# Transverse momentum-dependent parton distributions (TMDs)
# for longitudinally polarized nucleons
# from domain wall fermion calculations at the physical pion mass

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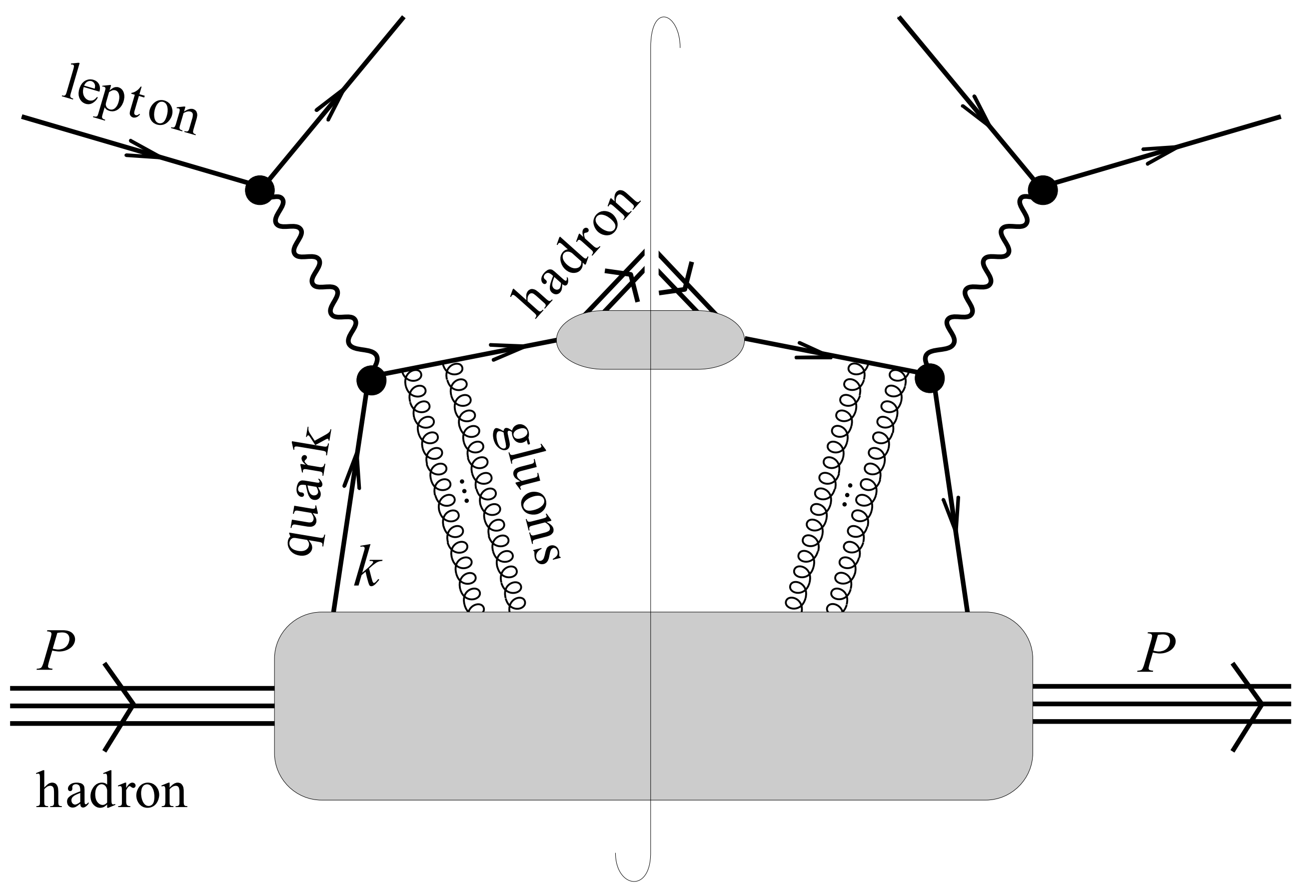
## Fundamental TMD correlator

$$\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b,P,S,\ldots) \equiv \frac{1}{2}\langle P,S|\ \bar{q}(0)\ \Gamma\ \mathcal{U}[0,\ldots,b]\ q(b)\ |P,S\rangle$$

$$\Phi^{[\Gamma]}(x,k_T,P,S,\ldots) \equiv \int \frac{d^2 b_T}{(2\pi)^2} \int \frac{d(b\cdot P)}{(2\pi)P^+} \exp\left(ix(b\cdot P) - ib_T\cdot k_T\right) \frac{\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b,P,S,\ldots)}{\widetilde{\mathcal{S}}(b^2,\ldots)}\Bigg|_{b^+=0}$$

- "Soft factor" $\widetilde{\mathcal{S}}$ required to subtract divergences of Wilson line $\mathcal{U}$
- $\widetilde{\mathcal{S}}$ is typically a combination of vacuum expectation values of Wilson line structures
- Here, will consider only ratios in which soft factors cancel

## Gauge link structure motivated by SIDIS



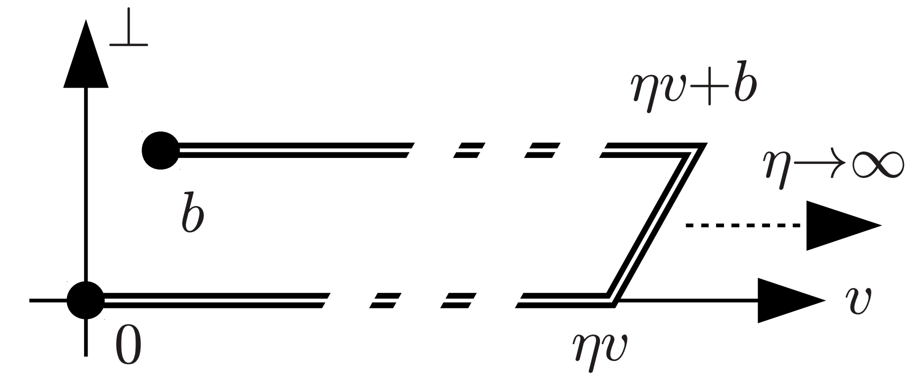

$l + H(P) \longrightarrow l' + h(P_h) + X$

Gauge link structure:

In matrix element $\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \ldots) \equiv$

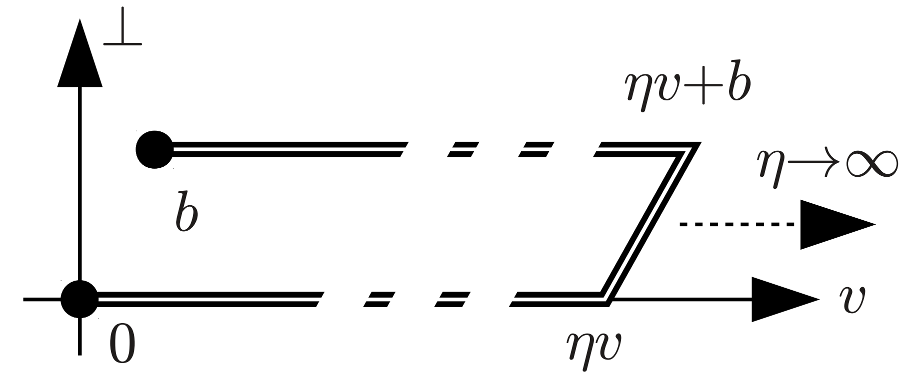
$\frac{1}{2}\langle P, S|\ \bar{q}(0)\ \Gamma\ \mathcal{U}[0, \ldots, b]\ q(b)\ |P, S\rangle$

Staple-shaped gauge link $\mathcal{U}[0, \eta v, \eta v + b, b]$




incorporates SIDIS final state effects

## Gauge link structure motivated by SIDIS




Beyond tree level: Rapidity divergences suggest taking staple direction slightly off the light cone. Approach of Aybat, Collins, Qiu, Rogers makes $v$ space-like. Parametrize in terms of Collins-Soper parameter

$$\hat{\zeta} \equiv \frac{P \cdot v}{|P||v|}$$

Light-like staple for $\hat{\zeta} \to \infty$. Perturbative evolution equations for large $\hat{\zeta}$.

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## Decomposition of $\Phi$ into TMDs

All leading twist structures:

$$\Phi^{[\gamma^+]} = f_1 - \left[\frac{\epsilon_{ij} k_i S_j}{m_H} f_{1T}^{\perp}\right] \text{odd}$$

$$\Phi^{[\gamma^+\gamma^5]} = \Lambda g_1 + \frac{k_T \cdot S_T}{m_H} g_{1T}$$

$$\Phi^{[i\sigma^{i+}\gamma^5]} = S_i h_1 + \frac{(2k_i k_j - k_T^2 \delta_{ij}) S_j}{2m_H^2} h_{1T}^{\perp} + \frac{\Lambda k_i}{m_H} h_{1L}^{\perp} + \left[\frac{\epsilon_{ij} k_j}{m_H} h_1^{\perp}\right] \text{odd}$$

## TMD Classification

All leading twist structures:

| $q \rightarrow$ / H $\downarrow$ | U | L | T |
|---|---|---|---|
| U | $f_1$ | | $h_1^{\perp}$ |
| L | | $g_1$ | $h_{1L}^{\perp}$ |
| T | $f_{1T}^{\perp}$ | $g_{1T}$ | $h_1$ $h_{1T}^{\perp}$ |

$\longleftarrow$ Boer-Mulders (T-odd)

$\uparrow$ Sivers (T-odd)

## Decomposition of $\widetilde{\Phi}$ into amplitudes

$$\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \hat{\zeta}, \mu) \equiv \frac{1}{2}\langle P, S|\ \bar{q}(0)\ \Gamma\ \mathcal{U}[0, \eta v, \eta v + b, b]\ q(b)\ |P, S\rangle$$

Decompose in terms of invariant amplitudes; at leading twist,

$$\begin{aligned}
\frac{1}{2P^+}\widetilde{\Phi}^{[\gamma^+]}_{\text{unsubtr.}} &= \widetilde{A}_{2B} + i m_H \epsilon_{ij} b_i S_j \widetilde{A}_{12B} \\
\frac{1}{2P^+}\widetilde{\Phi}^{[\gamma^+\gamma^5]}_{\text{unsubtr.}} &= -\Lambda \widetilde{A}_{6B} + i[(b \cdot P)\Lambda - m_H (b_T \cdot S_T)]\widetilde{A}_{7B} \\
\frac{1}{2P^+}\widetilde{\Phi}^{[i\sigma^{i+}\gamma^5]}_{\text{unsubtr.}} &= i m_H \epsilon_{ij} b_j \widetilde{A}_{4B} - S_i \widetilde{A}_{9B} \\
&\quad - i m_H \Lambda b_i \widetilde{A}_{10B} + m_H[(b \cdot P)\Lambda - m_H (b_T \cdot S_T)] b_i \widetilde{A}_{11B}
\end{aligned}$$

(Decompositions analogous to work by Metz et al. in momentum space)

## Fourier-transformed TMDs

$$\tilde{f}(x, b_T^2, \ldots) \equiv \int d^2k_T \, \exp(ib_T \cdot k_T) f(x, k_T^2, \ldots)$$

$$\tilde{f}^{(n)}(x, b_T^2, \ldots) \equiv n! \left( -\frac{2}{m_H^2} \partial_{b_T^2} \right)^n \tilde{f}(x, b_T^2, \ldots)$$

In limit $|b_T| \to 0$, recover $k_T$-moments:

$$\tilde{f}^{(n)}(x, 0, \ldots) \equiv \int d^2k_T \left( \frac{k_T^2}{2m_H^2} \right)^n f(x, k_T^2, \ldots) \equiv f^{(n)}(x)$$

ill-defined for large $k_T$, so will not attempt to extrapolate to $b_T = 0$, but give results at finite $|b_T|$.

In this study, only consider first $x$-moments (accessible at $b \cdot P = 0$), rather than scanning range of $b \cdot P$:

$$f^{[1]}(k_T^2, \ldots) \equiv \int_{-1}^{1} dx \, f(x, k_T^2, \ldots)$$

$\longrightarrow$ Bessel-weighted asymmetries (Boer, Gamberg, Musch, Prokudin, JHEP 1110 (2011) 021)

## Relation between Fourier-transformed TMDs and invariant amplitudes $\widetilde{A}_i$

Invariant amplitudes directly give selected $x$-integrated TMDs in Fourier ($b_T$) space, up to soft factors:

$$\tilde{f}_1^{[1](0)}(b_T^2, \hat{\zeta}, \ldots, \eta v \cdot P) = 2\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\widetilde{S}(b^2, \ldots)$$

$$\tilde{g}_1^{[1](0)}(b_T^2, \hat{\zeta}, \ldots, \eta v \cdot P) = -2\widetilde{A}_{6B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\widetilde{S}(b^2, \ldots)$$

$$\tilde{h}_{1L}^{\perp[1](1)}(b_T^2, \hat{\zeta}, \ldots, \eta v \cdot P) = -2\widetilde{A}_{10B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\widetilde{S}(b^2, \ldots)$$

## Generalized shifts

Form ratios in which soft factors, ($\Gamma$-independent) multiplicative renormalization factors cancel

$h_{1L}^{\perp}$ worm gear shift:

$$\langle k_x \rangle_{LT} \equiv m_N \frac{\tilde{h}_{1L}^{\perp[1](1)}}{\tilde{f}_1^{[1](0)}} = \left. \frac{\int dx \int d^2k_T \, k_x \Phi^{[s^j i\sigma^{j+}\gamma^5]}(x, k_T, P, S, \ldots)}{\int dx \int d^2k_T \, \Phi^{[\gamma^+]}(x, k_T, P, S, \ldots)} \right|_{\Lambda=1, s_T=(1,0)}$$

Average transverse momentum of quarks polarized in the same transverse ("$T$") direction in a longitudinally ("$L$") polarized nucleon; normalized to the number of valence quarks. "Dipole moment" in $b_T^2 = 0$ limit, "shift".

Issue: $k_T$-moments in this ratio singular; generalize to ratio of Fourier-transformed TMDs at *nonzero* $b_T^2$,

$$\langle k_x \rangle_{LT}(b_T^2, \ldots) \equiv m_N \frac{\tilde{h}_{1L}^{\perp[1](1)}(b_T^2, \ldots)}{\tilde{f}_1^{[1](0)}(b_T^2, \ldots)}$$

(remember singular $b_T \to 0$ limit corresponds to taking $k_T$-moment). "Generalized shift".

## Generalized shifts from amplitudes

Now, can also express this in terms of invariant amplitudes:

$$\langle k_x \rangle_{LT}(b_T^2, \ldots) \equiv m_N \frac{\tilde{h}_{1L}^{\perp[1](1)}(b_T^2, \ldots)}{\tilde{f}_1^{[1](0)}(b_T^2, \ldots)} = -m_N \frac{\widetilde{A}_{10B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

Analogously for generalized axial charge (here, no $k_T$ weighting):

$$\Delta\Sigma(b_T^2, \ldots) = \frac{\tilde{g}_1^{[1](0)}(b_T^2, \ldots)}{\tilde{f}_1^{[1](0)}(b_T^2, \ldots)} = -\frac{\widetilde{A}_{6B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

## Lattice setup

- Evaluate directly $\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \hat{\zeta}, \mu) \equiv \frac{1}{2}\langle P, S|\ \bar{q}(0)\ \Gamma\ \mathcal{U}[0, \eta v, \eta v + b, b]\ q(b)\ |P, S\rangle$
- Euclidean time: Place entire operator at one time slice, i.e., $b$, $\eta v$ purely spatial
- Since generic $b$, $v$ space-like, no obstacle to boosting system to such a frame!
- Parametrization of correlator in terms of $\widetilde{A}_i$ invariants permits direct translation of results back to original frame
- Form desired ratios of $\widetilde{A}_i$ invariants
- Extrapolate $\eta \to \infty$, $\hat{\zeta} \to \infty$ numerically.
- Use variety of $P$, $b$, $\eta v$; here $b \perp P$, $b \perp v$ (lowest $x$-moment, kinematical choices/constraints)

## Ensemble details

DWF ensemble provided by the RBC/UKQCD collaboration

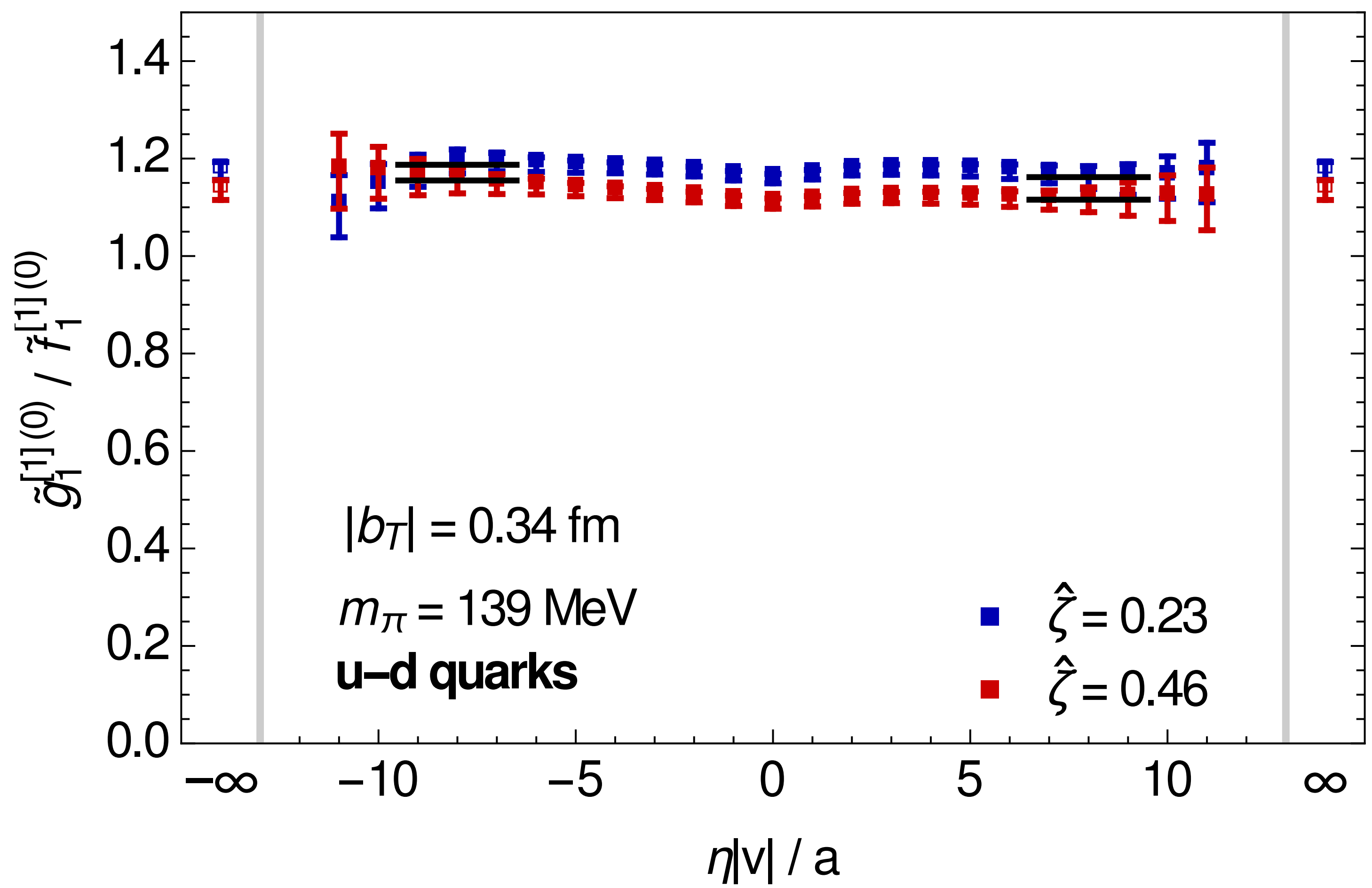
| $L^3 \times T$ | $a$(fm) | $m_\pi$ (MeV) | #conf. | #meas. |
|---|---|---|---|---|
| $48^3 \times 96$ | 0.114 | 139 | 130 | (33280 sloppy + 520 exact) AMA |

WARNING: Data obtained at one fairly small source-sink separation, $8a = 0.91\,\mathrm{fm}$

$\longrightarrow$ significant systematic uncertainty due to excited state contamination!

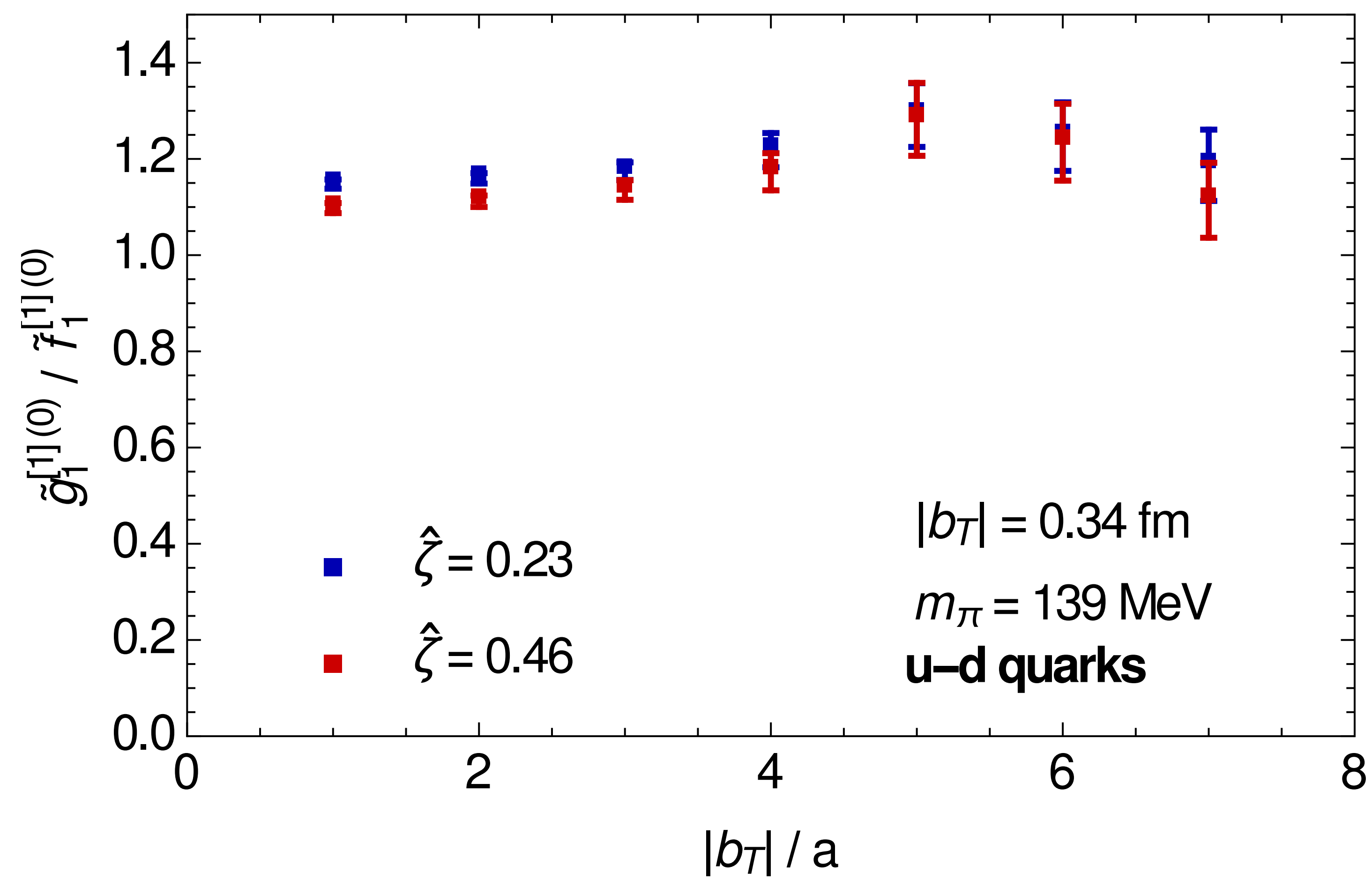
## Results: Generalized axial charge

Dependence on staple extent, at $|b_T| = 0.34\,\mathrm{fm}$



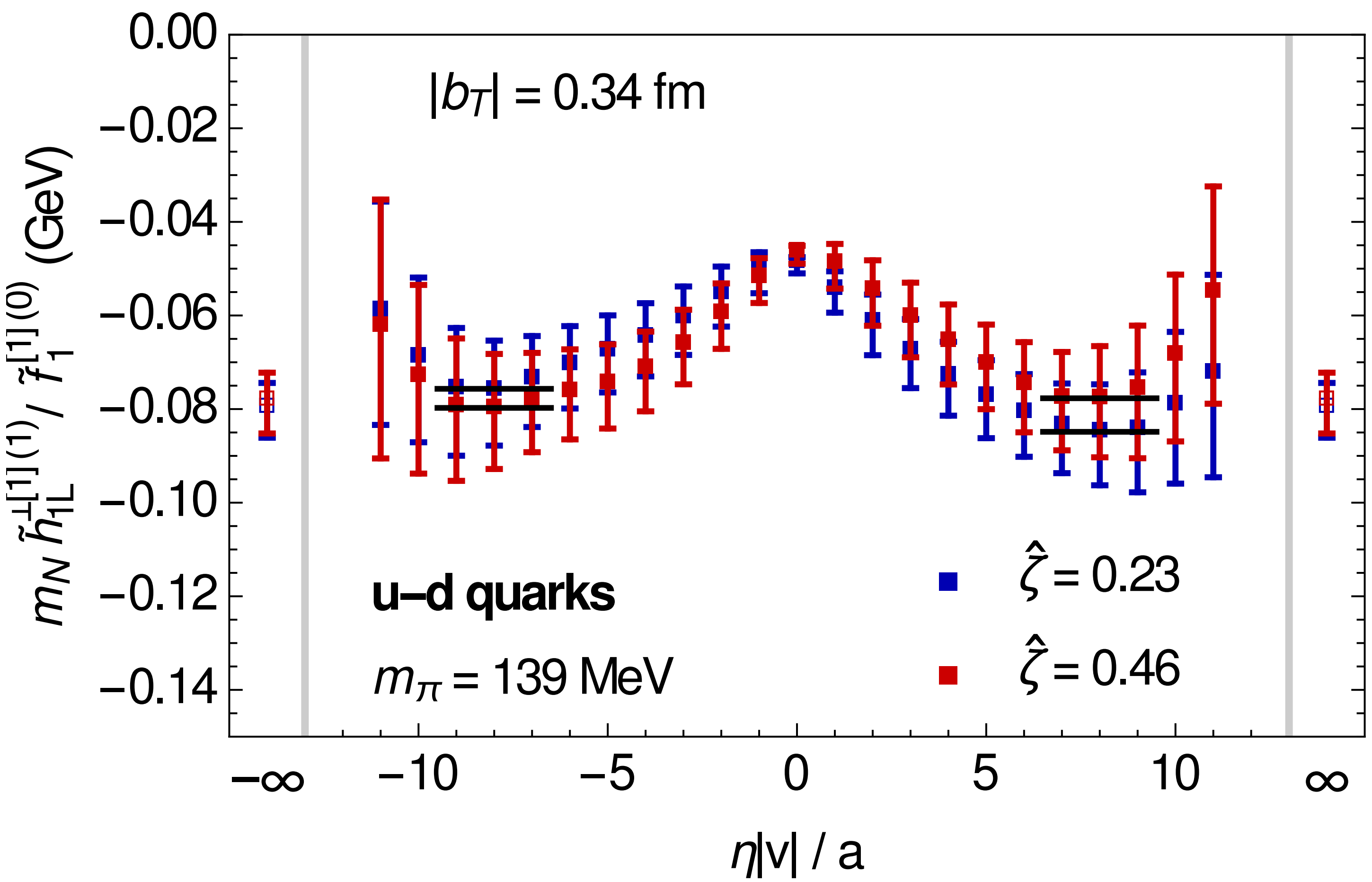
# Results: Generalized axial charge

Dependence of SIDIS/DY limit on $|b_T|$



## Results: $h_{1L}^{\perp}$ worm gear shift

Dependence on staple extent, at $|b_T| = 0.34\,\mathrm{fm}$



# Results: $h_{1L}^{\perp}$ worm gear shift

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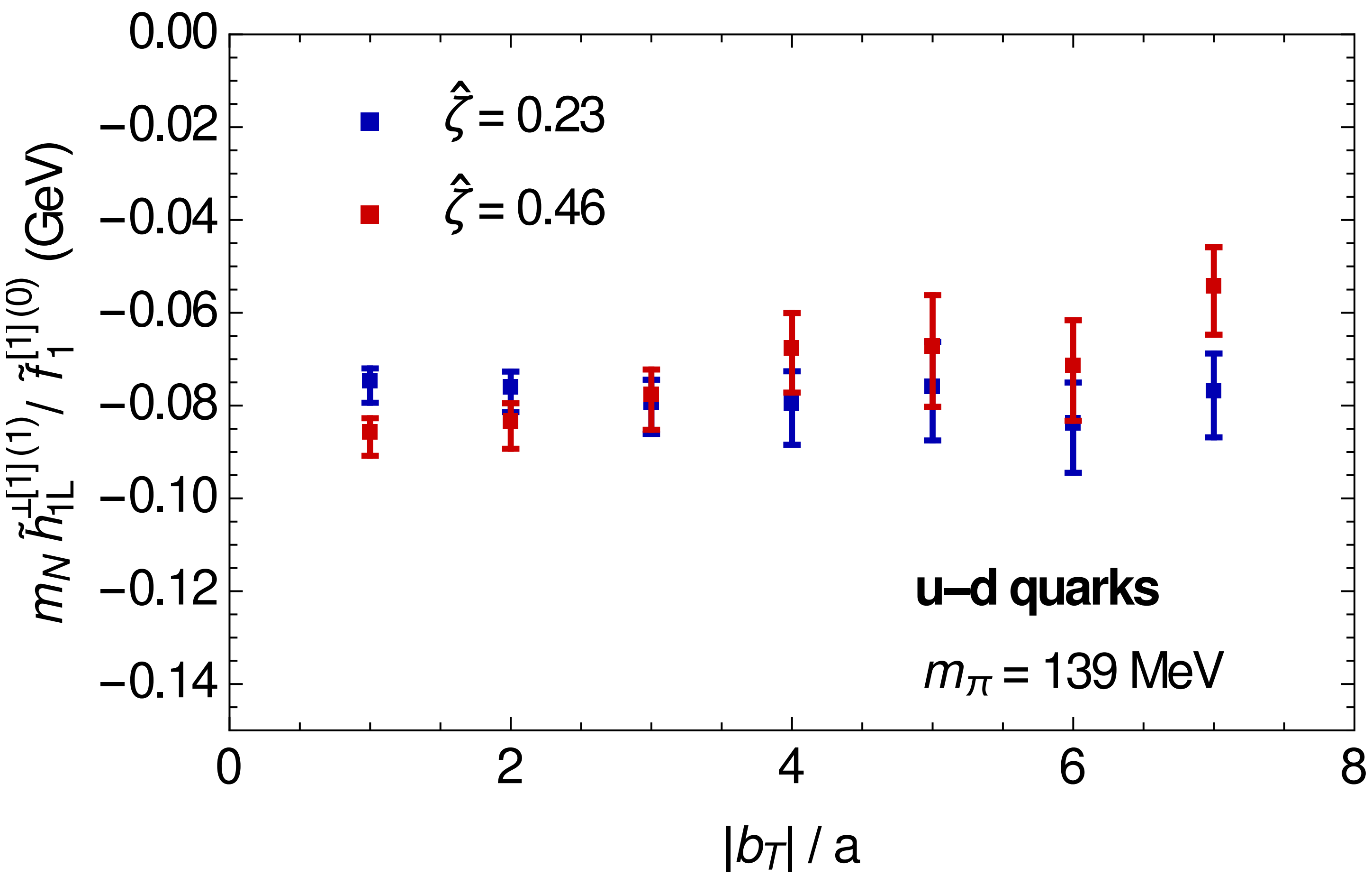
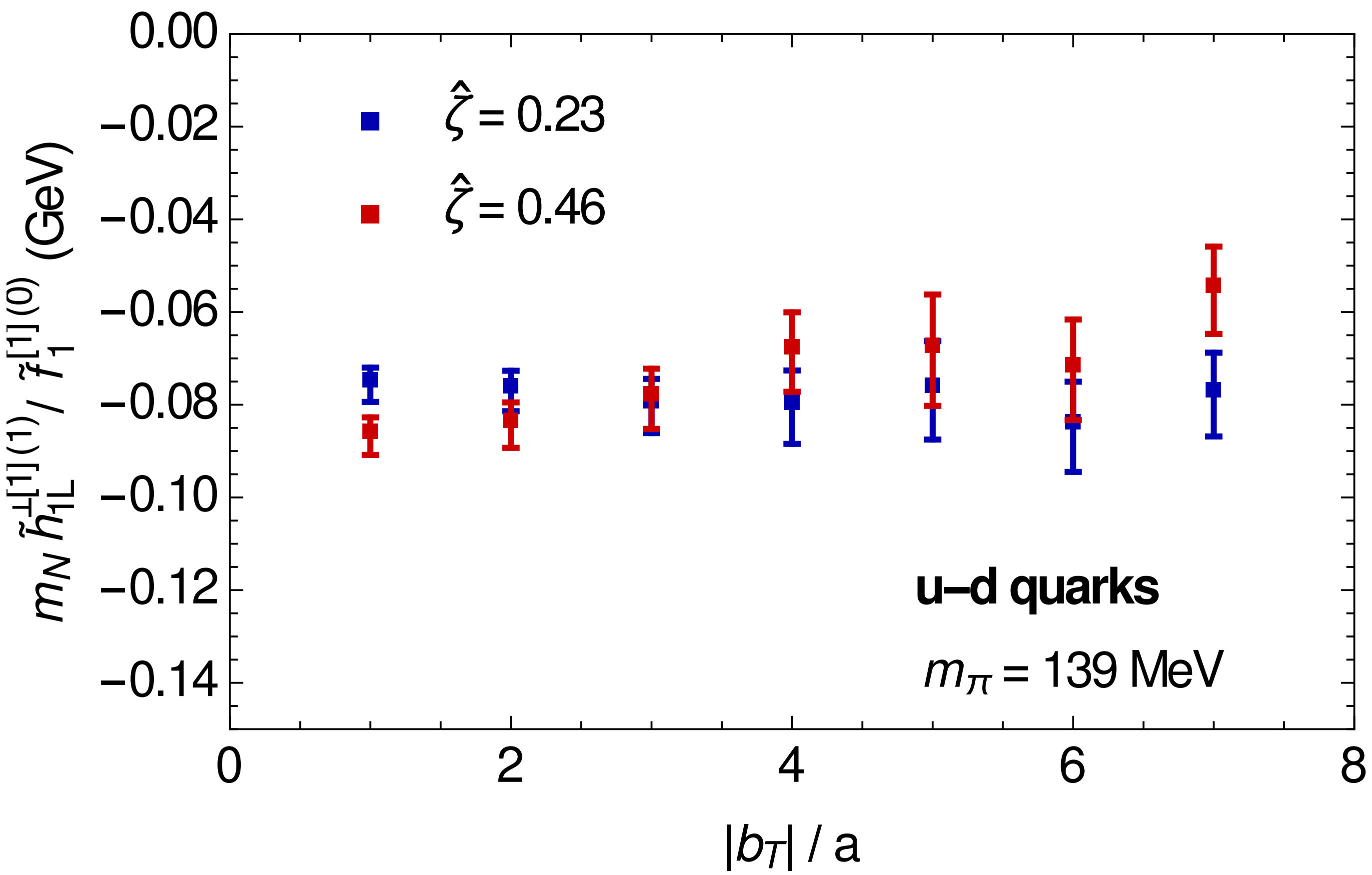
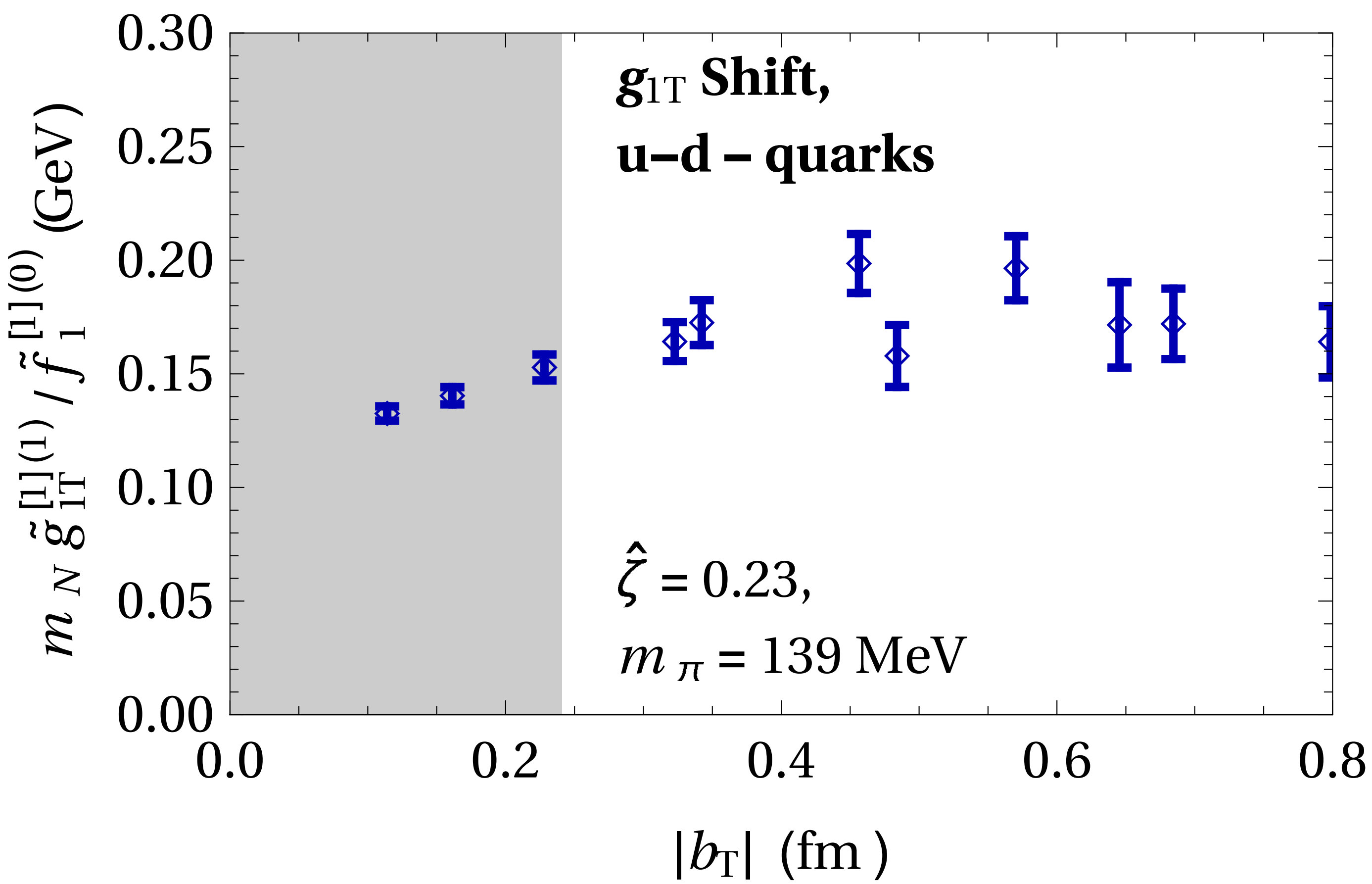


# Comparison: $h_{1L}^{\perp}$ vs. $g_{1T}$ worm gear shift

SIDIS/DY limit as a function of $|b_T|$




A wide variety of models predicts $h_{1L}^{\perp}$ and $g_{1T}$ to have the same magnitude (and opposite sign):
Spectator model, light-front constituent quark model, covariant parton model, bag model, light-front quark-diquark model, light-front version of the chiral quark-soliton model, nonrelativistic quark model . . .
Significant QCD effects not captured by models!

## Conclusions

- Calculations of TMD observables using bilocal quark operators with staple-shaped gauge link structures have reached the physical pion mass.
- To avoid soft factors, multiplicative renormalization constants, considered appropriate ratios of Fourier-transformed TMDs ("shifts").
- Here, observables in longitudinally polarized nucleons are extracted: generalized axial charge and $h_{1L}^{\perp}$ worm gear shift.
- $h_{1L}^{\perp}$ worm gear shift considerably smaller in magnitude compared to its counterpart $g_{1T}$ worm gear shift, exhibiting significant QCD effects not captured by a wide variety of commonly used models.