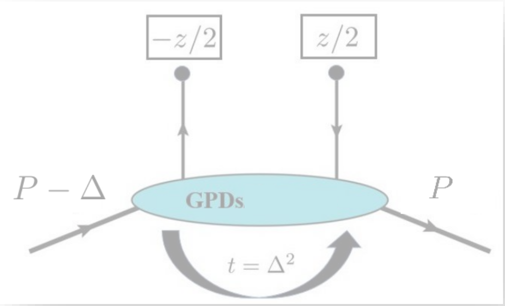
Symmetric frame:



$$H_{Q(0)}(z, P_s, \Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0} A_3 - \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3} A_4 + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0 \Delta_s^3 z^3 P_s^0}{2M^2 (P_s^3)^2} - \frac{z^3 \Delta_\perp^2}{2M^2 P_s^3}\right) A_6$$
$$+ \left(\frac{(\Delta_s^0)^3 z^3}{2M^2 P_s^0 P_s^3} - \frac{(\Delta_s^0)^2 \Delta_s^3 z^3}{2M^2 (P_s^3)^2} - \frac{\Delta_s^0 z^3 \Delta_\perp^2}{2M^2 P_s^0 P_s^3}\right) A_8$$

Contamination from frame-dependent power corrections

Asymmetric frame:



$$H_{Q(0)}\big|_a(z, P_a, \Delta_a) = A_1 + \frac{\Delta_a^0}{P_{avg,a}^0} A_3 - \left(\frac{\Delta_a^0 z^3}{2P_{avg,a}^0 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{\Delta_a^0 \Delta_a^3 z^3}{4P_{avg,a}^0 (P_{avg,a}^3)^2}\right) A_4$$
$$+ \left(\frac{(\Delta_a^0)^2 z^3}{2M^2 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^2 \Delta_a^3 z^3}{4M^2 (P_{avg,a}^3)^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{P_{avg,a}^0 \Delta_a^0 \Delta_a^3 z^3}{2M^2 (P_{avg,a}^3)^2} - \frac{z^3 \Delta_\perp^2}{2M^2 P_{avg,a}^3}\right) A_6$$
$$+ \left(\frac{(\Delta_a^0)^3 z^3}{2M^2 P_{avg,a}^0 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^3 \Delta_a^3 z^3}{4M^2 P_{avg,a}^0 (P_{avg,a}^3)^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^2 \Delta_a^3 z^3}{2M^2 (P_{avg,a}^3)^2} - \frac{z^3 \Delta_\perp^2 \Delta_a^0}{2M^2 P_{avg,a}^0 P_{avg,a}^3}\right) A_8$$

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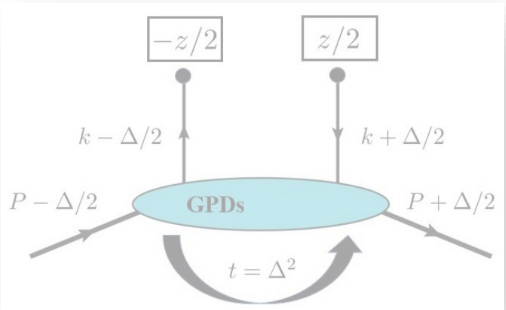
Lorentz covariant formalism

Quasi-GPDs & Form Factors: (Sample results)

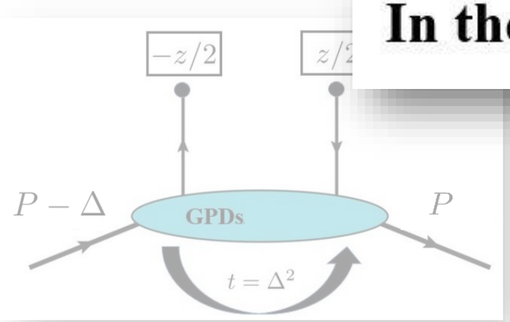
Symmetric frame:



$$H_{Q(0)}(z,P_s,\Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0}A_3 - \frac{\Delta_s^0 z^3}{2P_s^0P_s^3}A_4 + \left(\frac{(\Delta_s^0)^2z^3}{2M^2P_s^3} - \frac{\Delta_s^0\Delta_s^3z^3P_s^0}{2M^2(P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2P_s^3}\right)A_6 + \left(\frac{(\Delta_s^0)^3z^3}{2M^2P_s^0P_s^3} - \frac{(\Delta_s^0)^2\Delta_s^3z^3}{2M^2(P_s^3)^2} - \frac{\Delta_s^0z^3\Delta_\perp^2}{2M^2P_s^0P_s^3}\right)A_8$$

Contamination from frame-dependent power corrections

Asymmetric frame:



In the large-momentum limit, these expressions reduce to light-cone results

$$+\left(\frac{(\Delta_a^0)^2z^3}{2M^2P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^2\Delta_a^3z^3}{4M^2(P_{avg,a}^3)^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{P_{avg,a}^0\Delta_a^0\Delta_a^3z^3}{2M^2(P_{avg,a}^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2P_{avg,a}^3}\right)A_6$$

$$+\left(\frac{(\Delta_a^0)^3z^3}{2M^2P_{avg,a}^0P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^3\Delta_a^3z^3}{4M^2P_{avg,a}^0(P_{avg,a}^3)^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^2\Delta_a^3z^3}{2M^2(P_{avg,a}^3)^2} - \frac{z^3\Delta_\perp^2\Delta_a^0}{2M^2P_{avg,a}^0P_{avg,a}^3}\right)A_8$$

Main results

Relation between light-cone GPD H & Form Factors:

$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = A_1 + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z} A_3$$

Lorentz covariant formalism

Interlude:

Quasi-GPDs & Form Factors: (Sample results)

Let's go back to PDFs

Symmetric frame:

$$H_{Q(0)}(z, P_s, \Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0} A_3 - \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3} A_4 + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0 \Delta_s^3 z^3 P_s^0}{2M^2 (P_s^3)^2} - \frac{z^3 \Delta_\perp^2}{2M^2 P_s^3}\right) A_6$$
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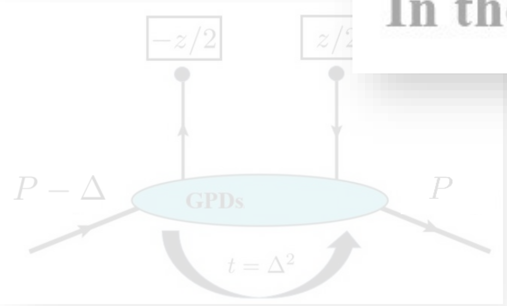
Contamination from frame-dependent power corrections

Asymmetric frame:

In the large-momentum limit, these expressions reduce to light-cone results



$$+ \left(\frac{(\Delta_a^0)^2 z^3}{2M^2 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^2 \Delta_a^3 z^3}{4M^2 (P_{avg,a}^3)^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{P_{avg,a}^0 \Delta_a^0 \Delta_a^3 z^3}{2M^2 (P_{avg,a}^3)^2} - \frac{z^3 \Delta_\perp^2}{2M^2 P_{avg,a}^3}\right) A_6$$
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Lorentz covariant formalism

Interlude:

Quasi-GPDs & Form Factors: (Sample results)

Let's go back to PDFs

Symmetric frame:

arXiv: 1705.01488

Quasi-PDFs, momentum distributions and pseudo-PDFs

A. V. Radyushkin

Old Dominion University, Norfolk, VA 23529, USA and
Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA

$$\mathcal{M}^\alpha(z,p) \equiv \langle p|\bar\psi(0)\,\gamma^\alpha\,\hat E(0,z;A)\psi(z)|p\rangle \qquad (12)$$

type, where $\hat E(0,z;A)$ is the standard $0 \to z$ straight-line gauge link in the quark (fundamental) representation. These matrix elements may be decomposed into p^α and z^α parts:

$$\mathcal{M}^\alpha(z,p) = 2p^\alpha \mathcal{M}_p(-(zp), -z^2) + z^\alpha \mathcal{M}_z(-(zp), -z^2)\,. \qquad (13)$$

The $\mathcal{M}_p(-(zp), -z^2)$ part gives the twist-2 distribution when $z^2 \to 0$, while $\mathcal{M}_z((zp), -z^2)$ is a purely higher-twist contamination, and it is better to get rid of it.

2 Form factors

Contamination from frame-dependent power corrections

... limit, these expressions reduce to light-cone results

Main results

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Contamination from frame-dependent power corrections

If one takes $z = (z_-, z_\perp)$ in the $\alpha = +$ component of $\mathcal{M}^\alpha$, the z^α-part drops out, and one can introduce a

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Quasi-GPDs & Form Factors: (Sample results)

Let's go back to PDFs

arXiv: 1705.01488

Quasi-PDFs, momentum distributions and pseudo-PDFs

A. V. Radyushkin
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Contamination from frame-dependent power corrections

If one takes $z=(z_-,z_\perp)$ in the $\alpha=+$ component of $\mathcal{M}^\alpha$, the z^α-part drops out, and one can introduce a

formula (6). For quasi-distributions, the easiest way to remove the z^α contamination is to take the time component of $\mathcal{M}^\alpha(z=z_3,p)$ and define

$$\mathcal{M}^0(z_3,p) = 2p^0\int_{-1}^{1} dy\, Q(y,P)\, e^{iyPz_3}\,. \qquad (14)$$

Therefore, γ^0 is better behaved than γ^3 with respect to power corrections

Main results

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Lorentz covariant formalism

Interlude:

Quasi-GPDs & Form Factors: (Sample results)

Let's go back to PDFs

arXiv: 1705.01488

Quasi-PDFs, momentum distributions and pseudo-PDFs

Old Domini
Thomas Jefferson Natio

Statement needs a qualifier: Situation more complicated for quasi-GPDs

(See next slide)

$$\mathcal{M}^\alpha(z,p) \equiv \langle p|\bar\psi(0)\,\gamma^\alpha\,\hat E(0,z;A)\psi(z)|p\rangle \quad (12)$$

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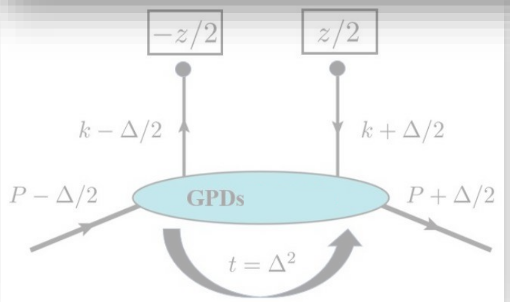
Lorentz covariant formalism

Contrary to quasi-PDFs, γ^0 operator for quasi-GPDs is plagued with (frame-dependent) power corrections



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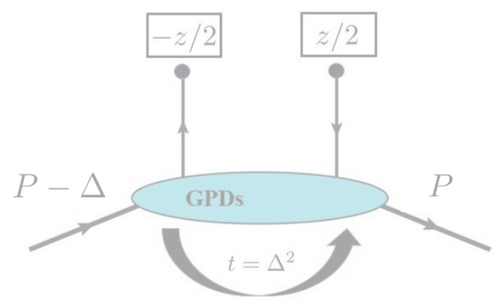
Asymmetric frame:



$$H_{Q(0)}\Big|_a(z, P_a, \Delta_a) = A_1 + \frac{\Delta_a^0}{P^0_{avg,a}} A_3 - \left(\frac{\Delta_a^0 z^3}{2P^0_{avg,a} P^3_{avg,a}} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})} \frac{\Delta_a^0 \Delta_a^3 z^3}{4P^0_{avg,a}(P^3_{avg,a})^2}\right) A_4$$
$$+ \left(\frac{(\Delta_a^0)^2 z^3}{2M^2 P^3_{avg,a}} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})} \frac{(\Delta_a^0)^2 \Delta_a^3 z^3}{4M^2 (P^3_{avg,a})^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})} \frac{P^0_{avg,a}\Delta_a^0 \Delta_a^3 z^3}{2M^2 (P^3_{avg,a})^2} - \frac{z^3 \Delta_\perp^2}{2M^2 P^3_{avg,a}}\right) A_6$$
$$+ \left(\frac{(\Delta_a^0)^3 z^3}{2M^2 P^0_{avg,a} P^3_{avg,a}} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})} \frac{(\Delta_a^0)^3 \Delta_a^3 z^3}{4M^2 P^0_{avg,a}(P^3_{avg,a})^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})} \frac{(\Delta_a^0)^2 \Delta_a^3 z^3}{2M^2 (P^3_{avg,a})^2} - \frac{z^3 \Delta_\perp^2 \Delta_a^0}{2M^2 P^0_{avg,a} P^3_{avg,a}}\right) A_8$$

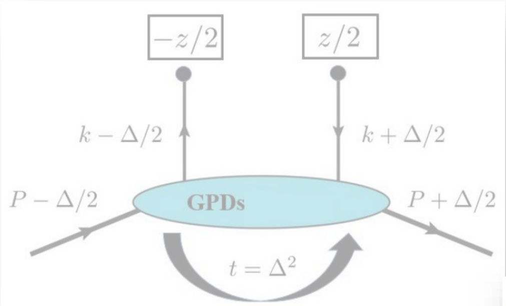
Main results

Relation between light-cone GPD H & Form Factors:

$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = A_1 + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z} A_3$$

Lorentz covariant formalism

Quasi-GPDs & Form Factors: (Sample results)



Symmetric frame:

$$H_{Q(0)}(z,P_s,\Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0}A_3 - \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3}A_4 + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0\Delta_s^3 z^3 P_s^0}{2M^2(P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2 P_s^3}\right)A_6$$

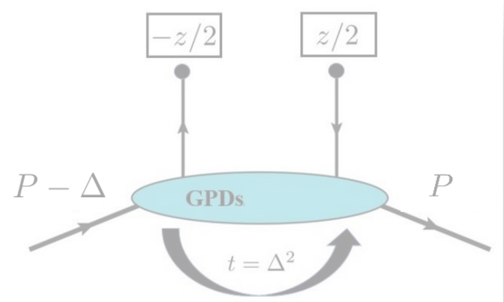
$$+\left(\frac{(\Delta_s^0)^3 z^3}{2M^2 P_s^0 P_s^3} - \frac{(\Delta_s^0)\ldots}{2M^2\ldots}\right)\ldots$$

You can think of eliminating power corrections by the addition of other operators:

$$(\gamma^1,\gamma^2)$$

In spirit of what's done for PDFs:

Asymmetric frame:



$$H_{Q(0)}\big|_a(z,P_a,\Delta_a) = A_1 + \frac{\Delta_a^0}{P_{avg,a}^0}A_3 - \left(\frac{\Delta_a^0 z^3}{2P_{avg,a}^0 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{\Delta_a^0\Delta_a^3 z^3}{4P_{avg,a}^0(P_{avg,a}^3)^2}\right)A_4$$

$$+\left(\frac{(\Delta_a^0)^2 z^3}{2M^2 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^2\Delta_a^3 z^3}{4M^2(P_{avg,a}^3)^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{P_{avg,a}^0\Delta_a^0\Delta_a^3 z^3}{2M^2(P_{avg,a}^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2 P_{avg,a}^3}\right)A_6$$

$$+\left(\frac{(\Delta_a^0)^3 z^3}{2M^2 P_{avg,a}^0 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^3\Delta_a^3 z^3}{4M^2 P_{avg,a}^0(P_{avg,a}^3)^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^2\Delta_a^3 z^3}{2M^2(P_{avg,a}^3)^2} - \frac{z^3\Delta_\perp^2\Delta_a^0}{2M^2 P_{avg,a}^0 P_{avg,a}^3}\right)A_8$$

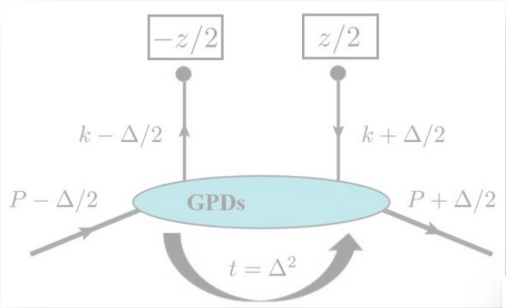
Main results

Relation between light-cone GPD H & Form Factors:

$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = \boxed{A_1} + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}\boxed{A_3}$$

Lorentz covariant formalism

Quasi-GPDs & Form Factors: (Sample results)



Symmetric frame:

$$H_{Q(0)}(z, P_s, \Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0}A_3 - \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3}A_4 + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0\Delta_s^3 z^3 P_s^0}{2M^2 (P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2 P_s^3}\right)A_6$$

$$+ \left(\frac{(\Delta_s^0)^3 z^3}{2M^2 P_s^0 P_s^3} - \frac{(\Delta_s^0)}{2M^2}\cdots\right.$$

In spirit of what's done for PDFs:

You can think of eliminating power corrections by the addition of other operators:

$$(\gamma^1, \gamma^2)$$

$$\cdots \frac{\Delta_a^0\Delta_a^3 z^3}{4P_{avg,a}^0 (P_{avg,a}^3)^2}\Big) A_4$$

$$\cdots \frac{\Delta_a^0 \Delta_a^3 z^3}{\cdots (P_{avg,a}^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2 P_{avg,a}^3}\Big) A_6$$

Main finding:

Lorentz covariant definition of quasi-GPDs:

Schematic structure:

$$H_Q \to c_0\langle\bar\psi\gamma^0\psi\rangle + c_1\langle\bar\psi\gamma^1\psi\rangle + c_2\langle\bar\psi\gamma^2\psi\rangle$$

Note: Here c's are frame-dependent kinematic factors that cancel frame-dependent power corrections to project quasi-GPD to the light-cone result

See Martha's talk for rigorous expressions

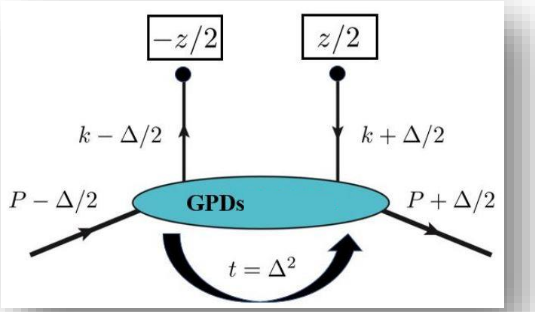
Main results

Relation between light-cone GPD H & Form Factors:

$$H(z \cdot P, z \cdot \Delta, t = \Delta^2, z^2) = \boldsymbol{A_1} + \frac{\Delta_{s/a} \cdot z}{P_{avg,s/a} \cdot z} \boldsymbol{A_3}$$

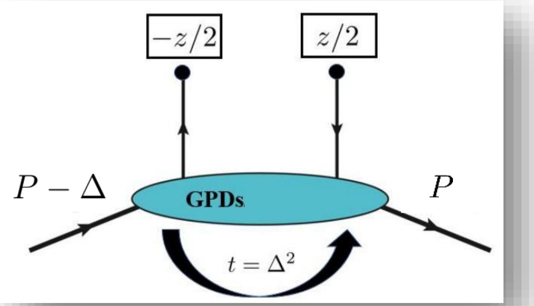
Lorentz covariant formalism

Quasi-GPDs: (Sample results)



Lorentz covariant definition of quasi-GPDs:

$$H_Q(z \cdot P, z \cdot \Delta, t = \Delta^2, z^2) = \boldsymbol{A_1} + \frac{\Delta_{s/a} \cdot z}{P_{avg,s/a} \cdot z} \boldsymbol{A_3}$$



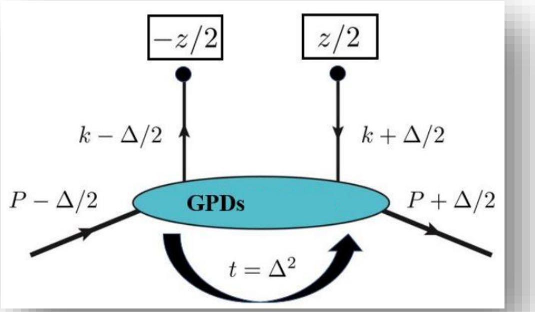
Main results

Relation between light-cone GPD H & Form Factors:

$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = \boldsymbol{A_1} + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}\boldsymbol{A_3}$$

Lorentz covariant formalism

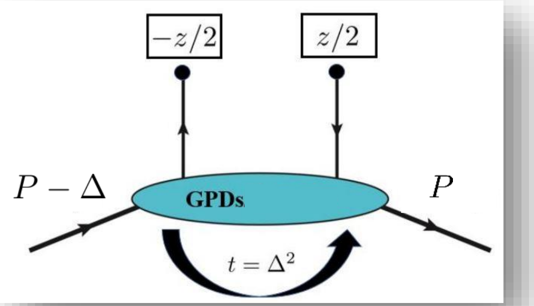
Quasi-GPDs: (Sample results)



Lorentz covariant definition of quasi-GPDs:

$$H_Q(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = \boldsymbol{A_1} + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}\boldsymbol{A_3}$$

Key point:



Think in terms of the matching coefficient at LO:

$$H(x,\ldots) = \int_{-\infty}^{\infty}\frac{d\xi}{|\xi|}C\left(\xi,\frac{\mu^2}{P_3^2},\ldots\right)H_Q\left(\frac{x}{\xi},\ldots\right)$$

(Xiong, Ji, Zhang, Zhao, 2013/ Stewart, Zhao, 2017/ Izubuchi, Ji, Jin, Stewart, Zhao, 2018/ …)

(Schematic structure)

$$C^{(0)} = \delta(1-\xi)$$

$$= \boldsymbol{A_1} + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}\boldsymbol{A_3}$$

Lorentz covariant definition of quasi-GPD allows fastest convergence to light-cone GPD at leading order

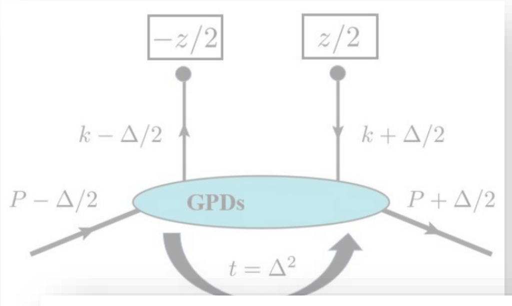
Main results

Relation between light-cone GPD H & Form Factors:

$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = \boldsymbol{A_1} + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}\boldsymbol{A_3}$$

Lorentz covariant formalism

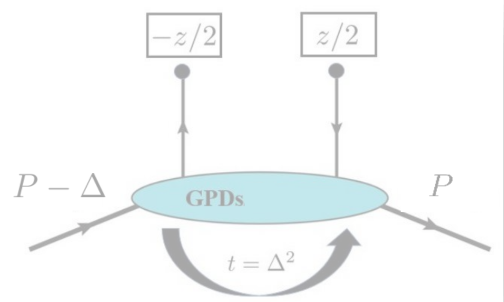
Quasi-GPDs: (Sample results)



Lorentz covariant definition of quasi-GPDs:

$$H_Q(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = \boldsymbol{A_1} + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}\boldsymbol{A_3}$$

Martha's talk: Numerical comparison between Lorentz covariant & non-covariant definitions of quasi-GPDs



$$H(x,\ldots) = \int_{-\infty}^{\infty}\frac{d\xi}{|\xi|}C\left(\xi,\frac{\mu^2}{P_3^2},\ldots\right)H_Q\left(\frac{x}{\xi},\ldots\right)$$

(Schematic structure)

$$C^{(0)} = \delta(1-\xi)$$

$$= \boldsymbol{A_1} + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}\boldsymbol{A_3}$$

(Xiong, Ji, Zhang, Zhao, 2013/
Stewart, Zhao, 2017/
Izubuchi, Ji, Jin, Stewart, Zhao, 2018/ …)

Lorentz covariant definition of quasi-GPD allows fastest convergence to light-cone GPD at leading order

Summary

Connecting dots: Ending with what I started with

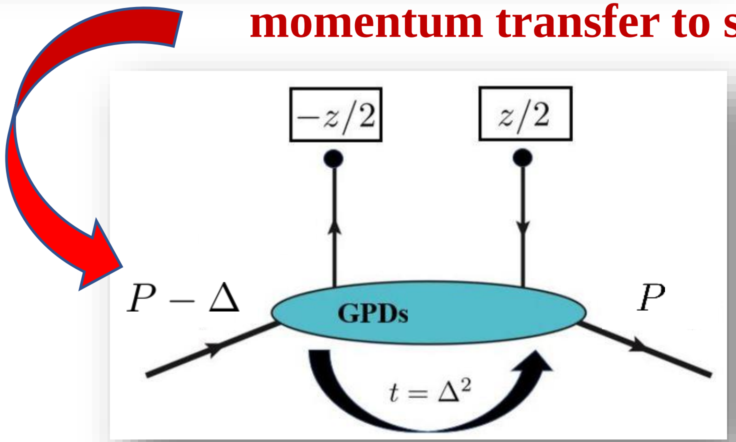
Summary

Goal:

Connecting dots: Ending with what I started with

Perform Lattice QCD calculations of GPDs in asymmetric frames

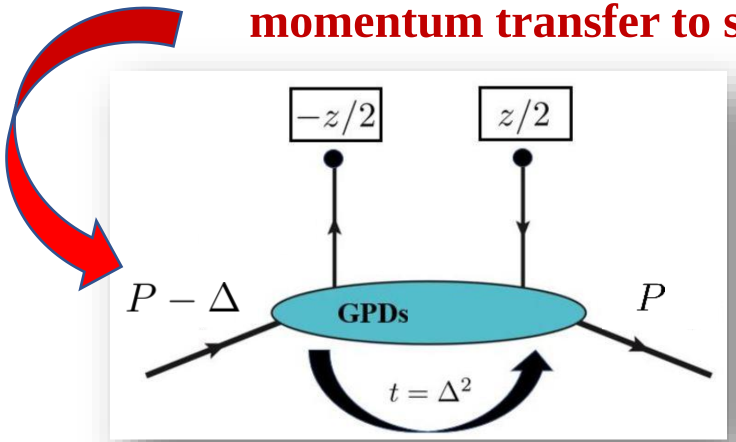
All momentum transfer to source



Summary

Connecting dots: Ending with what I started with

All
momentum transfer to source



Approach 1: Can we calculate a quasi-GPD in symmetric frame through an asymmetric frame?

Transverse boost: This Lorentz transformation allows for an exact calculation of quasi-GPDs in symmetric frame through matrix elements of asymmetric frame

Summary

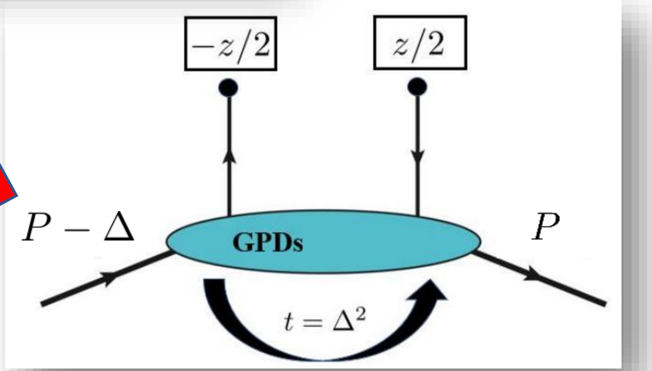
Connecting dots: Ending with what I started with

Approach 2: Why go through a calculation in asymmetric frame?

Why not calculate directly in asymmetric frame?

All

...ntum transfer to source



Key findings: QCD calculations of G...

1) **Historic definitions of H & E quasi-GPDs are not manifestly Lorentz covariant**

$$H_Q \to c\langle\bar{\psi}\gamma^0\psi\rangle$$

Symmetric frame:

$$H_{Q(0)}(z,P_s,\Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0}A_3 - \frac{\Delta_s^0 z^3}{2P_s^0P_s^3}A_4 + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2P_s^3} - \frac{\Delta_s^0\Delta_s^3 z^3 P_s^0}{2M^2(P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2P_s^3}\right)A_6 + \left(\frac{(\Delta_s^0)^3 z^3}{2M^2P_s^0P_s^3} - \frac{(\Delta_s^0)^2\Delta_s^3 z^3}{2M^2(P_s^3)^2} - \frac{\Delta_s^0 z^3\Delta_\perp^2}{2M^2P_s^0P_s^3}\right)A_8$$

Contamination from frame-dependent power corrections

Summary

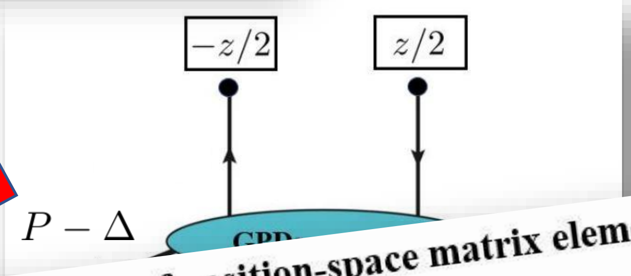
Connecting dots: Ending with what I started with

Approach 2: Why go through a calculation in asymmetric frame?

Why not calculate directly in asymmetric frame?

All

...ntum transfer to source



2) **Novel parameterization of position-space matrix element: (Vector operator)**

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^\mu}{M}\mathbf{A_1} + \frac{z^\mu}{M}\mathbf{A_2} + \frac{\Delta^\mu}{M}\mathbf{A_3} + \frac{i\sigma^{\mu z}}{M}\mathbf{A_4} + \frac{i\sigma^{\mu\Delta}}{M}\mathbf{A_5} + \frac{P^\mu i\sigma^{z\Delta}}{M^3}\mathbf{A_6} + \frac{z^\mu i\sigma^{z\Delta}}{M^3}\mathbf{A_7} + \frac{\Delta^\mu i\sigma^{z\Delta}}{M^3}\mathbf{A_8}\right]u(p,\lambda)$$

Key findings: QCD ... Frames

Summary

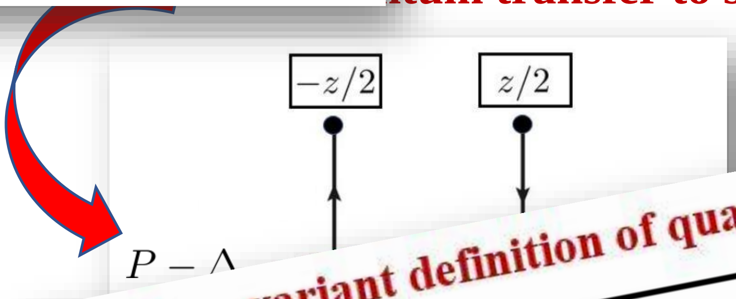
Connecting dots: Ending with what I started with

Approach 2: Why go through a calculation in asymmetric frame?

Why not calculate directly in asymmetric frame?

All

ntum transfer to source



3) **Lorentz covariant definition of quasi-GPDs:**

$$H_Q(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = A_1 + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z} A_3$$

$$H_Q \to c_0\langle\bar{\psi}\gamma^0\psi\rangle + c_1\langle\bar{\psi}\gamma^1\psi\rangle + c_2\langle\bar{\psi}\gamma^2\psi\rangle$$

Key findings: QCD calculatio ... in asymmetric fr

- **Lorentz covariant formalism for calculating quasi-GPDs in any frame**
- **Elimination of (frame-dependent) power corrections allowing faster convergence to light-cone GPDs at LO**

Backup slides

Main results

Renormalization: Sketch

Few words on operators:

- **Schematic structure of Lorentz non-covariant quasi-GPD:** $H_{\rm Q} \to c\langle\bar{\psi}\gamma^0\psi\rangle$
- **Schematic structure of Lorentz covariant quasi-GPD:** $H_{\rm Q} \to c_0\langle\bar{\psi}\gamma^0\psi\rangle + c_1\langle\bar{\psi}\gamma^1\psi\rangle + c_2\langle\bar{\psi}\gamma^2\psi\rangle$

See Martha's talk for rigorous expressions

How to renormalize?

Main results

Renormalization: Sketch

<u>Few words on operators</u>:

- **Schematic structure of Lorentz non-covariant quasi-GPD:** $H_{\mathrm{Q}} \to c\langle\bar{\psi}\gamma^0\psi\rangle$
- **Schematic structure of Lorentz covariant quasi-GPD:** $H_{\mathrm{Q}} \to c_0\langle\bar{\psi}\gamma^0\psi\rangle + c_1\langle\bar{\psi}\gamma^1\psi\rangle + c_2\langle\bar{\psi}\gamma^2\psi\rangle$

<u>Few words on renormalization</u>:

RI-MOM

- **Renormalization factors are different for** $\langle\bar{\psi}\gamma^0\psi\rangle$, $\langle\bar{\psi}\gamma^1\psi\rangle$, $\langle\bar{\psi}\gamma^2\psi\rangle$
  --- UV-divergent terms same
  --- Finite terms different
- **Matching:**
  --- Frame-independent
  --- Available for only γ^0
  --- Takes care of finite terms for γ^0
- **<u>Strategy to renormalize</u>: Use Renormalization factor for operator whose matching is known**